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**MODIFIED WEIBULL THEORY
AND STRESS-CONCENTRATION FACTORS
OF POLYCRYSTALLINE GRAPHITE**

**BY
F. H. HO**

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ABSTRACT

Stress concentration factors (SCF) due to geometric discontinuities in graphite specimens are observed to be much less than the theoretical SCF in an elastic material. In fact, the experimental SCF is always less than two and sometimes even less than one. A four parameter Weibull theory which recognizes the grain size effect is found to give an adequate explanation of the above observed discrepancies.

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1. INTRODUCTION

Statistical analysis, in particular, Weibull theory, has become increasingly popular in studying the nature of fracture of brittle materials. Fruitful results have been obtained when Weibull theory is applied to ceramics and glasses. However, it is not quite successful when applied to granular, porous, and brittle materials, such as polycrystalline graphite (Ref. 1). Price and Cobb (Ref. 2) observed that the Weibull theory accommodates the effect of nonuniform stress distribution when applied to results of tensile, bending and tube tests on H-327 graphite. They concluded that the theory overestimates the volume effect. The same conclusion was also obtained by Amesz, et al., (Ref. 3). The Weibull theory was applied to different tests of near isotropic graphite in Ref. 4 and failed to yield consistent values of material parameters, especially the Weibull modulus m . Mason (Ref. 5) attributed the size effect observed for the small specimens to the coarse-grained structure of the material relative to the specimen size.

Ho (Ref. 6) quantified Mason's idea by employing an empirical relation to account for grain size effect in a four parameter Weibull theory. He was able to interpret the strength data of H451 (Ref. 7) and AGOT (Ref. 8) graphite. The slightly lower m value obtained for the tensile strength as compared to the flexural strength is ascribed to various eccentricities encountered in the tensile test in Ref. 9. The theory is further tested in analyzing thermal stress test specimens (Ref. 10) and extended to biaxial tensile fracture in a porous medium. It also successfully explains the biaxial thermal stress test data (Ref. 11).

Stress concentration factor (SCF) in graphite due to geometric discontinuities, such as notches, have attracted little attention in recent

years. Both Brocklehurst and Brown (Ref. 12) and Bazaj and Cox (Ref. 13) observed experimental SCF, K_e , always less than the theoretical SCF, K_t . The K_e was found to be always smaller than two. Bazaj and Cox also reported an increase in K_e with decrease in the grain size of the material and cases wherein geometric discontinuity increases the strength of a structural member. All these data are reviewed here. The four parameter Weibull theory is then applied to resolve the observed difference between K_e and K_t .

2. THE FOUR PARAMETER WEIBULL THEORY

The four parameter Weibull theory (Ref. 6) will be reviewed briefly. The theory can be written as

$$S_V = \exp \left[- \int_V \left(\frac{\sigma - \sigma_u}{\sigma_o f} \right)^m dV \right]$$

where S_V = cumulative survival probability associated with the volume V ,
 σ = uniaxial tensile stress state in V ,
 σ_o = characteristic strength (a scale parameter),
 m = Weibull modulus (a shape parameter),
 σ_u = lower bound of the strength (a location parameter), usually taken to be zero,
 f = grain size effect correction factor .

An empirical expression for the factor, f , is proposed as

$$f(h_0; h_1, h_2) = [2/\pi \text{Cos}^{-1}(h_0/h_1)] [2/\pi \text{Cos}^{-1}(h_0/h_2)]$$

where h_0 = characteristic grain size, usually the maximum grain size,
 h_1, h_2 = width and thickness of the specimen.

3. APPLICATION TO GRAPHITE

Bazaj and Cox's data (Ref. 13) are re-examined here in light of the above modified Weibull theory. Geometrically identical specimens, including both plain and grooved cylindrical specimens, were machined from three types of graphite with different grain sizes. The experimental SCFs, K_e , under tensile load, were obtained. This is an ideal case to check the validity of the empirical correction for the grain size effect. Results are summarized in Table 1. The measurement and the prediction are close and within one standard deviation of the test data.

Experimental SCF were also determined from the median values of the tensile strength of the plain and grooved H205 graphite specimen. The D/d ratios (= larger diameter/small diameter of the grooved specimen) and five groove radii ranging from 1/64 to 1/4 in. were examined. The volume of the grooved specimen at the peak tensile stress location is much smaller than the volume of the plain specimen subjected to uniform tension. The volume effect is estimated using $m = 8$ in the Weibull theory. The experimental SCF is adjusted for the volume effect. Results are shown in Fig. 1. When dealing with graphite, the agreement between the theoretical and the experimental SCFs is excellent.

Measurements on the experimental SCF, K_e , were made at elevated temperatures of 1000°C and 2000°C for H205 graphite. It was found that the specimens with large groove radii had a higher median tensile strength (six to seven specimens tested at each point) than the plain specimens. These data were then eliminated from further consideration in Ref. 13 on the ground that stress raisers cannot increase the strength of a structural member. This is an unacceptable explanation. For if the volume effect outweighs the SCF due to geometric changes, such as groove, etc., the net

TABLE 1
EFFECT OF GRAIN SIZE ON STRESS CONCENTRATION

| Type of Graphite | Maximum Grain Size, h_0 (in.) | D/d = 1.25 | | D/d = 1.5 | | Predicted Ratio |
|------------------|---------------------------------|------------|----------------|-----------|----------------|-----------------|
| | | K_e | Relative Ratio | K_e | Relative Ratio | |
| H205 | 0.03 | 1.54 | 1 | 1.61 | 1 | 1 |
| Nipple | 0.10 | 1.35 | 0.877 | 1.41 | 0.876 | 0.911 |
| Electrode | 0.35 | 1.03 | 0.669 | 1.09 | 0.677 | 0.620 |

D = larger diameter of the grooved specimen.

d = small diameter of the grooved specimen.

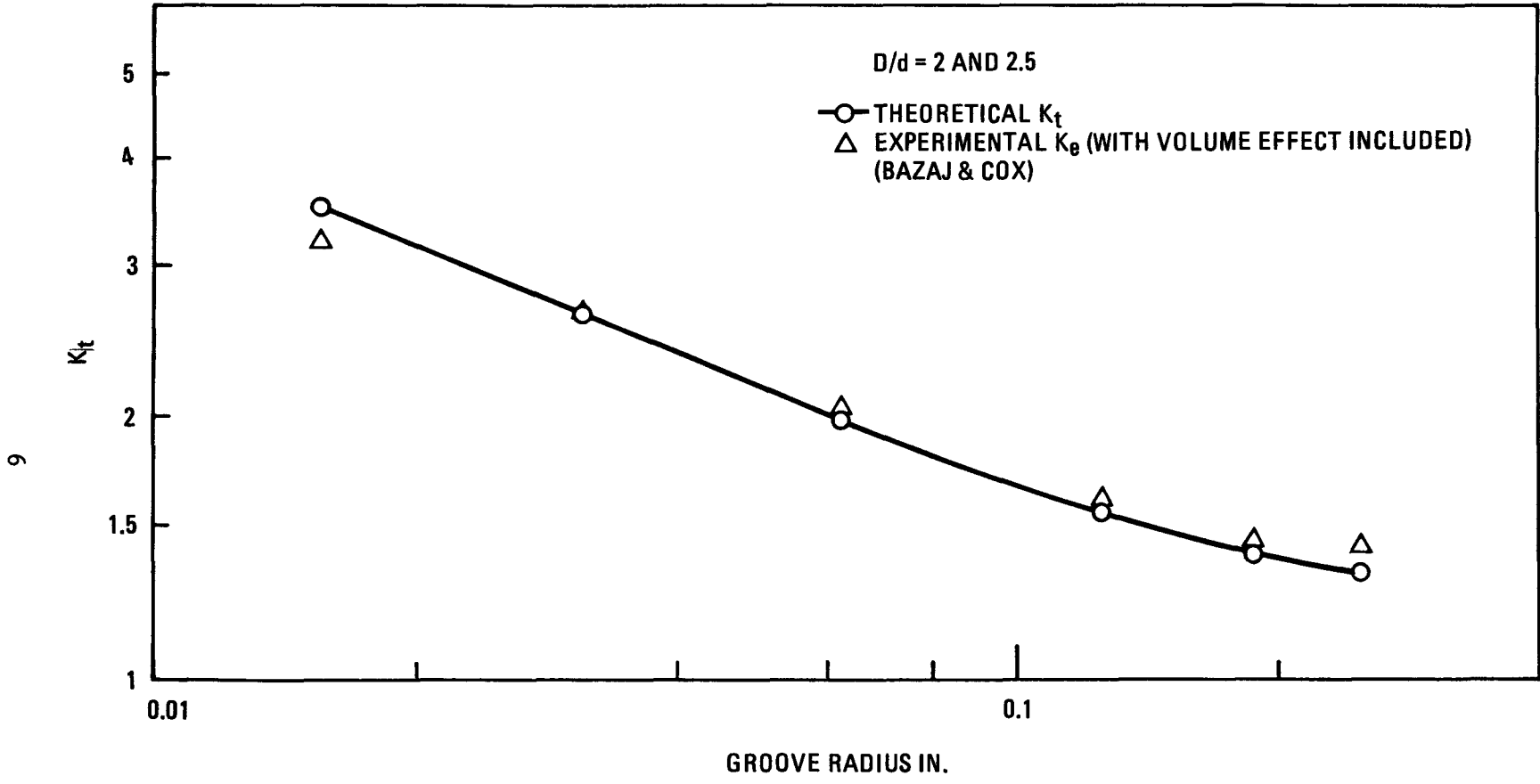


Fig. 1. Correlation of the experimental and theoretical SCFs by modified Weibull theory for H205 graphite

result is an increase in strength. The data at large groove radii may still be valid. To prove the point, the theoretical SCF, K_t , is adjusted with its volume to that of a corresponding grooved specimen. The resulting SCF, as shown in Fig. 2, is reasonably close to the experimental values; generally within one standard deviation.

Brocklehurst and Brown (Ref. 12) performed fracture tests on notched specimens of graphite. The test showed that graphite material is not as sensitive to notches with small root radii as theory of elasticity would predict for brittle materials. The experimental SCF are always less than theoretical predictions and do not exceed 2.0 for the graphites used. Their notch sensitivity test results would be perfect for testing the validity of the modified Weibull theory. Unfortunately, insufficient geometric information on specimen sizes and notch shapes hampers correlation. However, the four-point bending test data on NI graphite suffices for this purpose.

Rectangular beams of different thicknesses, various notch depths and several notch radii were tested in four-point bending. The control specimens were unnotched while all other specimens had a U-shaped edge notch at the mid-point of the center span. Test data and theoretical SCF are given in Table 4 in Ref. 12. Grain size quoted for NI graphite is 1 mm. This is used in the following correlation as the maximum grain size. Results would change by no more than 2% if 1.5 mm were the maximum grain size.

Table 2 summarizes the correlation. A Weibull modulus $m = 5.5$ was used for the volume effect correction. The experimental SCFs were corrected for volume and grain size effect to yield K_e . When K_e is compared to K_t , good agreement is obtained. The control specimens of two different sizes are also compared. Their strengths are within 4% of the theoretical prediction.

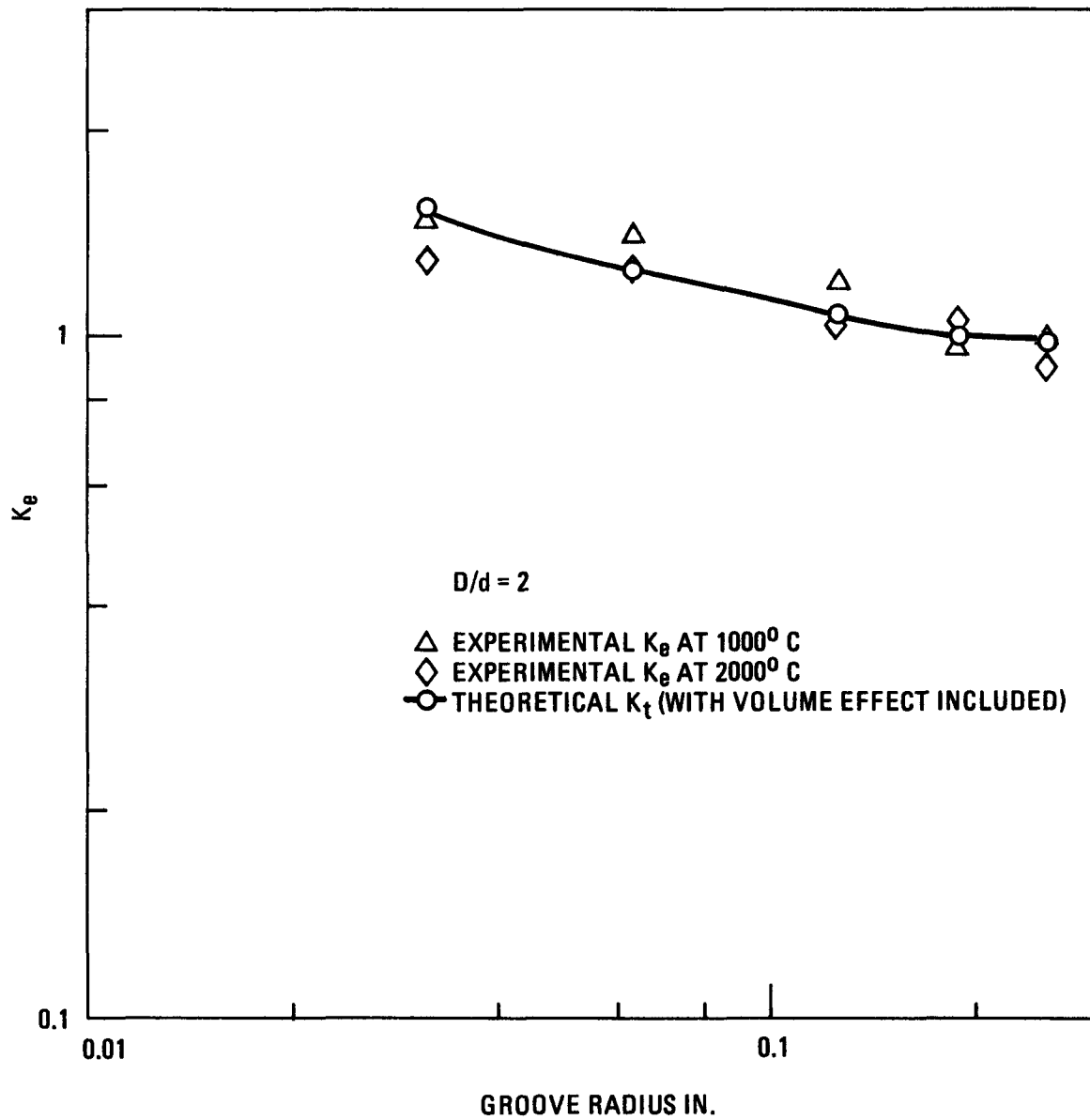


Fig. 2. Correlation of K_e and K_t at elevated temperatures for H205 graphite by modified Weibull theory

TABLE 2

| Notch Depth (mm) | 12.5 mm x 12.5 mm Cross Section Beam | | | | | |
|-------------------------------|--------------------------------------|------|------|------|------|------|
| | 2.5 | | | 5.0 | | 1.25 |
| Notch radius, mm | 1.0 | 2.5 | 0.6 | 2.5 | 0.6 | 2.5 |
| K_e | 1.20 | 1.92 | 2.87 | 1.71 | 2.73 | 2.03 |
| K_t | 1.2 | 1.8 | 3.1 | 1.7 | 2.9 | 1.9 |
| $[K_e - K_t)/K_t] \times 100$ | 0 | +6.7 | -7.5 | 0 | -5.9 | +6.8 |

6.25mm thick x 12.5 mm deep beam

| 1.25 | 2.5 | 3.75 | 5.0 |
|------|------|------|------|
| 2.5 | 2.5 | 2.5 | 2.5 |
| 1.85 | 1.83 | 1.76 | 1.62 |
| 1.9 | 1.8 | 1.75 | 1.7 |
| -2.7 | +1.7 | 0 | -4.3 |

4. CONCLUSIONS

The four-parameter Weibull theory which includes the grain size effect is used to analyze the stress concentration factor (SCF) in graphite specimen. The theory ascribes the difference between the experimental SCF and the theoretical SCF mainly to the volume effect. The volume effect usually results in a lower SCF. This is observed in all the tests performed so far. The SCF predicted by the theory is in good agreement with the experimental value. If the volume effect outweighs the effect of a geometric stress raiser, the experimental SCF can be less than unity.

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