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AN IMPEDANCE CAMERA: A SYSTEM FOR DETERMINING THE SPATIAL VARIATION OF ELECTRICAL CONDUCTIVITY

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DISCLAIMER

FOREWORD

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CONTENTS

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ABSTRACT

A data collection and data interpretation method is presented for predicting, from measurements made on the periphery of the core sample, the electrical conductivity distribution within core samples. This method uses an array of electrodes on the periphery to probe through the core sample. Surprisingly accurate detail can be seen in the estimated conductivity distribution. This method has been demonstrated using synthetic examples modeled and analyzed on a computer. Extensions of the procedure may be useful in subsurface geophysical probing and remote probing methods using physical phenomena satisfying Laplace's equation.

INTRODUCTION

There is a need for an "impedance camera"; i.e., a diagnostic device that enables one to "see" the variation of electrical conductivity σ within a region. Such a camera would be useful in geophysical diagnostics (performed on core samples, performed at the surface of the earth, and performed in boreholes) and in biomedical diagnostics (performed on human and animal limbs), among other applications. The camera should have a good resolution capability; i.e., be able to locate accurately where changes in the conductivity profile ("picture") occur. This camera should also be capable of providing accurate absolute values of conductivity. This capability would be an asset because, from the absolute conductivity, one may sometimes infer additional physical information, such as fluid permeability, porosity, and fluid type (e.g., water or oil) within the region of interest.

Remarkable advances have been made in recent years [1–4] for imaging the interior of the head and torso by using probes external to the body. We have found that, by modifying some of the approaches used in biophysical probing, it is possible to use low-frequency electromagnetic probing to achieve a noninvasive, moderate resolution impedance camera. This report describes the experimental and data interpretation procedures useable for probing of core samples, probing from boreholes in the ground, probing of the human limbs, and other applications. Computer simulation results are presented below that demonstrate the success of the method. The procedure is novel in that many electrodes are used at one time to "view" the region of interest, rather then using a small number of electrodes (as in the conventional methods).

The remainder of this report is primarily presented in the geophysical application area of viewing the interior of a core sample. It could alternatively be presented in terms of impedance plythesmography [1] (biophysical imaging). We expect the biophysical method to closely follow the methods used in this geophysical application.

HISTORICAL DEVELOPMENT

There are now finite-difference computer codes [5] that can model physical situations involving two- and three-dimensional variations in electrical conductivity. If the number of cells within the model is typically less than ten thousand, this modeling can be computationally efficient. For situations requiring much more than ten thousand cells in the physical model, no technique has been adequately demonstrated to have the required computational efficiency. Therefore, we concern ourselves herein with using the finite-difference modeling approach (which effectively limits our models to ten thousand cells or less) to do the forward problem of analyzing the probing response to excitations of a medium with two- and three-dimensional conductivity variations.

The manner by which we do the inverse problem (i.e., given the data, infer the model that produces these data) follows a conventional method [6,7]—an iterative linearization of the nonlinear equations governing the physical process. The manner by which we collect the data, however, is a nonstandard method. To help to explain the method, we outline its historical development.

Most core sample measurements are performed after an elaborate preparation of the sample, usually requiring sawing the cylindrical sample's uneven ends to furnish a relatively smooth right circular cylinder sample. Measurements of the bulk core properties can then easily be performed by probing between the two parallel planar faces at the ends of the cylinder. Unfortunately, the process of preparing the core (i.e., sawing off the ends) can change the properties of the core from those present after it was obtained. It has, therefore, been desirable to have a measurement method not requiring access to the ends of a core and not sensitive to the shape of the ends of the core. A first cut at providing such a system was recently proposed and used. [8] An extra advantage of this first system was that, with little sensitivity to the ends of the core, fluids could be introduced into and extracted from the core ends to assess the interrelationship of fluid flow and permeability on electrical conductivity. In the process of performing this work, it became apparent that core samples that externally appear homogeneous could be inhomogeneous in their electrical conductivity.

At that time we became aware of the impedance camera idea for probing the human torso. [3] For this biophysical application, the camera appeared to provide a reasonable qualitative picture but was lacking in providing detail. It occurred to us that, by modifying the manner in which this camera collected its data and also the way in which these data were interpreted, an improved image might be obtainable.

Our original idea was to use the same approach as adopted in following high-frequency electromagnetic rays bending through a region. [9] It appeared that, by using the duality [10] of the continuity of currents (conduction or displacement currents) at a discontinuity, the high-frequency ray-bending approach* might be modified to describe properly the low-frequency ray-bending approach. †

We thought of replacing the ϵ's of each cell in our successful high-frequency ray-bending code with the σ's of each cell to create a low-frequency code. This argument is attractive, but fallacious. The ray-bending idea is based upon solutions to the Helmholtz equation, whereas the low-frequency probing concept satisfies Laplace's equation. Unfortunately, as is well-known, the influence of an anomaly in a field governed by Laplace's equation is of limited spatial extent; i.e., regardless of the contrast of the anomaly with the host medium its range of influence is limited by its characteristic dimensions. This limitation is illustrated [11] in Figs. 1 and 2. Note that the void and metal sphere anomalies distort the applied field differently.

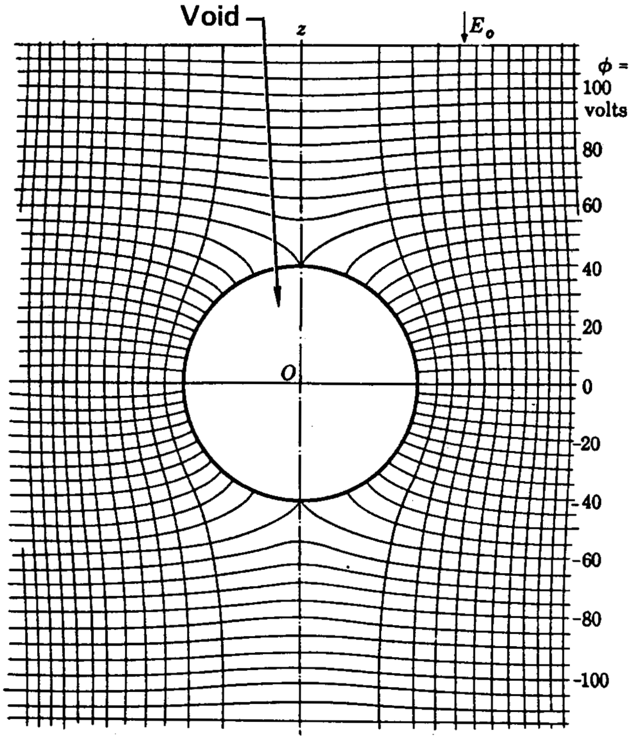


Fig. 1. Electrical conduction field about a spherical cavity in a conductor (from Fig. 8.10, p. 230, of P. Moon and D. E. Spencer, *Field Theory for Engineers*, D. Van Nostrand Company, 1961).

*The high-frequency ray-bending approach requires displacement current continuity, or

$$\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$$

and

$$E_{t1} = E_{t2},$$

where n and t denote the normal and tangential components of the electric field E at the boundary between regions 1 and 2.

†The low-frequency ray-bending approach requiring conduction current continuity, or

$$\sigma_1 E_{n1} = \sigma_2 E_{n2}$$

and

$$E_{t1} = E_{t2}.$$

However, at distances several body sizes removed from the sphere, it would be difficult to detect not only whether the anomaly was a void or metal, but also whether there was an anomaly present at all. This condition is not the case for high-frequency probing, where the influence of an anomaly of finite size can sometimes be seen even at arbitrarily large distances from the anomaly.

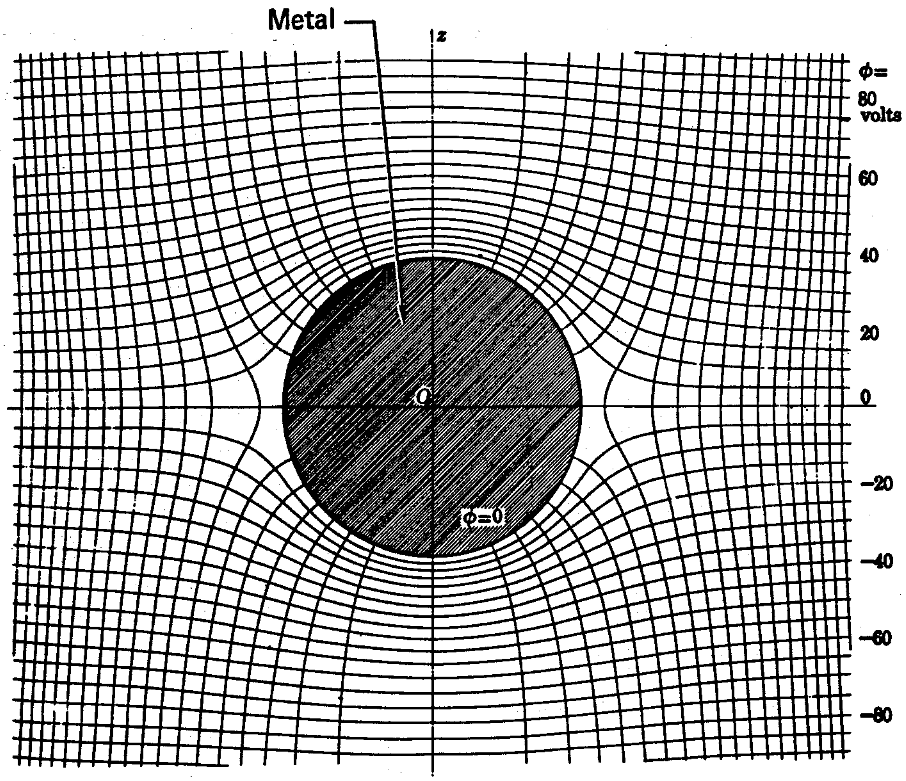


Fig. 2. **Field map showing distortion of electrical field caused by a metal sphere (from Fig. 8.07, p. 227, of P. Moon and D. E. Spencer, *Field Theory for Engineers*, D. Van Nostrand Company, 1961).**

PROPOSED MEASUREMENT AND INTERPRETATION APPROACH

This lack of success with the continuity-of-current approach led us to consider alternative measurement methods. For prior applications, we have seen great utility in the approach of using views to collect and to interpret the data. We next tried this view approach, using a variation of back-projection to interpret the results. This approach yielded qualitatively accurate pictures but with fuzzy images. The fuzziness of the image was attributed to the change in anomaly signature with distance from the anomaly; i.e., the rapid decrease in amplitude and broadening spatial extent of the anomaly for each view, as evidenced in Figs. 1 and 2.

This view approach obviously had some merit, but an improvement was needed to meet our objective of a moderate resolution impedance camera. We then tried using an iterative linearization of the nonlinear equations relating the observed currents to the applied voltages and model conductivities.[6] This procedure is standard although, to the authors' knowledge, it has not been used for a large number of views collected with a large number of electrodes. This procedure was quite successful, as discussed below.

SYNTHETIC MODELING RESULTS

For purposes of assessing the validity of the data collection and data interpretation procedure, synthetic examples have been considered using computer modeling. It is well-known that core samples are typically circular, not square. However, for ease in finite-difference mathematical modeling for the proof of the concept, we have simulated square core samples rather than circular core samples. The authors envisage no fundamental problems in applying the data collection and interpretation technique to circular cores.

Two conductivity profiles interior to synthetic core samples were simulated on the computer. An "easy" profile is shown in Fig. 3, and a "difficult" profile is shown in Fig. 4. An array of electrodes on the periphery of the synthetic core sample was used to probe the interior of the core. The array used in the simulation is shown in Fig. 5. A number of voltage configurations applied to the electrode array were considered to obtain different views of the core sample interior. Representative applied voltages, which give a particular view, are depicted in Fig. 6. Views were taken sufficient to make it possible to see the interior from all angles possible with the electrode array.

The synthetic measurements consisted of "measuring" the current entering or leaving each electrode, knowing the voltages impressed upon the array electrodes. The measuring consisted of computing the current that would enter or leave each electrode, knowing the impressed voltages surrounding the interior region with a spatially varying

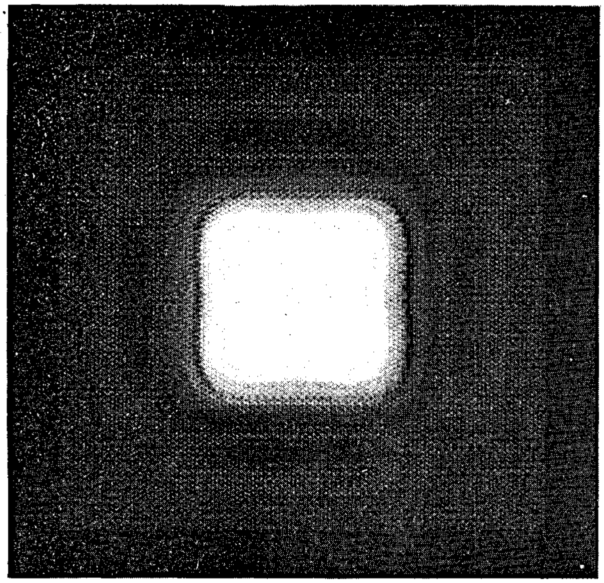


Fig. 3. "Easy" conductivity profile within synthetic square core sample (note that conductivity range is white (σ = 0.01 S/m), black (σ = 0.005 S/m); dimensions are arbitrary).

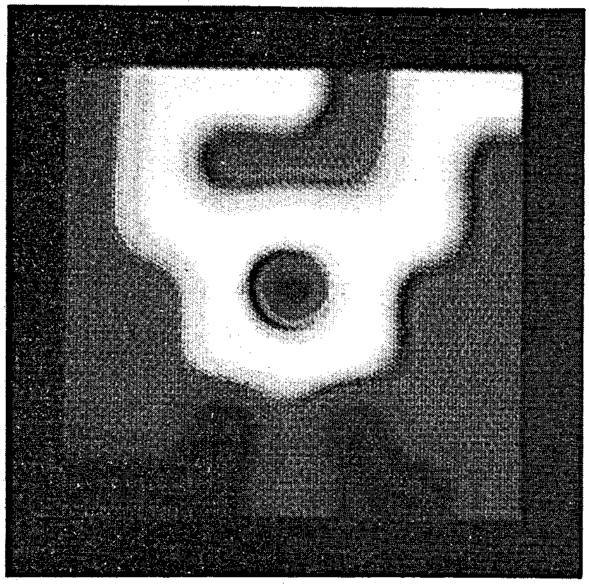


Fig. 4. "Difficult" conductivity profile within synthetic square core sample (note that conductivity range is white (σ = 0.015 S/m), black (σ = 0.005 S/m); dimensions are arbitrary).

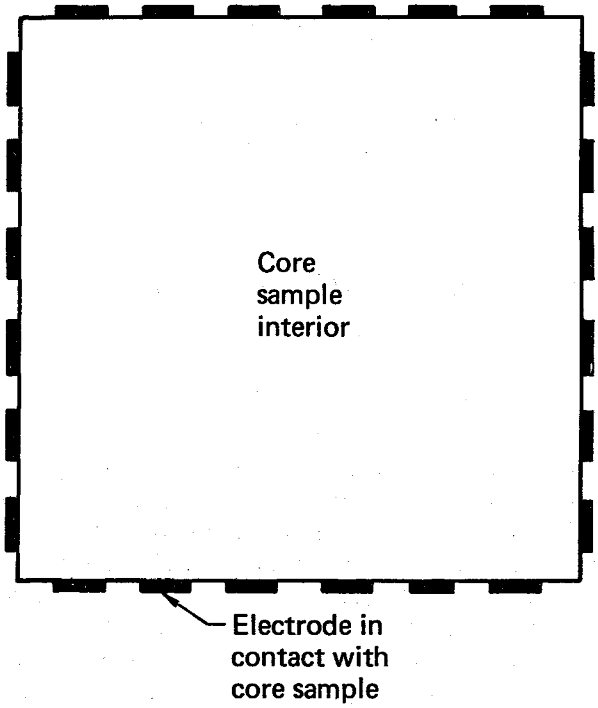


Fig. 5. Probing interior of core sample by means of array of electrodes (for various voltages applied to electrode array, currents entering or leaving each electrode are measured).

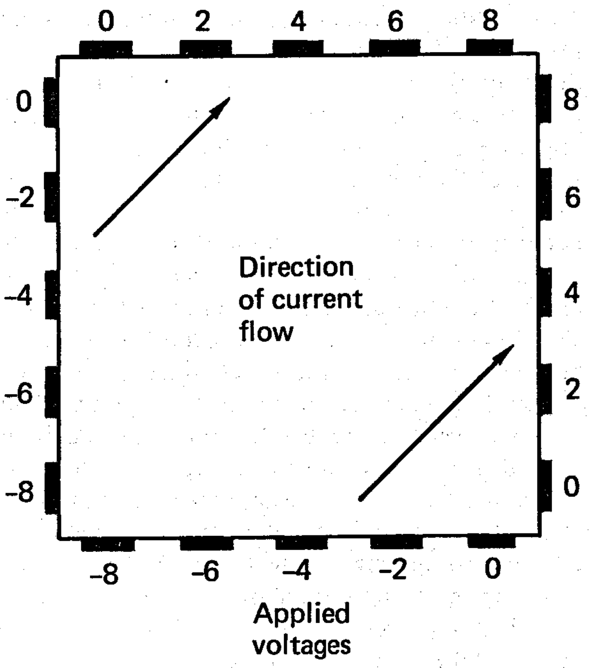


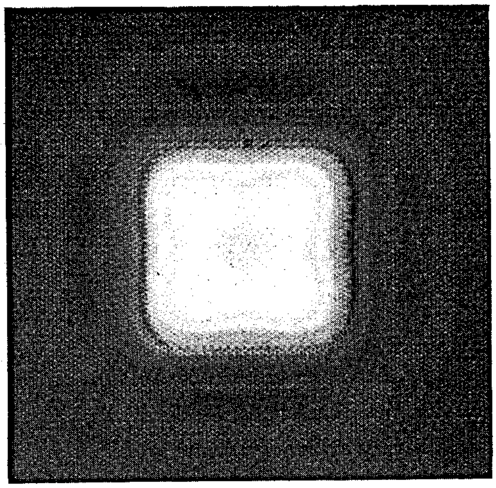
Fig. 6. Particular voltage sequence (applied to electrodes) shown directing current in constant direction when core sample interior is homogeneous (for each "view," voltages applied are chosen to direct current flow in a certain direction interior to sample).

conductivity. This computation was easily performed using a finite-difference modeling procedure. This procedure is based upon satisfying Laplace's equation, $\nabla \cdot (\sigma \nabla V) = 0$, where σ is the conductivity and V is the voltage interior to the core sample. Standard methods were used to solve the resultant set of finite-difference equations with the known boundary conditions on the periphery of the region being probed. This measurement by computer was used both for the actual specified conductivity profile and each conductivity profile produced in the iterative solution procedure.

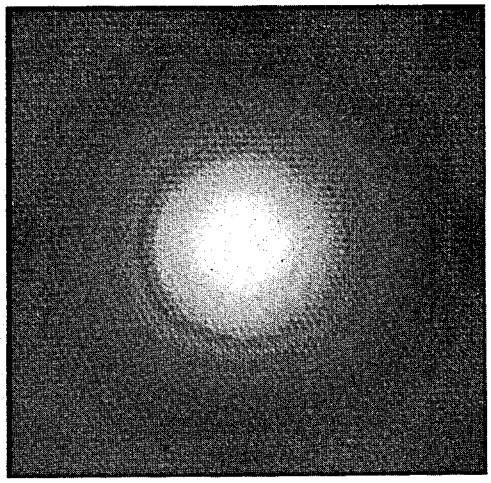
For the easy profile, Fig. 7 shows (a) the true synthetic conductivity profile, (b) the first iteration of the solution procedure (one iteration improvement beyond an initial estimate of the conductivity profile as being uniform throughout the core sample interior), and (c) the estimate of the interior conductivity profile after the tenth iteration.

Surprisingly, the first iteration result illustrates a qualitative variation similar to the actual conductivity profile. The tenth iteration results has almost a one-to-one correspondence with the actual conductivity profile.

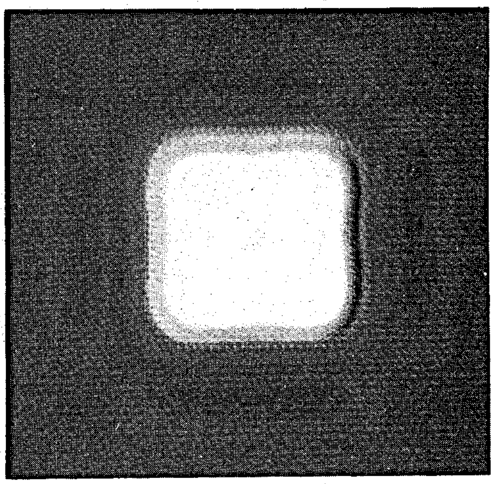
These successful results encouraged the authors to apply the method to the difficult profile of Fig. 4.



(a)

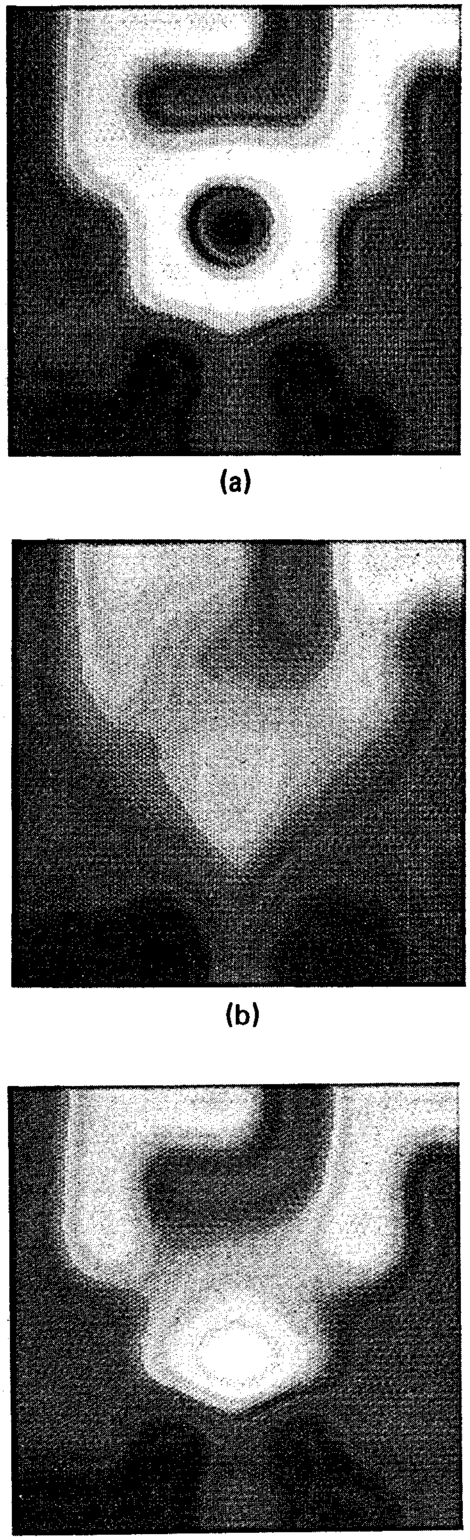


(b)



(c)

Fig. 7. The easy profile: (a) as it is (same as Fig.3), (b) as interpreted after one iteration, and (c) as interpreted after ten iterations.



(a)

(b)

(c)

Fig. 8. The difficult profile: (a) as it actually is (same as Fig. 4), (b) as interpreted after one iteration, and (c) as interpreted after ten iterations.

This profile is considered difficult in that the variation in conductivity presents both a detailed conductivity profile and a low conductivity region (in the middle) surrounded by a high conductivity region. Figure 8 depicts the difficult profile, the estimate of the profile after one iteration, and the estimate after the tenth iteration. Note that the first iteration result is again qualitatively similar to the actual profile. The result after ten iterations has much of the fine detail present in the actual profile. This result was beyond our expectations. It is, however, noticed that the low conductivity region in the middle surrounded by a high conductivity region present in the true synthetic profile is indicated as a high conductivity region in the tenth iteration result. This unsuccessful imaging result was not unexpected and illustrates the problem of effectively sampling (equivalent to having current flow through) a low conductivity region surrounded by a high conductivity region. This is because the current takes the path of least resistance. This limitation is present in all inverse methods based upon physical interactions based upon Laplace's equation.

OTHER POSSIBLE APPLICATIONS

This report specifically addresses the problem of obtaining low-frequency probing data when the electrodes completely surround the region of interest. Such an approach is admirably suited to studying the interior of core samples. This suitability is especially true if guard electrodes are used above and below the array electrodes used to collect data. An example of an engineering prototype of such a system is shown in Fig. 9. In terms of geophysical probing, the idea of electrodes completely surrounding the region of interest will likely see few applications due to the expense involved. One possible application should, however, be mentioned: assessing the conductivity structure within a volcano extending above the earth's surface.

Although the validity of this data collection and interpretation method has not as yet been demonstrated for the following problems, the approach has enough promise that it should be in-

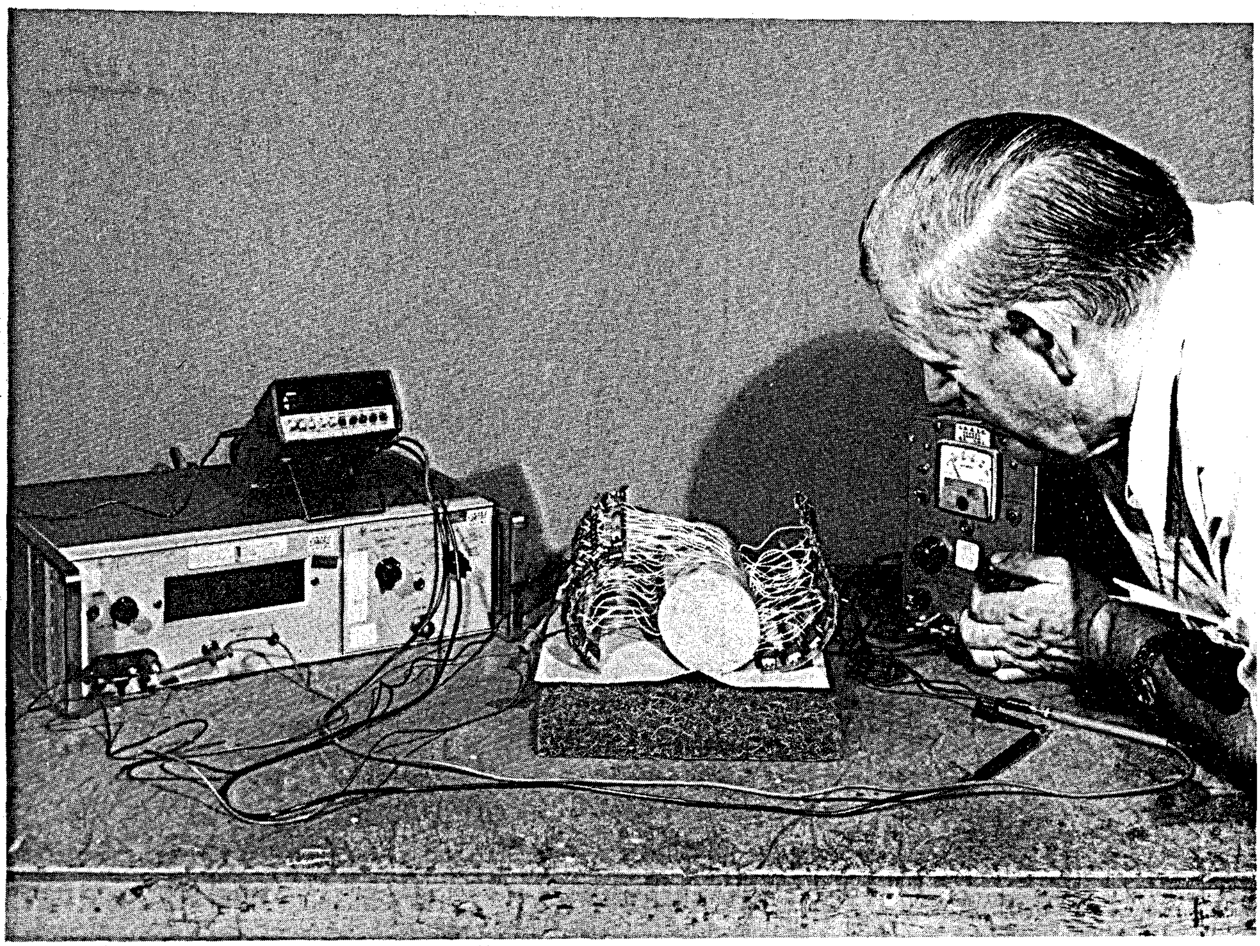


Fig. 9. Engineering prototype consisting of array of electrodes attached to side of core sample, with guard ring electrodes above and below array of electrodes from which measurements are taken.

vestigated for the probing geometries presented in Figs. 10 through 12. Figure 10 shows an adaptation of this method to cross-borehole low-frequency probing.

Note that by appropriately changing the impressed voltages on the multielectrode array, the equipotential contours (and thus the current flow directions) can be shifted to obtain views of the region of interest. A limitation in the number of views is evident. This limitation would lead to a resultant profile of lesser resolution than obtainable with the synthetic core samples. The lack of guard rings also causes problems due to the current flowing in three dimensions, rather than two dimensions.

Figure 11 shows an adaptation of the method to surface-to-borehole probing. Again, by varying the impressed voltages on the electrodes, a number of views could be obtained. It is evident that a higher quality picture would likely be obtained near the top of the borehole than near the bottom, due to the larger number of views possible near the top.

Figure 12 shows an adaptation of the method to surface probing. For this method to work well, guard electrodes on both sides of the sampled electrode array would be necessary to attempt to constrain the ground currents to flow within a plane beneath the sampled electrode array. This same approach would be useable within a single borehole; however, all azimuths would be sensed at the same time. This application has an even more limited number of views than the prior application and thus would likely be of even less resolution. The region nearest the surface of the borehole would be sampled at the highest density and probably determined with the most accuracy.

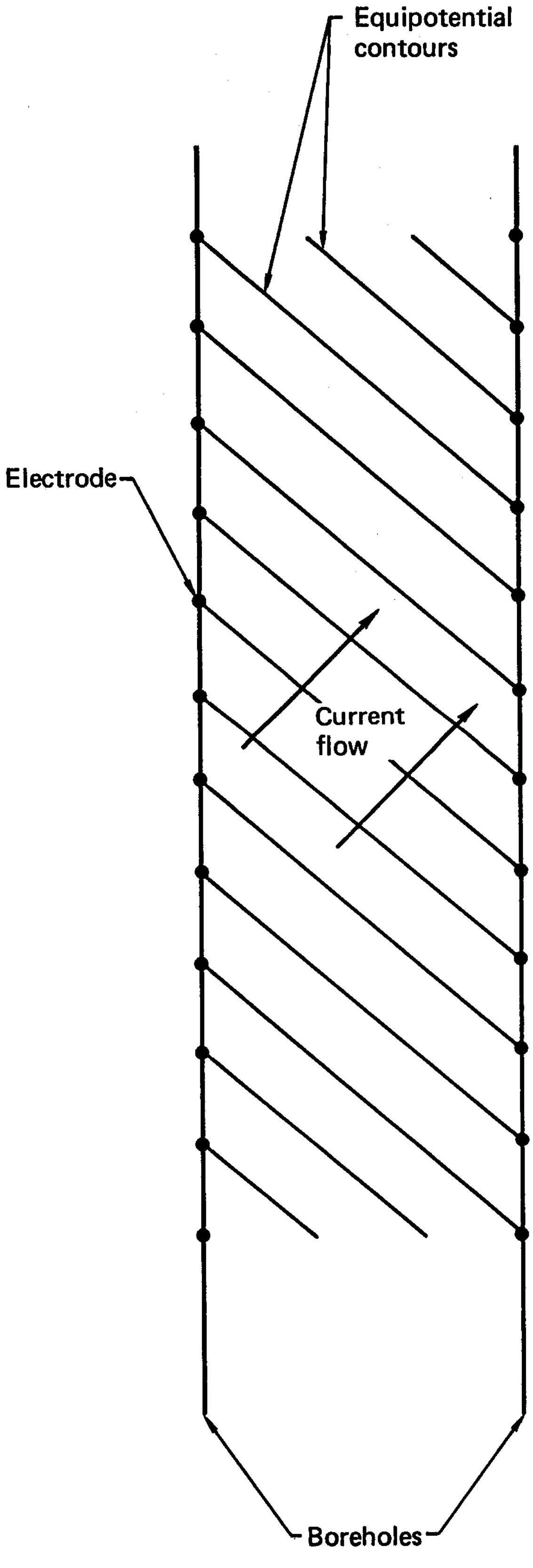


Fig. 10. Representative view illustrating how to preserve idea of using "views" within the ground — by using numerous electrodes within two boreholes and applying appropriate sequence of voltages to the electrode.

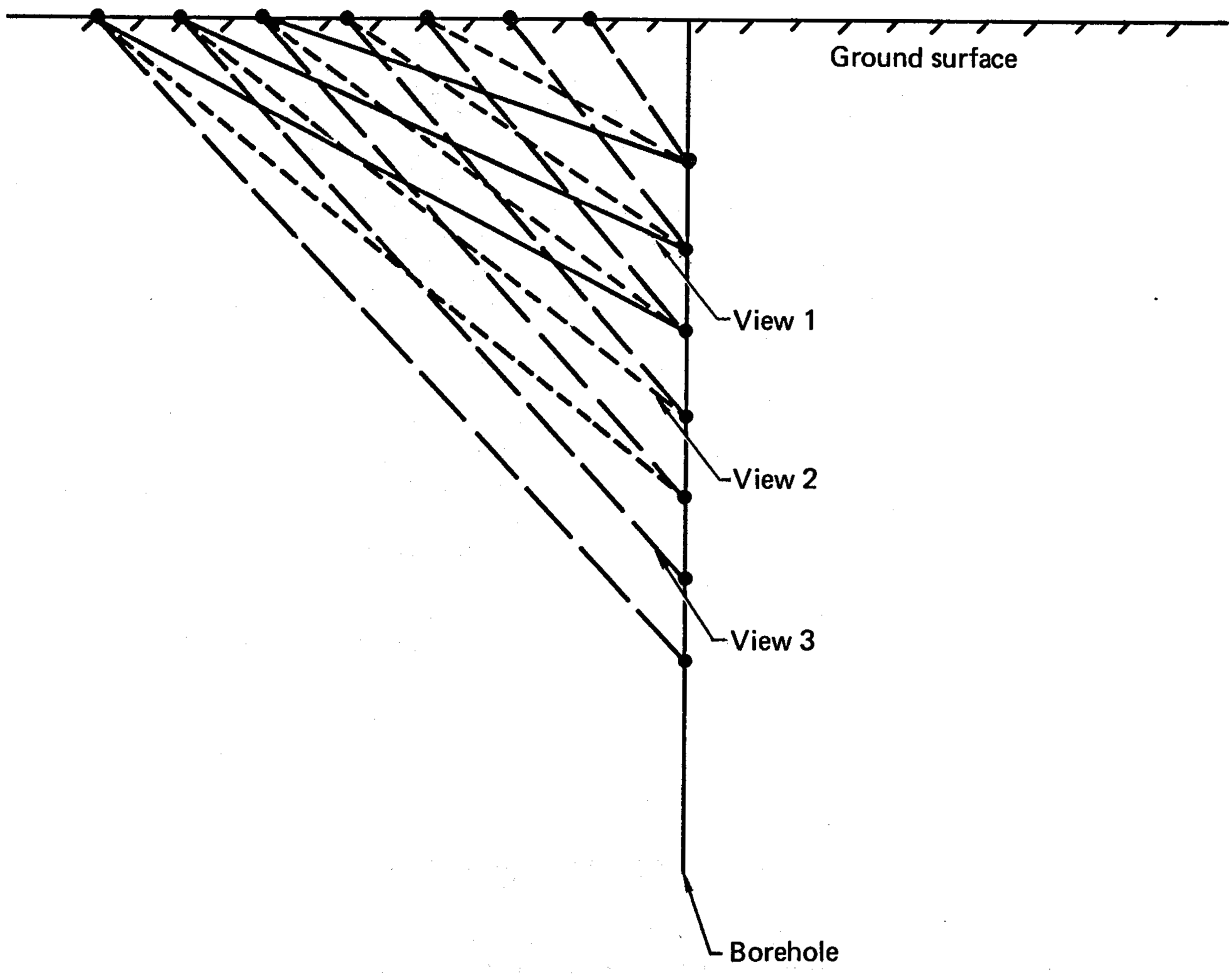


Fig. 11. Three representative current flows illustrating how to preserve idea of using "views" within the ground — by using a number of electrodes on the surface and in a borehole and applying appropriate sequence of voltages to electrodes (guard electrodes could be used on the surface to help define plane of current flow).

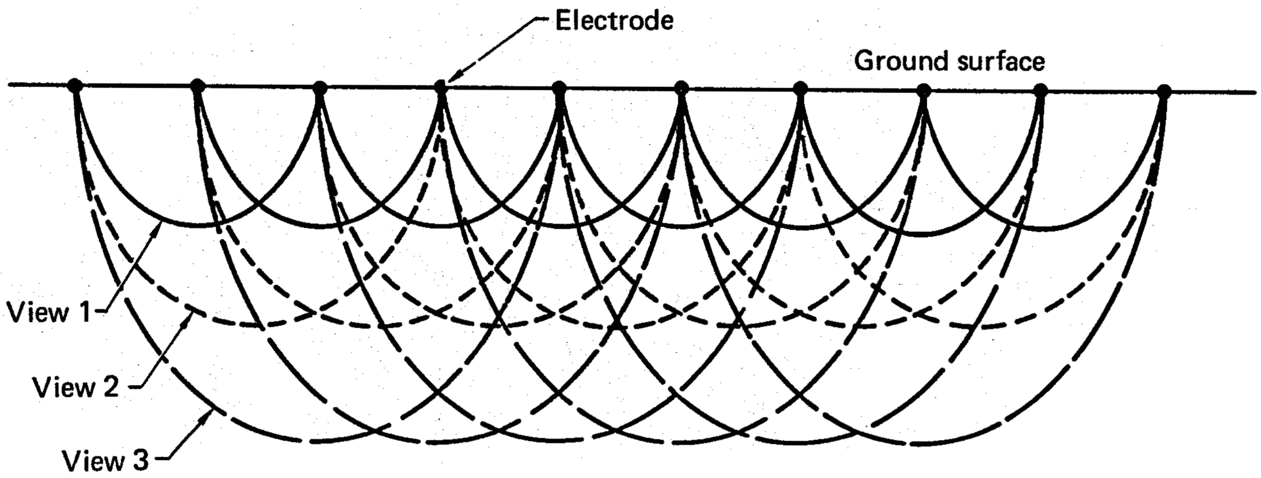


Fig. 12. Representative views showing how focusing a current into the ground at different angles can be created to provide a view — by using a number of electrodes at the surface and applying appropriate sequence of voltages to all electrodes in the array.

CONCLUSIONS AND RECOMMENDATIONS

A data collection and data interpretation approach has been developed, implemented, and applied to the problem of determining the electrical conductivity distribution within core samples. Low-frequency electromagnetic probing is performed using an array of electrodes on the sides of the core sample. The results obtained using this impedance camera procedure indicate that one can correctly infer many significant electrical conductivity variations interior to the core sample. This conclusion is based upon synthetic examples examined using a finite-difference model on a computer.

Extensions of the basic concept may also be helpful in geophysical probing from the surface and/or boreholes, or in other remote sensing methods based upon Laplace's equation. An automated data collection and interpretation apparatus usable for standard core samples should be built and routinely used. This apparatus could provide detailed information on the interior conductivity profile of core samples, and thus indicate which cores (and, consequently, regions around boreholes) are worthy of more detailed investigation.

Items worthy of future research include an assessment of the influence of noise in the data, a study of the accuracy of the reconstruction and its spatial dependence, an evaluation of the degree of dependence of various measurement configurations, an analytic study of the resolution limit, and a determination of the extent to which the use of *a priori* knowledge affects the interpretation.

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