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THE 50-MeV POLARIMETER

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Abstract

A description is given of the construction, operation and calibration of the 50 MeV polarimeter which was used at the ZGS. The dependence of the observed counts on various parameters, including the beam polarization, beam intensity and the solid angle in the two polarimeter arms is also discussed.

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The 50 MeV polarimeter was one of the cornerstones of the knowledge of beam polarization at the ZGS. Some of the details of the construction and operation of this polarimeter have been reported in the literature.[1-3] The calibration constant for this instrument (A) depends on the polarization or analyzing power for p + C elastic scattering (P_{pC}) near 50 MeV kinetic energy of the proton. The calibration constant A has had a large uncertainty associated with it in the past. New high precision measurements of P_{pC} near 50 MeV have recently become available from Osaka.[4] These data permit a more accurate determination of the 50 MeV polarimeter calibration constant.

There are several goals for this paper: 1) a description of the construction and operation of the 50 MeV polarimeter, Section A; 2) a critical reevaluation of the 50 MeV calibration constant using the recent Osaka data, Section B; 3) a discussion of the sensitivity of the measured counts from the two arms of the polarimeter to various parameters, such as the difference between the beam-up and beam-down polarizations, in Section C; and 4) a comparison of various expressions relating the measured counts to the beam polarization. It is anticipated that this paper is the first in a series of reports on the 50 MeV, CERN and Beam 22 polarimeters and their calibration constants. It is desired to obtain the best possible estimate of the beam polarization over the full range of ZGS energies.

A) Construction and Operation of the 50 MeV Polarimeter

There are three references in the literature (to my knowledge)[1-3] about the 50 MeV polarimeter and its calibration constant, A. Unfortunately, these three references quote different operating conditions and values of A.

The first reference[1] was one of the first papers reporting on data taken with the polarized beam at the ZGS. It was a measurement of $\Delta\sigma_T$ at 3.5 GeV/c. "The polarization at 50 MeV is measured using a polarimeter which continuously measures the left-right asymmetry in proton-carbon elastic scattering at 60°, where the measured asymmetry parameter is (85 ± 7)%," (Ref. 1).

The second reference[2] was an article about polarized proton acceleration in the ZGS. Most of the authors were from the Argonne Accelerator Research Facilities (ANL-ARF) Division. Details of the 50 MeV polarimeter construction are given, as well as the calibration constant 0.85 ± .07. However, the laboratory angle of 55° is mentioned. In addition, measurements of the polarimeter which I performed disagreed with some numbers quoted in Ref. 2, although there was general agreement on most of the construction parameters.

The third reference[3] was a recent article by the Michigan group on C_{NN}(90°, cm) as a function of energy. "This (50 MeV polarimeter) measured p-carbon elastic scattering at 55° in the lab where the analyzing power is 88 ± 5%." This value of A was used for most of the recent measurements of beam polarization at the 50 MeV polarimeter.

All three papers (Refs. 1-3) refer to the same article (Ref. 5) for the measurement of the analyzing power. However, Ref. 5 contains fits to data which were published in another conference proceedings.[6] The original data were from a Birmingham group and were also reported in Ref. 7. The P_{pC} values actually used in this paper are from Ref. 7.

According to L. Ratner[8], the 50 MeV polarimeter construction and angle were unchanged since installation. This statement agrees with Refs. 2 and 3, where $\theta_{lab} = 55°$ is quoted. My measurements gave $\theta_{lab} = 55.0 \pm 0.4°$, with the error reflecting measurement uncertainty, not angular acceptance. However, there may be a contradiction with Ref. 1 ($\theta = 60°$). Unfortunately, it is not clear in Ref. 1 whether the laboratory or c.m. angle is specified ($\theta_{lab} = 55°$, kinetic energy $T_{lab} = 50$ MeV corresponds to $\theta_{cm} = 59°$).

The polarimeter consisted of two identical arms (L, R) of three plastic scintillators with a needle-like carbon target. Scattered protons passed through a vacuum window into the air, through two scintillators (L_1, L_2 or R_1, R_2), through a degrader and into a third scintillator (L_3 or R_3). The size of scintillators L_1, L_2, R_1, R_2 was 1-1/4" x 2-1/4" x 1/8" and of L_3, R_3 was 1-1/2" x 3" x 1/16". (The individual counters were labeled with these dimensions; I only roughly checked them.) The separation of L_1 and L_2 was nominally 3-1/4" and of L_2 and L_3 was 10-3/4", with the same dimensions for the R arm. The actual scintillators were allowed some freedom to "tilt" closer to or further from the carbon target; they were not constrained to the same separations as the counter mounting holes, into which the photomultiplier and base fit. When I measured the polarimeter, the L_1, L_2 separation at the scintillators was 3-1/4" as specified above, but the L_2, L_3 separation was 3/8" less than specified. The R arm was not measured, since the polarimeter was being disassembled and the R_i counters had been removed. The absorber thickness was found to be 1/4" for the L arm.

In order to obtain the distances to the carbon target and the angle, the separation between the center of the counter mounting holes for L_1 and R_1 and for L_3 and R_3 were measured. From these values, $\theta_{lab} = (55.0 \pm .4)^\circ$ was found (assuming the beamline bissected the angle between the L and R arms) and the distance from the carbon target to L_3 or R_3 was estimated to be (24.72 ± 0.12)". The counters L_3 or R_3 defined the solid angle of the polarimeter arms. The 1-1/2" width of L_3 or R_3 corresponded to $\Delta\theta_{lab} = 3.5^\circ$.

There were several discrepancies noted between my measurements and the values quoted in Ref. 2, however none of them seem to be of significant importance to the polarimeter operation. The laboratory angle agreed with the 55° value in Refs. 2 and 3. The size of L_3 agreed with the value in Ref. 2, but the distance of L_3 or R_3 to the carbon target did not (24.7" compared to 26" from Ref. 2). The flight path after the absorber to L_3 also differed (10" compared to 14" from Ref. 2), as did the absorber thickness (1/4" compared to 3/16" from Ref. 2). These differences predominantly affect the numerical values in Table III of Ref. 2 and perhaps some of the proton energies after the absorber, but basically the polarimeter operated as described in Ref. 2. The only parameter of interest for this paper that differs is the θ-acceptance of the polarimeter (3.5° compared to 3.3°). It will be shown later that the difference in the calibration constant caused by this change is negligible.

The vertical position of the needle-like carbon target could be adjusted to intercept a smaller or larger fraction of the beam. Normally, the vertical position was adjusted so that the rates in the arms of the polarimeter were roughly 20% of the rates when the carbon target was fully in the beam.[9] Even in the latter case, the fraction of the beam intercepted by the target was quite small. In particular, the beam polarization was not uniformly sampled across the beam spot, but was selectively sampled near the top edge of the beam spot.

Although the nominal beam energy was 50 MeV, the actual beam energy was not measured recently. The assumption will be made that the energy was 50.0 MeV, and had not changed for many years. This is a plausible assumption because the settings for various equipment affecting injection of the beam into the ZGS were unchanged. Variations in the operating conditions of the LINAC might change the beam energy at the 50 MeV polarimeter by as much as ± 0.4 - 0.5 MeV.[9] Also, there is an energy spread in the beam, which was measured to be ± 0.15 MeV. Lack of knowledge of the beam energy leads to an uncertainty in the 50 MeV polarimeter calibration constant A because of the energy dependence of P_{pC}. This will be discussed further in Section B.

The rates were quite high in the counters during the 0.4 - 0.5 msec pulses from the LINAC. The instantaneous rates for scintillation counters L_1 or R_1, L_2 or R_2, L_3 or R_3 were typically 5, 1.7 and 0.7 Mhz respectively. The ratio of accidental to true coincidences was roughly 3% at 10μA and 5-10% at 35 μA from the polarized ion source. The coincidence rate was normally about 100-200 per pulse.

B) <u>The 50 MeV Polarimeter Calibration Constant</u>

If the number of coincidences for one beam pulse from counters L_1, L_2, L_3 are denoted L and from R_1, R_2, R_3 are denoted R, then the formula used to determine the beam polarization P is

$$P = \frac{1}{A} \cdot \frac{L - R}{L + R} = \frac{1}{A} \cdot \varepsilon \tag{1}$$

where A is the calibration constant and ε is the raw asymmetry in the counts from the two arms. The value of A is the p + C elastic scattering polarization or analyzing power P_{pC} averaged over the angular acceptance of the polarimeter, the variation in the beam energy or the energy spread in the beam, the finite target size, multiple scattering etc. In principle, corrections could be included to account for the difference in the beam polarization for the portion of the beam actually sampled compared to the average over the whole beam.

According to L. Ratner,[8] the change in the quoted calibration constant from 0.85 ± .07 to 0.88 ± .05 in Refs. 1-3 occurred as the result of a test at Argonne. The asymmetry ε was measured at 50 MeV as usual with the polarimeter. Then the beam kinetic energy was degraded to 40 MeV and the asymmetry was measured again. This time, a different set of p + C elastic scattering polarization data[10] were used to give the beam polarization. It was assumed that no depolarization of the beam occurred between 50 and 40 MeV,[11,12] as indicated by theoretical calculations and some experimental results. The revised value of A was given in Ref. 3.

The recent Osaka data[4] combined with earlier measurements from Birmingham,[7] Minnesota[10] and Oak Ridge,[13] permit a more accurate determination of the calibration constant A. The Osaka data consist of measurements of p + C elastic scattering polarization at one to five angles at about a dozen kinetic energies between 40 and 75 MeV and a large number of angles at 65 MeV.

The following technique was used to estimate the value of A for the 50 MeV polarimeter. The existing data near 50 MeV were corrected for the energy dependence of P_{pC}, were fit with a polynomial, and then were averaged over the angular acceptance of the polarimeter. The assumption was made that the shape of the p + C elastic scattering polarization curve remained the same except for a shift in angle and in polarization that was dependent only on energy

$$P_{pC}(T, \theta) \simeq P_{pC}(50\text{ MeV}, \theta + \delta\theta(T)) + \delta P(T) \tag{2}$$

where the angle shift $\delta\theta$ and polarization shift δP do not depend on angle. This expression was assumed valid near the peak in the polarization and to slightly larger angles.

A more detailed description of the determination of A follows. First, the Osaka data were fit with a quadratic using the three points nearest the peak at all energies between 40 and 60 MeV where at least 3 angles were measured. Several bits of information were available from these fits: a) the angle of the maximum p + C polarization, θ_{MAX}, as a function of kinetic energy (Fig. 1); b) the value of the maximum polariation, P_{MAX}, as a function of kinetic energy (Fig. 2); and c) a measure of the width of the quadratic, α, where

$$\begin{aligned} P &= \alpha\, \theta^2 + \beta\theta + \gamma \\ &= P_{MAX} + \alpha(\theta - \theta_{MAX})^2 . \end{aligned} \tag{3}$$

Note

$$P_{MAX} = \alpha(\theta_{MAX})^2 + \beta\, \theta_{MAX} + \gamma \tag{4}$$

The value of α was independent of energy within the uncertainties from the fits. This fact supports the assumption given by Eq. (2). Plots of P_{pC} at 40 MeV,[4,10,13] 50 MeV[4,7] and 65 MeV[4] over a wide angular range also seem consistent wth Eq. (2).

The second step in the determination of A was to fit the data in Figs. 1 and 2 with quadratics in kinetic energy T. The best fits are plotted. From these fits, the slopes at 50 MeV were determined

$$\left.\frac{\Delta\theta}{\Delta T}\right)_{50\ \mathrm{MeV}} \cong \left.\frac{\Delta\theta_{MAX}}{\Delta T}\right)_{50\ \mathrm{MeV}} = -0.362 \pm .016\ \text{degrees/MeV}$$

$$\left.\frac{\Delta P}{\Delta T}\right)_{50\ \mathrm{MeV}} \cong \left.\frac{\Delta P_{MAX}}{\Delta T}\right)_{50\ \mathrm{MeV}} \cong +\ (0.010 \pm .001)/\mathrm{MeV} \cong +\ (1.0 \pm 0.1)\%/\mathrm{MeV} \qquad (5)$$

where the first approximate equality in each equation above is based on the assumption of Eq. (2). Then

$$\delta\theta(T) \cong \left.\frac{\Delta\theta}{\Delta T}\right)_{50\ \mathrm{MeV}} \cdot (T - 50\ \mathrm{MeV})$$

$$\delta P(T) \cong \left.\frac{\Delta P}{\Delta T}\right)_{50\ \mathrm{MeV}} \cdot (T - 50\ \mathrm{MeV}) \qquad (6)$$

The third step to estimate A was to use Eqs. 6 to correct the 49 MeV Birmingham[7] and 48.9 and 49.7 MeV Osaka[4] results so they all corresponded to 50.0 MeV. For the 48.9 MeV results, this involved a shift of 0.40° to the angle and an increase of 1.1% to the polarization at all angles. The corrected points are shown in Fig. 3. These data were fit with various order polynomials in θ_{lab}, with the best cubic and quartic fits plotted in Fig. 3. The results of the fits are summarized in Table 1. The calibration constant A was found by averaging P_{pC} from the fits over the angular acceptance of the 50 MeV polarimeter ($\Delta\theta_{lab}$ = 3.5°, indicated by arrows in Fig. 3, and $\Delta\phi$ = ± 4.2°) and over the approximate differential cross section read from the graph in Ref. 2. I recommend the value

$$\boxed{A \simeq 0.845 \pm .02} \tag{7}$$

This same value is obtained if the differential cross section is assumed to be constant and if $\phi = 0$ is used, so that A would be an average of P_{pC} over the θ_{lab}-acceptance of the polarimeter. Also, changing the θ-acceptance from 3.5° to 3.3° (see Section A) causes a correction to A on the order of .0001 or 0.01%, which is much smaller than other errors.

The uncertainty in A was estimated by adding the following errors in quadrature: a) statistical uncertainty on the Osaka data, typically $\gtrsim$ .008; b) systematic error associated with the normalization of the Osaka data, ± 0.012; c) an uncertainty associated with lack of knowledge of the precise beam energy, typically ± .005 for an uncertainty of ± 0.5 MeV from Eq. (5); and d) an error for the uncertainty in the absolute average laboratory angle of the polarimeter arms, typically ± .006 for an angle uncertainty of ± 0.4° (see Section A). The error in d) was estimated by using the fits to the data in Fig. 3 and calculating from these fits

$$\frac{\Delta A}{\Delta\theta}_{lab} \begin{array}{l} \cong -0.016 \pm .003/\text{degree} \\ \\ \cong -(1.6 \pm .3)\ \%/\text{degree} \quad . \end{array} \tag{8}$$

The uncertainty computed from the errors listed above (± .017) did not take into account various other errors such as multiple scattering and the finite acceptance of the detectors used for the Osaka, etc. data.

The recommended calibration constant (Eq. (7)) is somewhat smaller than the value that had been used (0.88 ± .05), though still within the quoted errors. This causes an increase in the beam polarization measured at the 50 MeV polarimeter. Several independent bits of information indicate that such an increase is desirable. Polarized ion sources of the type used at the ZGS have typically given higher beam polarizations than have been observed using the larger calibration constant.[9] Measurements performed in Beam 1 at several momenta below 6 GeV/c have indicated systematic errors in some p + p elastic scattering polarization data in this momentum region and possibly a higher polarization at the 50 MeV polarimeter as well.[3] Finally, preliminary results from the Beam 22 polarimeter at many momenta between 1 and 3 GeV/c have indicated beam polarizations larger than those measured by the 50 MeV polarimeter, similar to the problems encountered in Ref. 3.

There is a possibility that the calibration constant value of 0.845 ± .02 may be systematically high. As described in Section A, the 50 MeV polarimeter does not uniformly sample the beam spot. Rather, it only scatters particles near the top edge of the beam spot. There is evidence from the 1.5 GeV/c linear accelerator at Los Alamos that the beam polarization after acceleration is from 1-4% lower near the edge than at the center of the beam spot.[14] A similar effect may have existed from the polarized ion source at Argonne, but the magnitude of such a drop in beam polarization near the edge of the beam spot was never measured to my knowledge. In addition, E. Parker[9] has estimated that the depolarization in the LINAC at Argonne (50 MeV) may be the same order of magnitude as that observed at Los Alamos (<1% at 1.5 GeV/c). Given the lack of a direct measurement, no correction has been applied to the calibration constant (Eq. (7)) for the nonuniform sampling of the beam polarization. It is also likely that the sampling changed on a day to day basis as the LINAC and polarized ion source were tuned for intensity and polarization.

In the spirit of the determination of the calibration constant above, a 10-parameter fit was performed to the 57 data points between kinetic energies 38 and 65 MeV, and angles 45° to 60° for the Osaka,[4] Birmingham,[7] Minnesota[10] and Oak Ridge[13] polarization data. A quadratic energy dependence was assumed. The shape of the polarization curves were assumed the same except for a shift in angle,

$$P_{pC}(T,\theta) \cong (A + B\alpha + C\alpha^2 + D\alpha^3 + E\alpha^4) + (FT + GT^2) \tag{9}$$

where

$$\alpha(\theta,T) = \theta + H + IT + JT^2 \quad .$$

The fit was quite good, as shown in Fig. 4. The value of chi-squared was 51.2 for 47 degrees of freedom and the value of P_{pC} at T = 50 MeV averaged over the θ-acceptance of the 50 MeV polarimeter was 0.852. This fit confirms the recommended value for the calibration constant Eq. (7).

C. Other Information from the 50 MeV Polarimeter Counts

There were four quantities measured by the 50 MeV polarimeter. These were the coincidences in the left and right arms for beam polarization up and down ($L\uparrow, L\downarrow, R\uparrow, R\downarrow$). In principle, four independent quantities can be obtained from these measured numbers. However, one quantity is taken up with a relative normalization. It is proportional to the total beam intensity, the effective solid angle, the differential cross section and the carbon target thickness. This reduces the useful information to three independent quantities.

In the Appendix, a discussion is given of various commonly used expressions for the beam polarization at the ZGS. Other expressions are also considered, and it is shown that the three independent quantities are:

$$PA + 0(\varepsilon^2)$$

$$\varepsilon_\Omega + \frac{PA}{1 - P^2A^2}\,\varepsilon_P - \frac{P^2A^2}{1 - P^2A^2}\,\varepsilon_A + 0(\varepsilon^3) \tag{10}$$

$$\varepsilon_B + \frac{PA}{1 - P^2A^2}\,\varepsilon_A - \frac{P^2A^2}{1 - P^2A^2}\,\varepsilon_P + 0(\varepsilon^3) \quad .$$

Of course, linear combinations of these quantities could also be used, such as

$$PA + O(\varepsilon^2)$$

$$\varepsilon_\Omega + PA(\varepsilon_B + \varepsilon_P) + O(\varepsilon^3) \tag{11}$$

$$\varepsilon_B + PA(\varepsilon_\Omega + \varepsilon_A) + O(\varepsilon^3) \quad .$$

The small quantities ε_A, ε_B, ε_P, ε_Ω are asymmetries of the beam polarization and intensity between beam-up and beam-down ($\varepsilon_P = (P_\uparrow - P_\downarrow)/(P_\uparrow + P_\downarrow)$, etc.) and of the calibration constant and effective solid angle between the left and right arms $\varepsilon_A = (A_L - A_R)/(A_L + A_R)$, etc.). The value of PA is given by

$$PA = \left(\frac{P_\uparrow + P_\downarrow}{2}\right) \cdot \left(\frac{A_L + A_R}{2}\right) \tag{12}$$

and the recommended value for A for the 50 MeV polarimeter was discussed in Section B and is given in Eq. (7). The quantity PA is _not_ assumed to be small in Eq. (10).

There are several expressions given in the Appendix which can be used to determine each of the quantities in Eq. (10). Numerical tests were performed to compare each of these expressions and the appropriate error. It was found that the same value _and_ error were computed from the different expressions to order ε or ε^2 (see Eq. (10)). The differences were generally much smaller than the statistical errors for runs on the order of 1-2 hours, at least for the 50 MeV polarimeter. The observed equality of the errors for a single quantity in Eq. (10) computed from the different expressions is not completely understood.

Under these circumstances, the following expressions have been chosen for numerical computation in this paper:

$$\tfrac{1}{2}\alpha_4 = \tfrac{1}{2}\left(\frac{L_\uparrow - R_\uparrow}{L_\uparrow + R_\uparrow} - \frac{L_\downarrow - R_\downarrow}{L_\downarrow + R_\downarrow}\right)$$

$$\cong PA + O(\varepsilon^2)$$

$$\tfrac{1}{2}\alpha_7/(1 - P^2A^2) = \tfrac{1}{2}\left(\frac{L_\uparrow - R_\downarrow}{L_\uparrow + R_\downarrow} + \frac{L_\downarrow - R_\uparrow}{L_\downarrow + R_\uparrow}\right) /(1 - P^2A^2)$$

$$\cong \varepsilon_\Omega + \frac{PA}{1 - P^2A^2}\ \varepsilon_P - \frac{P^2A^2}{1 - P^2A^2}\ \varepsilon_A + O(\varepsilon^3) \qquad (13)$$

$$\tfrac{1}{2}\alpha_5/(1 - P^2A^2) = \tfrac{1}{2}\left(\frac{L_\uparrow - L_\downarrow}{L_\uparrow + L_\downarrow} - \frac{R_\downarrow - R_\uparrow}{R_\downarrow + R_\uparrow}\right) /(1 - P^2A^2)$$

$$\cong \varepsilon_B + \frac{PA}{1 - P^2A^2}\ \varepsilon_A - \frac{P^2A^2}{1 - P^2A^2}\ \varepsilon_P + O(\varepsilon^3)$$

Numerical values for these three expressions are plotted for one particular time period in Fig. 5 (this corresponds to the 1.1 GeV/c polarized proton run in 1978).

It is not possible in general to separate the effects of asymmetries from the effective solid angle, calibration constants, beam intensity and polarization. Some additional assumptions are required. At high energies, it may become possible to make statements about ε_B and ε_Ω from Eq. (10). The reason is that PA is quite small at high energy for the Beam 1, Beam 22 and CERN polarimeters (at the ZGS). However, for the 50 MeV polarimeter, PA ≈ 2/3 and all terms in Eq. (10) may be important.

In particular, consider the asymmetry ε_P. Historically, the quantities

$$"P_\uparrow A" = \frac{L_\uparrow - R_\uparrow}{L_\uparrow + R_\uparrow}$$
$$"P_\downarrow A" = \frac{L_\downarrow - R_\downarrow}{L_\downarrow + R_\downarrow} \qquad (14)$$

were computed by the ZGS computer and transmitted to the experimenters. From the discussion above, it can be seen that additional assumptions are needed to isolate ε_P. In other words, the difference between "$P_\uparrow$A" and "$P_\downarrow$A" from Eq. (14) may have originated from ε_A, ε_B or ε_Ω as well as ε_P. Even a comparison from different polarimeters at the same time on the accelerator cannot solve this problem. There are three independent quantities that can be determined for each polarimeter (Eq. (10)), one of which is PA. There are also two asymmetries peculiar to each polarimeter that must be determined, namely ε_A and ε_Ω. This leaves no equations to solve for ε_P or ε_B. Furthermore, it is not clear that ε_P or ε_B would be the same for all polarimeters. For example, the beam transmission through the ZGS and beamlines might have depended on spin direction if the average beam phase space differed or if the current in one or more magnets varied sufficiently to have affected beam transmission. The conclusion is that too little information is available to determine $\varepsilon_P = (P_\uparrow - P_\downarrow)/(P_\uparrow + P_\downarrow)$ unless additional assumptions are made.

A solution for ε_p can be found if two or three other ε's are assumed to be zero. Two cases in particular will be considered: A) In this case, it will be assumed that averaging over all runs in Fig. 5,

$$\langle\varepsilon_A\rangle \cong 0 \qquad \langle\varepsilon_B\rangle \cong 0 \qquad \langle\varepsilon_\Omega\rangle \cong 0 \quad . \tag{15}$$

With these assumptions, using Eq. (13),

$$\begin{aligned} \langle\alpha_5\rangle &\cong - PA\ \langle\alpha_7\rangle \\ \langle\varepsilon_p\rangle &\cong \tfrac{1}{2}\,\frac{1}{PA}\ \langle\alpha_7\rangle \cong -\tfrac{1}{2}\,\frac{1}{P^2A^2}\ \langle\alpha_5\rangle \quad . \end{aligned} \tag{16}$$

This case would hold if the 50 MeV polarimeter was properly aligned in θ, had the same effective solid angle for both arms, and if the beam polarization direction was not correlated with the beam intensity on the average. B) This case uses slightly weaker assumptions. Again it is assumed that the beam direction and intensity were uncorrelated. Also, it assumes that the polarimeter was properly aligned in θ (both arms at $\theta_{lab} = 55°$), but that the counters L_3, R_3 may have been "tilted", causing an asymmetry in the effective solid angle. From the calculations in Section B, it was shown that A was not significantly affected by changes in solid angle if θ_{lab} was fixed. Hence, this assumption gives

$$\langle\varepsilon_B\rangle \cong 0 \qquad \langle\varepsilon_A\rangle \cong 0 \tag{17}$$

and from Eq. (13),

$$\langle\varepsilon_p\rangle \cong - \tfrac{1}{2} \frac{1}{P^2A^2} \langle\alpha_5\rangle$$
$$\langle\varepsilon_\Omega\rangle \cong \tfrac{1}{2} \frac{1}{(1 - P^2A^2)} (\langle\alpha_7\rangle + \frac{1}{PA} \langle\alpha_5\rangle) \quad . \tag{18}$$

On the other hand, there are limits to the size of ε_Ω that are acceptable on the basis of the 50 MeV polarimeter geometry. In my opinion, a tilting of counters L_3,R_3 by greater than ± 1/2" from the nominal positions would not be very plausible. With this constraint, the largest expected value for $|\varepsilon_\Omega|$ would be roughly 0.04. If there had been certain types of problems with the electronics for the 50 MeV polarimeter, larger effects might have occurred. It will be assumed that these problems did not exist during the time corresponding to Fig. 5.

Now consider the data plotted in Fig. 5. The average values are $PA \cong \tfrac{1}{2} \langle\alpha_4\rangle \cong 0.63$, $\tfrac{1}{2} \langle\alpha_7/(1 - P^2A^2)\rangle \cong 0.00$ and $\tfrac{1}{2} \langle\alpha_5/(1 - P^2A^2)\rangle \cong 0$. These values are consistent with Eqs. (16) and (18) with $\langle\varepsilon_p\rangle \cong 0$. Looking in more detail, large variations are seen in α_5, whereas α_4 and α_7 are more nearly constant. These observations suggest sizeable changes in ε_β as a function of time. Studies over a much longer period of time would be required to make truly general statements about the behavior of ε_p, etc.

D) Conclusions

1) The recommended calibration constant for the 50 MeV polarimeter, based on new p + C elastic scattering polarization data is 0.845 ± .02, somewhat less than the previous value of 0.88 ± .05. This recommended value

may be systematically high. Table 2 contains a summary of the parameters derived which affect the 50 MeV polarimeter calibration constant.

2) A number of different expressions can be used to compute the product PA, where P is the average beam polarization and A is the average calibration constant. The answers are the same to first order in small quantities and the errors are also the same.

3) In principle, it is impossible to determine $P_\uparrow A$ and $P_\downarrow A$ separately. There are too few equations for the number of unknowns in the problem.

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Table 1

Summary of Fits to 50 MeV p + C Elastic Scattering Polarization

Data Near $\theta_{lab} = 55°$

Order of Fit	Number of Parameters	χ^2	Degrees of Freedom	A
1. (linear)	2	44.2	10	0.846
2. (quadratic)	3	28.1	9	0.851
3. (cubic)	4	14.6	8	0.845
4. (quartic)	5	9.6	7	0.840

A is the calibration constant obtained by averaging the fitted curve over the angular range θ_{lab} = 53.25° - 56.75°. The value of A is unchanged if a linear approximation to the differential cross section (from Ref. 2) is used as a weight and if the averaging is performed over the range ϕ = - 4.2° → + 4.2° as well as over the θ_{lab} range above.

Table 2

Summary of Parameters Pertinent to the 50 MeV Polarimeter Calibration Constant Determination

θ_{lab} acceptance	$\pm$ 1.75°
$\langle\theta_{lab}\rangle$	(55.0 $\pm$ .4)°
$\Delta A/\Delta\theta_{lab}$	-(1.6 $\pm$ .3) %/degree
ϕ acceptance	$\pm$ 4.2°
Shift in P_{pC} with energy	
$\Delta\theta/\Delta T$	-(0.362 $\pm$ .016)degrees/MeV
$\Delta P/\Delta T$	(1.0 $\pm$ .1)%/MeV
Recommended Calibration Constant A	0.845 $\pm$ .02

Figure Captions

Figure 1 Laboratory angle corresponding to the maximum polarization for p + C elastic scattering as a function of kinetic energy for the Osaka data.[4] The curve is a quadratic fit to the points.

Figure 2 Maximum polarization for p + C elastic scattering as a function of kinetic energy for the Osaka data.[4] The curve is a quadratic fit to the points.

Figure 3 p + C elastic scattering polarization as a function of laboratory angle at 50 MeV from Refs. 4 and 7. The data were corrected in angle and magnitude for the energy dependence as described in the text. The best third and fourth order fits to these points are shown.

Figure 4 p + C elastic scattering polarization as a function of laboratory angle. The data at 38.0 MeV (■, □) are from Minnesota,[10] at 40.0 MeV (▲, △) are from Oak Ridge,[13] at 49.0 MeV (▼, ▽) are from Birmingham[7], and at 39.9, 44.6, 48.9, 49.7, 52.4, 54.6, 59.6, 64.5, and 65.0 MeV (●, 0) are from Osaka.[4] The curves shown are from a global fit to the filled points (■, ▲, ▼, ● see the text). The open points were not used in the fit, but are shown as an indication of the quality of the fit away from the maximum in the polarization.

Figure 5 Plot of three asymmetries as a function of time for the 50 MeV polarimeter. These asymmetries give the three independent quantities from Eq. (10) as discussed in the text. The data shown correspond to the 1.1 GeV/c polarized proton run in 1978.

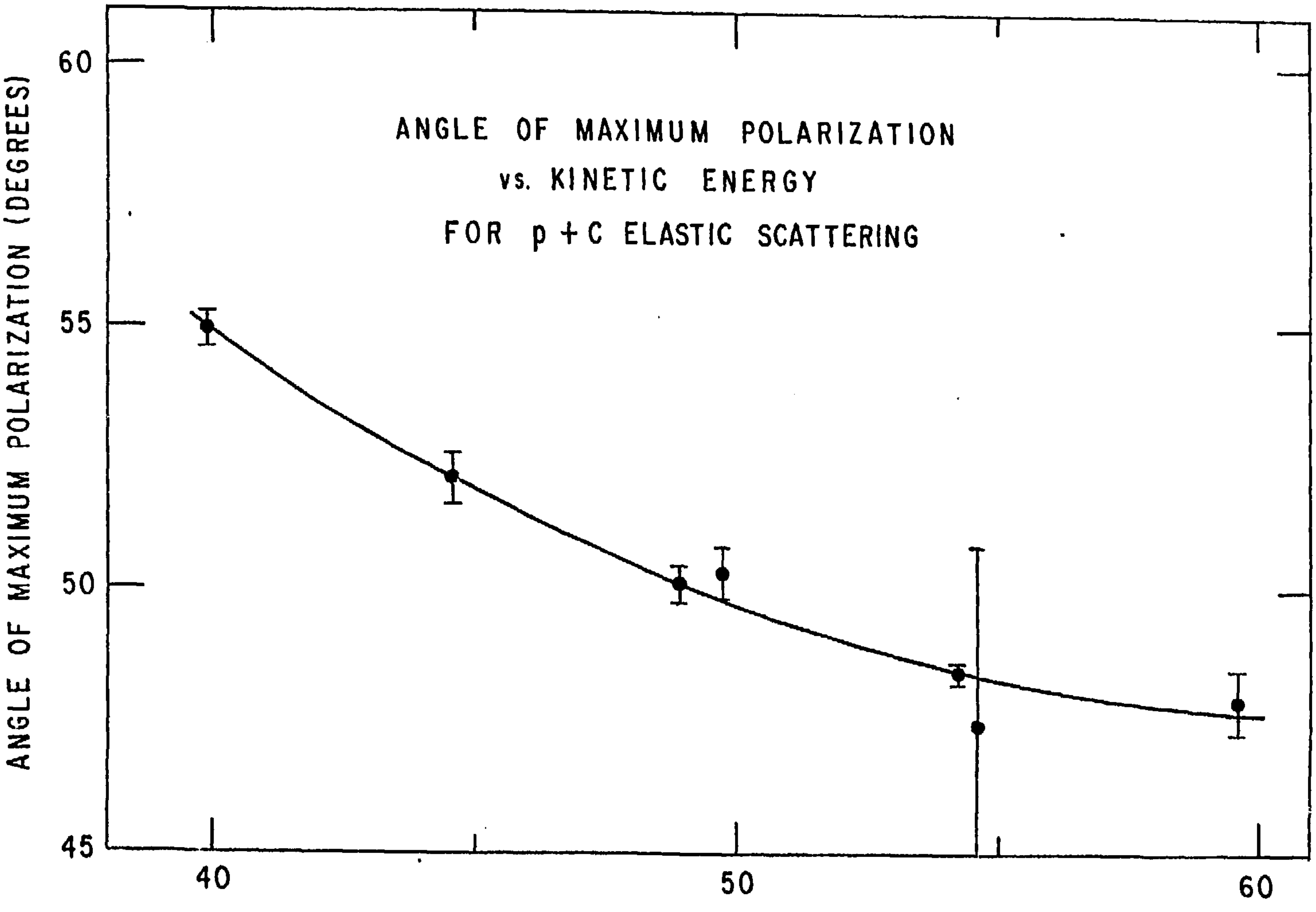

ANGLE OF MAXIMUM POLARIZATION
vs. KINETIC ENERGY
FOR p + C ELASTIC SCATTERING
ANGLE OF MAXIMUM POLARIZATION (DEGREES)
60
55
50
45
40
50
60
KINETIC ENERGY (MeV)

Fig. 1

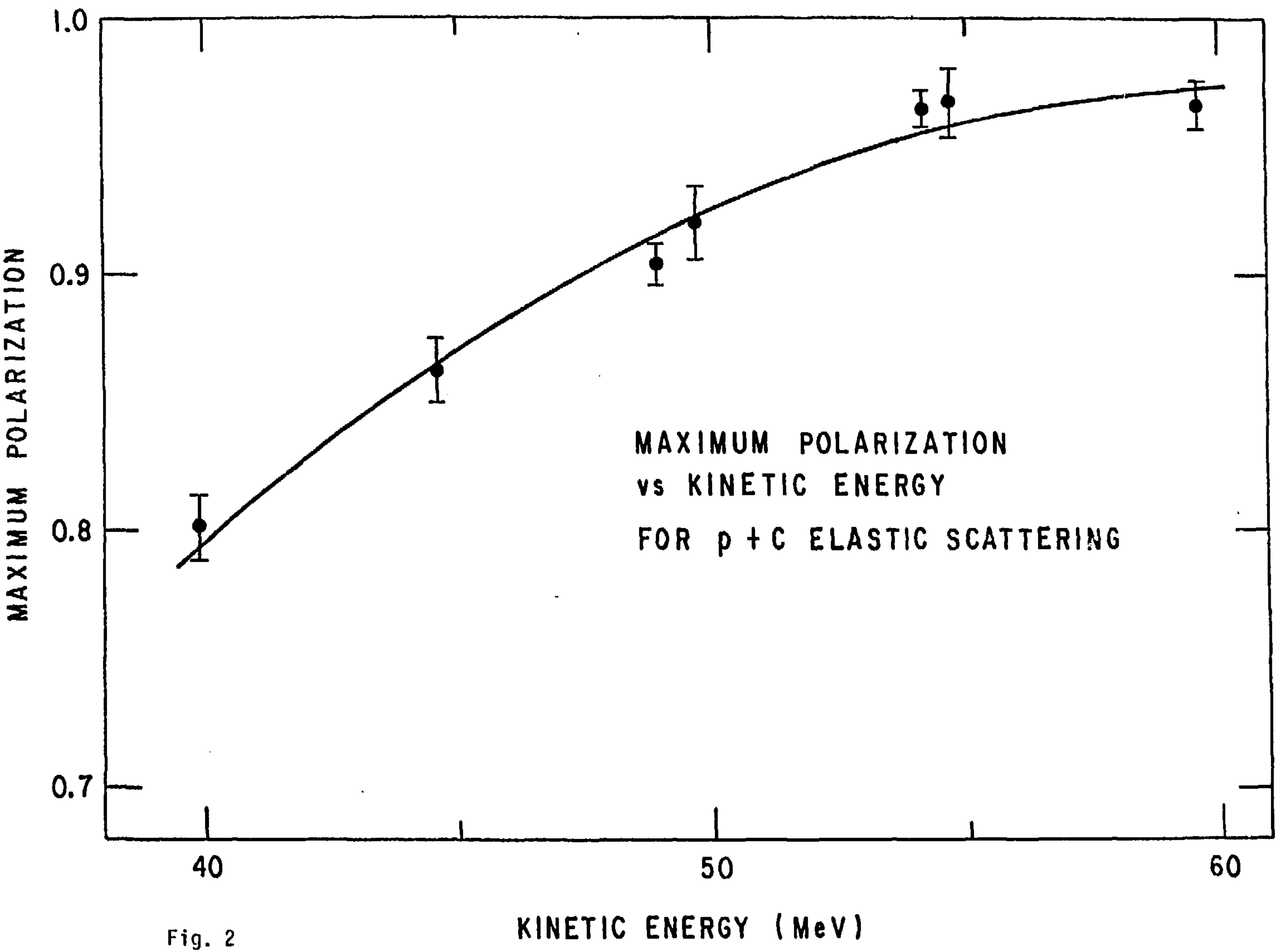
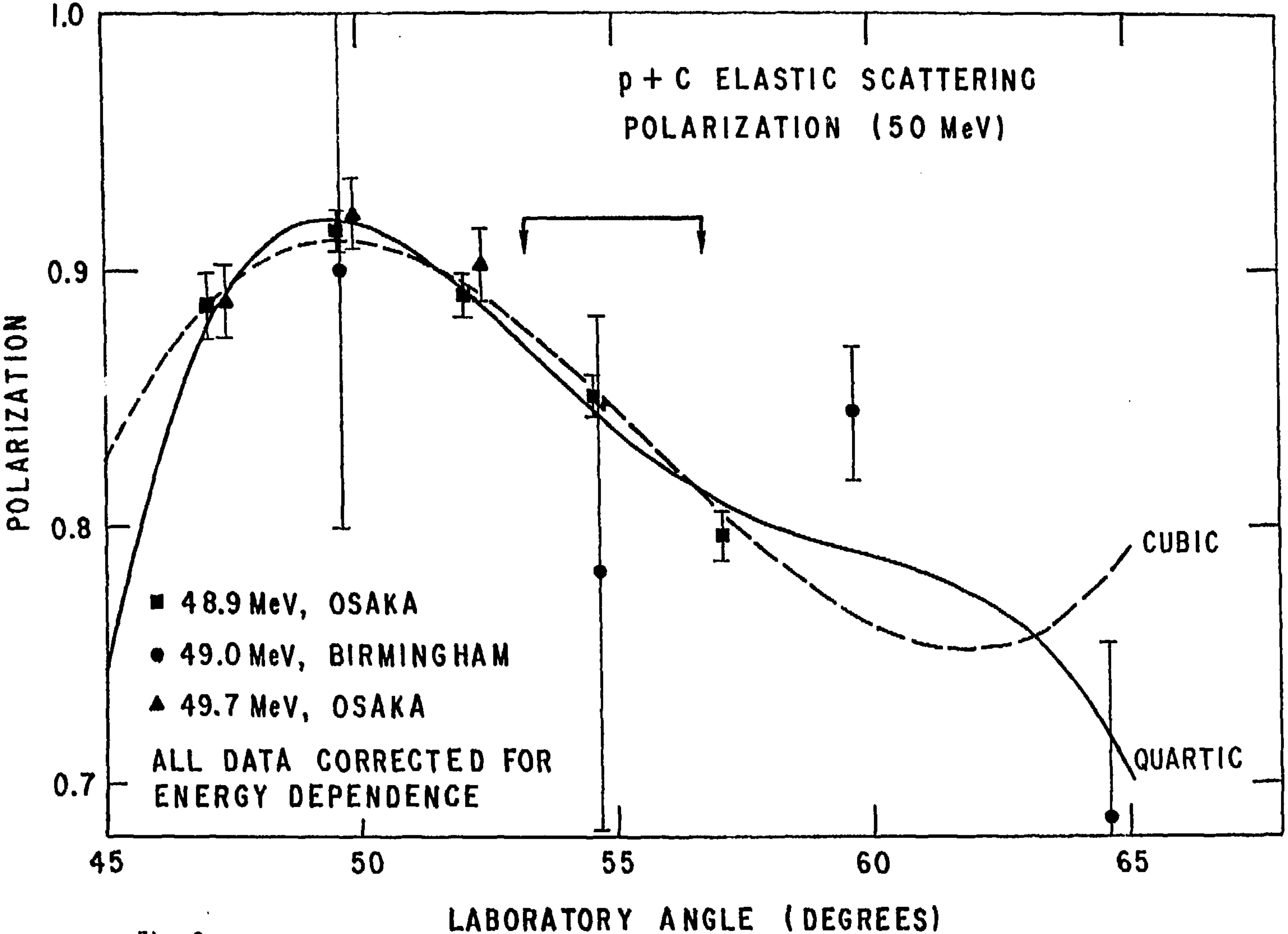

1.0
0.9
0.8
0.7
MAXIMUM POLARIZATION
40
50
60
KINETIC ENERGY (MeV)
MAXIMUM POLARIZATION
vs KINETIC ENERGY
FOR p + C ELASTIC SCATTERING

Fig. 2



p + C ELASTIC SCATTERING
POLARIZATION (50 MeV)
1.0
0.9
0.8
0.7
POLARIZATION
45
50
55
60
65
LABORATORY ANGLE (DEGREES)
■ 48.9 MeV, OSAKA
● 49.0 MeV, BIRMINGHAM
▲ 49.7 MeV, OSAKA
ALL DATA CORRECTED FOR ENERGY DEPENDENCE
CUBIC
QUARTIC

Fig. 3

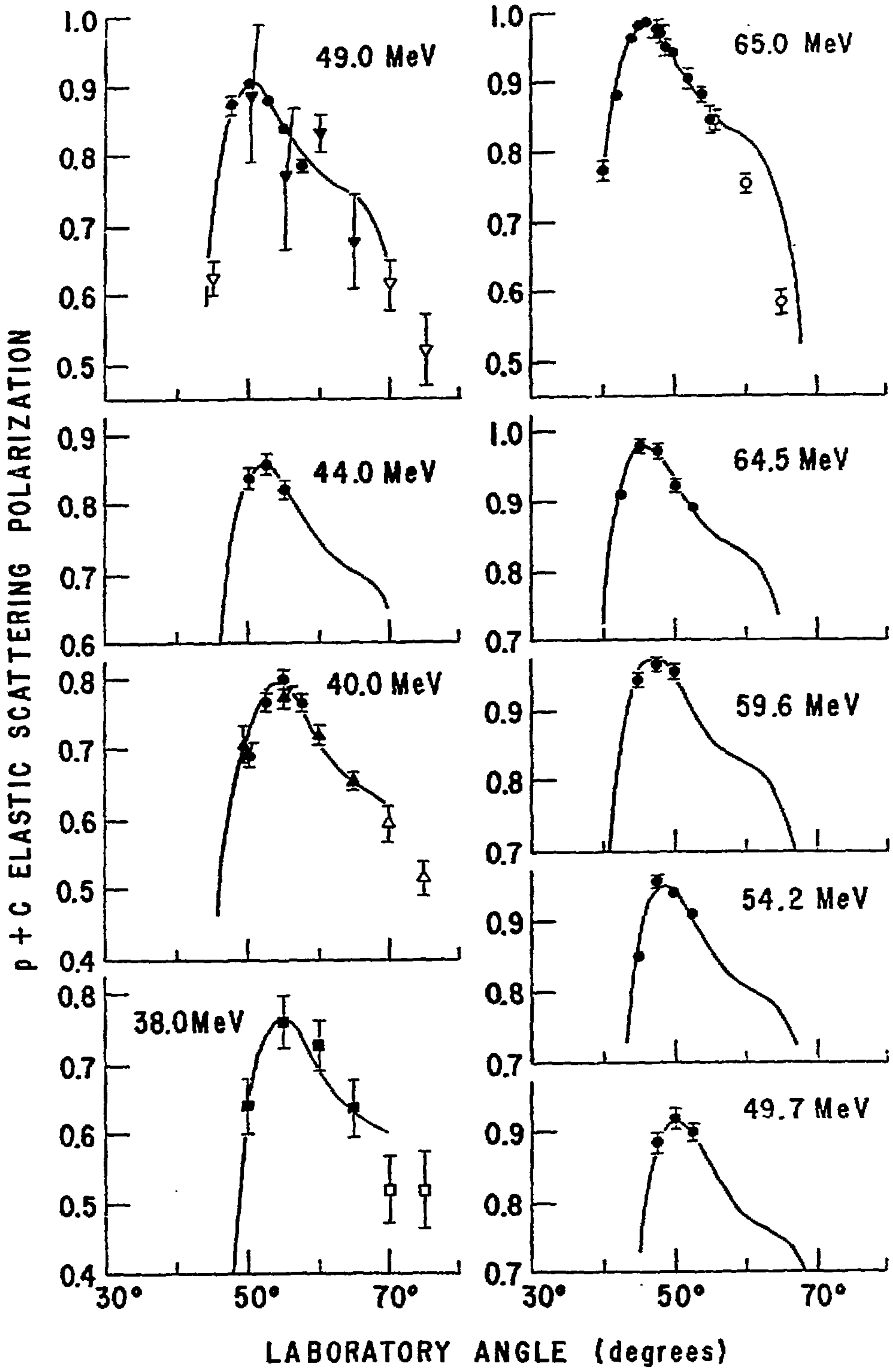
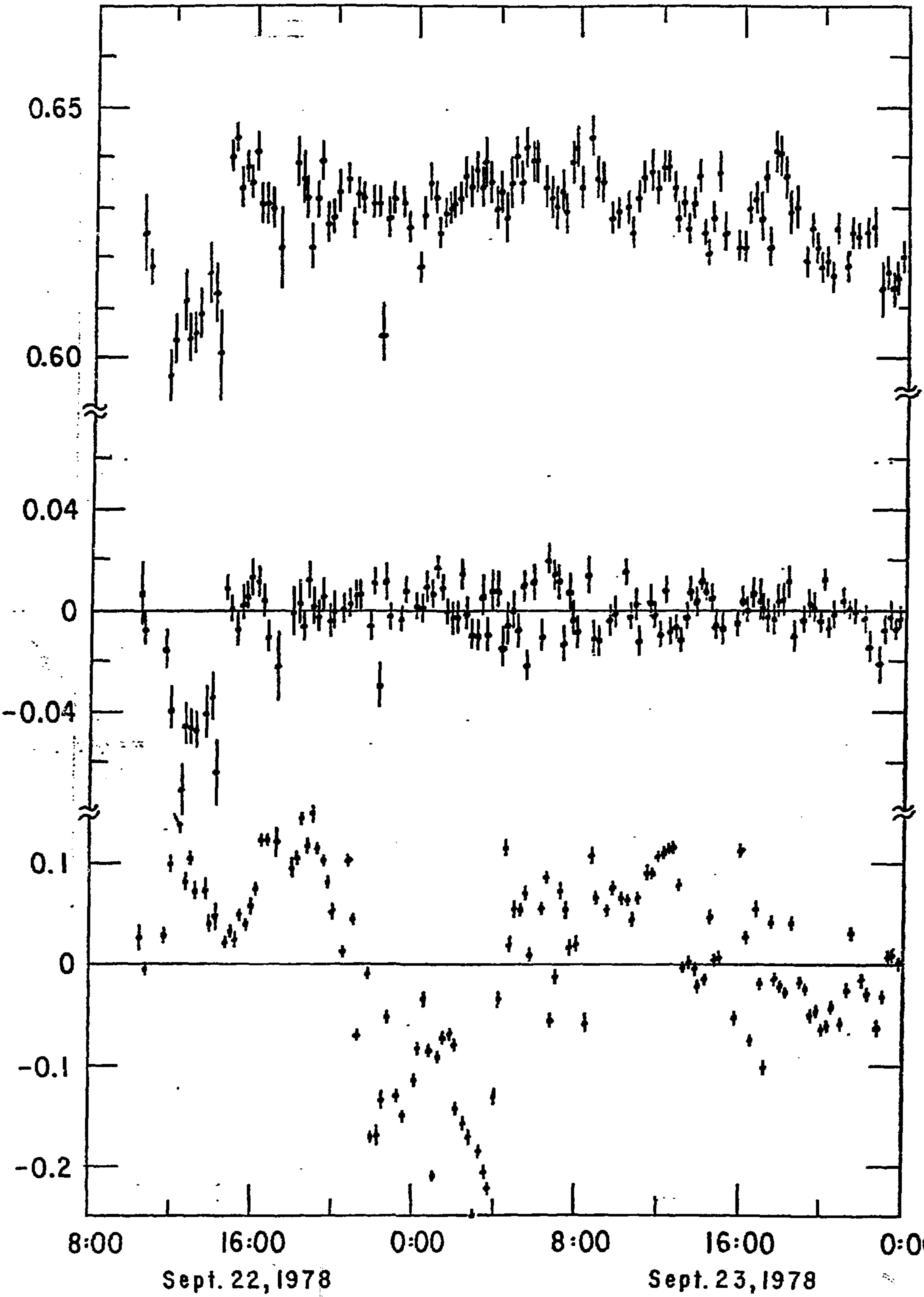

p + C ELASTIC SCATTERING POLARIZATION
49.0 MeV
44.0 MeV
40.0 MeV
38.0MeV
65.0 MeV
64.5 MeV
59.6 MeV
54.2 MeV
49.7 MeV
LABORATORY ANGLE (degrees)

Fig. 4



0.65
0.60
$a_4/2$
0.04
0
-0.04
$a_7/2(1-P^2A^2)$
0.1
0
-0.1
-0.2
$a_5/2(1-P^2A^2)$
8:00
16:00
0:00
8:00
16:00
0:00
Sept. 22, 1978
Sept. 23, 1978

Fig. 5

APPENDIX: ASYMMETRIES

A number of different asymmetries have been used to calculate the beam polarization for the 50 MeV, CERN and BEAM 22 polarimeters at the ZGS. These are discussed and their sensitivity to various parameters are computed. Following Hanna[15],

$$
\begin{aligned}
L_\uparrow &= B_\uparrow \, d\Omega_L \, (1 + P_\uparrow A_L) \\
R_\uparrow &= B_\uparrow \, d\Omega_R \, (1 - P_\uparrow A_R) \\
L_\downarrow &= B_\downarrow \, d\Omega_L \, (1 - P_\downarrow A_L) \\
R_\downarrow &= B_\downarrow \, d\Omega_R \, (1 + P_\downarrow A_R)
\end{aligned}
\qquad \text{(A1)}
$$

where L, R refer to the number of events detected by the polarimeter corresponding to a forward scattered particle to the left or right respectively; $\uparrow$,$\downarrow$ refer to beam polarization up or down; B gives the number of beam particles; $d\Omega$ is the "effective solid angle" of the polarimeter; A refers to the "effective analyzing power"; and $P_\uparrow$, $P_\downarrow$ are the beam polarizations for up and down pulses. Deadtime and accidentals are ignored in these equations. There are a total of 8 unknowns ($B_\uparrow$, $B_\downarrow$, $d\Omega_L$, $d\Omega_R$, $P_\uparrow$, $P_\downarrow$, A_L, A_R) and 4 measured quantities ($L_\uparrow$, $L_\downarrow$, $R_\uparrow$, $R_\downarrow$). (The Beam 22 polarimeter recorded the singles counts from a plastic scintillator in the direct beam. Also, ion chambers were used near the CERN polarimeter. There are serious problems with both these intensity monitors, and therefore they are ignored in this appendix.)

The equations (A1) can be reduced to a total of 6 unknowns ($B \cdot d\Omega$, $P \cdot A$, ε_A, ε_P, ε_Ω, ε_B) by the following expressions:

$$
\begin{aligned}
A &= (A_L + A_R)/2 \\
\varepsilon_A &= (A_L - A_R)/(A_L + A_R) \\
B &= (B_\uparrow + B_\downarrow)/2 \\
\varepsilon_B &= (B_\uparrow - B_\downarrow)/(B_\uparrow + B_\downarrow) \\
P &= (P_\uparrow + P_\downarrow)/2 \\
\varepsilon_P &= (P_\uparrow - P_\downarrow)/(P_\uparrow + P_\downarrow) \\
d\Omega &= (d\Omega_L + d\Omega_R)/2 \\
\varepsilon_\Omega &= (d\Omega_L - d\Omega_R)/(d\Omega_L + d\Omega_R)
\end{aligned}
\tag{A2}
$$

In addition, if ratios are computed, for example $L_\uparrow/L_\downarrow$, then the uninteresting normalization $B \cdot d\Omega$ drops out. This leaves <u>3 equations</u> and <u>5 unknowns</u>.

It is not clear that any of the ε's can be ignored compared to the others. Rough estimates of the possible size of each ε follow: ε_P) Based on the 50 MeV polarimeter readout, $P_\uparrow = .75$, $P_\downarrow = .71$ is not unusual. This gives $\varepsilon_P \simeq .03$. ε_B) The 50 MeV and CERN numbers were updated every 15 minutes. It would not be unusual to have two more pulses of spin-up than spin-down polarization (or vice versa). The number of pulses in a 15 minute period for spin up is roughly 100. This gives $\varepsilon_B \approx .01$. ε_Ω) For the 50 MeV polarimeter, the nominal distance from the target to the last counter was about 25". However, the measured distance for one counter was actually 3/8" less because the counter was not normal to the scattered particles, but was "tilted". This gives $\varepsilon_\Omega \approx .015$. ε_A) An estimate of the change in the average analyzing power at 50 MeV for a misalignment of 0.5° (8.7 mrad) in one arm gives $\varepsilon_A \approx .005$. Finally, the parameter $P \cdot A$ cannot be assumed small in all cases. In particular, for the 50 MeV polarimeter, $P \sim .73$ and $A \sim .88$ gives $P \cdot A \sim .64$. Therefore, in the expressions that follow, it will be assumed that $P \cdot A$ is not a small quantity and that all the ε's are roughly the same size (~ .01 - .03).

The three polarimeters at the ZGS that this study concerns all used different expressions for the beam polarization:

$$\text{50 MeV} \quad \begin{cases} "P_\uparrow A" = (L_\uparrow - R_\uparrow)/(L_\uparrow + R_\uparrow) \\ "P_\downarrow A" = (R_\downarrow - L_\downarrow)/(R_\downarrow + L_\downarrow) \end{cases}$$

$$\text{CERN} \quad "PA" = 1/2(L_\uparrow R_\downarrow - L_\downarrow R_\uparrow)/(L_\uparrow R_\downarrow + L_\downarrow R_\uparrow) \tag{A3}$$

$$\text{BEAM 22} \quad "PA" = (\sqrt{L_\uparrow R_\downarrow} - \sqrt{L_\downarrow R_\uparrow})/(\sqrt{L_\uparrow R_\downarrow} + \sqrt{L_\downarrow R_\uparrow})$$

These asymmetries are expanded in terms of ε_A, ε_B, ε_P, ε_Ω below:

$$\alpha_0 = \frac{L_\uparrow R_\downarrow - L_\downarrow R_\uparrow}{L_\uparrow R_\downarrow + L_\downarrow R_\uparrow}$$

$$= \frac{2PA}{1 + P^2A^2} \left[1 - \frac{2PA}{1 + P^2A^2} \varepsilon_P \varepsilon_A + \frac{P^2A^2}{1 + P^2A^2} (\varepsilon_P{}^2 + \varepsilon_A{}^2)\right] + O(\varepsilon^4)$$

$$= \frac{2PA}{1 + P^2A^2} + O(\epsilon^2) \tag{A4}$$

$$\alpha_1 = \frac{L_\uparrow L_\downarrow - R_\uparrow R_\downarrow}{L_\uparrow L_\downarrow + R_\uparrow R_\downarrow}$$

$$= 2[\epsilon_\Omega + \frac{PA}{1 - P^2A^2} \epsilon_P - \frac{P^2A^2}{1 - P^2A^2} \epsilon_A] + O(\epsilon^3)$$

$$\alpha_2 = \frac{L_\uparrow R_\uparrow - L_\downarrow R_\downarrow}{L_\uparrow R_\uparrow + L_\downarrow R_\downarrow}$$

$$= 2[\epsilon_B + \frac{PA}{1 - P^2A^2} \epsilon_A - \frac{P^2A^2}{1 - P^2A^2} \epsilon_P] + O(\epsilon^3)$$

The above expressions are all of the "CERN" type of asymmetry. As mentioned before, these are three independent equations in five unknowns. Fortunately, α_o can be used to give PA to order ϵ^2 by the equation

$$PA = \frac{1}{\alpha_o} - \sqrt{\frac{1}{\alpha_o^2} - 1} + O(\epsilon^2) \tag{A5}$$

This leaves 2 equations (α_1, α_2) in terms of 4 unknowns (ϵ_A, ϵ_B, ϵ_P, ϵ_Ω).

Similar equations for the "50 MeV" type asymmetries are given below:

$$\alpha_3 = \frac{L_\uparrow - R_\uparrow}{L_\uparrow + R_\uparrow} + \frac{L_\downarrow - R_\downarrow}{L_\downarrow + R_\downarrow}$$

$$= (1 - P^2A^2) \alpha_1 + O(\epsilon^3) \tag{A6}$$

$$\alpha_4 = \frac{L_\uparrow - R_\uparrow}{L_\uparrow + R_\uparrow} - \frac{L_\downarrow - R_\downarrow}{L_\downarrow + R_\downarrow}$$

$$= 2PA[1 - \epsilon_\Omega^2 - 2PA\, \epsilon_P(\epsilon_A + \epsilon_\Omega) + P^2A^2(\epsilon_A + \epsilon_\Omega)^2] + O(\epsilon^3)$$

$$= 2PA + 0(\varepsilon^2)$$

$$= (1 + P^2A^2)\ \alpha_0 + 0(\varepsilon^2)$$

$$\alpha_5 = \frac{L_\uparrow - L_\downarrow}{L_\uparrow + L_\downarrow} + \frac{R_\uparrow - R_\downarrow}{R_\uparrow + R_\downarrow}$$

$$= (1 - P^2A^2)\ \alpha_2 + 0(\varepsilon^3)$$

$$\alpha_6 = \frac{L_\uparrow - L_\downarrow}{L_\uparrow + L_\downarrow} - \frac{R_\uparrow - R_\downarrow}{R_\uparrow + R_\downarrow}$$

(A6 cont.)

$$= 2PA[1 - \varepsilon_B{}^2 - 2PA\ \varepsilon_A(\varepsilon_B + \varepsilon_P) + P^2A^2(\varepsilon_B + \varepsilon_P)^2] + 0(\varepsilon^3)$$

$$= (1 + P^2A^2)\ \alpha_0 + 0(\varepsilon^2)$$

$$\alpha_7 = \frac{L_\uparrow - R_\downarrow}{L_\uparrow + R_\downarrow} + \frac{L_\downarrow - R_\uparrow}{L_\downarrow + R_\uparrow}$$

$$= \alpha_1 + 0\ (\varepsilon^3)$$

$$\alpha_8 = \frac{L_\uparrow - R_\downarrow}{L_\uparrow + R_\downarrow} - \frac{L_\downarrow - R_\uparrow}{L_\downarrow + R_\uparrow}$$

$$= \alpha_2 + 0(\varepsilon^3)$$

In particular, no new information is available from these expressions.

Finally, the expression for PA using the "Beam 22" type asymmetry gives

$$\alpha_9 = \frac{\sqrt{L_\uparrow R_\downarrow} - \sqrt{L_\downarrow R_\uparrow}}{\sqrt{L_\uparrow R_\downarrow} + \sqrt{L_\downarrow R_\uparrow}}$$

$$= PA[1 - \frac{2PA}{1 - P^2A^2} \varepsilon_P \varepsilon_A + \frac{P^2A^2}{1 - P^2A^2} (\varepsilon_P{}^2 + \varepsilon_A{}^2)] + 0(\varepsilon^4)$$

$$= 1/2(1 + P^2A^2) \ \alpha_o + 0(\varepsilon^2)$$

$$= 1/2 \ \alpha_4 + 0(\varepsilon^2) \tag{A7}$$

$$= 1/2 \ \alpha_6 + 0(\varepsilon^2)$$

and also

$$\alpha_{10} = \frac{\sqrt{L_\uparrow L_\downarrow} - \sqrt{R_\uparrow R_\downarrow}}{\sqrt{L_\uparrow L_\downarrow} + \sqrt{R_\uparrow R_\downarrow}}$$

$$= 1/2 \ \alpha_1 + 0(\varepsilon^3)$$

$$\alpha_{11} = \frac{\sqrt{L_\uparrow R_\uparrow} - \sqrt{L_\downarrow R_\downarrow}}{\sqrt{L_\uparrow R_\uparrow} + \sqrt{L_\downarrow R_\downarrow}}$$

$$= 1/2 \ \alpha_2 + 0(\varepsilon^3)$$

Again, no new information is available from these expressions. Moreover,the equations for α_4, α_6, and α_9 lead to numerically equal values and errors for PA compared to equation (A5) to order ε. Therefore, to first order in ε, it doesn't make any difference which expression is used to determine PA. Similar statements apply to the determination of the quantities

$$\varepsilon_\Omega + \frac{PA}{1 - P^2A^2} \varepsilon_P - \frac{P^2A^2}{1 - P^2A^2} \varepsilon_A$$

$$\varepsilon_B + \frac{PA}{1 - P^2A^2}\ \varepsilon_A - \frac{P^2A^2}{1 - P^2A^2}\ \varepsilon_P$$

to second order in ε.

Returning to the question of deadtime and accidentals, it is clearly possible to add these effects into the equations. However, there are already more unknowns than equations, so that it is not possible to separate out ε_A, ε_B, ε_P, ε_Ω (these four unknowns are generally expected to be comparable in magnitude). From Section A, it is clear that accidentals are not negliqible for the 50 MeV polarimeter. Unfortunately, the accidentals were not recorded as a function of time.

In the particular case of the CERN polarimeter, some of the above comments are not correct. At times, there was a large asymmetry in the effective solid angle of the two arms, so that the L counts were always higher that the R counts (or vice versa). In these cases ε_Ω was not as small as ε_A, ε_B and ε_P. To minimize the effect of ε_Ω on the calculation of PA, the expression for α_4 should be avoided because of the $\varepsilon_\Omega{}^2$ term (see equation (A6)). The expressions for α_6, α_9 and equation (A5) have no $\varepsilon_\Omega{}^2$ term.