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RELATIVISTIC MEAN-FIELD THEORIES AND NUCLEAR PROPERTIES

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The nucleus is described as consisting out of relati-
vistic nucleons and explicit mesonic degrees-of-free-
dom which are considered to be the relativistic genera-
lisation of the Skyrme force. The meson-parameters can
be adjusted such that the model gives an excellent
description of spherical nuclear ground states. First
axially symmetric deformed calculations are presented.
Dynamic calculations of relativistic ^{16}O - ^{16}O scatter-
ing are also done; they show pronounced effects of the
mesonic degrees-of-freedom.

1. INTRODUCTION

Classical Mean-Field theories, e.g. Skyrme-Hartree-Fock,
/QF78/, have proven to be a useful basis for static and dy-
namic properties of nuclei. It is interesting to investigate
also a relativistic Mean-Field model where the nucleons
obey the Dirac-equation and the forces are mediated by ex-
plicit mesonic degrees-of-freedom. In fact such a relati-
vistic treatment is almost as old as classical models
/MG72, Wa74/ and it has since been studied extensively; for
a recent review see /SW85/.

In this contribution we look at the relativistic Mean-
Field model from many aspects. First, we investigate the
ability of the theory to describe nuclear ground-state
properties /RR86/; this is done systematically by means of
least-squares fits to experimental data, and, of course,
we restrict this study to spherical nuclei in order to re-
duce the expenses. Second, we look at the properties of de-
formed nuclei within the relativistic approach; these are
first exploratory calculations for light nuclei /Le86/.
Third, we study dynamic properties for the example of a re-
lativistic ^{16}O - ^{16}O collision /CR85/.

The paper is outlined as follows: In section 2 we ex-
plain the theory. In section 3, we explain the computation
of observables. In section 4, we present the fits to
(spherical) nuclear data. In section 5, we show the result
of deformed calculations. And in section 6, we discuss the
relativistic heavy-ion collision.

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2. THE RELATIVISTIC MESON-FIELD MODEL

We want to describe the nucleus as a relativistic system
consisting of nucleons which obey the Dirac equation, and of
mesons which mediate the internucleon forces. From the many
possible mesons we choose a few with simple internal quan-
tum numbers (spin 0 or 1, isospin 0 or 1, no strangness)
and with low mass; these are

    an isoscalar-scalar (σ) meson field ϕ ,
    an isoscalar-vector (ω) meson field V_μ ,
    an isovector-vector (ρ) meson field R_μ ,
    and the photon field A_μ .

One misses the π-field in that list as well as the η-mesons.
These are pseudoscalar mesons, and therefore they have
vanishing expectation value for the ground state of nuclei
and nuclear matter (unless parity were broken which is very
unlikely). Thus pseudoscalar mesons will not contribute in
a Mean-Field approximation and we neglect them in the fol-
lowing.

    We choose always the simplest coupling of the mesons,
selected above, to the nucleons. For the scalar meson field
we also include a cubic and quartic nonlinear self-coupling
in order to give the model sufficient flexibility to de-
scribe nuclear response features /BB77/. Altogether the
model Lagrangian density reads (in units $\hbar=c=1$)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_N)\psi$$
$$+ \frac{1}{2}[\partial^\mu \phi \partial_\mu \phi - m_s^2 \phi^2] - g_s \phi \bar{\psi}\psi - \frac{1}{3}b_2\phi^3 - \frac{1}{4}b_3\phi^4$$
$$- \frac{1}{2}[(\partial_\nu V_\mu - \partial_\mu V_\nu)\partial^\nu V^\mu - m_v^2 V_\mu V^\mu] - g_v V^\mu \bar{\psi}\gamma_\mu \psi \quad (1)$$

$$- \frac{1}{2}[(\partial_\nu R_\mu - \partial_\mu R_\nu)\cdot \partial^\nu R^\mu - m_R^2 R_\mu \cdot R^\mu] - \frac{1}{2}g_R R^\mu \cdot \bar{\psi}\gamma_\mu \tau\psi$$
$$- \frac{1}{2}[(\partial_\nu A_\mu - \partial_\mu A_\nu)\partial^\nu A^\mu] - \frac{1}{2}e A^\mu \bar{\psi}\gamma_\mu(1+\tau_0)\psi \quad (1)$$

where ψ is the nucleon field and $\bar{\psi}=\psi^+\gamma_0$; the γ^μ are the usu-
al Dirac matrices /BD 64/. The first line describes the
free nucleon Lagrangian and the other lines are the meson
Lagrangians with their couplings to the nucleons (and the
self-couplings of the scalar meson). It is a formidable
task to determine the full quantal dynamics of the Lagran-
gian (1). It would have to include all many-body-effects,
namely

    self consistent field,
    exchange diagramms,
    two body correlations (ladder diagramms),
    core polarisation (bubble diagramms), etc,

and all quantum field effects, namely

    vacuum polarisation,
    mass renormalisation,
    charge renormalisation, etc..

In fact, that is an almost impossible task; and it is use-
less as well, because we know that nucleons and mesons are
not the basic constituents of matter. The Lagrangian (1) is
to be considered as being just a phenomenological Lagran-
gian for a relativistic nuclear model and it makes most
sense in connection with approximations.

    We employ first the
    Mean-Field approximation: $\hat{\phi} \to \phi$, $\hat{V}_\mu \to V_\mu$,

$$\hat{R}_\mu \to R_\mu, \quad \hat{A}_\mu \to A_\mu, \quad (2)$$

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i.e. all meson fields are treated as classical c-number fields. Accordingly the nucleon currents become c-number currents, $\bar{\Psi}\Psi \to \rho_s = <\bar{\Psi}\Psi>$ etc, and the ground state expectation values $<...>$ can be expressed by the nucleon single particle wave functions φ_α, e.g.

$$\rho_s = <\bar{\Psi}\Psi> = \sum_{-\infty}^{+\infty} \chi_\alpha \; \bar{\varphi}_\alpha \varphi_\alpha \qquad (3)$$

where $\chi_\alpha = + \frac{1}{2}$ below the Fermi-energy and $\chi_\alpha = -\frac{1}{2}$ above the Fermi-energy. With the Mean-Field approximation we retain from all the many-body effects only the self consistent field and from all quantum field effects just the vacuum polarisation. That is a most dramatic simplification of the task.

The vacuum polarisation is hard to evaluate since it involves a sum over a continuum of states, see the schematic plot of the nucleon spectrum in fig. 1. The vacuum polarisation is an accumulated effect from the infinity of negative-energy states /CH 77/. The predominant self-consistent field is build from the few occupied positive-energy bound states which correspond to the conventional nuclear shell model states. We come down to a manageable set of equations by neglecting the vacuum polarisation, i.e. we employ the

No-Sea approximation: $\sum_{\alpha=-\infty}^{+\infty} \chi_\alpha ... \to \sum_{a=1}^{A} ... \qquad (4)$

where all sums accumulating nucleon densities run over the shell model states only.

By variation of the model Lagrangian (1) and employing the Mean-Field approximation (4) we obtain the coupled equations for the nucleon wave functions φ_α and for the
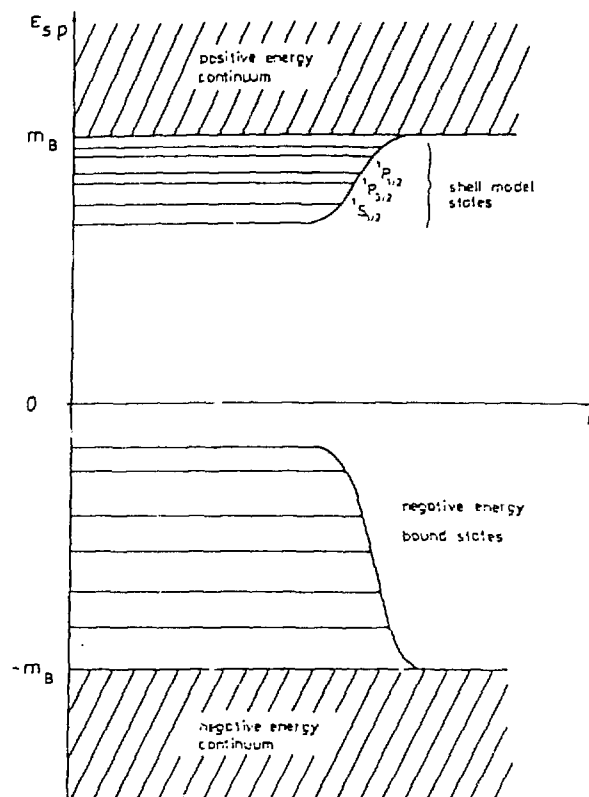
Fig. 1. Schematic spectrum of the Dirac equation for the nucleons. The positive energy bound state exhibit a one-to-one correspondence to the states of a classical nuclear shell model. This is indicated by denoting these states with the usual spectroscopic quantum numbers. The potential for negative energy states can also support bound states as indicated

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meson fields ϕ, V_μ, R_μ and A_μ:

$$i\gamma_0 \frac{\partial}{\partial t}\varphi_\alpha = (-i\gamma\cdot\nabla + m_N + g_s\phi + g_v V^\mu\gamma_\mu + \frac{1}{2}g_R R^\mu \cdot \tau\gamma_\mu$$

$$+eA^\mu \frac{1+\tau_0}{2} \gamma_\mu)\varphi_\alpha \qquad (5a)$$

$$(\frac{\partial^2}{\partial t^2} - \Delta + m_s^2)\phi = -g_s\rho_s - b_2\phi^2 - b_3\phi^3, \qquad \rho_s = \sum_{\alpha=1}^{A} w_\alpha \bar\varphi_\alpha \varphi_\alpha \qquad (5b)$$

$$(\frac{\partial^2}{\partial t^2} - \Delta + m_v^2)V_\mu = +g_v\rho_\mu, \qquad \rho_\mu = \sum_{\alpha=1}^{A} w_\alpha \bar\varphi_\alpha \gamma_\mu \varphi_\alpha \qquad (5c)$$

$$(\frac{\partial^2}{\partial t^2} - \Delta + m_R^2)R_\mu = +\frac{1}{2}g_R\rho_\mu, \qquad \rho_\mu = \sum_{\alpha=1}^{A} w_\alpha \bar\varphi_\alpha \gamma_\mu \tau\varphi_\alpha \qquad (5d)$$

$$(\frac{\partial^2}{\partial t^2} - \Delta)A_\mu = e\rho_{p,\mu}, \qquad \rho_{p,\mu} = \sum_{\alpha=1}^{A} w_\alpha \bar\varphi_\alpha \gamma_\mu \frac{1+\tau_0}{2} \varphi_\alpha \qquad (5e)$$

Note that an occupation weight w_α has been implemented in the sums for the nucleon densities. This is to give the model the flexibility to describe also non-magic nuclei. In practice we are using a schematic pairing within the constant gap approach with a gap $\Delta = 11.2$ MeV $A^{-1/2}$. For closed shell nuclei, of course, we set $w_\alpha = 1$ throughout. Note furthermore that proton- and neutron-wave-functions usually do not mix; then only the τ_0 - component in the isovector current ρ_μ exists and only the $R_{0,\mu}$ field component needs to be retained.

The eqs. (5) still embrace all possible applications. For dynamical calculations, e.g. of relativistic heavy ion collisions, they are to be treated to full extend. For static application, the eqs. (5) can be simplified:

the meson fields are independent of time, thus $\frac{\partial^2}{\partial t^2}\phi = 0$ etc and the nucleon wave functions are stationary, thus $i\frac{\partial}{\partial t}\varphi_\alpha = \epsilon_\alpha\varphi_\alpha$ with ϵ_α being the single particle energy; furthermore the space-vector parts of meson fields vanish, i.e. only $V_0, R_{0,0}$ and A_0 need to be treated. For spherical static calculation the problem becomes even simpler: the fields are rotational invariant, $\phi(r) \to \phi(|r|)$ etc, and the nucleon wave functions separate into radial function and spinor spherical harmonies.

Altogether the relativistic Meson-Field model consists out of the Lagrangian (1) together with the approximations (2) and (4). Thus the Lagrangian is an effective Lagrangian for relativistic Mean-Field calculation. This is a similar situation as with the Skyrme force which is an effective force for classical nuclear Mean-Field calculations: the structure of the Lagrangian is inspired by theoretical considerations; the parameters therein, however, are considered as being free parameters of the model to be adjusted to observable nuclear properties. The free parameters of the relativistic Lagrangian (1) are the meson-masses m_s, m_v and m_R and the couplings g_s, g_v, g_R, b_2 and b_3. The photon coupling e is fixed to the known value by requiring that the nuclear Coulomb field has the right asymptotics. Due to the Mean-Field approximation (2) there is no nucleon-mass renormalisation and we can fix the experimental value

$$m_N = 938.9 \text{ MeV} \qquad (6)$$

as an average of proton and neutron mass. The free model parameters are to be fixed by fits to observable nuclear ground-state properties. We make the experience, that the

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masses of the ω- meson and of the ρ-meson are rather insensitive to the data. Thus we fix them at nearly the OBEP values /Ho 81/

$$m_v = 780. \text{ MeV} \qquad , \tag{7}$$

$$m_R = 763. \text{ MeV} \qquad , \tag{8}$$

and we are left with the 6 free model parameters m_s, g_s, g_v, g_R, b_2 and b_3 to be adjusted.

3. THE GROUND-STATE OBSERVABLES

The solution of the eqs. (5) delivers meson fields and nucleon wave functions. From these we have to evaluate some observables in order to compare with experimental data. The most obvious observable is the total energy of the system. It can be derived from the Lagrangian (1) by standard techniques /BD 65/; this yields

$$E_{MF} = \int d^3r \left\{ \sum_{\alpha=1}^{A} w_\alpha \bar{\varphi}_\alpha \; i \frac{\partial}{\partial t} \varphi_\alpha \right.$$

$$+ \frac{1}{2} \left[\dot{\phi}^2 + \nabla\phi \right)^2 + m_s^2\phi^2 \right] - \frac{1}{3}b_2\phi^3 + \frac{1}{4}b_3\phi^4$$

$$\tag{9}$$

$$- \frac{1}{2} \left[\dot{V}_\mu \dot{V}^\mu + \nabla V_\mu \cdot \nabla V^\mu + m_v V_\mu V^\mu \right]$$

$$- \frac{1}{2} \left[\dot{R}_\mu \cdot \dot{R}^\mu + \nabla R_\mu \cdot \cdot \nabla R^\mu + m_R R_\mu \cdot R^\mu \right]$$

$$\left. - \frac{1}{2} \left[\dot{A}_\mu \dot{A}^\mu + \nabla A_\mu \cdot \nabla A^\mu \right] \right\} \qquad .$$

For the static case one can omit again $\dot{\phi}, \dot{V}_\mu, \dot{R}_\mu$ and $\dot{A}_\mu$, and let $i \frac{\partial}{\partial t} \varphi_\alpha \rightarrow \epsilon_\alpha \varphi_\alpha$. The energy (9) is the contribution from the Mean-Field theory. If we determine the occupation numbers w_α by pairing we have to add the pairing energy. Finally we have to account for a spurios centre-of-mass motion of the nucleus in its own Mean-Field. This is done by subtracting a centre-of-mass correction to the energy

$$E_{c.m.} = \frac{\langle \hat{P}^2_{c.m.} \rangle}{2mA} \qquad .$$

Altogether the total energy is composed from three pieces as

$$E_{total} = E_{MF} + E_{Pair} - E_{c.m.} \qquad .$$

The energy is the only observable which can be derived consistently from a given effective Lagrangian (and the implied approximations). All other observable, as e.g. formfactors or transition moments, would need to know the exact transformation from an "ab initio" Lagrangian to the effective one, because the same transformation should be applied to the observable (which usually is a given "ab initio" object). This transformation includes all the neglected many body and quantum field effects (see previous section); they are often called correlations as it is everything beyond the Mean-Field approximation. We do not know this correlation - transformation since we just parametrize the effective Lagrangian. Thus the only solution we have is that we select observables which can be expected to be insensitive to correlations. We think that the bulk properties of the nuclear charge and mass distribution, as e.g. the radius, the surface thickness, or the quadrupole deformation, are such observables.

Radius and surface thickness are derived from the charge formfactor of the nucleus. For the spherical calculations, the formfactor is obtained from the nucleon density (zero component of the vector current) by a Fourier-Bessel transformation

$$F_t(q) = 4\pi \int_0^\infty dr \; r^2 j_0(qr) \rho_{c,t}(r) \tag{12}$$

where t = proton or t = neutron. The nuclear charge formfactor is obtained by folding in the proton- and neutron-formfactor /SS80/ and unfolding the spurios centre-of-mass motion /QF78,FR86/. The radius R is then given by the first zero of the charge formfactor

$$R = 4.493/q^{(1)} \quad , \qquad F_C(q^{(1)}) = 0 \tag{13}$$

and the surface thickness σ is determined from the suppression of the first maximum compared to the formfactor of a hard sphere with radius R,

$$\sigma^2 = \frac{2}{q_m^2} \; \log \left(\frac{3j_1(q_m R)}{q_m R F_C(qm)} \right) \quad , \tag{14}$$

$$F_C(q_m) = 1 \; \text{Maximum} \quad .$$

Note that the radius R is the diffraction radius and not the r.m.s. radius. It is more related to the box-equivalent radius and it has the advantage that the diffraction radius is most insensitive to correlation effects (folding does not shift the zeroes of F_C) /FV 82, RF 85/. Both fourier components, $q^{(1)}$ for R and q_m for σ, occur at lower q and low q are less influenced by correlation effects than high q. Thus we hope that R and σ are appropriate observables for

a Mean-Field theory.

The quadrupole moment is used as an observable for deformed calculations. It is

$$Q_{20} = \int d^3r \, r^2 Y_{20} \, \rho_0(r) \; / \; \int d^3 r \rho_0(r) \tag{15a}$$

where ρ_0 is given by eq. (5c). The Q_{20} has dimensions fm^2. The dimension less quadrupole deformation is derived from Q_{20} by

$$\beta = \bar{\beta}(1 + \frac{5}{4\pi} \bar{\beta}^2), \quad \bar{\beta} = \frac{4\pi}{5Ar^2} Q_{20} \tag{15b}$$

where r is the r.m.s. radius for the nucleus.

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4. OPTIMISING GROUND STATE PROPERTIES OF SPHERICAL NUCLEI

The solution of the coupled field equations becomes fairly
simple for the ground-state (static limit) of spherical nu-
clei. With the usual spherical representation /BD64/ and
employing the inverse gradient step /RG82/ we have set up
a very fast program. Such that one can affort extensive and
systematic investigations of the model parameters.

As explained in Sect. 2, the model has the free para-
meters g_s, b_2, b_3, g_v, g_R, m_s, m_v and m_R. We want to adjust
these parameters such that the model r roduces as good as
possible measured ground state properties of nuclei: binding
energies and formfactors. We do this by minimizing the
squared deviation $\chi^2 = \sum_n (O_n^{exp} - O_n^{theory})^2 / \Delta O_n^2$ with re-
spect to the parameters of the theory. There the sum runs
over all chosen observables O_n. In experimental data analy-
sis the ΔO_n is the statistical error on the data. In our
case, we cannot use just the experimental error because both
observabels, binding energy and formfactor, are measured
with a much higher precision than any theory can reach. The
systematical error is, so to say, too large. Therefore we
insert for ΔO_n what we expect to be the ability of the theory
to describe that observable O_n. This way we regulate the
relative weights of the contributions.
In table 2 we show the selection of the observables, their
experimental values and the adopted errors; their theoreti-
cal definition is outlined in section 3. The selection cov-
ers a wide range in mass number, and some isotopic trends by
the isotopes of Ca and Sn In contrast to similar fits of
the Skyrme force /FR86/ we do not include any explicit in-
formation about the spin-orbit splitting. A relativistic
theory should predict this properly without any further help.

TABLE 1

Nucleus	Energy E_{total} [MeV]	diffr. radius R [fm]	surface σ [fm]
^{16}O	- 127.6	2.777	0.839
^{40}Ca	- 342.1	3.845	0.978
^{48}Ca	- 416.0	3.964	0.881
^{58}Ni	- 506.5	4.356	0.911
^{90}Zr	- 783.9	5.040	0.957
^{116}Sn	- 988.7	5.537	0.947
^{124}Sn	-1.050.0	5.640	0.908
^{208}Pb	-1.636.4	6.806	0.900
adopted error	0.2%	0.5%	1.5%

Table 1: Selection of observables, their experimental values
and their adopted errors.

The χ^2 is minimized by standard techniques /Be69/. We have
performed the χ^2-fits under some varying conditions, as ex-
plained below. The results for the achieved precision in
E, R and σ are compiled in fig. 2. First we concentrate on
the "standard fit" including all data and computing all ob-
servables as explained above (denoted by a full circle ●).
The resulting parameters are: m_s=488.176 MeV=2.47805fm^{-1},
g_s=10.0409, b_2=13.3451fm^{-1}, b_3=-39.6023, g_v=12.9091 and
g_ρ=9.6991. The model is obviously capable to adjust a very
good description of nuclear ground state properties, with
an error of 0.35% in σ . This precision can compete with
the quality of the best adjusted Skyrme forces /BQ82, FR86/.
It is important to note that the nonlinear scalar self-
couplings are needed to obtain this precision . A similar
fit with fixed b_2=0 and b_3=0 yields an average error in E
of 2%, in R of 1.4% and in σ of 23%. It is obvious that the
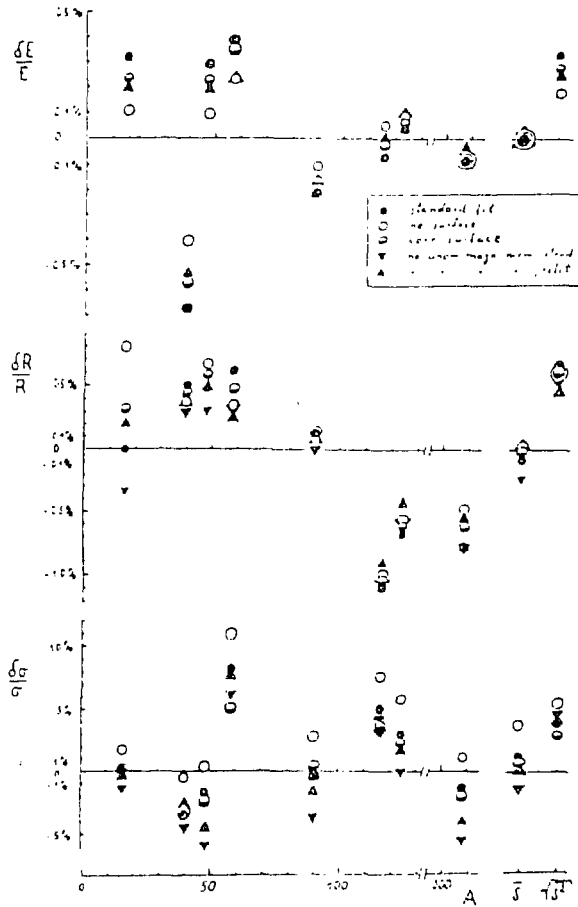surface σ is more critical in the fit than energy E and

Fig. 2: The relative errors in E, R and σ for the eight fit nuclei, drawn versus mass number A. The extra column $\bar{\delta}$ means the direct average of the deviation and $\sqrt{\bar{\delta^2}}$ the mean-square error. The various fits are denoted by different symbols as explained in the above box.

radius R. To estimate its effect we have performed a fit which includes only E and R as data, see the open circles (O) in fig. 2. This allows to fit the energy E much better; for the radius R the effect is mixed with a slight overall improvement, and the surface thickness σ is, of course less well described. Note, however, that the quality of σ has only moderatly worsened (from 3.5% to 5%) and that there is a systematic shift to smaller surfaces for all nuclei (see also the column $\bar{\delta}$ in fig 2). The Mean-Field model seems to prefer steeper surfaces than observed in experiment.

And indeed it is well known that effects beyond the Mean-Field approximation soften the surface, in particular the ground-state correlations due to nuclear collective vibrations /RF85/. The dominant effect on the surface thickness σ comes from the collective surface vibrations, the low lying 2^+ states. Their vibrational width and with it their effect on σ can be estimated from the B(E2) value for the $0_1^+ \rightarrow 2_1^+$ transition /RF85/. We have corrected the experimental σ for that effect and performed the fit with the corrected data. This yields the half-full circles (◓) in fig. 2. There is an overall improvement in the fit, in particular for the precision on σ. Note that this improvement comes mainly from the three nuclei with large surface vibration, namely ^{58}N, ^{116}Sn and ^{124}Sn. Further improvement is conceivable if one also accounts for the correlations from giant resonances. In any case we see that the fits exhaust the Mean-Field model to the limits where correlation effects are inevitable.

Some other kind of correlations have already been implemented in the way the charge formfactor is composed: we fold in the proton and the neutron formfactor and we add

the contribution of the nucleonic anomalous magnetic moment
to the Coulomb field. The latter piece can be counted as a
quantum field correlation effect (meson clouds). In order to
investigate the influence of the anomalous magnetic moment
we first have left out this contribution in a calculation
with the "standard fit", given above; this yields the inver-
ted triangles ($\blacktriangledown$) in fig. 2. Nothing changes in E, of
course. There is systematic shift to larger radii and larger
surfaces. In the whole, the precision of the observables
has not suffered very much. It can be restored completely
by fitting anew without the anomalous magnetic moment, see
the triangles ($\blacktriangle$) in fig. 2. Here we have an example of a
type of correlations which is just at the boundary of the
precision of the model.

All fits show a similar tendency. The average error
column 3 in fig 2, is adjusted to almost zero and thus the
mean squared error, column$\sqrt{\overline{\delta^2}}$, lives from the fluctuation
about the average. Nevertheless there is some systematics
in these fluctuations: the energy trend along isotopic
chains cannot be fully reproduced (see here the example Ca
and Sn, more in /RR86/), and there in an overall trend in
R with A. Both unresolved trends hint that something is
missing in the model. These will be some correlations; it
is yet to be found out which ones.

It is interesting to look what the model predicts for
quantities which have not been included fits. The most pro-
minent of them are the properties of symmetric nuclear
matter. We show them, together with the predictions of some
other models, in table 2. The techniques of a least-squares
fit provides us also with an estimate of the statistical
error on the prediction; this is given, pars pro toto, for
the "standard fit" in the table 2. Note that all the vari-

ations of the nonlinear relativistic model give virtually
the same prediction which agrees fairly well with the clas-
sical models, Skyrme-Hartree-Fock and liquid drop; except
for the effective mass m_N^*/m_N which comes out always very
low in the relativistic models (including the rel. Brueck-
ner-Hartree-Fock). The linear relativistic model ($b_2=0$,
$b_3=0$) deviates strongly, in particular for energy E and in-
compressibility K. It is interesting to note that the re-
lativistic B.H.F. calculations /BM86/ agree fairly well with
the extrapolated Mean-Field predictions; except for the bin-
ding energy which seems to be a bit small.

TABLE 2

	$\cdot E/A$ [MeV]	ρ_c [fm^{-3}]	K [MeV]	m_N^*/m_N
Standard fit	16.20	0.1507	177.8	0.584
	±0.05	±0.0009	±8	±0.004
Fit with corr. σ	16.19	0.1502	190.	0.585
Fit without σ	16.17.	0.1493	209.	0.584
Fit without anom. magn. moment	16.20	0.1502	199.	0.584
Fit with $b_2=0,b_3=0$	17.04	0.1429	584.	0.529
Skyrme fit Z_σ/FR86/	15.81	0.1627	233.	0.783
Relativ. Brueckner H.F. /BM86/	15.32	0.161	201.	0.57
Liquid drop model	15.95	0.145	212.	–

Table 2: The bulk properties of symmetric nuclear matter
at equilibrium for the various fits and for some other model.
E/A is binding energy per particle, ρ_0 is the nucleon den-
sity, K is the compressibility, and m_n^*/m_n is the effective
mass.

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It is not enough space here to outline other features of
the model. We summarize them shortly, more details may be
found in ref. /RRB6/. The full model predicts 6 MeV spin
orbit splitting of the 1p-level in ^{16}O which is exactly the
experimental value. This is the great advantage of the rela-
tivistic model compared to the classical models that the
spin properties are automatically implied and that no fur-
ther terms and parameters are needed to adjust them. Note,
however, that only the full model gives this good descrip-
tion. The linear model fails badly predicting 11 MeV for
the spin-orbit splitting in ^{16}O. There are two other fea-
tures where the classical Mean-Field models failed: the le-
vel density near the Fermi-energy is systematically to low,
and the amplitude of the shell fluctuations on the nuclear
charge density is by a factor 2-3 to large. We have looked
at this in the relativistic model and find that they fail
exactly the same way as the classical models do. These are
very probably again effects which call for an inclusion of
correlations.

Altogether we see that the relativistic model is as
flexible and powerful as the classical Skyrme-Hartree-Fock
description. It allows the same marvelous reproduction of
nuclear ground-state data, and it shares many of the
failures of the classical model, which all hint at corre-
lations (beyond Mean-Field). However, it is clearly superior
in predicting spin properties. Furthermore it provides a
good basis for any application where relativistic wave
functions are needed.

5. DEFORMED STATIC CALCULATIONS

The success of the spherical applications of the model moti-
vates to do the next step in evaluating nuclear properties:
the evaluation of deformed ground states. Before we can har-
vest interesting results on the relativistic effects in de-
formation properties, we have to work out some involved
numerical techniques.

TABLE 3

Nucleus	E (MeV)	r (fm)	β	comment
12C	-94.1	1.99	-0.005	
16O	-128.9	2.29	0.006	
16O	-120.6	2.31	0.003	grid 0.25 fm
16O	-119.9	2.32	--	spherical code
20Ne	-142.2	2.62	-0.54	
20Ne	-161.5	2.45	-0.20	various
20Ne	-166.4	2.54	0.42	initial
20Ne	-141.3	4.21	1.51	deformations
20Ne	-121.9	7.20	2.39	
24Mg	-208.9	2.56	-0.20	
40Ca	-373.8	3.06	0.002	
40Ca	-358.2	3.09	--	spherical code
48Ca	-490.4	3.18	-0.0003	
48Ca	-466.7	3.21	--	spherical code

Table 3.: Results of calculations with the linear parametri-
sation of ref.2: $g_s=10.3$, $g_v=12.6$, $m_s=550$, $m_v=783$ and
$m_b= 938$ MeV, including Coulomb force. If not commented
otherwise an axial grid with a spacing of 0.5fm has been
used. The β is the dimensionless quadrupole deformation.
The r is the r.m.s. radius.

We represent the four components of the nucleon wave func-
tion and the meson fields on an axial space grid. In order

to avoid the fermion doubling /BM83/ we have used shifted
grids for the four different components of the wave func-
tions, employing a shift by half a mesh spacing either in
z-direction or in r-direction or in both directions. As in
similar classical calculations /DK81/ we derive consistent
field equations on the grid by first discretising the ac-
tion on the grid and then performing the variation. The
energy of the system is computed from the (discretized) ac-
tion by standard techniques of field theory. We restrict
all further considerations to the static case where all ob-
jects are time-independent and in particular where all
space-vector parts of the vector fields vanish. In order
to employ the well established iteration techniques of
classical calculations /RC82/ we transform the Dirac equa-
tion into an effective Schroedinger equation (having an
energy-dependent mass), and we use then the damped gradient
iterations /RC82/ to find the (local) minima of the action.
The details of the numerical procedure will be published
elsewhere /Le86/.

In table 3 we show the results of the relativistic
axially deformed calculations for various light nuclei.
Here the old linear parametrisation of Walecka /Wa76/ has
been used throughout, and the dimensionless quadrupole de-
formation is defined as in eq. (15). First we note that the
"standard" spherical nuclei, namely 16O, 40Ca, and 48Ca,
come out to be spherical; the remaining small deformation
indicates the precision of the numerical representation.
For 16O we have compared two grid spacings, the standard
grid with 0.5fm and a finer grid with 0.25fm spacing. The
residual deformation is much smaller on the finer grid. We
see also a dramatic change in the binding energy. This is
similar to classical calculation. The fine grid gives

already an excellent discription as one sees from the com-
parison with the spherical results. Nevertheless we prefer
to use the "coarse" grid with 0.5fm spacing for reasons of
computing time. The overall effect is mainly an energy shift
which may be estimated from comparing spherical and de-
formed results for 16O, 40Ca and 48Ca.

The interesting results are those for the deformed nu-
clei. 20Ne and 24Mg show substantial deformations and they
are comparable to classical calculations /QF78/ and to data.
For 12C we obtain negligible deformation. That is not so
surprising because 12C is rather a soft nucleus than a de-
formed system, and the position of the actual deformation
minima depend sensitively on the forces used. In fact, we
know that the spin orbit splitting tends to restore spheri-
city and that the parametrisation used in table 3 has a
large spin-orbit force.

For 20Ne we have explored the whole range of axially
deformed local minima. This was achieved by starting the
iteration from various differently deformed initial config-
urations. The prominent minimum comes out at $\beta = 0.42$ in
accordance with classical results. The occurrence of a,
less favorite, oblate minimum s also quite common. The next
oblate and prolate deformtions are a bit surprising. They
may be due to the fact that we compute always closed shell
nuclei (with proper reordering of course); a pairing force
may smoothen some of these ex. emely deformed minima. The
most prolate solution, however, may survive because it re-
presents quite clearly a chain of five α-particles loosely
bound to each other. This configuration may be an effect of
the parametrisation which produces a rather large binding
for the α-particle.

6.DYNAMICS OF 16O-16O COLLISIONS

A relativistic Mean-Field model is a natural starting point
for a microscopic description of energetic heavy-ion col-
lisions. We have applied the model, as outlined in section
2, to time dependent calculations of 16O-16O scattering
/CR85/. For these first exploratory calculations we have
used a slightly simplified Lagrangian: we omit the photon,
the ρ-meson and the nonlinear self-couplings of the
σ-meson, i.e. we use the linear model of ref /Wa72/. The
parameters are given in the table caption of table 3.

A few technical explanations are necessary:
We represent the meson fields and the baryon wave functions
on a three-dimensional (24x24x24) mesh in coordinate space.
Quantities in momentum space are obtained by means of a fast
Fourier transform method. We employ isospin-degenerate
nucleon wave functions and treat the spin degrees of freedom
in the usual four-component spinor formalism. Thus we pro-
pagate eight independent fourcomponent wave functions for
each ^{16}O nucleus. We use the relativistic static Hartree
wave functions for each of the ^{16}O initial states. The nu-
clei are located at a separation distance of 10 fm from each
other on the mesh, and are Lorentz boosted to the appropri-
ate initial energy in the center-of-velocity frame. The
time evolution for the baryon wave functions is done with
a fifth-order predictor-corrector in momentum space and the
meson equations are solved simultaneously by Green's func-
tions techniques. With the above mesh, the momentum-space
truncation occurs at about 2 GeV/c. As a test of the ac-
curacy of our numerical techniques we evaluate the total
energy at each time step in the evolution, and note that
it is constant to better than 1×10^{-5}. We have also

evaluated the continuity equation at each point in time to
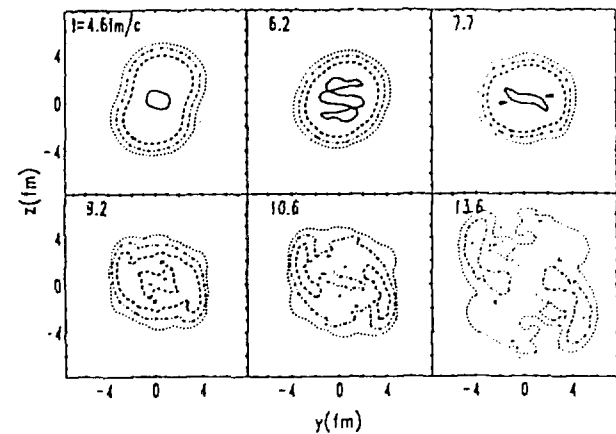further guarantee the correctness of our solutions.

FIG. 3. Contour snapshots of the *coordinate*-space
baryon density $\rho_B(x=0,y,z,t)$, in the scattering plane for
^{16}O(600A MeV, $b=2$ fm) + ^{16}O. The maximum density is
0.29 fm^{-3} and is attained at 6 fm/c. The values of the den-
sity contours are as follows: solid curve, 0.26 fm^{-3}; dashed
curve, 0.10 fm^{-3}; dot-dashed curve, 0.04 fm^{-3}; large-dotted
curve, 0.014 fm^{-3}; and small-dotted curve, 0.005 fm^{-3}.

We have studied with this approach collisions of
^{16}O(E_{1ab}=300, 600, and 1200 MeV/nucleon) + ^{16}O at various
impact parameters. The time evolution is followed until the
expanding matter reaches the edges of the mesh. We show in
Fig. 3 the time evolution of the spatial baryon density for

E_{lab}=600 MeV/nucleon and b=2 fm. The early time behavior in
the approach is similar to that obtained by use of non
relativistic TDHF. Subsequently, a compression zone and
sideward flow develop (between t=6 and 10 fm/c), after
which the system proceeds to undergo spallation. This result
can be contrasted with TDHF calculations at medium energies
where projectilelike and targetlike fragments emerge after
the reaction /SC80/. The sideward flow observed in the pre-
sent approach resembles the large transverse-momentum trans-
fer predicted by nuclear fluid dynamics /SM80/ and the VUU
approach,/MS85/.

Both, the total spallation of the nuclei and the strong
collective sideward flow, are caused by the meson field be-
ing own degrees-of-freedom. We find that the σ- and ω-meson
fields develop strong out-of-phase oscillations in space
and time, soon after the maximum compression point. The nu-
cleons are deflected by these oscillations in fields. The
strong space – time oscillations destroy the nucleon-nucleon
binding and produce the spallation. The strongest meson
fields are concentrated near the origin and that gives rise
to the sideward flow.

Altogether the result of the relativistic Mean-Field
dynamics are half-way between classical TDHF calculations
and fluid dynamics. The mesonic degrees-of-freedom seem to
procuce effects which are usually expected from two-body
correlations. They do not produce enough to reach the fluid
dynamics limit. Probably some two-body correlations have
still to be added. But certainly the relation between two –
body effects and Mean-Field will be different in relati-
vistic and in classical models. That is an interesting task
for further studies.

7. CONCLUSION

It has been shown that the relativistic Mean-Field theory
is as capable in reproducing the bulk properties of nuclei
as the classical Skyrme-Hartree-Fock model. A very precise
description can be achieved with an error of 0.35% for the
energies, of 0.6% for the radii, and of 3.9% for the sur-
face thicknesses. Studying modifications of the model (sur-
face vibrations, anomalous magnetic moments) we could show
that the fit has exhausted the possibilities of a Mean-
Field model, and any step further needs to consider cor-
relations of various sorts. We want to point out the advan-
tage of relativistic over classical models that all spin
properties (in particular the spin-orbit force) are implicit
in the theory without requiring extra terms and extra para-
meters.

The relativistic model produces reasonable deformations
roughly comparable to results of classical calculations.
There are differences in detail which deserve further stu-
dies of model and force dependence of the deformation-ener-
gy surfaces.

The dynamic calculations show the most pronounced re-
lativistic effect namely that the mesons, being now dynami-
cal degrees-of-freedom of its own, produce part of the
effects which are usually expected from two-body collisions.
We observe complete spallation of the nuclei and pronounced
collective sideward flow.

RELATIVISTIC MEAN-FIELD THEORIES...

REFERENCES

/BB77/ J. BOGUTA, A.R. BODMER, Nucl.Phys., A 292 /1977) 414

/BD64/ J.D. BJORKEN, S.D. DRELL. "Relativistic Quantum Mechanics", McGraw-Hill, New York 1964

/BD65/ J.D. BJORKEN, S.D. DRELL, "Relativistic Quantum Fields", McGraw-Hill, New York, 1965

/BE69/ P.R. BEVINGTON, "Data reduction and error analysis for the physical sciences", McGraw-Hill, New York 1969

/BM83/ C.M. BENDER, K.A. MILTON, D.M. SHARP, Phys. Rev. Lett. 51 (1983) 1815

/BQ82/ J. BARTEL, P. QUENTIN, M. BRACK, C. GUET, M. B. HÄKANSSON, Nucl. Phys., A 386 (1982) 79

/Ch77/ S.A. CHIN, Ann. Phys., 83 (1977) 301

/CR85/ R.Y. CUSSON, P.-G. REINHARD, J.J. MOLITORIS, M. STÖKKER, M.R. STRAYER, W. GREINER, Phys. Rev. Lett., 55 (1985) 2786

/DK81/ K.T.R. DAVIES, S.E. KOONIN, Phys. Rev., C 23 (1981) 2042

/FR86/ J. FRIEDRICH, R.-G. REINHARD, Phys. Rev., C 33 (1986) 335

/FV82/ J. FRIEDRICH, N. VOEGLER, Nucl. Phys, A 373 (1982)191

/Ho81/ K. HOLINDE, Phys. Rep., 68 (1981) 122

/Le86/ S.J. LEE et al., to be published

/MG72/ L.D. MILLER, A.E.S. GREEN, Phys. Rev., C5 (1972) 241

/MS85/ J.J. MOLITORIS, H. STÖCKER, Phys. Rev., C 32 (1985) 346

/QF78/ R. QUENTIN, M. FLOCARD, Ann. Rev. Nucl. Part.Sci.,28 (1978) 523

/RC82/ P.- G. REINHARD, R.Y. CUSSON, Nucl. Phys., A 378 (1982) 418

/RF85/ P.-G. REINHARD, J. FRIEDRICH, Z. Phys., A 321 (1985) 619

/RR86/ P.-G. REINHARD, M. RUFA, J. MARUHN, W. GREINER, J. FRIEDRICH, Z. Phys., A 323 (1986) 13

/SC80/ H. STÖCKER, R.Y. CUSSON, J. MARUHN, W. GREINER, Z. Phys., A 294 (1980) 125

/SM80/ H. STÖCKER, J. MARUHN, W. GREINER, Phys. Rev. Lett., 44 (1980) 725

/SS80/ G.G. SIMON; Ch. SCHMITT, F. BORKOWSKI, V.M. WALTER, Nucl. Phys., A 333 (1980) 381

/SW85/ B.D. SEROT, J.D. WALECKA, Adv. Nucl. Phys.,16(1985) 1

/Wa74/ J.D. WALECKA, Ann. Phys., 83 (1974) 491