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NEW RESULTS IN HIGH BETA MHD THEORY*

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New results are described in the following three areas of high MHD theory: (1) equilibrium and stability of diffuse high-β stellarators, (2) MHD equilibrium and stability of minimum-B mirror traps, and (3) simulation of simple and reversed field mirror machines.

I. High Beta Stellarator Studies (LASL)

*Work performed under the auspices of the U.S. Department of Energy.

Both analytic and numerical studies have been performed which demonstrate the existence of high-β stellarator equilibria and make a practical connec ith experiments.

A. Analytic Stu

An analytic formulation has been derived for calculating equilibria in a diffuse high-β stellarator configuration. Although analytic sharp boundary equilibria have been known for some time,[1-4] the extension to diffuse profiles is non-trivial. The reason for this is associated with the misleading intuition from sharp boundary theory which suggests that diffuse high-β stellarator equilibria could be obtained by expansion about a straight cylindrically symmetric θ-pinch field, $B(r)\underset{\sim}{e}_z$. Expansions of this type turn out to be incompatible with toroidal periodicity constraints and no such equilibria exist.

The expansion described here overcomes this difficulty by considering the basic θ-pinch field to be of the form $B(r,\theta)\underset{\sim}{e}_z$; that is, one must consider the leading order system to consist of noncircular flux surfaces with finite toroidal shifts. Under this assumption diffuse high-β stellarator equilibria can be found.

The leading order field is the θ-pinch field just described. The first order fields are a combination of helical fields, each one characterized by $\ell\theta + hz$. In principle there can be an arbitrary number of induced and applied helical fields with different ℓ values, but all with the same pitch number h. In practice, for well confined equilibria, two and only two helical fields (ℓ and $\ell+1$) can be applied. Also, no purely transverse fields are allowed in first order. The helical fields are determined from a potential function ϕ, which satisfies a second order partial differential equation with $B(r,\theta)$ appearing in the coefficients. Toroidal effects are first assumed to enter in second order. To calculate the second order fields, a periodicity condition (resulting from z independent terms) must be satisfied. This condition imposes a second constraint between ϕ and B. Thus, the problem of finding high-β stellarator equilibria can be cast in the form of two nonlinear, coupled partial differential equations for the unknowns ϕ and B.

These equations are given by

$$\nabla_\perp \cdot \left(\frac{\nabla_\perp \phi}{1-\beta}\right) - \frac{h^2 \phi}{1-\beta} = 0 \qquad (1)$$

$$|\nabla_\perp \phi|^2 + h^2 |\phi|^2 = G(\beta) - \frac{B_a^4}{R} (1-\beta)^2 \, r \cos\theta \qquad (2)$$

where B_a is the applied θ-pinch field, $\beta = 1 - B^2(r,\theta)/B_a^2$, R is the major radius of the torus, $G(\beta)$ is an arbitrary free function and the helical field is determined by $\underset{\sim}{B}_\perp = \nabla_\perp \phi / B(r,\theta)$. At $r = 0$, ϕ and β must be regular. At the outer boundary, $r = b[1+\sum \Delta_\ell \cos(\ell\theta+hz)]$ there is a shaped conducting shell on which $\underset{\sim}{n} \cdot \underset{\sim}{B} = 0$.

Equations (1) and (2) have been solved analytically in the small hr limit for two cases: (1) small, but finite β and $\Delta_0 \sim \Delta_1 \sim \Delta_2$ and (2) finite β and Δ_0, $\Delta_2 << \Delta_1$. The result is a diffuse analog of the well known sharp boundary equilibrium relationship corresponding to zero toroidal shift. The consequences of these analytic calculations are discussed in conjunction with the numerical studies in Sec. IB.

Finally, a comparison has been made with the low-β stellarator. In both cases, helical fields are required to provide closed flux surfaces. However, in a high-β stellarator the toroidal drift force is balanced by the interaction of two helical fields; ℓ, $\ell+1$. In a low-β stellarator a single helical field suffices. Here, the toroidal drift force is balanced, not by the helical fields, but by the interaction of a small vertical field with the small induced toroidal dipole current.

B. Numerical Studies

Numerical solutions of the diffuse, high-β stellarator equilibrium problem are also presented, which are obtained by solving finite difference approximations to the nonlinear, time-dependent equations describing ideal, magnetohydrodynamic flow in three dimensions as described in [5,6]. Such solutions have provided answers to practical design questions associated with existing experiments, which arise because, in contrast with tokamaks or even low-β stellarators, equilibria exist only for numerically correct helical field amplitudes.

Equilibria have been computed for diffuse profile plasmas with parameters corresponding to existing experiments. Similarly to the sharp boundary theory, the numerical calculations and the asymptotic theory described in Sec. IA give a diffuse equilibrium relation for ℓ=1,0 which scales as $hRB_{\ell=0}B_{\ell=1}/B_a^2 = g_o(\beta)$, and for ℓ=1,2 as $RB_{\ell=1}B_{\ell=2}/aB_a^2 = g_1(\beta)$, where $\Delta_o = B_o/2B_a$, and $\Delta_{\ell\neq 0} = (B_{\ell\neq 0}/B_a)\ (b/a)^{\ell-1}$, and the B_ℓ are evaluated at the plasma radius, a. For $B_\ell/B_a << 1$, the numerical results with ℓ=1,0 are compared in Fig. 1 with analytic theory. The numerical results were computed for a Gaussian-like profile and the analytic theory for the profiles $\beta = \beta_a/[1+(r/a)^4]^n$, n = 1,2. Both the analytic and numerical results show significant differences from the sharp boundary theory indicating that the equilibrium is sensitive to the profile.

The numerical calculations of diffuse equilibria for ha and B_ℓ/B_a finite, as in the Scyllac and Isar experiments, give similar scaling, and have been compared with experimental results.[7,8] For example, when adjustments to the equilibrium fields of the Scyllac feedback sector were made to compensate for the diffuse profile effects predicted by theory, an increase in confinement time from 25 μs to 45 μs was observed.[7]

An Isar-like configuration[9] with ℓ = 0,1,2 fields has also been studied, because of the potential for reducing the dependence of the equilibrium fields on β by using three helical fields. As a result of appropriately combining them, the different dependence on β of the stellarator force, $\underset{\sim}{F}_{10}$, (produced by the interaction of ℓ = 0 and ℓ = 1 fields) and that of $\underset{\sim}{F}_{12}$, (produced by the ℓ = 1 and ℓ = 2 fields) cancels the net β dependence without cancelling the total $\underset{\sim}{F}_{10} + \underset{\sim}{F}_{12}$ force. However, the interaction of $\ell = 0$ and $\ell = 2$ fields also produces a force, $\underset{\sim}{F}_{02}$, with $\ell = 2$ symmetry and no axial dependence. Numerical calculations for a straight system with parameters such that $\underset{\sim}{F}_{10} + \underset{\sim}{F}_{12} = 0$ indicate that F_{02} results in large, unbounded, and therefore unacceptable deformations of the plasma as shown in Fig. 2. This deformation can be suppressed by applying a relatively large $[\sim (\Delta_0\Delta_2)^{1/2}]$ vertical field.

Finally, wall stabilized equilibria have been calculated with plasma to wall ratio, $b/a = 3$. All diagnostics indicate an approach to stationary equilibrium in contrast to the unstable motion typically observed in cases with $b/a > 4$.

In summary we now have obtained analytic and numerical high-β-stellarator equilibria whose properties and scaling can be related to experiments and earlier theory.

II. MHD Equilibrium and Stability of Minimum-B Mirror Traps (LLL)

In a general high-beta guiding-center MHD equilibrium of an anisotropic minimum-B mirror-trapped plasma, stability is determined by the sign of the Kruskal-Oberman[10] energy variation. For near-marginally-stable flux-localized perturbations, and within a positive factor, the energy variation is[11]

$$\Delta W = \int d\ell \left\{ (h/2\pi B^2)(\nabla\psi + \mu\nabla\alpha)^2 (\underset{\sim}{b}\cdot\nabla\phi)^2 \right.$$

$$\left. + \left[(\mu T/B)\underset{\sim}{\kappa}\cdot\underset{\sim}{b} \times (\nabla\psi + \mu\nabla\alpha) + \Theta \right]\phi^2 \right\} \qquad (3)$$

where $\Phi(\ell)$ is the amplitude of the perturbation whose principal displacements are along $\underline{b} \times (\nabla\psi+\mu\nabla\alpha)$, μ is an arbitrary constant, and (ψ,α) re flux coordinates. The trap is supposed omnigenous,[12] allowing the pressures to be taken to depend only upon B and the principal flux coordinate ψ. For a well-designed and well-injected mirror trap, the quantities $h(\psi,B)$, $\partial h/\partial\psi$, and $T(\psi,B)$ are all positive, where $h = B+4\pi B^{-1}(p_\perp-p_\parallel)$, $T = (-4\pi/B)[(\partial p_\parallel/\partial\psi) + (h/B)(\partial h/\partial B)^{-1}(\partial p_\perp/\partial\psi)]$. Likewise, $F > 0$, where $F = B^{-2}[h^2(\partial h/\partial B)^{-1} + 12\pi p_\parallel - (B^2 + 8\pi p_\perp)^{-1}(B^2-4\pi p_\parallel)^2]$, and $0 < \Theta < (F/2\pi B)[\underset{\sim}{\kappa}\cdot\underset{\sim}{b}\times(\nabla\psi + \mu\nabla\alpha)]^2$, where $\underset{\sim}{b} = B^{-1}\underset{\sim}{B}$ and $\underset{\sim}{\kappa} = \underset{\sim}{b}\cdot\nabla\underset{\sim}{b}$. Hence necessary and sufficient conditions for the stability of flux-localized perturbations are given by the nonexistence of solutions of the Euler-Lagrange equations along a line of force which are obtained by use of the limiting forms of Θ in ΔW. Nevertheless, Eq. (3) may still be difficult to apply.

Under circumstances of considerable interest to the mirror program, it is frequently assumed that the principal radial density gradients can be relegated to a small boundary layer, $(1-\varepsilon)\Psi < \psi < (1+\varepsilon)\Psi$, near the surface of the plasma. When ε is very small, the angular flux coordinate α can exhibit rapid variation from flux surface to flux surface, $\nabla\alpha\cdot\nabla\psi = O(\varepsilon^{-1})$, causing considerable difficulty with the numerical analysis. Thus new analytic procedures have been derived for the solution of equilibrium in terms better suited to the examination of stability when ε is small. Stringent stability boundaries are found.

In small-curvature straight-axis mirror traps, conversion of the usual cylindrical coordinates (r,θ,z) into a mixed coordinate set (ψ,θ,z) permits the introduction of a single function $P(\psi,\theta,z)$, the coordinate potential, which fully defines the finite beta geometry. Thus, if partial derivatives in (ψ,θ,z) coordinates are denoted by subscripts, then

$$B^2 + 8\pi p_\perp = B_v^2 \ , \ \frac{1}{2} r^2 = P_\theta \ , \ \alpha = 2\pi B P_\psi \ , \tag{4}$$

where $B_v(z)$ is the known induction of the vacuum configuration on axis. If $\zeta = (h/B)\underset{\sim}{b}\cdot\nabla\underset{\sim}{b} = (4\pi h/cB^2)\underset{\sim}{j}\cdot\underset{\sim}{b}$, then the equilibrium equation[12] $B\zeta' = -T\underset{\sim}{\kappa}\cdot\underset{\sim}{b}\times\nabla\psi$ may be rewritten

$$(h/\alpha_\theta)\left[R_\theta(R'/2R)_\psi - R_\psi(R'/2R)_\theta - (2R\alpha_z/\alpha_\theta)_\psi\right]$$

$$= -\int_{-\infty}^{z} d\ell\,(T/\alpha_\theta)\{R_\theta(R'/2R)_z - R_z(R'/2R)_\theta - (2R\alpha_z/\alpha_\theta)_z$$

$$+ \left[R(\alpha_z/\alpha_\theta)^2 + (R')^2/4R\right]\}, \tag{5}$$

where $R' = \underset{\sim}{b}\cdot\nabla R = R_z - (\alpha_z/\alpha_0)R_\theta$, and we have used the boundary condition that $\zeta \to 0$ as $z \to -\infty$. The scalar function $c(z)$ measures the quadrupole induction in the interior of the plasma (where the pressures are independent of ψ), $\underset{\sim}{B} = B\underset{\sim}{e}_z - (r/2)(dB/dz)\underset{\sim}{e}_\theta + (r/2)B(dc/dz)(\underset{\sim}{e}_r\cos 2\theta - \underset{\sim}{e}_\theta\sin 2\theta)$. Likewise, $P = (\psi/2\pi B)\arctan(e^c\tan\theta)$ in the interior. When $\beta = 8\pi p_\perp/B_v^2$ is small, or at arbitrary β but near the inner boundary layer surface, Eq. (5) admits an analytic solution,

$$a_\psi - (\tfrac{1}{4})\sin 2\alpha \int_0^z dz_1\ \{(\cosh c - \sinh c\cos 2\alpha)^{-1}$$

$$\int_{z_1}^{\infty} dz_2 (T/B)(X \sinh c - 2Y \cosh c)\Big\}, \qquad (6)$$

where $X = (dc/dz)^2 + B^{-2}(dB/dz)^2 - 2(d/dz)(B^{-1}dB/dz)$ and $Y = B^{-1}(dB/dz)(dc/dz) - d^2c/dz^2$, with X and Y both evaluated on axis.

For large-aspect-ratio (small curvature) configurations, the most unstable perturbations satisfy $\underset{\sim}{\xi} = \underset{\sim}{b} \times \nabla\phi$,[13] and the energy variation becomes[11]

$$\Delta W = (8\pi)^{-1} \int d\psi d\alpha d\ell \ \Big\{(h/2\pi B^2)\big[(BD\phi/D\psi + \alpha_\psi BD\phi/D\alpha)'\nabla\psi + \alpha_\theta(BD\phi/D\alpha)'\nabla\theta\big]^2$$

$$+ M(BD\phi/D\alpha)^2 - 2\tilde{\zeta}(BD\phi/D\alpha)(BD\phi/D\psi + \alpha_\psi BD\phi/D\alpha)'\Big\}, \qquad (7)$$

where $\nabla\phi = (D\phi/D\psi)\nabla\psi + (D\phi/D\alpha)\nabla\alpha + \phi'\underset{\sim}{b}$. The function $\ddot{\zeta}$ vanishes at low β o near the inner boundary layer surface, and in this same regime

$$M = (T\alpha_\theta/B)\ \big[(\alpha_z/\alpha_\theta)^2 + (R'/2R)_z + (R'/2R)^2 + (BR'/2\alpha_\theta)(\alpha_\theta/BR)'\big]$$

$$- \big[(h/B)(R_\theta/2R)\alpha_\psi'\big]' - \pi T\ (R_z R_\theta/R\alpha_\theta)\alpha_\psi'. \qquad (8)$$

The system is flute unstable if $\int M\, d\ell < 0$, unless line tied. In any event ballooning is also possible for perturbations $|BD\phi/D\psi + \alpha_\psi BD\phi/D\alpha| \ll |BD\phi/D\alpha|$, provided solutions exist to the Euler-Lagrange equation

$$\big[(h\alpha_\theta^2/4\pi B^2 R)u'\big]' - Mu = 0, \qquad (9)$$

where $u = -BD\phi/D\alpha$. Thus, the nonexistence of solutions of Eq. (9)

necessary for stability. It is also sufficient, as can be shown by the reduction of an earlier condition due to Newcomb.[13]

When β/ε is large, the last term of Eq. (8), which is $O(\beta^2/\varepsilon^2)$, dominates the other terms, which are $O(\beta/\varepsilon)$. Hence, in wells of baseball symmetry, instability occurs away from the symmetry planes when β/ε exceeds a number of order unity. Thus, unless stabilized by finite orbit effects (which might permit the existence of a boundary layer of thickness comparable to a typical cyclotron orbit diameter), it appears that flat-topped pressure profiles are inconcistent with the achievement of high beta. Instability is associated with the presence of geodesic curvature, $\underset{\sim}{\kappa}\cdot\underset{\sim}{b} \times \nabla\psi \neq 0$. In turn, $\underset{\sim}{j}\cdot\underset{\sim}{b}$ does not vanish everywhere. Drift currents accumulate charge at one point on a line of force which must then flow off at another point.

III. Simulation of Simple and Reversed Field Mirror Machine (NYU)

The problem considered is the buildup of a mirror confined plasma including a transition with field reversal. A strong theoretical indication has been given that field reversal will occur "naturally" in a plasma without injection[14] if the mirror coils are properly programmed. To confirm this requires simulation with a transport code which includes the capability of following changes in topology. We have constructed an axially symmetric, macroscopic code which is a modification of earlier, very efficient transport codes for the Doublet[15] and Divertor[16]. Numerical results in the adiabatic limit (zero resistivity and heat conductivity) are presented here. Results with full transport and energy and mass sources will be presented elsewhere.

The model includes "classical" plasma diffusion and heat flow. We recall[17] that, in a symmetric mirror machine there is no skin effect. The Grad-Hogan formulation [18,19] is followed in order to eliminate the convection velocity and in this way rigorously separate out 1-D transport from 2-D geometry evolution. Proper choice of independent and dependent variables[19], [20] allows the 1-D transport to take the form of a standard diffusion system with second derivatives explicitly shown, allowing numerical stability and accuracy to be determined.

The independent (1-D) variable is taken to be the mass, $M = \int \rho dV$. We write $\dot{\phi}$ for $\partial\phi/\partial M$ and $d\phi/dt$ holding M fixed ($\phi' = \partial\phi/\partial V$). Conservation of mass, flux, and energy (with V as independent variable),

$$\rho_t + (\rho U)' = 0 \qquad , \qquad U = \oint u \cdot dS = \frac{dV}{dt}$$

$$\psi_t + U\psi' = \eta\langle r^2\rangle F \qquad , \qquad F = (K\psi')' \ , \ K = \langle|\nabla V|^2/r^2\rangle \quad (10)$$

$$p_t + Up' + 2pU' = (\lambda_* T')' + \eta\langle r^2\rangle F^2 \quad , \quad \lambda_* = \lambda(T)/\omega^2\tau^2$$

are rewritten in terms of M,

$$\sigma(M) = \dot{\psi} = \psi'/\rho \qquad , \qquad \mu(M) = p/\rho^2$$

$$\frac{d\sigma}{dt} = (\eta\langle r^2\rangle F)^{\cdot} \quad , \quad F = \rho(K\rho\sigma)^{\cdot} \qquad (11)$$

$$\frac{d\mu}{dt} = \frac{1}{\rho}\,[\lambda_{*}\rho(\mu\rho)]^{\cdot\cdot} + \eta\langle r^2\rangle F^2/\rho^2$$

where the auxiliary variable ρ is obtained from the pressure balance $(K\psi')' = -\,dp/d\psi$,

$$\dot{\rho}[K\sigma^2 + 2\mu] + \rho[\sigma(K\sigma)^{\cdot} + \dot{\mu}] = 0$$

$$\rho = \frac{a}{[K\sigma^2+2\mu]^{1/2}}\exp\left\{-\frac{1}{2}\int\frac{\sigma^2 dK}{K\sigma^2+2\mu}\right\}$$

The second derivatives, $\ddot{\rho}$, in (11) are eliminated explicitly in terms of $\ddot{\sigma}$ and $\ddot{\mu}$. The quantities σ and μ diffuse; ρ, etc., are carried along. The eigenvalues of the 2×2 matrix of second derivatives (diffusion coefficients, dimensionally $M^2(t)$) are

$$\lambda = \frac{1}{2}\,(\lambda_o + \rho\lambda_{*}) \pm \frac{1}{2}\,[\lambda_o^2 + \rho^2\lambda_{*}^2 - 2\beta\rho\lambda_{*}\lambda_o]^{1/2}$$

where

$$\beta = \frac{2\mu}{2\mu+K\sigma^2} = \frac{p}{p + \frac{1}{2}\,\langle B^2\rangle}, \qquad 0 < \beta < 1$$

$$\lambda_o = \eta K\langle r^2\rangle\rho^2\beta$$

If the energy equation is replaced by a simple source (as in a "Flux Conserving Tokamak"), $d\mu/dt = Q$, the single, classical diffusion coefficient, $\lambda = \lambda_o$ is obtained. To be precise, no skin effect is present with a plasma-vacuum interface. But a limiter, scraping off plasma, will give a very slowly diffusing skin at the classical diffusion rate λ_o rather than at the normal skin rate λ_o/β.

Figure 3 shows condensed information of an adiabatic run in which the mirror ratio is increased in steps. The current profiles (precisely, $F = \langle J/r \rangle$), originally flat, are deformed as shown. As in a Doublet [15], there is a current peak at the separatrix. The magnetic field profiles on axis are also shown.

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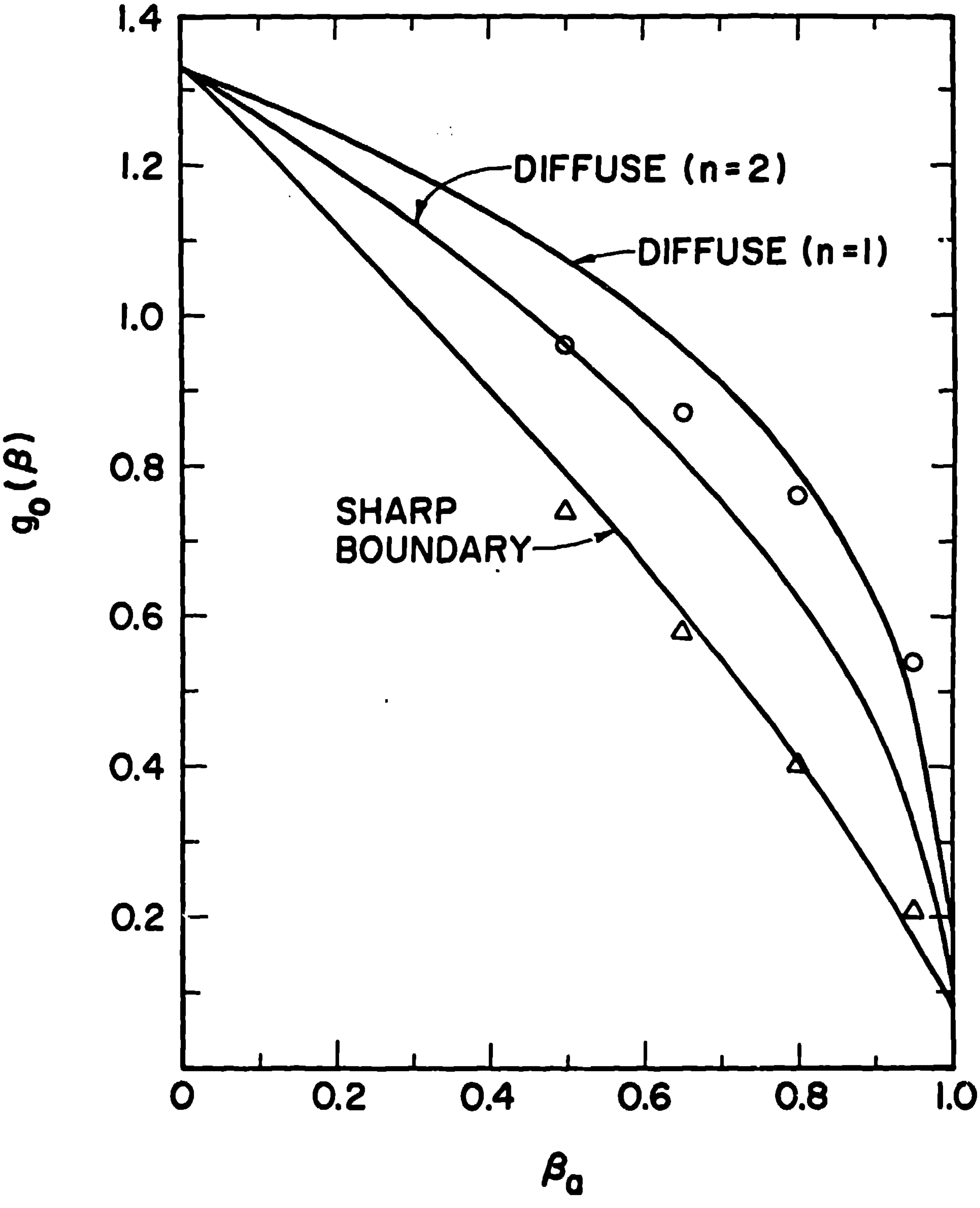

Fig. 1. The results of numerical equilibrium calculations are compared with analytic theory. The triangles and circles denote numerical equilibria with sharp boundary and Gaussian-like plasma profiles respectively. ..

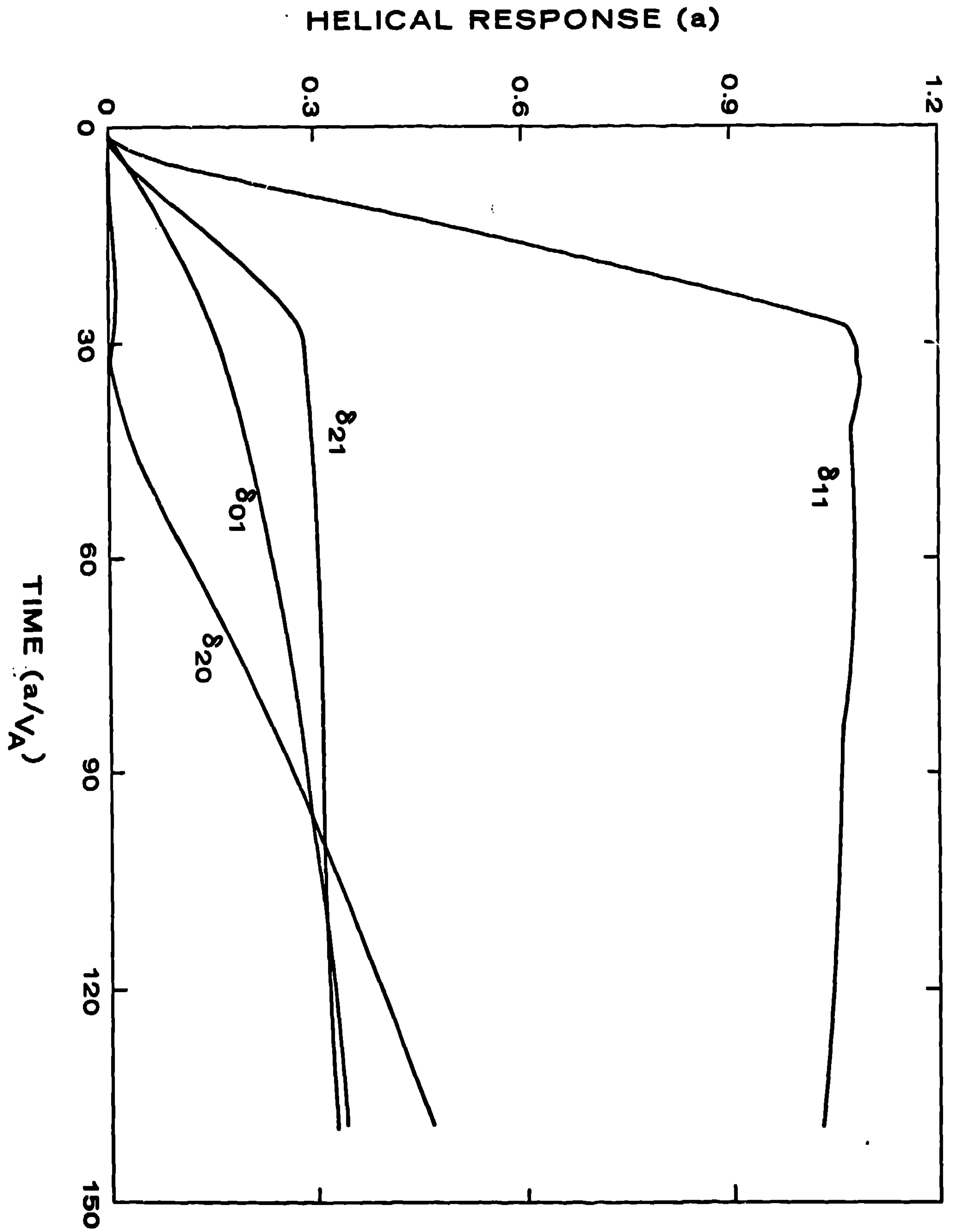

Fig. 2. The amplitude of the plasma responses $\delta_{\ell n}$, where n is the wave number in units h, to the simultaneous application of ℓ=0, ℓ=1, and ℓ=2 fields with n=1. Note that δ_{20}, the elliptical deformation of the column in response to F_{02}, is larger than δ_{21}, the response to the applied ℓ=2 field and still growing. Continued growth will bring the plasma into contact with the wall.