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DAMPING RATES AND FREQUENCY SHIFTS PRODUCED BY A FEEDBACK SYSTEM IN COHERENT OSCILLATIONS OF MULTI-BUNCHED BEAMS

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SUMMARY:

The matrix formalism developed in an earlier PEP Note[1] is generalized here to the case of motion of any number of bunches in each of two counter-rotating beams. The motion of the bunches in both beams is coupled through the feedback memory which arises from the finiteness of the feedback system bandwidth. The damping rates and the frequency shifts of one-dimensional coherent oscillations are calcualted. Numerical examples are given for the particular case of bunches uniformly spaced around the orbit.

I. INTRODUCTION:

In an earlier work[1], a matrix formalism was developed for solving feedback problems with memory. We apply here this approach to the case of a feedback system having memory and acting on one-dimensional coherent oscillations of a system of many bunches in two counter-rotating beams. For the limit of very short memory, one can neglect the coupling between different bunches since the feedback signal is damped out by the next bunch passage. The results for this case were obtained in Reference 2.

The schematic layout of the feedback system and the related notation are presented in Fig. 1. The kicker and detector are denoted by K and D. $M_{\pm}$ and $F_{\pm}$ are the transfer matrices for one revolution and from the kicker to the detector for the two beams respectively.

If the kicker signal is damped only slowly, it will act not only on the intended bunch but on the subsequent bunches (of both beams) as well. The action depends on the displacement of the first (intended) bunch thus establishing coupling between the motion of different bunches. In the system of k bunches in each of two beams, there are 4K eigenmodes of oscillation and correspondingly 4K eigenvalues for the normal modes of coupled oscillation.

In Section 2 we derive the general formula for the matrix which defines the eigenvalues of the motion. In Section 3 we apply it to the PEP transverse feedback system.

II. FULL TRANSFER MATRIX FOR ONE REVOLUTION

We describe the motion of bunches by two, 2K column vectors $\bar{X}$ and $\overset{+}{X}$

$$\overset{\pm}{X} = \begin{pmatrix} \overset{\pm}{x}_1 \\ \overset{\pm}{x}'_1 \\ \overset{\pm}{x}_2 \\ \overset{\pm}{x}'_2 \\ \cdot \\ \cdot \\ \cdot \\ \overset{\pm}{x}_k \\ \overset{\pm}{x}'_k \end{pmatrix} . \qquad (1)$$

The action of the kicker can be described by the matrix operators $\hat{\mathcal{L}}_-$ and $\hat{\mathcal{L}}_+$ defined as follows:

$$\hat{\mathcal{L}}_-\bar{x} = \hat{I}\,\bar{x} + \sum_{m=1}^{\infty}\hat{P}\,(\hat{\bar{f}}_m)\,\hat{F}_-\hat{M}_-^{-m}\,\bar{x} + \sum_{m=1}^{\infty}\hat{P}\,(\hat{\overset{+}{g}}_m)\,\hat{F}_+\hat{M}_+^{-m}\,\overset{+}{x}\ , \qquad (2)$$

$$\hat{\mathcal{L}}_+\overset{+}{x} = \hat{I}\,\overset{+}{x} + \sum_{m=1}^{\infty}\hat{P}\,(\hat{\bar{g}}_m)\,\hat{F}_-\hat{M}_-^{-m}\,\bar{x} + \sum_{m=1}^{\infty}\hat{P}\,(\hat{\overset{+}{f}}_m)\,\hat{F}_+\hat{M}_+^{-m}\,\overset{+}{x}\ , \qquad (3)$$

where the superscript caret denotes a matrix, and the symbol $\hat{I}$, stands for the 2k by 2k unit matrix.

Here we have introduced the "projection" operator of dimension 2k:

$$\hat{P}(\hat{h}_m) = \begin{pmatrix} 0 & 0 & 0 & 0 & . & . & . & . & . & 0 \\ h_{m11} & 0 & h_{m12} & 0 & . & . & . & . & . & h_{m1k} \\ 0 & 0 & 0 & 0 & . & . & . & . & . & 0 \\ h_{m21} & 0 & h_{m22} & 0 & & & & & & \\ . & & & & & & & & & . \\ . & & & & & & & & & . \\ . & & & & & & & & & . \\ . & & & & & & & & & . \\ h_{mk1} & 0 & h_{mk2} & 0 & . & . & . & . & . & h_{mkk} \end{pmatrix} \qquad (4)$$

The coefficients $\overset{\pm}{f}_{mjl}$ and $\overset{\pm}{g}_{mjl}$ describe the behavior of the kicker signal. These coefficients are referred to in Eq. (2) and Eq. (3) as matrix arguments of the projection operator. Their meaning can be clearly seen if we write explicitly the action of the kicker on the jth bunch on the nth passage:

$$\Delta\,\bar{x}_j(n) = 0 \qquad (5)$$

$$\Delta\,\bar{x}'_j(n) = \sum_{\ell=1}^{k}\sum_{m=1}^{\infty}\left[\overset{+}{g}_{mj\ell}\ \overset{+}{x}{}_\ell^{(D_+)}(n-m) + \bar{f}_{mj\ell}\,\bar{x}_\ell^{(D_-)}(n-m)\right] \qquad (6)$$

$$\Delta \overset{\pm}{x}_j(n) = 0 \tag{7}$$

$$\Delta \overset{\pm}{x}'_j(n) = \sum_{\ell=1}^{k} \sum_{m=1}^{\infty} \left[\overset{\pm}{f}_{mj\ell} \overset{\pm}{x}_{\ell}^{(D_+)}(n-m) + \bar{g}_{mj\ell} \bar{x}_{\ell}^{(D_-)}(n-m) \right] \quad . \tag{8}$$

where the superscript (D's) specify that the x's have been measured at the detectors.

The coefficients, $f_{mj\ell}$, bind bunches of the same beam together while the $g_{mj\ell}$ do the same for bunches of different beams. In the general case they all can be different, but for uniformly distributed equal number bunches in both beams $\hat{\bar{f}}_m = \hat{\bar{f}}_m$, and $\hat{\bar{g}}_m = \hat{\bar{g}}_m$. The 2k x 2k matrices $\hat{F}$ and $\hat{N}$ are quasi-diagonal with 2 x 2 sub-matrices F and M respectively and zeros elsewhere.

Let us now introduce 4k column vector $X = \begin{pmatrix} \bar{X} \\ \overset{+}{X} \end{pmatrix}$. For that vector, the full matrix of one revolution is

$$T = \begin{pmatrix} I + \sum_{m=1}^{\infty} \hat{P}(\hat{\bar{f}}_m)\, \hat{F}_- \hat{M}_-^{-m} & \sum_{m=1}^{\infty} \hat{P}(\hat{\overset{+}{g}}_m)\, \hat{F}_+ \hat{M}_+^{-m} \\ \sum_{m=1}^{\infty} \hat{P}(\hat{\bar{g}}_m)\, \hat{F}_- \hat{M}_-^{-m} & I + \sum_{m=1}^{\infty} \hat{P}(\hat{\overset{+}{f}}_m)\, \hat{F}_+ \hat{M}_+^{-m} \end{pmatrix} \begin{pmatrix} \hat{M}_- & \hat{0} \\ \hat{0} & \hat{M}_+ \end{pmatrix} \quad , \tag{9}$$

where $\hat{0}$ stands for the zero 2k x 2k matrix.

After some algebra, one gets the following expression for the matrix, T:

$$T = \begin{pmatrix} A_- & B_+ \\ B_- & A_+ \end{pmatrix} \quad , \tag{10}$$

where

$$A_{\mp} = \begin{pmatrix} M_{11}^{\mp} & M_{12}^{\mp} & 0 & 0 & - & - & 0 & 0 \\ (M_{21}^{\mp}+R_{11}^{\mp}) & (M_{22}^{\mp}+S_{11}^{\mp}) & R_{12}^{\mp} & S_{12}^{\mp} & - & - & R_{1k}^{\mp} & S_{1k}^{\mp} \\ 0 & 0 & M_{11}^{\mp} & M_{12}^{\mp} & - & - & 0 & 0 \\ R_{21}^{\mp} & S_{21}^{\mp} & (M_{21}^{\mp}+R_{22}^{\mp}) & (M_{22}^{\mp}+S_{22}^{\mp}) & - & - & R_{2k}^{\mp} & S_{2k}^{\mp} \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 0 & 0 & - & - & - & - & M_{11}^{\mp} & M_{12}^{\mp} \\ R_{k1}^{\mp} & S_{k1}^{\mp} & - & - & - & - & (M_{21}^{\mp}+R_{kk}^{\mp}) & (M_{22}^{\mp}+S_{kk}^{\mp}) \end{pmatrix} \tag{11}$$

$$B_{\mp} = \begin{pmatrix} 0 & 0 & 0 & 0 & - & - & 0 & 0 \\ P_{11}^{\mp} & Q_{11}^{\mp} & P_{12}^{\mp} & Q_{12}^{\mp} & - & - & P_{1k}^{\mp} & Q_{1k}^{\mp} \\ 0 & 0 & 0 & 0 & - & - & 0 & 0 \\ P_{21}^{\mp} & Q_{21}^{\mp} & P_{22}^{\mp} & Q_{22}^{\mp} & - & - & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 0 & 0 & - & - & - & - & 0 & 0 \\ P_{k1}^{\mp} & Q_{k1}^{\mp} & - & - & - & - & P_{kk}^{\mp} & Q_{kk}^{\mp} \end{pmatrix} \tag{12}$$

In these expressions, the new symbols we have used have the following meanings:

$$R^{\mp}_{j\ell} = \sqrt{\beta^{\mp}_D/\beta_k}\ \left(\Sigma^{\mp f}_{cj\ell} - \alpha_k\, \Sigma^{\mp f}_{sj\ell}\right) \ , \qquad (13)$$

$$S^{\mp}_{j\ell} = -\sqrt{\beta^{\mp}_D\ \beta_k}\ \ \Sigma^{\mp f}_{sj\ell} \ , \qquad (14)$$

$$P^{\mp}_{j\ell} = \sqrt{\beta^{\mp}_D/\beta_k}\ \left(\Sigma^{\mp g}_{cj\ell} - \alpha_k\, \Sigma^{\mp g}_{sj\ell}\right) \ , \qquad (15)$$

$$Q^{\mp}_{j\ell} = -\sqrt{\beta^{\mp}_D\ \beta_k}\ \ \Sigma^{\mp g}_{sj\ell} \ , \qquad (16)$$

where $\beta^{\mp}_D$, β_k and α_k are the values of the β- function and half of its slope ($\alpha_k = -\beta'_k/2$) at the positions of the monitors $D^{\mp}$ and the kicker, respectively. The symbols, $\Sigma^{\mp f}_{cj\ell}$, $\Sigma^{\mp f}_{sj\ell}$, $\Sigma^{\mp g}_{cj\ell}$ and $\Sigma^{\mp g}_{sj\ell}$ stand for the following sums:

$$\Sigma^{\mp f}_{cj\ell} = \sum_{m=1}^{\infty} f^{\mp}_{mj\ell} \cos \left[(m-1)\ \mu_o - \Delta\Psi_{\mp}\right] \ , \qquad (17)$$

$$\Sigma^{\mp f}_{sj\ell} = \sum_{m=1}^{\infty} f^{\mp}_{mj\ell} \sin \left[(m-1)\ \mu_o - \Delta\Psi_{\mp}\right] \ , \qquad (18)$$

$$\Sigma^{\mp g}_{cj\ell} = \sum_{m=1}^{\infty} g^{\mp}_{mj\ell} \cos \left[(m-1)\ \mu_o - \Delta\Psi_{\mp}\right] \ , \qquad (19)$$

$$\Sigma^{\mp g}_{sj\ell} = \sum_{m=1}^{\infty} g^{\mp}_{mj\ell} \sin \left[(m-1)\ \mu_o - \Delta\Psi_{\mp}\right] \ , \qquad (20)$$

where

$$\Delta\Psi_{\mp} = \mu^{\mp}_D - \mu_k = \mu_o + \mu^{\mp}_D - \mu_{k+1} \qquad (21)$$

is the phase advance of the betatron oscillation from the previous kicker K to the detector $D^{\mp}$. The second expression in Eq. (21) contains the more convenient quantity $(\mu_{k+1} - \mu_D^{\mp})$, i.e., the phase advance from the detector to the following kicker. (The index k+1 is put there to stress the difference.)

III. CALCULATION OF DECREMENTS AND TUNE SHIFTS FOR PEP.

Let us now apply the general formula Eq. (10) to our storage ring. PEP has three equally spaced bunches in each beam. The feedback system for each transverse oscillation has one kicker and one detector positioned practically at the same place, where $\beta' = 0$. Hence, we have $k = 3$, $\mu_{k+1} - \mu_D^{\mp} = \mu_o$, $\alpha_k = 0$, $\Delta\gamma^{\mp} = 0$, and $\beta_D^- = \beta_D^+ = \beta_k$. The system acts on the bunches through a tuned radiofrequency amplifier driven at the 72-nd harmonic of the orbital frequency. For this case the coefficients $\bar{f}_{m1\ell} = \overset{+}{f}_{m1\ell}$ and $\bar{g}_{m1\ell} = \overset{+}{g}_{m1\ell}$ can be approximated by the following expressions

$$f_{m1\ell} = f_o \, e^{-\alpha\left(m-1+\frac{\ell-1}{k}\right)} \cos\left[\left(m-1+\frac{\ell-1}{k}\right) \Delta\phi + \delta\right] \quad , \tag{22}$$

$$g_{m1\ell} = f_o \, e^{-\alpha\left(m-1+\frac{\ell-1}{k}+\frac{1}{2k}\right)} \cos\left[\left(m-1+\frac{\ell-1}{k}+\frac{1}{2k}\right) \Delta\phi + \delta\right] \quad . \tag{23}$$

In these formulae, α is the decrement of the tuned amplifier in one orbital period, while both $\Delta\phi$ and δ are related to its detuning, i.e., the difference between the resonant frequency of the amplifier's tank circuit and the feedback system's carrier frequency, the latter being maintained always at the 72-nd harmonic of the orbital frequency, and the former being subject to thermal drift, aging of components and deliberate detuning. The tank circuit of the PEP feedback system includes as its inductive component the kicker magnet. When the resonant frequency of the tank differs from the driving frequency (the 72-nd harmonic of the orbital frequency), the phase of the kicker signal on a subsequent passage of the bunch consists of two terms. The initial phase δ can be thought of as being caused by the fact that the equilibrium phase of the kicker current relative to the 72-nd-harmonic driving current varies with tank tuning. The feedback system is designed so that the kicker current is at a peak of the driven 72-nd-harmonic waveform when the bunch passes the kicker if the tank is tuned exactly to the driving frequency. If the tank is detuned, the phase of the driven kicker current is shifted, and the bunch passes off the peak. After the bunch passes, the drive current is removed, and the resonant circuit rings down from this phase-shifted state. The phase slippage of the kicker signal, $\Delta\phi$, arises, because the ringing frequency is determined solely by the tank circuit and differs from the 72-nd-harmonic driving frequency if the tank is detuned with the result that the ringing kicker-current waveform migrates in phase relative to the time of successive bunch passages. The formulae for δ and $\Delta\phi$ in terms of the detuning of the tank circuit $\Delta F/FRES$ are

$$\Delta\phi = 2\pi\, T_o\, \Delta F \qquad (24)$$

and

$$\tan\delta = \frac{(\Delta\phi/\alpha) - e^{-\alpha/6}\sqrt{1+(\Delta\phi/\alpha)^2}\,\sin(\Delta\phi/6)}{1 - e^{-\alpha/6}\sqrt{1+(\Delta\phi/\alpha)^2}\,\cos(\Delta\phi/6)} \qquad (25)$$

The other coefficients are obtained by cyclic permutation, as follows. Equation (22) defines the first row $f_{m11}, f_{mj2}, \ldots f_{m1k}$. Any other row j ($f_{mj1}, f_{mj2}, \ldots, f_{mjk}$) is produced by (j-1) cyclic permutations of the first row, and the same procedure applies to the g's. With these expressions in hand, we can perform the summations in Equation (17) through (20).

$$R^{+}_{j\ell} = R^{-}_{j\ell} = \frac{f_o}{2} \Sigma_c \left(\frac{\ell-1}{k}\right) \quad , \tag{24}$$

$$S^{+}_{j\ell} = S^{-}_{j\ell} = + \frac{f_o}{2} \beta_k \Sigma_s \left(\frac{\ell-1}{k}\right) \quad , \tag{25}$$

$$P^{+}_{j\ell} = P^{-}_{j\ell} = \frac{f_o}{2} \Sigma_c \left(\frac{\ell-1}{k} + \frac{1}{2k}\right) \quad , \tag{26}$$

$$Q^{+}_{j\ell} = Q^{-}_{j\ell} = + \frac{f_o}{2} \beta_k \Sigma_s \left(\frac{\ell-1}{k} + \frac{1}{2k}\right) \quad , \tag{27}$$

where

$$\Sigma_{c,s}(p) = e^{-\alpha p}(E^{+}_{c,s}(p) + E^{-}_{c,s,}(p)) \tag{28}$$

with

$$E^{\pm}_{c,s}(p) = \frac{\begin{matrix}\cos\\ \sin\end{matrix}(\mp p\ \Delta\phi \mp \delta) - e^{-\alpha} \begin{matrix}\cos\\ \sin\end{matrix}(\mu_o \mp (p-1)\ \Delta\phi \mp \delta)}{1+e^{-2\alpha} - 2e^{-\alpha} \cos\ (\mu_o \pm \Delta\phi)} \tag{29}$$

Using these expressions, we calculate the eigenvalues λ_i of the matrix Eq (10) which, in this case, has dimensions 12x12 ($i = 1,2....12$). Let us introduce, instead of λ_i, the more helpful quantities θ_i through the relation

$$\lambda_i = e^{i\theta_i} \tag{30}$$

The real part of θ_i is the perturbed phase advance, and the imaginary part of it gives the decrement (if it is positive) or increment (if it is negative) of oscillations damped by the feedback system. Since the matrix, T, is real, all θ_i fall into complex conjugate pairs. Hence, in general, we have 6 different decrements and tunes.

IV. DUSCUSSION AND CONCLUSIONS

We have calculated the eigenfrequencies θ_i for the PEP transverse feedback system for four different values of the decrement α , and the results are presented in Fig. 1 through Fig. 8 as pairs of graphs giving the DECREMENT (imaginary part of θ_i) produced in the coherent beam motion by the feedback system and the corresponding FREQUENCY SHIFT (real part of $(\theta_i - \mu_o)/2\pi$) as functions of the detuning ΔF/FRES of the tank circuit. The parameters listed at the bottom of each graph are identified as follows:

P	=	$f_o \beta_k / 2$
ALPHA	=	α
DNU	=	Fractional part of $\mu_o/2\pi$
DPSI	=	Fractional part of $(\mu_{k+1} - \mu_D)/2\pi$

The case which corresponds to the design of the PEP feedback system, referred to in Reference 2, is the one presented in Figures 3 and 4, and we address our attention to that case first. The parameter, P, has been chosen according to the parameters given in Section 3 of Reference 2, which include a beam energy of 4 GeV. The choices DNU = DPSI = 0.25 correspond to the location of the detector adjacent to the kicker and to a betatron tune (fractionl part) of one quarter. The value chosen for ALPHA corresponds to the specifications for the feedback amplifier given in the PEP Technical Memo, PTM-191, viz. a bandwidth of 0.82 MHz or equivalently a Q of 12.[3]

Since there are six normal modes, there are six curves in each figure. In Fig. 3, they fall into two groups of three which are scarcely distinguishable. The value of the DECREMENT for the ideally tuned system (DF/FRES=0) agrees well with that predicted in Ref.2. The FREQUENCY SHIFTS shown in Fig. 4 are completely negligible, as expected. As can be seen from Fig. 3, tolerances on the amplifier tuning ΔF/FRES of the order of 10^{-2} are more than adequate for keeping the decrements of all modes within $\pm$ 5% change.

Now, returning to the other figures, in each case the values of P and ALPHA have chosen to hold constant their product. P*ALPHA=constant. This procedure corresponds to holding the power available from the feedback amplifier constant while varying its bandwidth or equivalently its decay time. In Figures 1 and 2, the decay time of the amplifier is one-third that in the design case so that the bandwidth is 2.5 MHz - - very large compared to the bunch-passage frequency of 0.82 MHz. As expected, all modes are equally damped, because the feedback system treats the bunches virtually independently. The FREQUENCY SHIFTS are miniscule. The magnitudes of the DECREMENTS are, however, about three times smaller than those of the design case.

Figures 5 and 6 show a very interesting case in which the decay time of the amplifier has been increased approximately to equal the interbunch period. The corresponding bandwidth is 0.25 MHz, considerably smaller than the bunch-passage frequency, and the corresponding Q is 40. With DF/FRES=0, some modes are damped at around three times the rate achieved by the design case, while others are damped at lower rates but in no case lower than the design case. Of special interest is the fact that by deliverately detuning the tank circuit to DF/FRES = $\pm$ 0.005 we can achieve DECREMENTS for all modes which are about equal and which are approximately twice those obtainable with the system as designed, or, alternatively, the same DECREMENTS can be achieved with half the power.

Finally, Figures 7 and 8 show what happens when the bandwidth is made ten tiems smaller than in the design case. Strong damping can be produced in the lowest frequency mode but only at intolerable sacrifices in damping for some of the highest frequency modes. This case corresponds to a bandwidth of 82 KHz which is comfortably larger than the expected frequency of the lowest mode viz. DNU/I_o = 34 KHz but smaller than that of the next mode which should appear about 136 KHz higher at 170 KHz.

Our main conclusions are: first, that the PEP feedback system as designed will produce damping rates which differ somewhat from normal mode to normal mode because of the memory (finite bandwidth) of the system, but the differences are at the $\pm$ 5-percent level and may not be measureable and, second, that substantially higher damping rates might be achieved with the same amplifier power by reducing the bandwidth and detuning the tank circuit.

REFERENCES

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2. J.-L. Pellegrin, J. Rees, "Beam Excitation and Damping with the Transverse Feedback System," PEP Note-315, August 1979.

3. L. Genova _et al._, "PEP Transverse Beam Feedback Power Amplifiers - Specifications." PEP Technical Memo, PTM-191, January 9, 1979 .

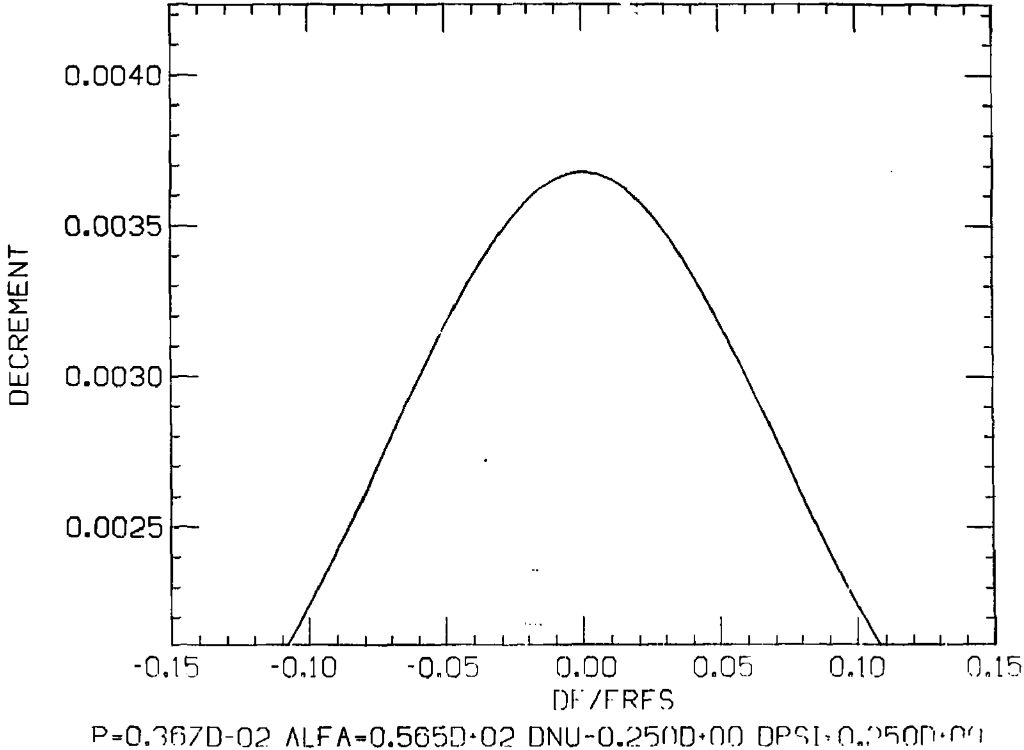

FIGURE 1: Very wide band system. Interbunch period is 9.4 times decay time of tuned amplifier.

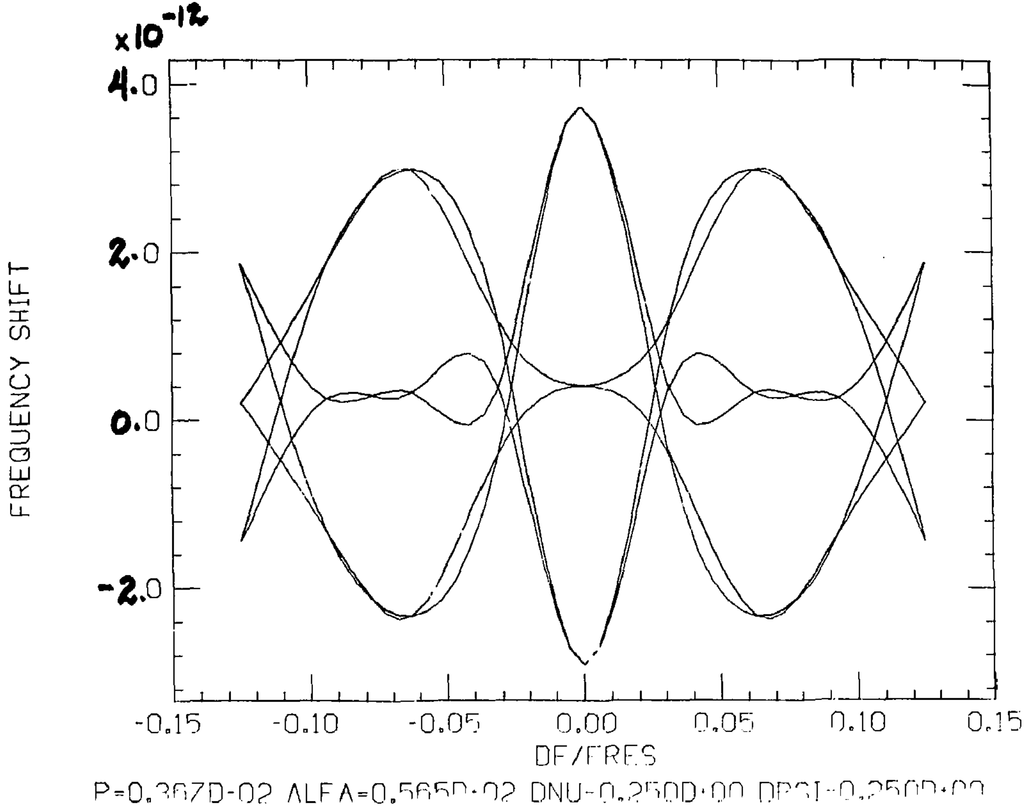

FIGURE 2: Same conditions as Fig. 1.

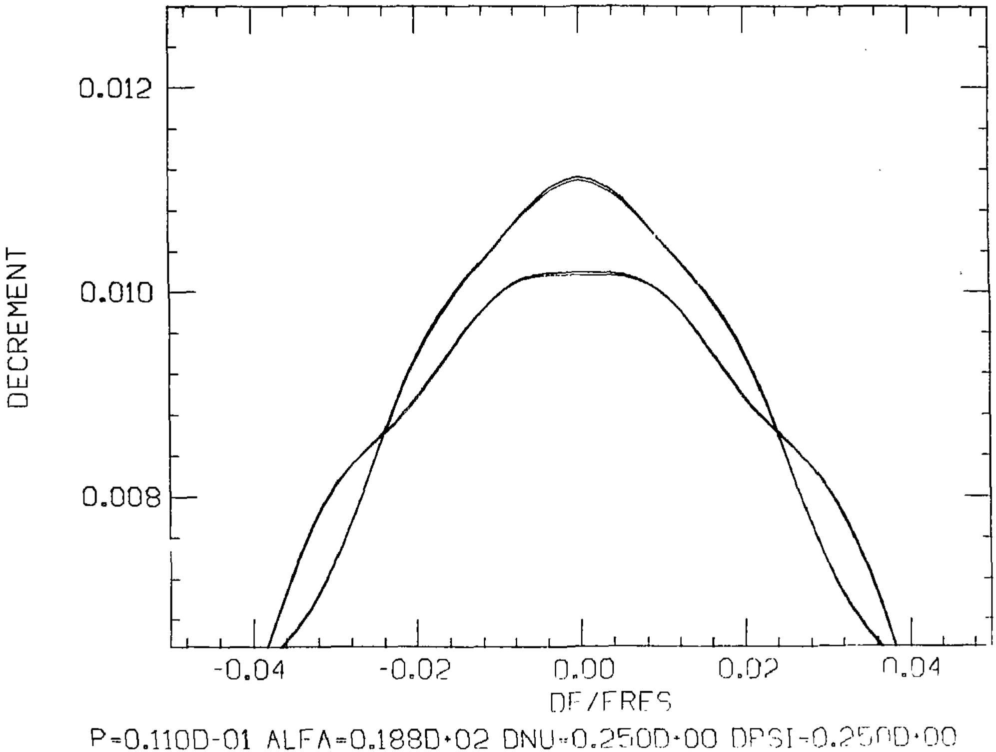

FIGURE 3: Feedback system design conditions. Wide band case. Interbunch period is 3.1 times decay time of tuned amplifier.

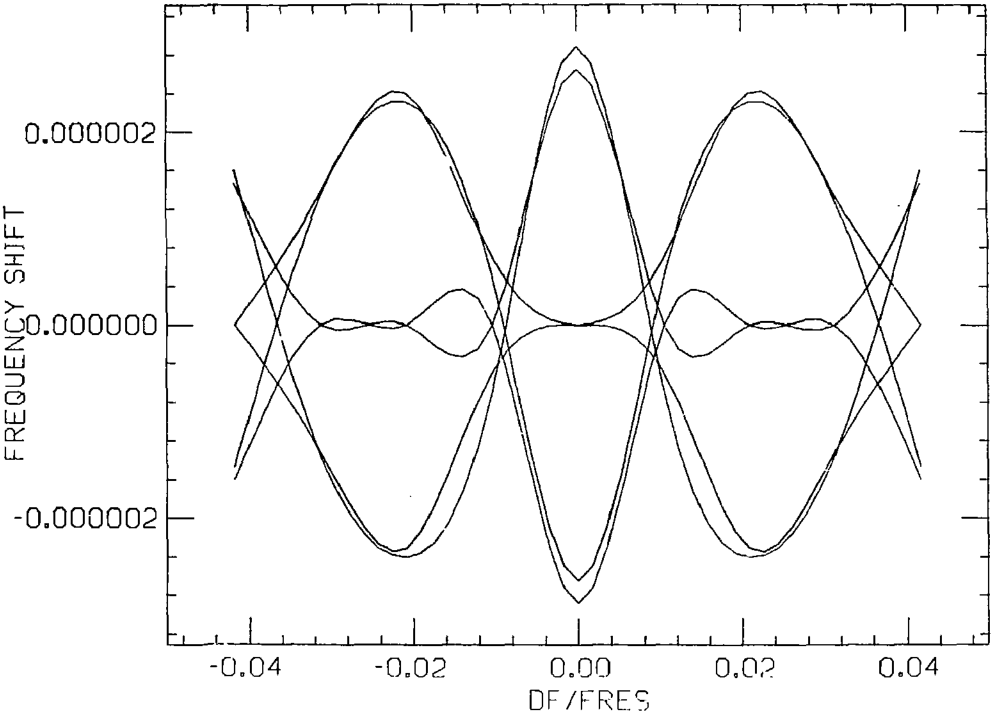

FIGURE 4: Same conditions as Fig. 3.

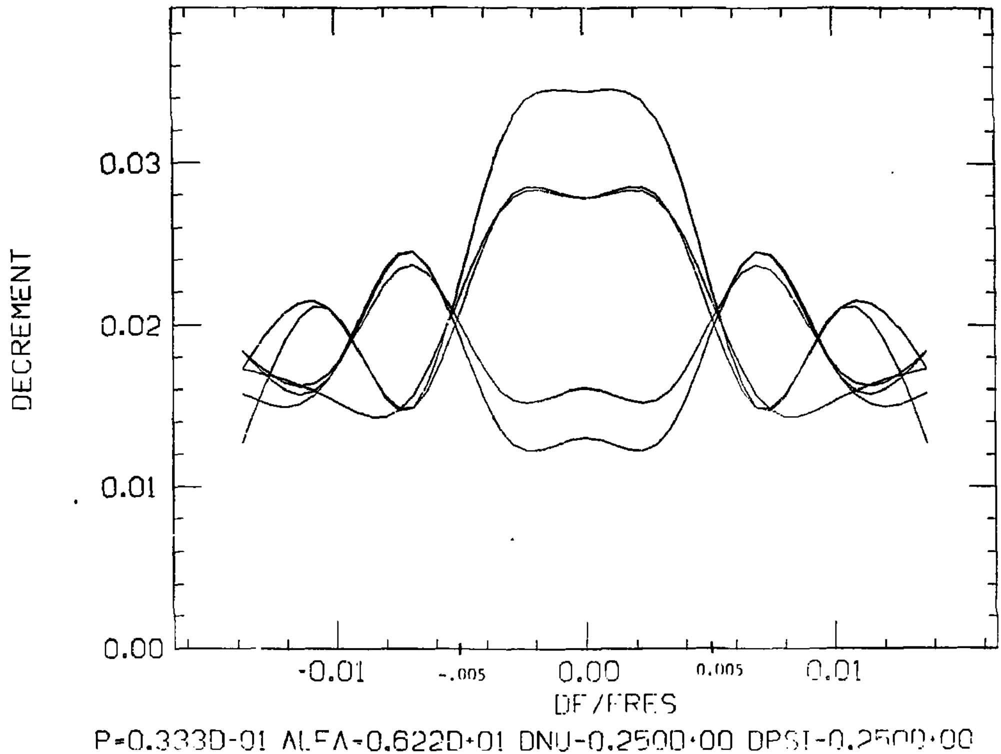

FIGURE 5: Intermediate band system. Interbunch period is about equal to feedback amplifier decay time.

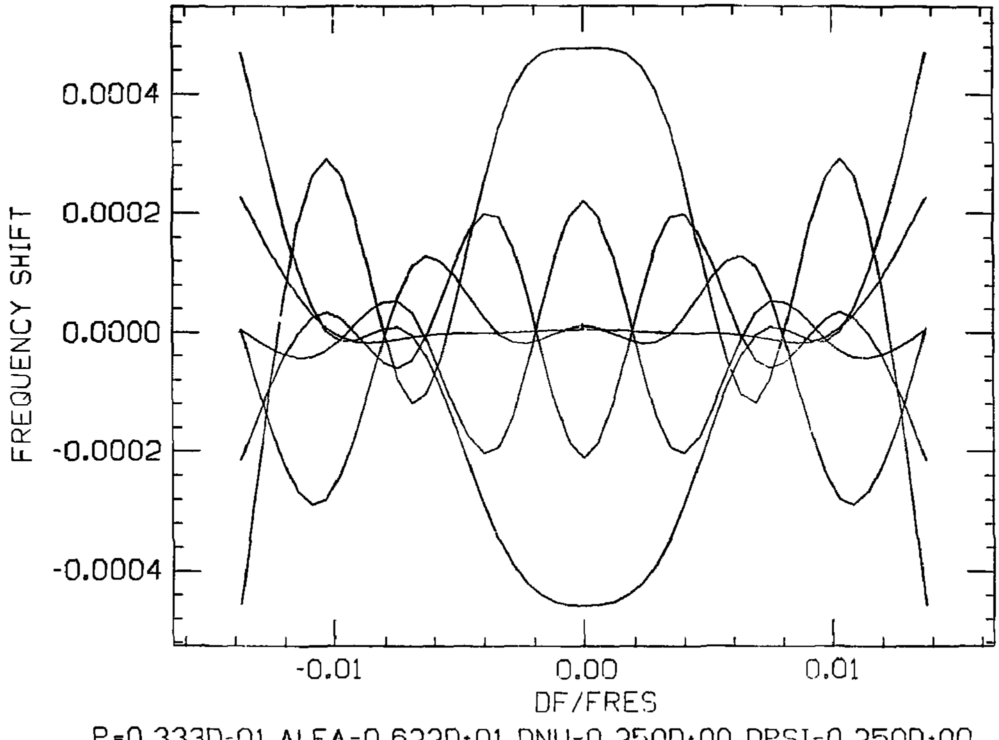

FIGURE 6: Same conditions as Fig. 5.

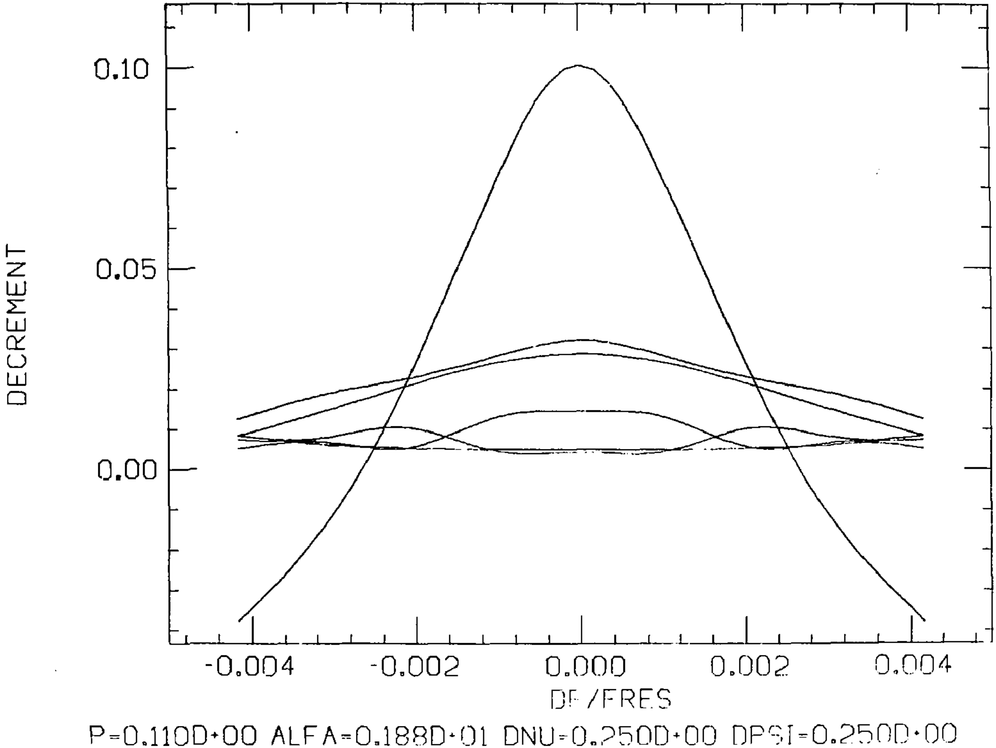

FIGURE 7: Narrow Band System. Interbunch period is 0.31 times feedback amplifier decay time.

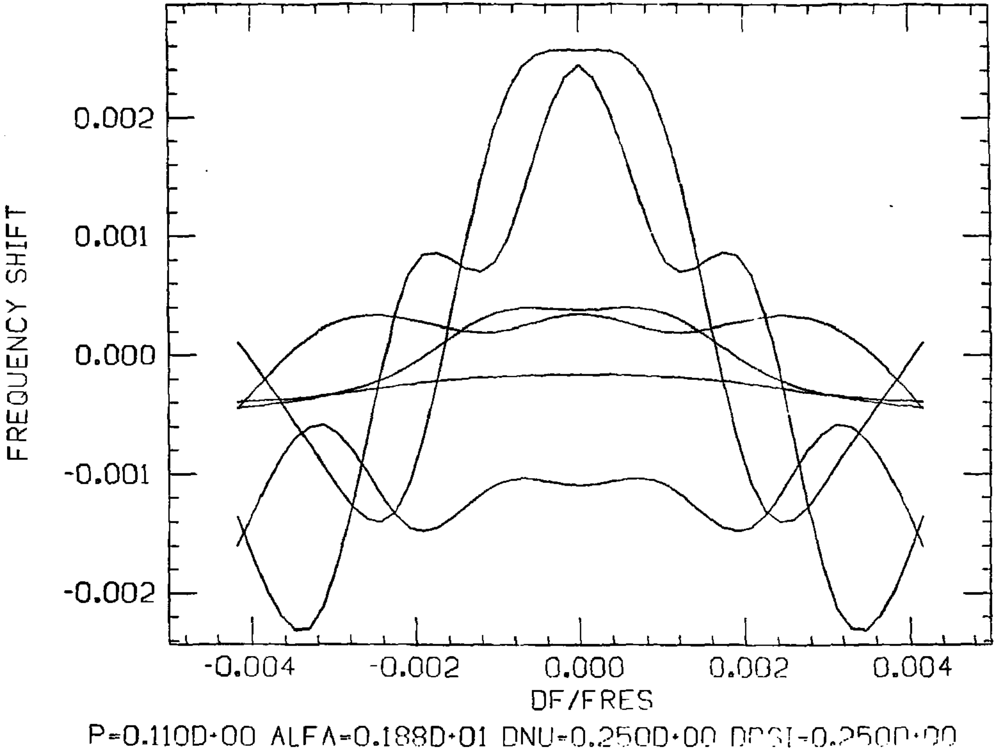

FIGURE 8: Same conditions as Fig. 7.