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Bubble Growth in Superheated He-II*

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Abstract

Bubble growth in superheated He-II is controlled by the transfer of heat to the surface of the growing bubble by nonlinear Gorter-Mellink counterflow. The present work presents analytic formulas for the bubble radius as a function of time in the limiting cases of small and large superheats. The formulas include the effect of the inertial reaction of the surrounding liquid to the expansion of the bubble. A numerical example shows that bubble velocities of the order of meters per second are possible. A related problem, involving only heat transfer but no movement of the liquid, is the motion of the free surface of superheated He-II in a very long tube. This problem has a similarity solution. The interfacial velocity in the tube is much smaller than the bubble growth velocity.

Introduction

Suppose we have a sample of liquid He-II at a pressure P and a temperature $T_b = T_s(P) + \Delta T$, where $T_s(P)$ is the saturation temperature corresponding to the pressure P and ΔT is a small superheat. (A list of symbols used in this work is included at the end of the article.) Such a sample is thermodynamically unstable and will tend to change into vapor. The rate of this conversion depends on the density of nucleation sites initially present and the rate of growth of the bubbles arising at these sites.

The rate of bubble growth is controlled by the transfer of heat from the superheated liquid to the surface of the growing bubble. In He-II this transfer is by nonlinear Gorter-Mellink counterflow (heat flux proportional to the cube root of the temperature gradient); see eq. (16). For superheats ΔT that are not too large, the temperature distribution in the liquid surrounding a bubble will be very close to its steady-state distribution $T = T_b - \Delta T R^5/r^5$. This greatly simplifies the problem of bubble growth; but, as we shall see later, the superheats for which the quasi-static theory is valid are quite small. As will be shown, the theory also takes a simple form in the opposite extreme of large superheats, where bubble growth is so rapid that the liquid layer in which the temperature changes appreciably is thin compared with the bubble radius.

Bubble growth in He-II is explosive, bubble velocities of hundreds of centimeters per second easily being possible. The growth of the bubbles can be studied by high-speed photography if nucleation sites are present. A related situation, for which a complete theory is available, and which may be easier to study experimentally, is evaporation from the free surface of He-II in a long tube open at one end. The temperature of the liquid helium in the tube is again T_b, and at the start of the experiment the external pressure at the open end of the tube is dropped to a pressure P so that $T_b = T_s(P) + \Delta T$, where ΔT is the desired superheat. If no bubbles are nucleated in the bulk of the helium and phase change takes place only at the free surface, the motion of the free surface is calculable. Its velocity is nonuniform, varying as the three-fourths power of the elapsed time, and is of

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the order of centimeters per second. The reason for the much slower motion of the phase front in this case compared with the case of bubble growth is that in this case the vapor does not impart any motion to the liquid adjacent to it.

Characteristic Time to Reach Steady State

The characteristic time to reach the steady state is determined by the Gorter-Mellink conductance parameter K (W m$^{-5/3}$ K$^{-1/3}$), the heat capacity of He-II per unit volume S (J m^{-3} K^{-1}), the superheat ΔT (K), and the instantaneous bubble radius R (m). The only time that can be made out of these four variables is $\tau = S(\Delta T)^{2/3} R^{4/3}/K$. For the quasi-static theory developed here to be valid, the condition $\dot{R}\tau \ll R$ must hold. In such a case, the bubble radius will change little during the time it takes to reach thermal equilibrium, and the temperature distribution will be a quasi-static steady state.

Quasi-static Heat Balance for the Growing Bubble

The quasi-static heat balance for the growing bubble can be written

$$4\pi R^2 \dot{R} \rho_v L = 4\pi R^2 K (5\Delta T/R)^{1/3} \tag{1}$$

From eq. (1) and the preceding expression for τ, we find at once that

$$\frac{\dot{R}\tau}{R} = \frac{5^{1/3} S \Delta T}{\rho_v L} \tag{2a}$$

so that the allowable superheats ΔT must satisfy

$$\Delta T \ll \frac{\rho_v L}{5^{1/3} S} \tag{2b}$$

At T_s = 1.8 K, P_s = 1.64 kPa, and ρ_v (estimated from the perfect gas law) is 4.38×10^{-4} g/cm^3. Using S = 0.408 J cm^{-3} K^{-1} and L = 23 J/g, we find that ΔT must be $\ll$14.4 mK. According to the Clausius-Clapeyron equation, the slope $(dP/dT)_s$ of the saturation line at 1.8 K is 5.60 kPa/K. Thus, the limiting superheat of 14.4 mK corresponds to a pressure reduction of 80.6 Pa = 0.612 torr, which is extremely small.

The heat balance equation [eq. (1)] can be integrated with respect to time to give

$$R = \left(\frac{4}{3}\right)^{3/4} 5^{1/4} \left(\frac{Kt}{\rho_v L}\right)^{3/4} (\Delta T)^{1/4} \tag{3}$$

assuming the initial bubble radius is very small.

Transient Limit—Large Superheats

When the superheat is much larger than the limit of eq. (2b), the temperature distribution is never close to the steady distribution. But we can approximate it with the help of the assumption, justified later, that the layer in which the temperature changes is thin compared with the radius of the bubble. Then we can neglect the curvature of that layer. The temperature distribution in that layer in *a coordinate system moving with the bubble surface* is given by

$$T_b - T = \Delta T \left(1 - \frac{X}{\left(\frac{8}{3\sqrt{3}} + X^2\right)^{1/2}} \right) \tag{4a}$$

where

$$X = z(\Delta T)^{1/2}(S/Kt)^{3/4} \tag{4b}$$

and z is the distance outwards from the bubble surface. The solution of eq. (4) is the solution to the clamped-temperature problem.[1] It is applied in the moving coordinate system because the thin liquid film in which the temperature changes appreciably is being transported outwards by the expanding vapor.

The inward heat flux at the bubble surface is given by

$$K(\partial T/\partial z)^{1/3}_{z=0} = \left(\frac{3\sqrt{3}}{8}\right)^{1/6} K^{3/4}S^{1/4}(\Delta T)^{1/2}t^{-1/4} \tag{5}$$

The heat balance equation for the growing bubble is

$$4\pi R^2 \dot{R}\rho_v L = K(\partial T/\partial z)^{1/3}_{z=0} 4\pi R^2 \tag{6a}$$

or

$$\dot{R} = \left(\frac{3\sqrt{3}}{8}\right)^{1/6} \frac{K^{3/4}S^{1/4}(\Delta T)^{1/2}t^{-1/4}}{\rho_v L} \tag{6b}$$

This can be integrated at once to give

$$R = \left(\frac{8}{3\sqrt{3}}\right)^{1/2} \frac{K^{3/4}S^{1/4}(\Delta T)^{1/2}t^{3/4}}{\rho_v L} \tag{7a}$$

or

$$R = \left(\frac{8}{3\sqrt{3}}\right)^{1/2} \left(\frac{Kt}{\rho_v L}\right)^{3/4} (\Delta T)^{1/4} \left(\frac{S\Delta T}{\rho_v L}\right)^{1/4} \tag{7b}$$

which allows easy comparison with eq. (3).

The thickness of the temperature transition layer will be small compared with the radius given in eq. (7) if the value of X in eq. (4b) calculated for $z = R$ is large compared with 1. Substituting eq. (7a) into eq. (4b), we find

$$X(R) = \left(\frac{8}{3\sqrt{3}}\right)^{1/2} \frac{S\Delta T}{\rho_v L} \tag{8}$$

which will be much greater than 1 if ΔT greatly exceeds the limit given by the right-hand side of eq. (2b).

As an example, we take:

$T_s = 1.8$ K $\quad K = 11.6$ W cm$^{-5/3}$ K$^{-1/3}$ (1.85 K)
$\Delta T = 0.1$ K $\quad \rho_v = 4.38 \times 10^{-4}$ g/cm^3
$T_b = 1.9$ K $\quad t = 1$ s
$L = 23$ J/g $\quad S = 0.533$ J cm^{-3} K^{-1} (1.85 K)

Then, according to eq. (7), $R = 209$ cm and $\dot{R} = 157$ cm/s. At $t = 0.1$ s, $R = 37.2$ cm and $\dot{R} = 279$ cm/s. Such rapid bubble growth can be described as little less than explosive.

Consideration of Mechanical Terms

The expanding bubble pushes the surrounding liquid out of its way, and, in doing so, it does work on the liquid that must be subtracted from the energy available for vaporization. Furthermore, the inertial reaction of the liquid to being accelerated causes a rise in pressure at the bubble surface that increases the saturation temperature there. We estimate these effects now.

To calculate the pressure rise at the bubble surface, we treat the liquid as incompressible. Then the velocity field surrounding a bubble is $v = \dot{R}R^2/r^2$. The pressure rise at the bubble can then be calculated from Euler's equation

$$-\frac{1}{\rho_\ell}\frac{\partial P}{\partial r} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} \tag{9}$$

A short calculation gives

$$P(R) - P_s = \rho_\ell\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) \tag{10}$$

According to eq. (7*b*), $R \sim t^{3/4}$, in which case eq. (10) can be written

$$P(R) - P_s = \frac{21}{32}\rho_\ell \frac{R^2}{t^2} \tag{11}$$

The power $\dot{W}$ expended by the expanding vapor in pushing the liquid out of its way is

$$\dot{W} = \left(P_s + \frac{21}{32}\rho_\ell\frac{R^2}{t^2}\right)4\pi R^2\dot{R} \tag{12}$$

If we divide this by the left-hand side of eq. (6*a*), we get

$$\frac{\dot{W}}{4\pi R^2\dot{R}\rho_v L} = \frac{P_s + (21/32)\rho_\ell(R^2/t^2)}{\rho_v L} \tag{13}$$

In the numerical example considered at the end of the preceding section, this ratio is 0.294 at $t = 0.1$ s and 0.204 at $t = 1$ s. Thus, not all the heat transported to the bubble is available for vaporization. We can estimate the effect of this on bubble growth by using the complement of the ratio of eq. (13) as an ad hoc correction to K. Then the radii achieved at $t = 0.1$ s and $t = 1$ s are 28.7 cm and 176 cm, respectively.

If we use the pressure rise given in eq. (11) to correct the superheat in eq. (7*b*), we find, after rearrangement,

$$R = R_* \left[1 - \frac{21\rho_\ell R^2/32t^2}{(dP/dT)_s \Delta T}\right]^{1/2} \tag{14}$$

where R_* is the as yet uncorrected value of R given by eq. (7b). Here ΔT is still $T_b - T_s$. From eq. (14), it follows at once that

$$\frac{R}{R_*} = (1+\alpha)^{-1/2} \quad , \quad \alpha = \frac{21}{32}\frac{\rho_\ell (R_*/t)^2}{(dP/dT)_s \Delta T} \tag{15}$$

In the numerical example we have been following, $\alpha = 1.42$ and 0.534 at $t = 0.1$ s and 1 s, respectively, so that $R = 18.5$ cm and 142 cm, respectively.

Since $R_* \sim t^{3/4}$, it follows from eqs. (13) and (15) that the corrections will be large for small t and small for large t. Thus, the theory presented here is an asymptotic theory correct for large enough t.

Evaporation from a Free Surface

Let the tube containing the liquid He-II extend in the z-direction, and let the initial position of the free surface be $z = 0$. At $t = 0$, let the pressure above the free surface be dropped suddenly so that the liquid at temperature T_b becomes superheated by a temperature difference ΔT. The vapor produced at the free surface exhausts in the negative z-direction, while the free surface advances in the positive z-direction. The temperature distribution in the quiescent liquid obeys the Gorter-Mellink diffusion equation

$$S\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[K\left(\frac{\partial T}{\partial z}\right)^{1/3}\right] \tag{16}$$

subject to the following boundary and initial conditions:

$$T(z,0) = T_b \tag{17a}$$

$$T(\infty,t) = T_b \tag{17b}$$

$$T\left[Z(t),t\right] = T_s \tag{17c}$$

$$\rho_\ell L\dot{Z} = K(\partial T/\partial z)^{1/3}_{z=Z} \tag{17d}$$

Equations (17a) and (17b) state that the initial liquid temperature and the liquid temperature at the far end of the tube are both T_b. Equation (17c) says that the liquid temperature at the free surface, whose instantaneous position is $z = Z(t)$, is T_s. Equation (17d) is a heat balance equation that states that the heat transported to the free surface by Gorter-Mellink counterflow is expended in vaporizing liquid.

When the thermophysical properties K, S, and L are independent of temperature, the problem stated in eqs. (16) and (17) has a similarity solution, the details of which are given in Appendix 1. The final result for the displacement Z of the free surface is

$$Z = A(K/S)^{3/4}(\Delta T)^{-1/2}t^{3/4} \tag{18a}$$

where

$$A = (8/3\sqrt{3})^{1/2} B(1-B)^{-1/2} \qquad (18b)$$

and

$$B = S\Delta T/\rho_\ell L \qquad (18c)$$

When $B \ll 1$, which is usually the case,

$$Z = \left(\frac{8}{3\sqrt{3}}\right)^{1/2} \frac{K^{3/4} S^{1/4} (\Delta T)^{1/2} t^{3/4}}{\rho_\ell L} \qquad (B \ll 1) \qquad (18d)$$

Equation (18) differs from eq. (7*a*) only in the appearance of ρ_ℓ (which is 0.145 g/cm^3) in place of ρ_v. With the data given previously, $B = 0.0159$; eq. (18*d*) then gives $Z = 0.630$ cm and $\dot{Z} = 0.473$ cm/s at the end of 1 s.

Reference

1 **Dresner, Lawrence** Transient heat transfer in superfluid helium—Part II. *Adv Cryog Eng* (1984) **29** 323–333.

Appendix 1

In solving eqs. (16) and (17), we find it convenient to set

$$c = T_b - T \qquad (A\text{--}1c)$$

and work in special units in which

$$K/S = 1 \qquad (A\text{--}1b)$$

$$\Delta T = T_b - T_s = 1 \qquad (A\text{--}1c)$$

Then (16) and (17) become

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left(\frac{\partial c}{\partial z}\right)^{1/3} \qquad (A\text{--}2)$$

$$c(z,0) = 0 \qquad (A\text{--}3a)$$

$$c(\infty,t) = 0 \qquad (A\text{--}3b)$$

$$c(Z,t) = 1 \qquad (A\text{--}3c)$$

$$(\rho_\ell L/S)\dot{Z} = -(\partial c/\partial z)^{1/3}_{z=Z} \qquad (A\text{--}3d)$$

These equations have a similarity solution of the form

$$c = y(z/t^{3/4}) \qquad (A\text{--}4a)$$

$$Z = At^{3/4} \qquad (A\text{--}4b)$$

where y is a function and A is a constant yet to be determined.

If we substitute eq. (A-4*a*) into eq. (A-2), we find the following ordinary differential equation for y:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)^{1/3} + \frac{3}{4} x \frac{dy}{dx} = 0 \quad , \quad x = z/t^{3/4} \qquad (A\text{--}5)$$

The boundary and initial conditions become

$$y(\infty) = 0 \tag{A-6a}$$

$$y(A) = 1 \tag{A-6b}$$

$$(3/4)(\rho_\ell L/S)A = -\dot{y}^{1/3}(A) \tag{A-6c}$$

Equation (16) can be integrated readily if we set $\theta^3 = dy/dx$. We find

$$\dot{y} = \theta^3 = -\left(\frac{3}{4}x^2 + a^2\right)^{-3/2} \tag{A-7}$$

where a^2 is a constant of integration. If we integrate eq. (A-7) from A to ∞, we have, in view of eqs. (A-6a) and (A-6b),

$$\int_A^\infty \left(\frac{3}{4}x^2 + a^2\right)^{-3/2} dx = 1 \tag{A-8a}$$

With the help of eq. (A-7), eq. (A-6c) becomes

$$A^2\left(\frac{3}{4}A^2 + a^2\right) = \frac{16}{9}\left(\frac{S}{\rho_\ell L}\right)^2 = \frac{16}{9}B^2 \tag{A-8b}$$

By using the identity $\int dz(1+z^2)^{-3/2} = z(1+z^2)^{-1/2}$, we can rewrite eq. (A-8a) as

$$a^2\sqrt{3}/2 = 1 - (\sqrt{3}A/2a)(1 + 3A^2/4a^2)^{-1/2} \tag{A-8c}$$

If we set $\xi^2 = 3A^2/4a^2$ and $\eta = a^2\sqrt{3}/2$, eqs. (A-8b) and (A-8c) become

$$\eta\xi(1+\xi^2)^{1/2} = B \tag{A-9a}$$

and

$$\eta = 1 - \xi(1+\xi^2)^{-1/2} \tag{A-9b}$$

From eqs. (A-9a) and (A-9b), we find easily that

$$\xi = B(1-2B)^{-1/2} \tag{A-10a}$$

$$\eta = (1-2B)(1-B)^{-1} \tag{A-10b}$$

from which follows eq. (18b).

If we now convert to ordinary units, B becomes $S\Delta T/\rho_\ell L$, which is dimensionless. Thus, so is A. Then, eq. (A-4b) becomes eq. (18a). The function y can easily be calculated from eq. (A-7), but we have no need for its explicit form here.

Symbols

a	constant of integration introduced in eq. (A-7)
A	constant in eq. (18a) and eq. (A-4b) for the displacement Z of the free surface
B	$S\Delta T/\rho_\ell L$
c	$T_b - T$
K	Gorter-Mellink conductance parameter
L	heat of vaporization
P	pressure
P_s	saturation pressure
r	radial coordinate
R	bubble radius
$\dot{R}$	dR/dt
S	heat capacity per unit volume
t	time
T	temperature
T_b	bath or ambient temperature
T_s	saturation temperature
ΔT	superheat $T_b - T_s$
v	radial velocity of liquid
$\dot{W}$	power expended by expanding vapor
x	$z/t^{3/4}$
X	quantity defined in eq. (4b)
y	similarity solution introduced in eq. (A-4a)
z	distance from bubble surface; also length coordinate along the tube
Z	displacement of the free surface of the liquid
$\dot{Z}$	dZ/dt
α	dimensionless quantity defined in eq. (15)
ξ	$3A^2/4a^2$
η	$a^2\sqrt{3}/2$
ρ_v	vapor density
ρ_ℓ	liquid density
θ	$\dot{y}^{1/3}$
τ	characteristic time to reach temperature equilibrium

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