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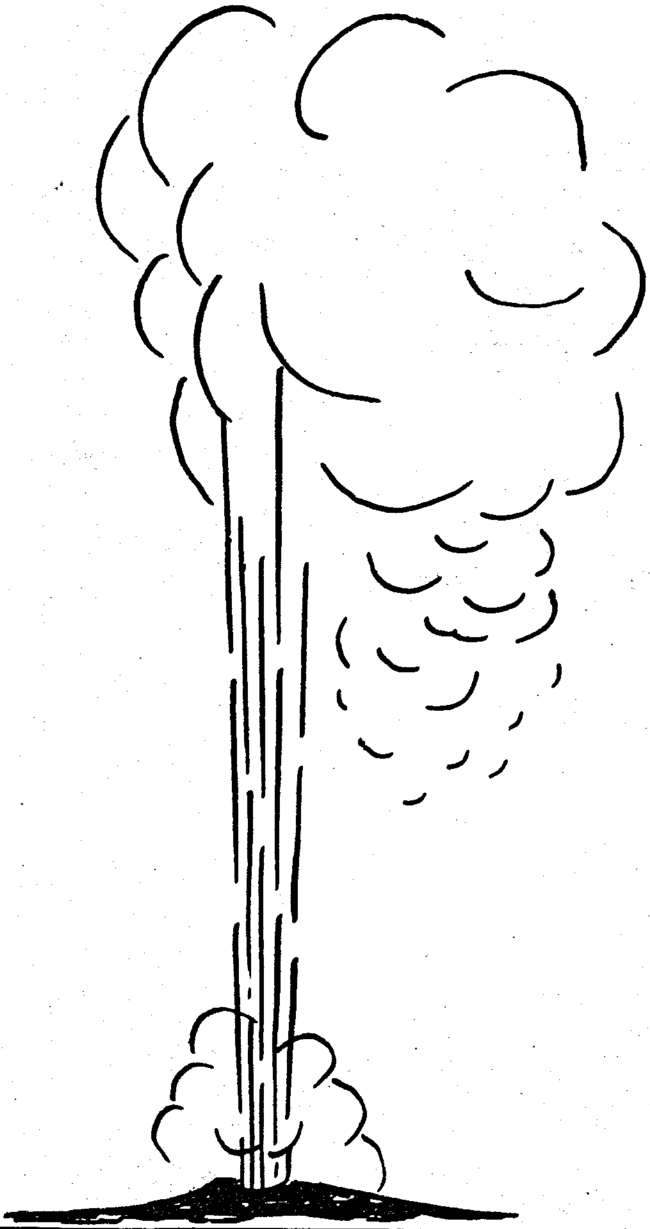

GEOTHERMAL RESERVOIR MANAGEMENT

By
Charles R. Scherer
Kamal Golabi

February 1978

Work Performed Under Contract No. EY-76-S-03-0034

University of California
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ABSTRACT

This study considers the optimal management of a hot water geothermal reservoir. The physical system investigated includes a three-dimensional aquifer from which hot water is pumped and circulated through a heat exchanger. Heat removed from the geothermal fluid is transferred to a building complex or other facility for space heating. After passing through the heat exchanger, the (now cooled) geothermal fluid is reinjected into the aquifer. This cools the reservoir at a rate predicted by an expression relating pumping rate, time, and production hole temperature.

The economic model proposed in the study maximizes discounted value of energy transferred across the heat exchanger minus the discounted cost of wells, equipment, and pumping energy. The real value of energy is assumed to increase at r percent per year. A major decision variable is the production or pumping rate (which is constant over the project life). Other decision variables in this optimization are production timing, reinjection temperature, and the economic life of the reservoir at the selected pumping rate.

Results show that waiting time to production and production life increases as r increases and decreases as the discount rate increases. Production rate decreases as r increases and increases as the discount rate increases. The optimal injection temperature is very close to the temperature of the steam produced on the other side of the heat exchanger, and is virtually independent of r and the discount rate. Sensitivity of the decision variables to geohydrological parameters was also investigated. Initial aquifer temperature and permeability have a major influence on these variables, although aquifer porosity is of less importance.

A penalty was considered for production delay after the lease is granted. Production timing is sensitive to this "incentive" and to the amount of royalty charged, although production rate is not. By manipulating these two incentives the onset of production by a net benefit-maximizing producer can be moved forward or backward in time.

ACKNOWLEDGMENTS

We are pleased to acknowledge the advice and assistance of the many individuals who have contributed to this study. Dr. Chin Fu Tsang of Lawrence Berkeley Laboratory was an active and enthusiastic participant in the gradual development of the project, rendering invaluable help in adapting the hydrothermal model used in this study. Dr. Tsang has contributed scores of hours to this study, for which we are most appreciative. We are grateful to Professor Paul Witherspoon of the University of California, Berkeley, for initially directing us to a specific hydrothermal reservoir model and for his useful suggestions when the project was in its formative stages.

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We are grateful to Professors Paul Kruger and Henry Ramey of Stanford University for providing an opportunity to present our ideas to practicing geothermal engineers at the First (1975), Second (1976), and Third (1977) Stanford Geothermal Workshops. Professors William Brigham of Stanford University and William Yeh of UCLA also contributed several valuable technical suggestions as our work progressed.

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Finally, we thank Cynthia Mothersole and Debra De Luca who patiently typed several versions of this manuscript, and Randy Kaack who edited and prepared the final draft.

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INTRODUCTION

Charles R. Scherer and Kamal Golabi

1.1 MANAGING HOT WATER GEOTHERMAL ENERGY

The plumes of steam issuing from the geothermal steam-electric generators near Geyserville, California only hint at the vast amount of energy stored in hot water geothermal reservoirs throughout the western United States. Most western geothermal energy resides in hot water aquifers from which it can be removed for space heating or electric power generation. Although energy production from this source has been technically feasible for decades, the relative costs and revenues have never favored extensive exploitation until recently. Now, due to the recent increases in the value of energy and the concern over the environmental impacts of fossil and nuclear energy sources, more attention has been focused on geothermal energy. Given that the overall economics have become more favorable, it is appropriate to consider reservoir management plans that maximize the economic value of a particular reservoir.

From a resource economic viewpoint, an optimal production policy will depend on, among other things, the relative costs and value of geothermal energy and other energy sources, both now and in the future. Recognizing that energy value will probably increase with time raises the question of when production of a particular reservoir should begin. If the real value of energy increases substantially over the next decades, then one might consider postponing production for a while. Intuition suggests that a non-zero waiting time may exist that maximizes present worth of net revenues.

Hence, a general approach to this problem would make production rate and timing contingent on pertinent information on alternative sources. A good example of this approach is given in Manne's (1976) <u>Energy Technology</u>

Assessment Model, where he points out that:

> "Each energy source has its own cost parameters and introduction date, but is interdependent with other components of the energy sector." [Manne (1976) p. 379]

But by their large dimensions, such macro analyses tend to divorce careful consideration of the technology of particular energy sources from the plans for their development. Accordingly, since we believe explicit treatment of the pertinent process technology is important in developing meaningful resource management models, in place of a macro model we substitute the increasing value-in-use over time of the energy produced and compare this with the costs of producing it.

The heat energy in the aquifers is derived from magmatic intrusions into the earth's crust, to which heat is conducted from the interior of the earth.* Although the heat source will be effectively infinite for the next few decades, the rate of heat transfer from the magma across the aquicludes to the aquifer matrix and fluid is governed by the rate of thermal conductivity of the aquicludes, which is relatively low, so the energy of the geothermal field is also effectively non-renewable in the future of economic relevance (given a positive discount rate).*

Although hot water geothermal reservoirs may provide energy for either electric power or non-electric steam generation, we are concerned only with the latter application in this study. Hot brine (water) is brought to the earth's surface (by its own pressure or pumped) and the heat energy is removed from the water either by a heat exchanger or by direct expansion through a

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* Although a thermal gradient exists everywhere from the center to the surface of the earth, a geothermal field, also referred to as an anomaly, may be detected (actually defined) by an unusually steep local thermal gradient near the earth's surface. By local, we mean a kilometer or two in all (horizontal) directions from the point where the gradient is greatest.

turbine. The spent brine is then dumped to waste, as in Wairakei, New Zealand, or it may be reinjected into the aquifer some distance from the production hole. If wasted, the chemical content of these brines can cause substantial environmental damage. Furthermore, since continuous pumping of the water at a rate faster than the natural recharge rate can cause land subsidence, it appears that reinjection will be required for all hot water geothermal development in the U. S.*

Reinjecting cooled brine into the aquifer will cool the aquifer. As the temperature drops, the quality of the remaining heat declines, and so does its energy value. Since the rate of cooling is directly proportional to the rate of heat extraction, intuitively it seems that there might be some economically "optimal" starting time and energy extraction rate for such a reservoir. There will also be a best lifetime and a best reinjection temperature associated with this optimum pumping rate, and these design parameters will be sensitive to several cost inputs, including royalties. In particular, it is possible that the "best" extraction policy may not be to start at once and produce (extract) the energy as fast as current technology will permit, even if the reservoir is not jointly owned.** However, for discount rates greater than the rate of increase in energy value, the present worth of deferred value rapidly diminishes as extraction is postponed.

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* Good examples of potential damage from subsidence are found in the California Imperial Valley where substantial subsidence could seriously disturb the vertical alignment of the irrigation canals, and in the Wairakei fields in New Zealand [Atherton et al. (1976), Stilwell and Hall (1975)].

** The problems of mutual exploitation of a jointly owned reservoir are well known among natural resource engineers, economists, and lawyers. Since water is a "migratory" resource (unlike, say, coal), it is possible for one joint owner to legally exhaust a resource held in common by several surface property owners. In the case of geothermal energy production, the problem is that one owner's extraction of the energy will cool the reservoir (even if all extracted fluid is reinjected), reducing temperature – and hence value – for all. This may be a potential problem on the horizon for geothermal energy use. Although we do not investigate joint management strategies in this report, the problem and its complications are discussed briefly as future work.

1.2 MOTIVATION FOR THIS STUDY

Since the first generation of studies has determined that hot water geothermal energy is "economically feasible," it now seems appropriate to consider such factors as the best production rate, the length of time production should continue, the best reinjection temperature, and the best time to begin production of a particular reservoir in order to obtain the maximum value from the resource. These factors are interrelated and can be investigated with the general type of analytical model demonstrated in this report. Hence the purpose of this work, as originally conceived by Scherer (1975), is to formulate an evaluative, computationally-oriented optimization framework that is of operational value to its users.

Anticipated users of this work can be divided into two groups. First, members of the private energy resource development sector may be interested in answering the basic questions raised above with a view toward profit-maximizing management. Second, agencies of state and federal governments charged with prudent management of geothermal energy resources on public lands are also concerned with these same basic questions. Geothermal resources under lands which were originally in the public domain constitute a significant fraction of the total known geothermal reserves. It is the responsibility of public resource management agencies such as the U. S. Geological Survey, U. S. Bureau of Land Management, and Department of Energy to determine which reservoirs (or public lands) shall be produced, when they shall be produced, and how fast. They are also in charge of determining – within the limits prescribed by law – what the royalties shall be on energy extracted. Since it has been held in recent court decisions (San Francisco Chronicle, November 4, 6, 1977 and Fogarty, 1977) that the state and federal governments retain rights to minerals and steam (presumably hot water as well), even though the surface use rights were deeded to private parties under the Homestead Act of 1916, the potentially substantial royalties from geothermal energy production on these lands will now be transferred to the federal and state treasuries. This will most likely heighten the interest of state and federal regulators in determining "appropriate" royalties. Moreover, they may wish to employ certain incentives to accelerate or postpone onset of extraction in light of other national energy objectives, and the evaluative methodology demonstrated in this report may therefore be useful in illustrating the potential of government-administered incentives to direct geothermal energy development. Hence the analytical methods that are of use to the private sector may be of equal value to government regulators in evaluating and managing public geothermal resources. Accordingly, the primary motivation of this research, supported by public resources, has been to contribute to the development of analytical methods which will advance the state of the art of geothermal resource evaluation and management in both the private and public sectors.

We have also been concerned with the demonstration of a conceptual approach, as well as development of operational analytics. During the last three decades — and especially more recently — there has been a great amount of conceptual work on the theory of socially optimal natural resource depletion. This literature includes work by Cummings and Burt (1969), Gordon (1967), Heal (1976), Hotelling (1938), Pearce and Rose (1975), Schulze (1974), Scott (1967), and Smith (1968), and has been summarized by Peterson and Fisher (1976), and in the proceedings of a symposium on exhaustible resources in the Review of Economic Studies (1974). However, as Peterson and Fisher point out:

> "In their current state, these models are excellent vehicles for teaching concepts and techniques of dynamic optimization, especially in the presence of externalities. Unfortunately, they cannot be used to manage actual natural resources, because their functional forms are too simple and their empirical content too low." [Peterson and Fisher (1976) p. 17]

Hence, a second motivation for this work is the introduction of more physical and empirical "content" into the theoretical optimal extraction literature in order to provide a link between theory and application. We have chosen non-electrical hot water geothermal energy production as the vehicle for this demonstration.

With the introduction of new technologies such as in hot water geothermal energy production, the need for technical and "engineering" data is often at least as great as the need for management investigation. A question then arises as to which technical data are most important and as such should be the objects of additional funded research. However, prior to management model investigations such as this one, it is not always clear how data needs rank in order of importance in determining how and when to produce a particular reservoir. Therefore, a third motivation of this research has been to demonstrate how the production model developed herein can be used to determine the information to which the planning and design process is most sensitive.

1.3 OBJECTIVES OF THIS STUDY

We now turn to a more specific statement of the objectives of this research. First we develop an extraction model that assumes production begins immediately (or never), and then we address the following questions:

a. At what rate and for what duration of time should a geothermal reservoir be produced, and to what degree should the brine be cooled before reinjection into the aquifer in order to maximize the present worth of profits?

b. To what extent are these decisions dependent on the economic parameters that influence value over time (interest rates and rates of growth in energy value)? In particular, how are these decisions affected by variations (uncertainty) in these parameters?

c. What is the economic worth of a reservoir, how is it assessed, and in what manner and to what extent is this value dependent on physical and economic parameters (such as initial temperature, permeability, growth rate in the value of energy, market interest rate, royalty and land rent, equipment and operational costs, and costs of wells and their expected lives)?

d. To what extent can regulatory agencies influence the rate of geothermal energy production by manipulating factors like royalties, lease terms, and land rents?

e. Which are the critical geohydrological parameters in the engineering design of the geothermal facility? How beneficial would it be to obtain additional information regarding these parameters before the extraction facilities are designed?

We then relax the requirement that production begin at once. Instead we investigate the relationship between waiting time until start of production and the other design parameters already identified. Specifically, the following questions are addressed:

a. At what time should production start, how fast, and how long should a hot water geothermal reservoir be exploited in order to maximize the net present worth of the resource?

b. Given that the entrepreneur can postpone production, what is the present worth of the associated profits, and in what manner and to what extent is this value dependent on parameters such as rate of growth of the value of energy, market interest rate, royalty, land rent, and penalties imposed by the government for delaying extraction?

c. To what extent can regulatory agencies influence the timing and rate of geothermal energy by manipulating incentives, such as royalty, lease terms, land rents, and penalties for delays in extraction?

1.4 OUTLINE OF REPORT

With these objectives in mind, we present some background information in the next chapter on hot water geothermal systems and describe the physical relationship between extraction rate and temperature over time for the reinjection case. Next, we review some fundamental principles of resource allocation and then present our economic model for selecting optimal steady-state production rate, reinjection temperature, and economic life of the reservoir when the extracted energy is used for non-electrical steam production. We then analyze the relationship between the cost of each component of the production and surface equipment and our decision variables. Using these costs, and data for a typical aquifer, we present the results of our optimization and attempt to answer the questions discussed above. Chapter 2 concludes with a discussion of these results.

In Chapter 3 we consider the best production program for a hot water geothermal reservoir with emphasis on the optimal time to commence production. Using production functions relating production rate to the quality of produced energy and functions describing the extraction cost of geothermal energy, we present an operational model that gives the best time to begin production, the optimal pumping rate, and the best planning horizon. We investigate the effect of economic parameters and incentives on profits, extraction rate and timing, and study the extent to which regulatory agencies can influence the timing and rate of exploitation by manipulating economic incentives. In Chapter 4 we discuss these models critically and suggest directions for further research. In the last chapter we summarize salient conclusions.

Chapter 2

OPTIMAL PRODUCTION OF GEOTHERMAL ENERGY

Kamal Golabi and Charles R. Scherer

2.1 INTRODUCTION

In this chapter we develop the basic economic model for optimal production of hot water geothermal energy for non-electric steam generators, assuming production commences immediately (or not at all). A central assumption and feature of our model is that cooled or "spent" brine is reinjected back into the aquifer, causing the aquifer to cool over time in proportion to pumping rate. To represent this physical phenomenon, we have used the hydrothermal expression of Gringarten and Sauty (1975). This relationship between heat energy extraction rate and production well temperature over time was formulated for a production-reinjection geothermal well doublet with homogeneous aquifer. Our optimization model can be modified to accommodate other hydrothermal models. However, the assumption of reinjection is essential.

As the work on this project progressed, it became necessary to report the results on the basic model at an early date. These are contained in a preliminary technical report by Golabi and Scherer (1977). This report has been revised slightly and appears here as the remainder of this chapter.

2.2 THE HOT WATER GEOTHERMAL SYSTEM

By hot water geothermal system we refer to a homogeneous saturated aquifer bounded top and bottom by impermeable aquicludes (see Figure 2.1). The water in the aquifer (before any pumping) is in thermal equilibrium with both aquicludes and with the aquifer matrix. The aquifer is horizontally unbounded in all directions; i.e., it is horizontally infinite.

The hydraulic operation of a hot water geothermal system may be described as follows. Water is pumped from the production well (at [1] in

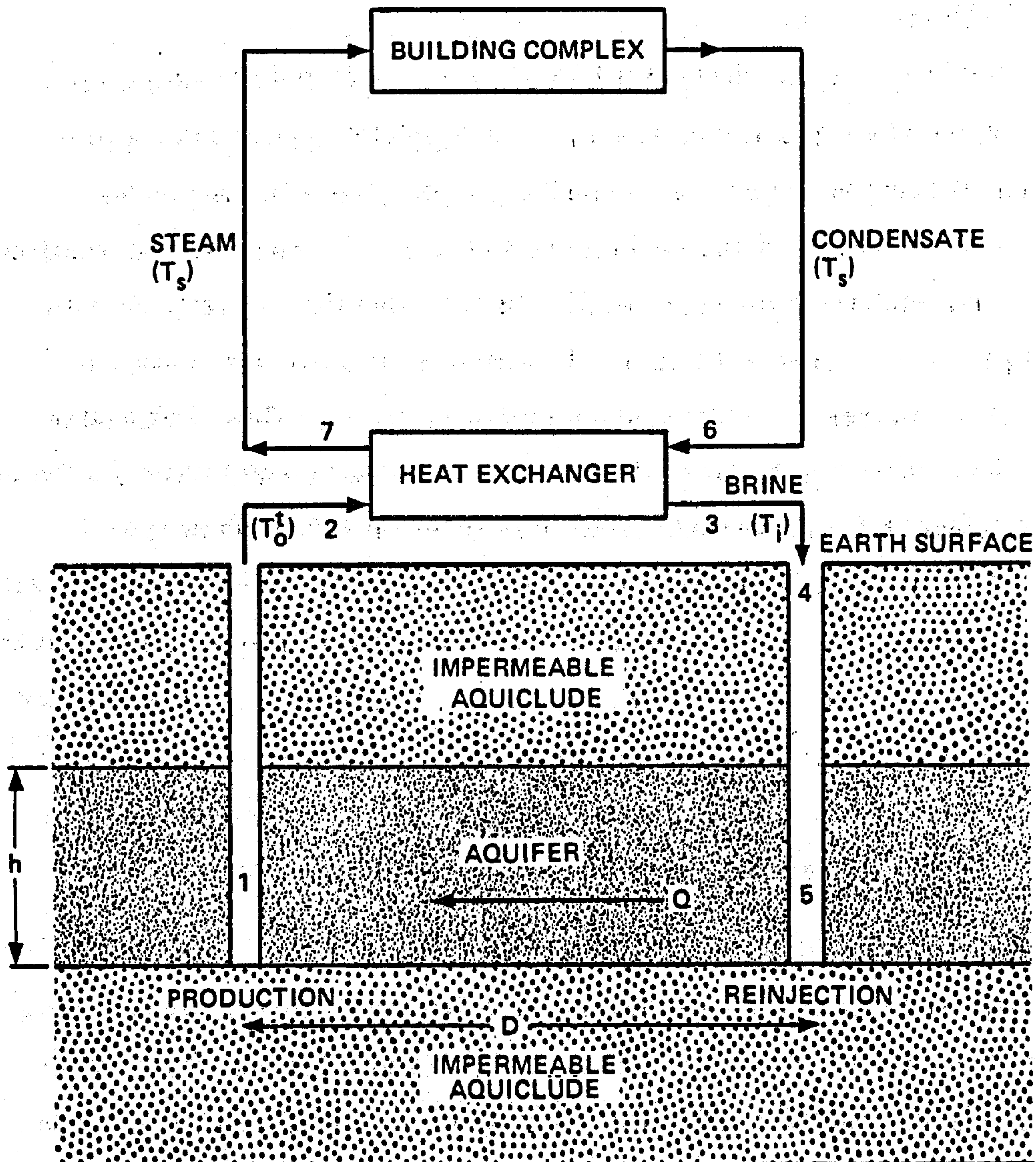

FIGURE 2.1 THE SYSTEM

Figure 2.1) up to the surface where it enters the heat exchanger [2]. After it leaves the heat exchanger [3], it is piped to the reinjection well [4] and pumped back into the aquifer [5]. It then moves through the aquifer toward point [1]. The flow is turbulent from point [1] to point [5], and laminar from [5] to [1]. Initially, when pumping begins, the entire aquifer is in equilibrium at temperature T_o. Water is pumped out at that temperature and cooled to T_i as the energy is extracted. After the water is pumped back into the aquifer, it is heated by the aquifer matrix as it moves from [5] to [1] along an infinite number of laminar streamlines, the shortest of which is a straight line between the injection well and the production well. For some period the water will be heated back to T_o by the time it arrives at point [1].

As heat is transferred from the aquifer matrix to the fluid, the temperature of the matrix decreases. It follows that a point in time, τ, will come when the matrix can no longer heat the fluid to T_o by the time the fluid reaches point [1]. When this happens, the production well temperature at [1] (and hence at [2]) will begin to drop. If we denote the time-variable production well temperature as T_o^t, this process of temperature degradation over time can be plotted as shown in Figure 2.2. The point in time, τ, when temperature begins to decline below T_o is called "breakthrough," referring to the time when the reduced fluid temperature breaks through to the production well. Breakthrough is inversely proportional to Q, the water flow rate, as we shall see shortly. The post "breakthrough" rate of decrease in temperature also depends on Q and reinjection temperature, T_i. The relationship between production well temperature and time can be specified for a given flow rate using the work of Gringarten and Sauty (1975). In their hydrothermal model, brine is withdrawn at the rate Q and reinjected at the same rate. The temperature of the

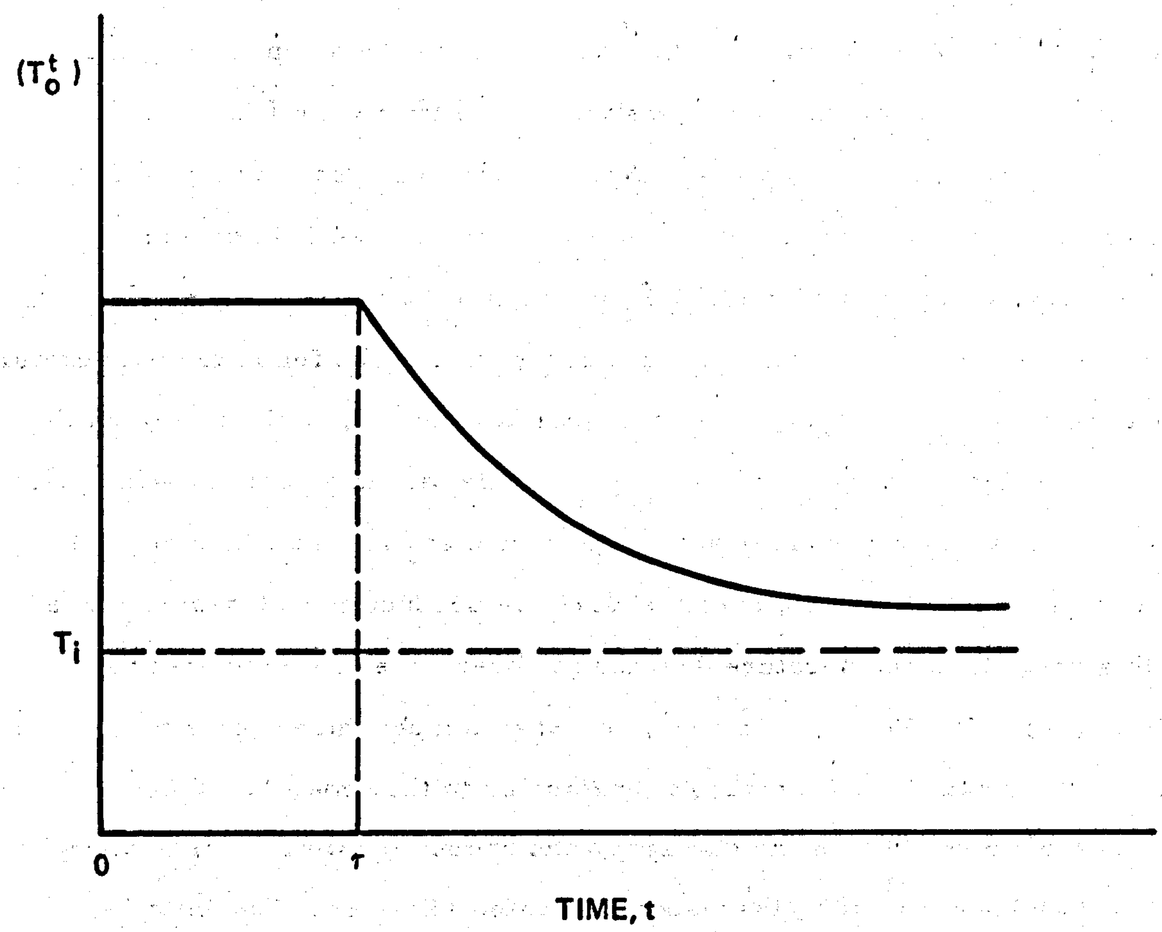

FIGURE 2. 2 THE TEMPERATURE vs. TIME PLOT FOR A GIVEN FLOW RATE

reinjected fluid at time t is denoted by T_i^t. For the first τ years $(0 \geq t \leq \tau)$, $T_o^t = T_o^o = T_o$ is the initial equilibrium temperature of the unexploited reservoir and τ denotes the breakthrough time. The breakthrough time is inversely proportional to Q and is described by the following relationship [see Tsang *et al.* (1976)]:

$$\tau(Q) = t_u/6 , \tag{2.1}$$

where t_u is a unit for time,

$$t_u = \frac{2\pi h D^2 \rho_a c_a}{8760 Q \rho_f c_f} , \tag{2.2}$$

and h is the aquifer thickness (m), D the well separation (m), Q the pumping rate (m^3/hr), and ρ_a, ρ_f, c_a, c_f the densities and specific heats of the aquifer matrix and the fluid, respectively. The relationship between the heat capacities of the aquifer matrix, rock structure, and the fluid is given by,

$$\rho_a c_a = \phi c_f \rho_f + (1-\phi) \rho_R c_R \tag{2.3}$$

where ϕ is the porosity of the aquifer.

For the purpose of this analysis, we neglect the temperature drop in surface pipes so that the temperature of the fluid entering the heat exchanger is T_o^t and the injection temperature, T_i^t, equals the temperature of the fluid leaving the heat exchanger. The temperature after breakthrough is determined by a function $\bar{g}\,(T_i^t,\ t/t_u)$ which gives the ratio of the temperature drop through the heat exchanger experienced by the brine at time t, to that at time zero:

$$\frac{T_o^t - T_i^t}{T_o - T_i^t} = \bar{g}\,(T_i^t,\ t/t_u) . \tag{2.4}$$

We will later show that the variation in T_i^t is small and hence $\bar{g}$ can be approximated by a function g which is valid for invariant T_i.

2.3 THE NON-ELECTRICAL STEAM CONSUMER

We assume the energy extracted from the geothermal reservoir will be used to generate the low pressure steam for process heat or institutional space heating (e.g. a hospital or industrial complex). Although the hot water could be "flashed" directly to steam, the chemical composition of this fluid is such that corrosion or scaling is anticipated. Since control of this problem for a large institutional steam system would be far more costly and complicated than for a heat exchanger, the latter is preferred for this application. In our model steam condensate from a building complex steam-heating system enters the heat exchanger at the saturation temperature of steam (at the desired pressure) T_s (at [6] in Figure 2.1), is heated and leaves the heat exchanger as steam at T_s [7], is circulated throughout the building complex losing heat to the building in the process of phase change, and returns as condensate to the heat exchanger [6].

Aside from piping heat losses, there are only two ways in which heat can leave the doublet system (includes reservoir as well as surface equipment): by transfer to the steam cycle, and by heat loss from the heat exchanger. A realistic model should incorporate this heat loss, and we shall return to this detail in section 2.5.6. For now we neglect heat losses in the heat exchanger and assume all the heat removed from the brine at any given time is used to generate steam.

The effectiveness, ε, of a heat exchanger is defined in terms of the hot and cold brine temperatures, T_o^t and T_i, and the temperature of the cold side of the exchanger, T_s. The maximum transferable heat (given an infinite exchange area) is $Q c_f \rho_f (T_o^t - T_s)$. However, since heat transfer becomes very expensive as T_i approaches T_s, the heat actually transferred is usually less than the maximum defined above. Accordingly ε is defined as the heat actually transferred divided by the maximum amount of heat that could be transferred given an infinite transfer surface area. [Edwards et al. (1973) p. 253]:

$$\varepsilon = \frac{Q c_f \rho_f (T_o^t - T_i)}{Q c_f \rho_f (T_o^t - T_s)} = \frac{T_o^t - T_i^t}{T_o^t - T_s} \quad . \tag{2.5}$$

For a given Q, eqn. 2.5 implies that ε increases to unity as $T_i \rightarrow T_s$. In addition, the effectiveness of a heat exchanger is a function of the number of transfer units NTU(t) at time t, [Edwards et al. (1973) p. 243],

$$\varepsilon = 1 - e^{-NTU(t)} , \tag{2.6}$$

where

$$NTU(t) = \frac{k(t)A}{Q} , \tag{2.7}$$

and

$$k(t) = 0.00488\ U(t)/c_f \rho_f \quad .$$

U(t) is the overall heat transfer coefficient at time t, (BTU/hr-ft^2 - $^\circ$F), $c_f \rho_f$ is the heat capacity (cal/cc $^\circ$C) and A is the heat exchanger area (m^2). The units of k are m/hr making NTU in eqn. 2.7 dimensionless.*

We are now ready to discuss the variation in T_i^t with time. Combining eqns. 2.5 and 2.6 yields

$$\frac{T_o^t - T_i^t}{T_i^t - T_s} = \frac{1 - e^{-NTU(t)}}{e^{-NTU(t)}} = e^{NTU(t)} - 1 \, . \tag{2.8}$$

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* We have generally preferred to use metric units. However, for cases where data are commonly available in British units, we have used this system of units and included the appropriate conversion factors in the equations as coefficients.

From eqn. 2.4 we have $T_o^t - T_i^t = (T_o - T_i^t)\bar{g}$. Substituting this in eqn. 2.8 gives

$$\frac{T_o - T_i^t}{T_i^t - T_s} = \frac{e^{NTU(t)} - 1}{\bar{g}}$$

which yields

$$\frac{T_o - T_i^t}{T_o - T_s} = \frac{e^{NTU(t)} - 1}{\bar{g} + e^{NTU(t)} - 1} \quad . \tag{2.9}$$

Since $\bar{g}$ is a monotone non-increasing function of t with the range of [0, 1], eqn. 2.9 yields the range of T_i^t. For $t \leq \tau$, $\bar{g} = 1$ and

$$T_i^t = T_s + (T_o - T_s)\, e^{-NTU(0)} \quad . \tag{2.10}$$

As $t \to \infty$, $\bar{g} \to 0$ and $T_i^t \to T_s$. The variation of T_i^t is therefore small and $\bar{g}$ can be approximated by a function g which assumes T_i does not vary with time. Using the results of the Gringarten-Sauty model, an expression for g has been developed [Tsang <u>et al</u>. (1976)] and is given by,

$$g(t/t_u) = \begin{cases} 1 & \text{if } t \leq \tau \\ \gamma_1 e^{-\phi_1 t/t_u} + \gamma_2 e^{-\phi_2 t/t_u} + \gamma_3 e^{-\phi_3 t/t_u} & \text{if } t \geq \tau \end{cases} \tag{2.11}$$

where $\phi_1 = 0.0138$, $\phi_2 = 0.656$, $\phi_3 = 8.006$, $\gamma_1 = 0.338$, $\gamma_2 = 0.337$, and $\gamma_3 = 1.368$.

In the remainder of this report we will assume a reinjection temperature T_i that is constant with time. Eqn. 2.4 can therefore be written as

$$\frac{T_o^t - T_i}{T_o - T_i} = g(t/t_u) \quad . \tag{2.12}$$

However, although T_i may be assumed constant with time, its value obviously affects heat removed per unit of time (for a given Q) and hence discounted net revenues. That is, lower values of T_i yield greater heat removals per time but cause the field to cool more rapidly. Furthermore, to achieve lower values of T_i (for a given T_s), larger and hence more expensive heat exchangers are required. To see this, note from eqns. 2.5, 2.6, and 2.7 that for a given Q,

$$\frac{T_o - T_i}{T_o - T_s} = 1 - e^{-k(0)A/Q} \tag{2.13}$$

yielding

$$A = \frac{Q}{k(0)} [\ln (T_o - T_s) - \ln (T_i - T_s)] \quad , \tag{2.14}$$

implying that $A \rightarrow \infty$ as $T_i \rightarrow T_s$.

2.4 ECONOMIC MODEL

2.4.1 Costs

We begin this section with the definition of the costs and benefits associated with extraction of geothermal heat energy. By "cost," we shall mean the "opportunity cost" to society of resources (steel, concrete, pumps, well drilling services, etc.) used in extracting geothermal energy, resources that could have been put to some other alternative use. The amount that some other party would be willing to pay in order to procure the services of these "social resources" will be called their "opportunity cost." We will further assume there are no primary "externalities" associated with this energy extraction. This is a good assumption, because spent brines will be re-injected, preventing subsidence and escape to the atmosphere of noxious gases. Secondary impacts, such as population influx to build and operate the geothermal system are assumed small and are neglected.

2.4.2 Benefits

Turning to the value or benefits of the extracted energy, there are at least two ways to proceed. The first is to assume demand for the energy is price sensitive, using the area under the demand curve as an index of "willingness to pay" and hence social benefit or value [Hotelling (1938)]. This is the appropriate approach if demand, as perceived by the energy producer, is at least somewhat price-elastic. Alternatively, if there are other sources of energy (including imports), then we define benefits as the cost to the customer of the next least expensive alternative energy source to geothermal, reasoning that he will be this much better off if he uses geothermal energy in lieu of this next best alternative. For example, a customer using geothermal energy to generate steam for space heating has the option of generating steam with an oil or coal-fired boiler, each of which also has some social opportunity cost. The lowest of these costs is therefore taken as the "price" or value of the geothermal energy. The geothermal energy consumer is willing to pay up to that amount in order to buy the geothermal energy. Of course in a purely competitive energy market, this least expensive price is the only market price, and the geothermal energy producing company is simply a price-taker attempting to maximize profits. Since there are alternative energy sources for space heating, we shall consider the optimal extraction of geothermal energy using the latter benefit measure (as opposed to price-sensitive demand).

2.4.3 Discount Rate

The essential factor in the theory of optimal extraction is time. Indeed, the major question is "how much now and how much later ?" In order to structure a framework in which to examine this question, we need to explicitly state how time affects the costs and benefits of the extraction process, and how time affects our perception of these costs and benefits. We now briefly discuss both of these topics, beginning with the latter.

Assuming we can establish some time series of costs and benefits associated with the extraction process, we can proceed toward answering the "how much now, how much later" question by first determining the relative values (as seen from the present) of a dollar now and a dollar later. It is generally accepted that a dollar today is worth more today than a dollar a year from now (we shall temporarily disregard the impact of inflation for now, and subsequently show that it need not be considered at all for the purposes of this report). In this sense, we tend to "discount" future value. Specifying the exact weighting between "now" and "later" is a matter of subjective judgment or preference of the individual. It is rooted in the individual's attitude toward the present and the future, and revealed in such actions as saving vs. spending (consuming). Identifying the rate at which the individual discounts future dollars is relatively straight forward.

Similarly, it is relatively easy for the firm to establish its discount rate as the highest possible rate of return on alternative investment. If the present worth of the revenues minus the costs of some activity is positive at this discount rate, then the firm should undertake this activity, in lieu of the next best alternative.

But things are not quite so simple when public resources are being expropriated and "used," as would be the case in the present study if the geothermal resources were in the public domain. Many authors have written extensively on the problem of specifying the correct weighting of dollars now vs. dollars later for public sector resource allocation. While this research has generated several useful insights into this problem, many of them seem to be contradictory, and the serious student of this problem is bound to be somewhat perplexed in his attempt to discover the "most sensible" approach to the correct social discount rate for public sector resource allocation.

We can approach the problem from the perspective of traditional economic theorists. Drawing on basic welfare theorems (and ultimately a whole philosophy on political economy), economists recognize that the "private market" does not always produce the "right" amount of all goods and services. Certain goods and services are best produced by the public sector (usually governments). The question then is how much of each resource should be diverted from the private sector to produce these "public goods." The concern here is that the social value of goods that could have been produced privately with these diverted resources would exceed the social value of the goods produced in the public sector. To prevent this, some economists argue that the social discount rate should be set equal to the private market rate of interest. Only if a project has a positive net present worth at this rate should it be undertaken in the public sector.

However, there are several problems with this simple rule. Beginning with the more mechanical, we note that there is no single market rate of interest. Indeed, the best we can find here is an arbitrary composite of rates of return on various forms of investment and debt, with differing risk and differing maturity (in time). Secondly, it can be shown that different perceptions of risk (Smith's vs. Jones' attitude toward risk, public vs. private attitude toward risk) influence attitudes toward the future. Thirdly, and perhaps of most importance to this study is the problem of inter-generational equity; if resources are limited, then it is always the present generation that determines how much they shall use and hence how much shall remain for future generations. And this is a subjective matter involving a trade-off, at a collective level, of altruistic vs. hedonistic attitudes. Even if a social discount rate could be estimated based on this consideration, there is no reason to believe it would conform to the rates estimated using the other approaches.

From the above discussion, it is clear that the problem of the "correct" discount rate is a difficult one indeed. Since we have not undertaken to solve this conundrum in this report, our only recourse is to incorporate the time value of money and resources into our study on a parametric basis. We shall consider a range of discount rates, presenting results for several values within this range.

2.4.4 Effect of Time on Costs and Benefits

We now consider the effect of time on the costs and benefits of the geothermal energy extraction problem, a subject far less frustrating and complicated than the question of perceptions of these costs and benefits. As Hanke, Carver, and Bugg (1975) point out, it is appropriate to disregard inflation in a dynamic analysis, if real (as opposed to inflated) costs and benefits are used, and the discount rate is not compensated for inflation. Conversely, if one of these is inflation compensated, then they must all be. Accordingly, we will use "real" benefits, costs, and discount rates. However, since we must now go further and deeper for each BTU of energy consumed, we shall formulate our model for the general case where the real value of energy is allowed to increase with time.*

---

* We are effectively assuming away the "technological fix" here. This is probably rather conservative. As the economic rent on remaining "in place" resources increases, potential returns to speculative capital investment in research and development market grow large, and substantial attempts at technical innovation are made. In this way, technical innovation is shown to be consistent with, and a natural outcome of, the model of a purely competitive economy [see Barnett and Morse (1963)]. Whether such a market exists and functions in the public interest is, of course, another matter.

We will use two kinds of relationships for the increase in real value of energy:

$$\text{a.} \quad P_t = P_o e^{rt} , \qquad\qquad (2.15)$$

$$\text{b.} \quad P_t = P_o(1 + rt)$$

where

P_t = price (value) of energy at some time, t,

P_o = price (value) of energy at time t = 0,

r = rate of increase of real energy price per year.

2.4.5 Economic Problem of Resource Extraction

We can now state the general economic problem of geothermal energy extraction. The question of when and how much energy to extract from a geothermal aquifer depends on the relative benefits and costs of the energy now and into the future. On one hand, the real value of the energy increases with time as outlined above. This suggests that extraction should be postponed to a time when the net social value (benefits minus costs) is greater. On the other hand, pumping energy costs increase at the same rate, and a positive discount rate discounts these greater future values, so the rate of increase in value of energy and the discount rate work against each other in determining when and how much energy to extract. Furthermore, the temperature-time profile for a particular pumping rate, Q, implies a significant trade-off between energy obtained now and later. If energy is extracted rapidly at first, the temperature will decrease rapidly, seriously diminishing the quality of the heat in the future. Moreover, for a given pumping rate, more heat can be extracted by lowering the reinjection temperature.* However, for achieving lower reinjection temperatures we require larger and hence more costly heat exchangers.

* Note that by eqn 2.12 $T_o^t = T_o g + T_i(1 - g)$. This implies that T_o^t decreases as T_i decreases. However, $T_o^t - T_i$, which determines the amount of heat recovered, increases as T_i decreases.

2.4.6 Production Model With Exponentially-Growing Energy Value (Exponential Growth Model)

In this section, we structure the production model assuming the value of energy increases exponentially with time, and that the price (value) of energy at time zero can be computed, based on the cost of alternative sources of energy. We shall refer to this version of the production model as the "Exponential Growth Model." We assume the producer is a price taker who can sell all extracted energy at a price just under the cost of the next most expensive steam alternative. From eqn. 2.15 we have

$$P_t = P_o e^{rt} ,$$

where P_t is the price (value) of energy at time t, $t > 0$, and r is the continuous annual rate of increase of real energy price with time. At the end of this section, we present a method for the computation of P_o.

The amount of heat removed from the reservoir per unit time is the product of the flow rate, heat capacity of the fluid and the temperature drop through the heat exchanger experienced by the hot brine. For the first τ years, this temperature drop is $T_o - T_i$. From that time until the termination of the project at time L, the temperature drop is governed by eqn. 2.12. Since a certain amount of heat is lost in the heat exchange, we limit the approach of the brine temperature at the heat exchanger inlet to δ °C of the brine outlet temperature ($\delta \geq 0$). This restricts the "optimal life" of the project to L_δ, where L_δ is such that

$$T_o^{L_\delta} - T_i = \delta \qquad (2.16)$$

Note that by eqn. 2.12, eqn. 2.16 yields

$$g(L_\delta/t_u) = \frac{\delta}{T_o - T_i}, \tag{2.17}$$

which implies that for given Q and T_i, L_δ is a decreasing function of δ. Our optimization problem is therefore

$$\underset{Q,\ T_i,\ L}{\text{Maximize}} \quad \pi = (1-\eta)\int_0^{\tau(Q)} 34.76\, P_o\, e^{rt}\, Q\, c_f\, \rho_f (T_o - T_i)\, e^{-it}\, dt$$

$$+ (1-\eta)\int_{\tau(Q)}^{L} 34.76\, P_o\, e^{rt} Q\, c_f\, \rho_f (T_o - T_i) g(t/t_u)\, e^{-it}\, dt \tag{2.18}$$

$$- C(Q,\ T_i,\ L)$$

subject to

$$g(L/t_u) \geq \frac{\delta}{T_o - T_i}$$

$$Q \geq 0,$$

where

η = royalty for geothermal lease paid as a percentage of the value of produced energy,

Q = extraction rate (m^3/hr),

c_f = specific heat of the fluid (cal/g °C),

ρ_f = fluid density (g/cm^3),

P_o = assumed energy price ($ / MBTU),

i = real discount rate

τ = breakthrough time (years),

L = project life (years),

$C(Q, T_i L)$ = cost function describing the present worth of total capital and operating costs, and 34.76 is a conversion factor to yield revenues in dollars per year. By taking time in days, we could have obtained a closer approximation to the discounted profits. However, for simplicity, we compute time in years. Let

$$\beta = \pi h D^2 \rho_a c_a / 26280\, c_f \rho_f$$

$$\tau = \beta / Q$$

$$t_u = 6\beta / Q$$

$$\psi_j = \phi_j / 6\beta$$

$$\alpha = i - r$$

$$a = 34.76\, P_o c_f \rho_f (1 - \eta)$$

where ϕ_j are the exponential parameters of g in eqn. 2.11. For $t > \tau$, eqn. 2.11 can therefore be written as

$$g(t/t_u) = \gamma_1 e^{-\psi_1 Q t} + \gamma_2 e^{-\psi_2 Q t} + \gamma_3 e^{-\psi_3 Q t} . \qquad (2.19)$$

Thus, the integral in eqn. 2.18 reduces to

$$\pi = aQ(T_o - T_i) \int_0^{\beta/Q} e^{-\alpha t} dt$$

$$+ aQ(T_o - T_i) \int_{\beta/Q}^{L} (e^{-\alpha t} \sum_{j=1}^{3} \gamma_j e^{-\psi_j Q t}) dt$$

$$- C(Q, T_i, L) ,$$

yielding

$$\pi = aQ(T_o - T_i) \left[(1 - e^{-\beta/Q})/\alpha + \sum_{j=1}^{3} \frac{\gamma_j (e^{-(\psi_j Q + \alpha)\beta/Q} - e^{-(\psi_j Q + \alpha)L})}{\psi_j Q + \alpha} \right]$$

$$- C(Q, T_i, L) \quad . \tag{2.20}$$

2.4.7 Production Model with Linearly-Growing Energy Value (Linear Growth Model)

Since the assumption of exponentially increasing value for energy may tend to overestimate this value after several decades, we present an alternative model for the rate of increase of energy. Specifically, we let

$$P_t = P_o(1 + rt) \ , \tag{2.21}$$

where P_o and r are the parameters defined in section 2.5.6. We shall refer to this variation of the model as the "Linear Growth Model." As in the previous section, we continue to assume that price (value) is a given value to the firm. In this case our optimization problem becomes:

$$\underset{Q,\, T_i,\, L}{\text{Maximize}} \ \pi = (1 - \eta) \int_0^{\tau(Q)} 34.76\, P_o(1 + rt)\, Q\, c_f\, \rho_f (T_o - T_i)\, e^{-it}\, dt$$

$$+ (1 - \eta) \int_{\tau(Q)}^{L} 34.76\, P_o(1 + rt) Q\, c_f\, \rho_f (T_o - T_i)\, g(t/t_u)\, e^{-it}\, dt \tag{2.22}$$

$$- C(Q, T_i, L) \ ,$$

subject to

$$g(L/t_u) \geq \frac{\delta}{T_o - T_i}$$

$$Q \geq 0 \ .$$

Invoking eqn. 2.19, we can write

$$\pi = aQ(T_o - T_i)\left[\int_0^{\beta/Q} (1+rt)\,e^{-it}\,dt + \int_{\beta/Q}^{L} (1+rt)\,g(t/t_u)\,e^{-it}\,dt\right] \tag{2.23}$$

$$- C(Q,\, T_i,\, L)\ .$$

Now we can evaluate eqn. 2.23 to obtain

$$\pi = aQ(T_o - T_i)\left\{\frac{1 - e^{-i\beta/Q}}{i} - r\left[\frac{e^{-i\beta/Q}}{i^2}\left(1 + \frac{i\beta}{Q}\right) - \frac{1}{i^2}\right]\right.$$

$$+ \sum_{j=1}^{3} \gamma_j \frac{e^{-(\psi_j Q + i)\beta/Q} - e^{-(\psi_j Q + i)L}}{\psi_j Q + i}$$

$$- r \sum_{j=1}^{3} \gamma_j e^{-(\psi_j Q + i)L} \frac{[(\psi_j Q + i)L + 1]}{(\psi_j Q + i)^2} \tag{2.24}$$

$$\left. + r \sum_{j=1}^{3} \gamma_j e^{-(\psi_j Q + i)\beta/Q} \frac{[(\psi_j Q + i)\beta/Q + 1]}{(\psi_j Q + i)^2}\right\}$$

$$- C(Q,\, T_i,\, L)\ ,$$

which gives π in terms of our decision variables.

Note that in eqns. 2.20 and 2.24 the lifetime L has been assumed to be greater than the breakthrough time, τ. When $L \leq \tau$, eqns. 2.20 and 2.24 have to be accordingly modified. This modification will be discussed in section 2.6.

2.4.8 Determining P_o

We have defined P_o as the cost of the least expensive alternative method of producing one million BTU's of low pressure steam. At the present time, this alternative is producing steam in a boiler heated by fuel oil or coal. The components comprising the cost are capital, fuel, and operating costs. Based on empirical cost data (Hayden, 1976):

$$\text{Original Capital Cost} = \$55/\text{Boiler HP} ,$$

which equals $1643/MBTU/hr, as each Boiler HP is equivalent to 0.033475 MBTU/hr. Taking the lifetime of the equipment as 25 years, the annual (fixed) capacity cost is obtained by multiplying the original capital cost by CRF (i, 25), where CRF (i, n) is defined for this study as the capital recovery factor (when the interest rate is i and the lifetime of the equipment is n) plus cost of insurance, and local taxes expressed as a fraction, m, of original capital costs:

$$CRF(i, n) = \frac{i(1+i)^n}{(1+i)^n - 1} + m , \tag{2.25}$$

Therefore:

$$\text{Annual Capital Cost} = 1643 \cdot CRF(i, 25) \ \$/\text{yr}/\text{MBTU}/\text{hr} .$$

The fuel cost (No. 2 fuel oil at 15¢ /gallon) is $.66 per thousand pounds of 5 psi steam. Thus:

$$\text{Fuel Cost} = \$.66/\text{hr}/\text{MBTU}/\text{hr} = 5782 \ \$/\text{yr}/\text{MBTU}/\text{hr} .$$

In addition, the operation of pump and boiler fan costs $125/Boiler HP per year. Hence:

$$\begin{aligned} \text{Operating Cost} &= 125 \ \$/\text{yr}/\text{Boiler HP} \\ &= 3734 \ \$/\text{yr}/\text{MBTU}/\text{hr} . \end{aligned}$$

The value of the energy P_o, is therefore

$$P_o = 1643 \, CRF(i, 25) + 5782 + 3734 \quad (\$/yr/MBTU/hr),$$

yielding

$$P_o = 0.1876 \, CRF(i, 25) + 1.086 \quad \$/MBTU. \tag{2.26}$$

2.5 COST FUNCTION

2.5.1 Preliminaries

In this section we develop the cost function, $C(Q, T_i, L)$. The components of the cost function are: 1) costs for wells and casing and their maintenance, 2) well assemblies, 3) pumps and their operation, 4) pipes and pipe cleaning, 5) heat exchangers, and 6) rent and salaries. In this section we develop the relationship between the costs of each piece of equipment and our decision variables.

We will denote the maximum flow rate from each production well by $\overline{Q}$. This upper limit is determined by two factors. The first is the assumption in the Gringarten-Sauty hydrothermal model that the flow into the production well be laminar. Laminarity is indexed by the Reynolds number

$$N_R = \frac{Vd}{\nu}, \tag{2.27}$$

where V is the specific discharge given by

$$V = \frac{Q}{2\pi r_w h}. \tag{2.28}$$

In the above relationships, r_w is the well radius, d the average grain diameter, and ν the kinematic viscosity. Hence, once the properties of the field are determined, the first limit on flow rate, $\overline{Q}_1$ will be known.

The second limit is a function of the technology of geothermal brine pumps. These vertical pumps are limited both by their technical capacities (maximum flow rate) and the drawdown generated in the production well, which is in turn dependent on the flow rate. The steady state drawdown for the production well is given by De Wiest (1967, p. 249):

$$\Delta P = \frac{Q}{2\pi Kh} \ln \frac{D}{r_w} \quad . \tag{2.29}$$

The hydraulic conductivity, K, is obtained from

$$K = 1.1653 \times 10^{-11} \frac{k\gamma}{\mu} , \tag{2.30}$$

where γ is the specific weight of water (lb_f/ft^3), μ the absolute viscosity ($lb_f\,sec/ft^2$), k the intrinsic permeability of the aquifer (millidarcies), and 1.1653×10^{-11} a conversion factor, so that the units of K are m/hour. Again, once the height of the aquifer, the intrinsic permeability, and the temperature of the hot brine are known, this second maximum flow rate, $\overline{Q}_2$, that would be consistent with current pump technology can be determined. $\overline{Q}$ is the minimum of $\overline{Q}_1$ and $\overline{Q}_2$.

We will denote by S the present value of total salaries and rents for the geothermal reservoir paid during the life of the project, i.e.

$$S = \int_0^L (\text{Annual Rents + Annual Salaries})\, e^{-it}\, dt \quad . \tag{2.31}$$

Note that S is independent of the extraction rate. Since there are certain fixed costs that must be paid for each doublet, the total cost function, $C(Q, T_i, L)$, is a step function of Q (see Figure 2.3) with jumps equal to the present value of well and overhead assembly costs plus fixed capital costs of pumps and heat exchangers. Let $q(Q, T_i, L)$, $0 \leq Q \leq \overline{Q}$, be the cost function describing the present value of total costs (excluding rents and salaries) associated with

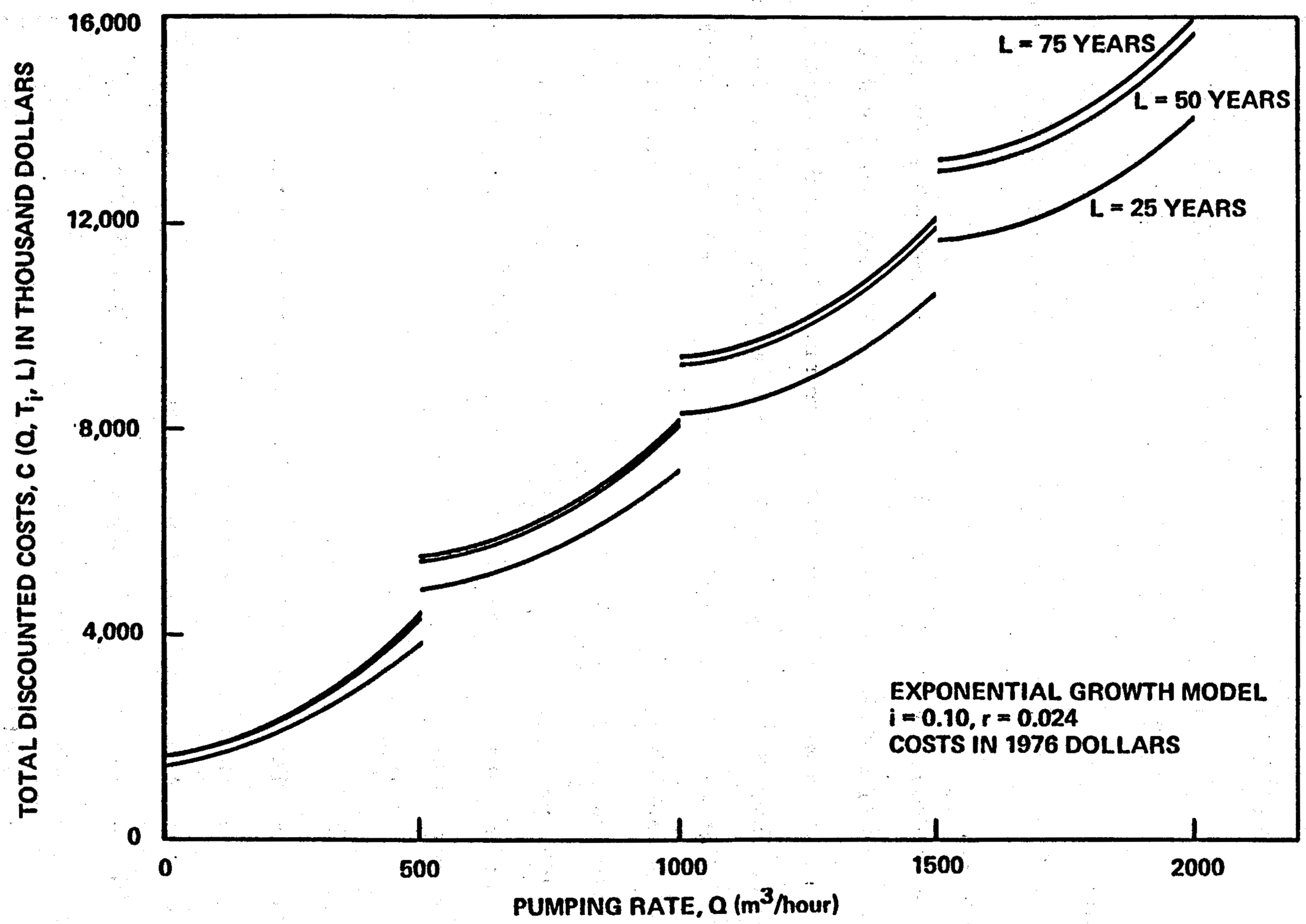

Figure 3. Total Discounted Costs vs Pumping Rate for Given Project Lives

FIGURE 2.3 TOTAL DISCOUNTED COSTS vs. PUMPING RATE FOR GIVEN PROJECT LIVES one doublet. Then, suppressing the dependence of C and q on T_i and L, we can write

$$C(Q) = 0 \qquad \text{if } Q = 0$$

$$= q(Q) + S \qquad \text{if } 0 < Q \leq \bar{Q}$$

$$= q(\bar{Q}) + q(Q - \bar{Q}) + S \qquad \text{if } \bar{Q} < Q \leq 2\bar{Q}$$

and in general

$$C(Q) = nq(\bar{Q}) + q(Q - n\bar{Q}) + S \qquad \text{if } n\bar{Q} < Q \leq (n+1)\bar{Q} \tag{2.32}$$

for n = 1, 2, ...

In our analysis, we take the useful life of pumps and heat exchangers as ten years and that of pipes and well assemblies as 25 years. We will later demonstrate that the well life is a crucial determining factor of the economic life of a reservoir. Since the life of a geothermal well may vary from field to field, we will let well life be an input parameter. We assume that payments for the cost of each type of equipment and accrued interest are distributed uniformly over the lifetime of the equipment. Furthermore, each piece of equipment (with the exception of the wells) has a salvage value equal to a percentage of its remaining unpaid costs, if it is sold before its lifetime is up.

For each doublet, let

WC = Cost of wells and casing,

WL = Useful life of wells,

PM = Total cost of the vertical and horizontal pumps,

WA = Cost of well assemblies,

WM = Annual well maintenance costs,

PO(t) = Operating costs of pumps as a function of time,

HE = Cost of a heat exchanger,

PP = Cost of pipes,

PC	=	Annual pipe cleaning costs,
L	=	Life of the project,
L_1	=	The smallest multiple of 10 containing L,
L_2	=	The smallest multiple of 25 containing L,
L_3	=	The smallest multiple of well life containing L,
s_1	=	Salvage value of pumps as a percentage of their remaining payments,
s_2	=	Salvage value of heat exchangers as a percentage of their remaining payments,
s_3	=	Salvage value of pipes as a percent of their remaining payments,
s_4	=	Salvage value of well assemblies as a percentage of their remaining payments.

The total cost function of one doublet is therefore

$$q(Q, T_i, L) = 0 \qquad \text{if } Q = 0$$

$$\begin{aligned} &= \int_0^L [\,(PM + HE)\,CRF(i, 10) + (WA + PP)\,CRF(i, 25) \\ &\quad + (WC)\,CRF(i, WL) + WM + PC + PO(t)\,]\, e^{-it}\, dt \\ &\quad + [\,(1 - s_1)\,PM + (1 - s_2)\,HE\,]\; CRF(i, 10) \int_L^{L_1} e^{-it}\, dt \\ &\quad + [\,(1 - s_3)\,PP + (1 - s_4)\,WA\,]\; CRF(i, 25) \int_L^{L_2} e^{-it}\, dt \\ &\quad + (WC)\,CRF(i, WL) \int_L^{L_3} e^{-it}\, dt \qquad \text{if } 0 < Q \leq \bar{Q}, \end{aligned} \tag{2.33}$$

where CRF (i, ·) is the capital recovery factor defined in eqn. 2.25. The last three terms give the present value of the extra costs associated with project termination prior to completion of lifetime cycles of various equipment components.

To better visualize these extra termination costs, suppose L, the life of the project is 23 years and the useful life of wells, WL, has been assumed to be 10 years. Then L_1 and L_3 are 30 years and $L_2 = 25$ years. The wells which have been drilled in the beginning of the 21st year still have a useful life of seven years. Since wells do not have any salvage value, we assume the remaining payments on the well cost become immediately payable. The present value of this cost is

$$WC \cdot CRF(i, 10) \int_{23}^{30} e^{-it}\, dt \quad .$$

The other pieces of equipment, however, have a salvage value. For example, consider pipes which have a life of 25 years. The present value of the unpaid cost is

$$PP \cdot CRF(i, 25) \int_{23}^{25} e^{-it}\, dt \quad .$$

Since a percentage of this cost, namely s_3, can be recovered by resale, the termination cost for pipes is

$$(1 - s_3)\, PP \cdot CRF(i, 25) \int_{23}^{25} e^{-it}\, dt \quad .$$

In the remaining parts of this section, we will obtain relationships describing the costs for pipes, pipe cleaning, pumps, operation of pumps and heat exchangers as functions of our decision variables Q and T_1.

2.5.2 Pipe Costs

The data in Table 2.1 have been supplied by a leading pipe manufacturer (Lupear, 1976) for the cost of steel pipe. We will assume a flow velocity v of 6 ft/sec. Both the flow velocity and the pipe specification are made in accordance with standard industrial practice which is based on suboptimization analyses (Lombard, 1976). Multiplying the flow rate Q (in m^3/hr) by 16.938 gives the flow rate in $in.^3$/sec. From

$$Q = \pi d^2 v/4 ,$$

and v = 72 in./sec, we obtain

$$\begin{aligned} d &= (4 \times 16.938\, Q/\pi \times 72)^{0.5} \\ &= 0.5473\, Q^{0.5} \text{ in.} \end{aligned} \tag{2.34}$$

A polynomial regression of degree two gives the following relationship between C^p, the cost/ft and d:

$$C^p = 0.1337\, d^2 + 0.737\, d - 1.33 \tag{2.35}$$

Substituting the value for d from eqn. 2.34 in 2.35, and multiplying by 3.28 (ft/m) and D (the separation distance in meters) yields the pipe cost:

$$\text{Pipe Cost (\$)} = k_p D[0.1313\, Q + 1.323\sqrt{Q} - 4.36] , \tag{2.36}$$

where k_p is a cost multiplier to reflect pipe support and installation costs.

We will assume an additional cost for pipe cleaning. The pipe cleaning cost, PC is proportional to the length of the pipeline.

Hence

$$PC = p_c \cdot D \tag{2.37}$$

where p_c is the estimated annual cost of cleaning one meter of pipe.

Table 2.1

PIPE COST DATA

d (Diameter, inches)	C^p (Cost, $/ft)
4	4.08
6	6.89
8	13.06
10	19.99
12	28.92
14	33.98
16	42.93
18	55.89
20	67.30

Cost of ASTM – Grade B Steel Pipe – Schedule 60

Source: H. Lupear, 1976

2.5.3 Pump Costs

Because of the very large drawdown generated in the production well at rather high production rates (see eqns. 2.29 and 2.30), it is not economical and probably infeasible for a single pump to lift the brine from the aquifer, pump it through the pipeline and heat exchanger to the injection well and overcome the pressure buildup in the injection well. We therefore require that two pumps be used for each doublet. The first will be a vertical turbine pump installed in the production well. This pump will lift the brine to the surface and send it through the piping system and heat exchanger to a second pump, the latter being a horizontal pump capable of pumping the brine back to the aquifer through the injection well. In this section, we obtain a function describing the relationship between extraction rate and total pumping costs.

2.5.4 Production Pump

The vertical turbine pump in the production well will discharge the brine to a surface piping system. The pump consists of five component assemblies: 1) the Drive, and electric motor, 2) the Discharge Assembly, on which the motor is mounted, 3) the vertical Lineshaft, 4) the Column Assembly, through which the lineshaft extends and 5) the Bowl Unit. In this section, we present our estimation of the cost of each individual component. The pump capacity cost, PM, is the sum of these costs and the cost of the horizontal injection pump discussed in 2.5.5.

(i) The Motor Cost

The cost of the electric motor is a function of its brake horsepower, as given in Table 2.2. A linear regression gives the following relationships between motor cost and horsepower (with a correlation coefficient of 0.998):

$$\text{Motor Cost } (\$) = 14.97\ \text{HP} + 1907.1 \quad . \tag{2.38}$$

Table 2.2

PUMP MOTOR COSTS

Horsepower, HP	Cost, $
250	5578
300	6387
350	7212
400	8039
450	8890
500	9622
600	10440
700	12024
800	13741
900	15459
1000	17176

Cost of 1760 rpm, 60 cycle, General Electric motors

Source: G. Crabtree, 1976

We next consider the horsepower requirement of the drive.

When the rotating energy of the drive is transmitted through the lineshaft, some of this energy is lost in the lineshaft bearings by mechanical friction. For each RPM, the shaft horsepower loss per 100 ft of lineshaft can be obtained from pump manufacturers' bulletins as a function of shaft diameter. On the other hand, once the total hydraulic downthrust and the horsepower requirement of the pumping unit are known, the shaft diameter can be determined. The horsepower in eqn. 2.38 is the sum of the horsepower requirement (brake horsepower to pump, BHP) and the horsepower loss.

The brake horsepower (BHP) is given by Peerless (Bulletin B-141, p. 15) :

$$BHP = \frac{\text{Capacity (gpm)} \cdot \text{Total Heal (ft)} \cdot \text{Specific Gravity}}{3960 \cdot \text{Vertical Pump Efficiency}} \tag{2.39}$$

The total head in eqn. 2.39 is the sum of the distance the brine has to be lifted and friction losses in the system. Friction losses consist of the loss caused by the skin friction as the water rises in the column pipe as well as friction losses in the heat exchanger and the pipeline system. We will denote the friction losses by b. Of course, for accurately estimating b, the flow rate Q (which determines the pipe and column diameter and indirectly the shaft size and hence friction losses in pipes and column) and heat exchanger area A (which with Q determines the head losses in the heat exchanger) must be known. Since we are seeking these quantities (and the magnitude of b is small compared to the total head), we assume a value for b in the cost function. Once the optimal Q and A are known, b can be more accurately estimated and if its value is

* The authors gratefully acknowledge the assistance of Craig Brown and George Crabtree of FMC Corporation, Poorless Pump Division, who supplied cost data and design features of Peerless pumps.

significantly different from the original b, Q, and A can again be computed based on the new value for b. Our experience indicates that the value for b can be safely estimated at 20 m. (The pressure drop in a 12-in. 1000 ft standard steel pipe with Q = 500 m^3/hr is 3.05m (Peerless, Brochure EM 77, p. 11), and the skin friction in a 12-in. column of length 765 ft containing a shaft of 2-3/16 in. diameter is 4.9 m (Peerless, Bulletin B-185, p. 81). The remaining 12.05 m is ample enough for the pressure drop in the heat exchanger (Perry, 1950, p. 391).

Let z be the static level of the brine, that is, the vertical distance in meters between the discharge and the free pool when no water is being pumped. The total head is therefore the sum of the drawdown ΔP, z, and b. From eqn. 2.39 therefore,

$$\begin{aligned} BHP &= \frac{Q \cdot 264/60[\Delta P(Q) + z + b]\, 3.28 \cdot \text{sp. gr.}}{3960 \cdot Eff_V} \\ &= \frac{0.003644\, Q \text{ sp. gr.}}{Eff_V} [\Delta P + z + b] \ , \end{aligned} \tag{2.40}$$

where ΔP is a function of Q and is given by eqn. 2.29.

Our next step is the computation of shaft horsepower losses. From the horsepower ratings table for AISI-1045 threaded lineshaft - 1760 RPM (Peerless, Bulletin B-185, p. 85), we note that the HP rating for a given shaft diameter does not significantly vary over a wide range of the thrust values (1,000-20,000 lb). We take the values corresponding to 10,000-lb thrust, and thus relate the shaft diameter to BHP. Since both HP loss and shaft and tubing costs are related to shaft diameter (Peerless, Bulletin B-185, p. 84, 87; Crabtree, 1976), we can also relate the latter two quantities to the brake horsepower. The data are given in Table 2.3.

Taking the mean value of the shaft HP loss column (2.34 HP/100 ft), the estimated HP loss is

$$\text{HP Loss} = 0.0234\,(\Delta P + z)\ , \tag{2.41}$$

which when added to the BHP obtained in eqn. 2.41 gives the total hosepower required of the drive. Hence, from eqns. 2.38, 2.40, and 2.41 we obtain

$$\text{Motor Cost} = 14.97\left[\frac{0.003644\,Q\ \text{sp. gr.}}{\text{Eff}_V}(\Delta P + z + b) + 0.0234\,(\Delta P + z)\right] + 1907.1 \tag{2.42}$$

(ii) Discharge Assembly Cost

This component constitutes a minor portion of the pump costs. In the cost estimation, we take the cost of a 10 x 10 F Standard Fabricated Steel Head, which is recommended by the pump manufacturer for our range of flow rates.

(iii) Cost of Shaft and Enclosing Tube

The vertical lineshaft is enclosed in a tube and extends downward through the column assembly to the bowl unit. As mentioned earlier, the shaft diameter (which determines the tube diameter) is a function of the horsepower requirement of the drive. The last column of Table 2.3 gives the combined cost of shaft and its enclosing tube per foot of lineshaft length. A linear regression yields the following relationship between the shaft and tube cost, and the horsepower requirement (with correlation coefficient of 0.967):

$$\text{Shaft Cost (\$/ft)} = 0.112\ \text{HP} - 3.089.$$

Therefore,

$$\text{Shaft Cost (\$)} = \left\{0.112\left[\frac{0.003644\,Q\cdot\text{sp. gr.}}{\text{Eff}_V}(\Delta P + z + b) + 0.0234\,(\Delta P + z)\right] - 3.089\right\}\left[3.28\,(\Delta P + z)\right] \tag{2.43}$$

Table 2.3

SHAFT AND TUBE COSTS

BHP	Shaft Diam.[a] inches	Tube Diam.[a] inches	Shaft HP Loss[a] BHP/100 ft	Shaft and Tube[2] Dollar/ft
50.9	1	1-1/2	.53	15.0
81.6	1-3/16	2	.72	19.5
188	1-1/2	2-1/2	1.25	26.2
281	1-11/16	3	1.4	34.6
443	1-15/16	3	1.9	36.9
560	2-3/16	3-1/2	2.3	50.8
767	2-7/16	4	2.9	92.0
1051	2-11/16	5	3.4	163.2
1387	2-15/16	5	4.2	163.2
1814	3-3/16	5	4.8	168.4

[a] Bulletin B-185, Peerless Pump Division, FMC Corp., pp. 84-87.

[b] G. Crabtree, 1976

which simplifies to

$$\text{Shaft Cost (\$)} = [\,0.001339\,Q\,\frac{sp.\,gr.}{Eff_V}\,(\Delta P + z + b)$$

$$+\,0.0768\,(\Delta P + z) - 10.132][\Delta P + z]\ .$$

(iv) Column Assembly Costs

As in the computation of pipe costs, the flow rate determines the size and hence the cost of the column assembly. The setting (length of column assembly) is the sum of the static level z, the drawdown and four meters of section pipe connected to the bowl unit. The cost per meter of the column assembly is given by the terms in brackets in eqn. 2.36. Hence Column Assembly Cost (CAC) is given by

$$\text{CAC (\$)} = [\,z + 4 + Q\ln(D/r_w)/2\pi Kh\,][\,0.1313\,Q + 1.323\sqrt{Q} - 4.36\,] \qquad (2.44)$$

(v) Bowl Unit Cost

The bowl unit or the pumping element consists of one or more pumping stages. Each stage lifts a given quantity of water by a given height and consists of a bowl case and an impeller which rotates at the speed of the drive. The cost of the bowl unit is proportional to the number of required pumping stages and hence to the lift and capacity.

Although any two reference points can be used in the extrapolation of bowl unit cost, we take as our reference points the cost of two bowl units for which data are readily available from pump manufacturers.

Let c_1 be the cost of a bowl unit capable of lifting 50 m^3/hr (220 gpm) of water a distance of $\Delta P(50) + z$ meters and c_2 the cost of a bowl unit capable of lifting 250 m^3/hr (1100 gpm) of water a distance of $\Delta P\,(250) + z$ meters. The Bowl Unit Cost (BUC) is therefore

$$BUC = c_1 + \frac{(c_2 - c_1) \cdot ([\Delta P(Q) + z] - [\Delta P(50) + z])(Q - 50)}{([\Delta P(250) + z] - [\Delta P(50) + z])(250 - 50)}$$

$$= c_1 + \frac{(c_2 - c_1)[(Q - 50)(\ln D/r_w)/2\pi Kh](Q - 50)}{[(250 - 50)(\ln D/r_w)/2\pi Kh](250 - 50)} \qquad (2.45)$$

$$= c_1 + (c_2 - c_1)(Q - 50)^2/40,000 \ .$$

2.5.5 Injection Pump

The cost of the injection pump is considerably less than the production pump. We require a horizontal pump with a discharge head of ΔP to overcome the pressure buildup in the injection well. The drive is supplied by an electric motor.

(i) Pump Cost

The pertinent data (Brown, 1976) is shown in Table 2.4. The discharge heads in the second column corespond to the drawdown generated by flow rates in the first column (based on typical field data). The last column indicates that the cost is proportional to the flow rate Q. Hence

$$\text{Horizontal pump cost (\$)} = 24Q \ . \qquad (2.46)$$

(ii) Motor Cost

The horsepower of the horizontal pump can be determined from eqn. 2.39 yielding

$$BHP_H = \frac{0.003644\, Q \cdot \text{sp. gr.}}{Eff_H} \Delta P \ . \qquad (2.47)$$

Substituting in eqn. 2.38 yields the cost of the horizontal motor, MC_H:

Table 2.4

HORIZONTAL PUMP DATA

Capacity (m^3/hr)	Discharge Hd. (feet)	Pump Spec. Number	Pump Cost (dollars)	Base Dollars	Total Dollars
50	76	AD-11	800	400	1200
250	380	TU-15	4000	2000	6000
500	760	TU-22	8500	4000	12500

Source: C. Brown, 1976

Table 2.5

HEAT EXCHANGER COSTS

Heat Exchanger Area (ft^2)	Cost (dollars)
5000	75000
2500	40000

$$MC_H(\$) = \frac{0.0546\,Q \cdot sp.\ gr.}{Eff_H}\left[\frac{Q \ln (D/r_w)}{2\pi Kh}\right] + 1907.1 \tag{2.48}$$

2.5.6 Pump Operating Costs

Let R_t be the price of electricity (\$/kwh) at time t, supplied to the motors of the vertical and horizontal pumps. Since the price of energy is assumed to increase with time, eqn. 2.15 yields

$R_t = R_o e^{rt}$ for the model of section 2.4.6

$R_t = R_o(1 + rt)$ for the model of section 2.4.7

Then pump operating cost at time t, PO(t), is given by

$$PO(t) = (HP_V + HP_H)\ 0.7457 \times 8760\ R_t\ ,$$

which, combined with eqns. 2.40, 2.41, and 2.47 gives

$$PO(t) = R_t k_m \left[\frac{23.8\,Q}{Eff_V} \cdot sp.\ gr.\ (\Delta P + z + b) + 152.86\,(\Delta P + z) + \frac{23.8\,Q \cdot sp.\ gr.}{Eff_H} \cdot \Delta P\right], \tag{2.49}$$

where k_m is a multiplier indicating annual maintenance costs of pumps and their motors.

2.5.7 Heat Exchanger Costs

The heat exchanger cost is proportional to heat exchanger area A. The cost estimates in Table 2.5 were supplied by a manufacturer (Breese, 1976) for heat exchangers with stainless tubes and cast iron shell. Based on these data, the relationship between the heat exchanger cost (HE) and area A is:

$$HE = 5000 + 150.7\,A$$

where A is in m^2. Combined with eqn. 2.14, we can write

$$HE = 5000 + 150.7 \frac{Q}{k(0)} [\ln (T_o - T_s) - \ln (T_i - T_s)] \quad . \qquad (2.50)$$

2.5.8 Well and Well Assembly Costs

The cost for wells and their casing has to be determined based on the thickness of the impermeable strata and the aquifer for each individual field. The well assemblies (Christmas tree valves, etc.) are to some extent a function of the capacity. In our computer program, we have allowed these costs as well as an annual cost for maintenance of each pair of production injection wells to be given as inputs.

2.6 RESULTS

2.6.1 Preliminaries

In this section we present the results of our analysis of the two economic models discussed in section 2.5. The objective is to find the extraction rate Q^*, project life L^*, and injection temperature T_i^* which maximize the functions described by eqns. 2.20 and 2.24, subject to the constraint that the difference between the production and injection temperatures would remain greater than a prescribed amount, δ degrees centigrade.

We begin by expressing T_i^* as a function of Q and L, thereby reducing the number of decision variables to two. It is easily seen from eqns. 2.20 and 2.24 that the total revenue is a decreasing linear function of T_i, i.e., the revenue increases as $T_i \longrightarrow T_s$. This follows from the fact that a lower injection rate implies that for any given pumping rate Q, a larger amount of heat can be extracted. However, it follows from eqn. 2.14 that for a given steam temperature T_s, achieving lower values of T_i requires larger and hence more expensive heat exchangers. Since both the cost and revenue functions are continuous in T_i, it follows that for any given value of Q and L, the optimal injection temperature T_i^*, is achieved at the point where the marginal revenue with respect to T_i equals the marginal cost of further reducing T_i. In other words

$$\left.\frac{\partial R_1}{\partial T_i}\right|_{T_i^*} = \left.\frac{\partial C}{\partial T_i}\right|_{T_i^*}$$

and

$$\left.\frac{\partial R_2}{\partial T_i}\right|_{T_i^*} = \left.\frac{\partial C}{\partial T_i}\right|_{T_i^*}$$

where R_1 and R_2 denote revenues for the models of sections 2.4.6 and 2.4.7, respectively.

Note that the dependence of the cost function $C(Q, T_i, L)$ on T_i is only through the cost for heat exchangers which is described by eqn. 2.50, and their salvage values. Denoting the sum of terms which do not contain the heat exchanger cost HE in eqn. 2.33 by $J(Q, L)$, the total cost function of one doublet can be written as

$$q(Q, T_i, L) = J(Q, L) + \int_0^L HE \cdot CRF(i, 10)\, e^{-it} dt + \int_L^{L_1} (1 - s_2) HE \cdot CRF(i, 10)\, e^{-it} dt$$

$$= J(Q, L) + HE \cdot CRF(i, 10)[(1 - e^{iL_1}) - s_2(e^{-iL} - e^{-iL_1})] / i, \qquad (2.51)$$

where J is independent of T_i. Let

$$M \equiv \frac{150.7}{k(0)} CRF(i, 10)[(1 - e^{-iL_1}) - s_2(e^{-iL} - e^{-iL_1})] / i.$$

Substituting the value for HE from eqn. 2.50 into eqn. 2.51 and utilizing eqn. 2.32 we obtain

$$C(Q, T_i, L) = nJ(\bar{Q}, L, i) + n\bar{Q}M[\ln(T_o - T_s) - \ln(T_i - T_s)] + J(Q - n\bar{Q}, L, i)$$

$$+ (Q - n\bar{Q})M[\ln(T_o - T_s) - \ln(T_i - T_s)]$$

$$= nJ(\bar{Q}, L, i) + J(Q - n\bar{Q}, L, i) + QM[\ln(T_o - T_s) - \ln(T_i - T_s)] \; .$$

Hence

$$\left.\frac{\partial C}{\partial T_i}\right|_{T_i^*} = -\frac{QM}{T_i^* - T_s} \; . \tag{2.52}$$

To obtain $\frac{\partial R_1}{\partial T_i}$, note that eqn. 2.20 can be written as

$$\pi = R_1(Q, T_i, L) - C(Q, T_i, L)$$

$$= aQ(T_o - T_i)\sigma_1 - C(Q, T_i, L) \tag{2.53}$$

and eqn. 2.24 as

$$\pi = R_2(Q, T_i, L) - C(Q, T_i, L)$$

$$= aQ(T_o - T_i)\sigma_2 - C(Q, T_i, L) \; , \tag{2.54}$$

where σ_1 and σ_2 denote the terms inside the brackets in eqn. 2.20 and eqn. 2.24, respectively, yielding

$$\frac{\partial R_1}{\partial T_i} = -aQ\,\sigma_1 \tag{2.55}$$

and

$$\frac{\partial R_2}{\partial T_i} = -aQ\,\sigma_2 \; . \tag{2.56}$$

Therefore equating eqns. 2.55 and 2.56 with eqn. 2.52 yields respectively

$$T_i^* = T_s + \frac{M}{a\sigma_1} \quad \text{for the exponential model} \tag{2.57}$$

and

$$T_i^* = T_s + \frac{M}{a\sigma_2} \quad \text{for the linear model .} \tag{2.58}$$

And so we conclude that T_i may appropriately be expressed as a function of Q and L for purposes of optimization.

2.6.2 Optimization Algorithm

The algorithm used for obtaining the optimal solution to the linear and exponential models is a grid search over values of L and Q. The lifetime, L, is varied from L_{min} to L_{max} in increments of L_{inc}. For each L, the pumping rate Q is varied from Q_{min} to Q_{max} in increments of Q_{inc}. These values of L_{min}, L_{max}, L_{inc}, Q_{min}, Q_{max}, and Q_{inc} are specified by the decision maker according to judgment and are inputs to the computer program. For each Q and L, the δ constraint is checked to make sure the difference between the production and injection temperatures does not fall below δ degrees centigrade. In this regard, note that by eqns. 2.17 and 2.19, the temperature drop is a function of $Q \cdot L$. Therefore if, for a given L and some $\hat{Q}$, $(\hat{Q} \leq Q_{max})$, $T_o^L(\hat{Q}) - T_i \leq \delta$, it follows that $T_o^L(Q) - T_i < \delta$ for all $Q > \hat{Q}$. Thus, the search for an optimal Q ceases at $\hat{Q}$ and resumes with $Q = Q_{min}$ at $L + L_{inc}$. For each L and feasible Q, $T_i^*(Q, L)$ is computed from eqn. 2.57 or 2.58, and the values for the present worth of profits, π , are determined. The set of values (Q^*, L^*, T_i^*) that yields the maximum π, are the optimal decision variables. For our example computations we use L_{min} = 0 years, L_{max} = 250 years, L_{inc} = 1 year, Q_{min} = 50 m^3/hr, Q_{max} = 5000 m^3/hr, and Q_{inc} = 5 m^3/hr.

To determine the profit for each set of decision variables, the present worths of total revenues and costs must be computed. To begin, we estimate the cost of the necessary equipment and their salvage values as well as operating costs according to the equations developed in section 2.5. These costs are, of course, functions of our decision variables as well as input data. The total discounted cost for each doublet is then computed from eqn. 2.33, and total costs $C(Q, T_i, L)$ from eqn. 2.32.

As mentioned in section 2.4.7, when lifetime L is greater than the breakthrough time τ, eqns. 2.20 and 2.24 yield the present worth of profits. However, when $L \leq \tau$, a modified version of these equations has to be used for the exponential and linear models. The present worth of profits when $L \leq \tau$ is given by

$$\pi = aQ(T_o - T_i) \cdot (1 - e^{-\alpha L}) / \alpha - C(Q, T_i, L) \tag{2.59}$$

for the exponential model and

$$\pi = aQ(T_o - T_i)[(1 - e^{-iL}) / i - r \left\{ e^{-iL}(1 + Li) - 1 \right\} / i^2] - C(Q, T_i, L) \tag{2.60}$$

for the linear model. These expressions are derived from eqns. 2.18 and 2.22 by setting the upper limits of the first integrals equal to L since $L \leq \tau$. Of course the second integrals are equal to zero.

The fortran program used to execute this algorithm can be readily utilized by other users. Technical geothermal and economic data are input to the program. The cost subroutine can be easily modified by different users to accomodate the particular costs involved in the exploitation of each individual field. The computer program, the cost subroutine and instructions for their use are given in Appendix A. The approximate time required for an optimum solution (for a particular i and r) was under 10 sec cpu time on the UCLA IBM 360/91 computer.

In section 2.6.4 we discuss the results of our computation for both the linear and exponential models. The computation is carried out for a particular set of data which to our best judgment reflects the current value of pertinent costs. The geohydrological data have generally been chosen in the midrange of values associated with known geothermal resources. We will call these data the "basic data" and the models using these data the "basic models."

2.6.3 Basic Data

The following data have been used in the analysis of the basic models (all costs are in 1976 dollars).

Thickness of Aquifer, h	100 m
Doublet Separation, D	300 m
Well Radius, r_w	0.15 m
Well Capacity, $\bar{Q}$	500 m^3/hr
Porosity of Aquifer, ϕ	0.20
Intrinsic Permeability, k	200 m.d.
Initial Equilibrium Temperature, T_o	150°C
Temperature of Generated Steam, T_s	109°C
Heat Capacity of Fluid, $\rho_f c_f$	0.92 cal/cc$^\circ$C
Heat Capacity of Rock, $\rho_R c_R$	0.50 cal/cc$^\circ$C
Specific Gravity of Fluid	0.9173
Overall Heat Transfer Coefficient of Fluid, $U(0)$	1000 BTU/hr ft^2 $^\circ$F
Friction Losses, b	20 m
Static Level of Fluid, z	0 m
Vertical Pump Efficiency, Eff_V	0.75

Horizontal Pump Efficiency, Eff_H	0.75
Pump Salvage Value as Fraction of Remaining Payments, s_1	0.40
Heat Exchanger Salvage Value as Fraction of Remaining Payments, s_2	0.40
Pipe Salvage Value as Fraction of of Remaining Payments, s_3	0.40
Well Assembly Salvage Value as Fraction of Remaining Payments, s_4	0.40
Pipe Cleaning Cost, P_c	10 \$/m/year
Pipe Support Multiplier, k_p	1.25
Cost of 50 m^3/hr Bowl Unit, c_1	1250 dollars
Cost of 250 m^3/hr Bowl Unit, c_2	3941 dollars
Pump Maintenance Cost Coefficient, k_m	1.10
Well Cost per Doublet, WC	600,000 dollars
Well Maintenance Cost, WM	6000 \$/yr/doublet
Useful Life of Wells, WL	25 years
Well Assembly Cost, WA	35,000 dollars
Electricity Cost in 1976, R_o	3 ¢/kwh
Annual Salaries	50,000 \$/year
Annual Rents	4,000 \$/year
Royalty, η	0.10
Minimum Allowable Temperature Difference, δ	6°C

The absolute viscosity η (in poises) of the fluid is directly computed from the Bingham formula (Bingham, 1922, p. 340) :

$$\frac{1}{\mu} = 2.1482 \left[(T_o - 8.435) + \sqrt{8078.4 + (T_o - 8.435)^2} \right] - 120 \qquad (2.60a)$$

Multiplying μ in poises by 0.00209 gives the viscosity in $lb_f\ sec/ft^2$.

2.6.4 Results for the Basic Models

In this section we present our results for a set of interest rates i, and a set of rates of increases in the price of energy, r, using the basic data. As we mentioned in section 2.4.3, the determination of a single market interest rate is very difficult. In fact, it is more likely that different investors would use different interest rates, depending on their perception of risk and their best alternative investment opportunities. Accordingly, we present our results for a range of interest rates from i = 0.06 to i = 0.15. We also vary the value for r within a range that we feel corresponds to likely futures, namely, from zero to three percent, enabling us to examine the sensitivity of our decision variables to changes in this important parameter as well. In addition to this general survey we discuss in some detail the results when r = 0.024 and i = 0.10, which we have chosen as our basic case. Energy prices are forecast to increase at 2.4 percent per year in constant dollars according to the 1977 National Energy Outlook (Federal Energy Administration, 1977), and 10 percent is the discount rate designated by the Office of Management and Budget (OMB Circular A-94).

2.6.5 Profits

We begin with the present worth of maximum profits, π for the exponential and linear models respectively as shown in Tables 2.6 and 2.7. By profits we mean the difference between total discounted revenues and costs. The values across the top row represent discount rates, while those in the bottom denote values for P_o, the 1976 value of one million BTU of 5 psi pipeline steam (which depends on interest rate by eqn. 2.26). The left column contains the values for r. For each i and r the optimal profit is given in the table.

As expected, the tables show that optimal profits increase as r increases and decrease as i increases. That this would always be true can be seen by the

Table 2.6

PRESENT WORTH OF MAXIMUM PROFITS, π^* ($1976, $1000) FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	374.	317.	266.	220.	160.
0.010	554.	456.	375.	308.	226.
0.020	830.	620.	505.	412.	303.
0.024	1065.	728.	561.	457.	337.
0.030	1503.	938.	688.	537.	389.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.7

PRESENT WORTH OF MAXIMUM PROFITS, π^* ($1976, $1000) FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	374.	317.	266.	220.	160.
0.010	543.	447.	369.	304.	223.
0.020	717.	584.	479.	393.	291.
0.024	787.	639.	523.	429.	318.
0.030	914.	721.	588.	482.	358.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

following argument. Obviously for fixed Q and L, π is decreasing in i and increasing in r. Let $i_1 < i_2$ and suppose Q_1, L_1 maximize $\pi(i_1; Q, L)$ and Q_2, L_2 maximize $\pi(i_2; Q, L)$. Then,

$$\pi(i_1; Q_1, L_1) \geq \pi(i_1; Q_2, L_2) > \pi(i_2; Q_2, L_2) .$$

Similarly let $r_1 < r_2$ and suppose Q^1 and L^1 maximize $\pi(r_1; Q, L)$ and Q^2, L^2 maximize $\pi(r_2; Q, L)$. Then

$$\pi(r_2; Q_2, L_2) \geq \pi(r_2; Q_1, L_1) > \pi(r_1; Q_1, L_1) .$$

For each i and r, the profits for the exponential model are higher than the linear model. This follows from the fact that $\int f(t)\, e^{rt}\, dt \geq \int f(t)\,(1 + rt)\, dt$ with the equality holding when r = 0. Note that the profits in tables 2.6 and 2.7 are identical when r = 0.

2.6.6 Optimal Pumping Rate

We come now to the most important decision variable of this chapter, Q. Tables 2.8 and 2.9 present the optimal pumping rates, Q^*, for the two models. The optimal pumping rates increase as i increases and decrease as r increases, with the values for the linear model slightly larger than those of the exponential model (though not by much, except when r = 3.0). Furthermore, the difference between the optimal pumping rates for the two models increases as r increases. These results are consistent with intuition whereby as r is increased, we tend to extract heat more slowly, thus reserving a larger amount for the future when value is higher. However, as i increases we tend to extract heat at a higher rate, leaving less for the future when energy value (although rapidly increasing) is heavily discounted. The fact that the energy values are higher for the exponential model than the linear model accounts for the decision to extract less heat when the growth rate is exponential, especially when r is large.

Table 2.8

OPTIMAL PUMPING RATE, Q^* (cubic meters/hr) FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	380.	390.	400.	405.	420.
0.010	365.	375.	385.	395.	410.
0.020	325.	375.	385.	395.	410.
0.024	320.	345.	385.	395.	410.
0.030	310.	340.	360.	380.	405.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.9

OPTIMAL PUMPING RATE, Q^* (cubic meters/hr) FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	380.	390.	400.	405.	420.
0.010	365.	375.	385.	395.	415.
0.020	365.	375.	385.	395.	410.
0.024	365.	375.	385.	395.	410.
0.030	325.	375.	385.	395.	410.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

2.6.7 Optimal Project Life

Tables 2.10 and 2.11 present the optimal project lives for the two models. The economic lives are nonincreasing in i and nondecreasing in r, with the values for the exponential model larger than those of the linear model. Thus, as future profits are discounted more heavily, we tend to extract a greater amount of heat per unit of time over a shorter period in comparison with when the discount rate is not as high. On the other hand, when the energy is expected to rapidly increase in value with time, extraction of heat over a longer period of time is more profitable and, given the definition of profits in this case, a better use of the resource.

The cross signs on some of the lifetimes in Tables 2.10 and 2.11 indicate the beginning of a range of optimal values for L. That is, the present worth of maximum profits remains constant for lifetimes greater than the numbers indicated by a cross, until a time L_δ, after which it decreases. Although the project lives within the range of $L^\dagger$ to L_δ are equally desirable from a profit perspective, they are all associated with the same values of Q^* and A^*. Accordingly which L^* is actually "best" is of no operational importance.

For the basic model and $\delta = 6$, $L_\delta \geq L_{max}$ which we have chosen as 250 years. Therefore π^* is effectively constant from $L^\dagger$ to 250 years for these results. We shall return to L_δ and its relationship with δ in section 2.6.10. However, we continue now with our discussion of the 'profit plateau' phenomenon.

2.6.8 Response of the Profit Function

It is now possible to discuss the response of the profit function to values of L and $Q^*(L)$, the pumping rate which maximizes $\pi(L, Q)$ for a given L. We begin by noting that the objective functions in eqns. 2.20 and 2.24 tend to become independent of L as L gets large. That is, for a given Q, there exists a $\pi'(Q)$ such that

Table 2.10

ECONOMIC RESERVOIR LIFE, L^* (years)
FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	20.	20.	20.	20.	20.
0.010	25.	25.	25.	25.	25.
0.020	250.†	25.	25.	25.	25.
0.024	250.†	215.†	25.	25.	25.
0.030	250.†	220.†	165.†	125.†	25.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

† Denotes the start of an optimal range of values. (See section 2.6.7.)

Table 2.11

ECONOMIC RESERVOIR LIFE, L^* (years)
FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	20.	20.	20.	20.	20.
0.010	25.	25.	25.	25.	20.
0.020	25.	25.	25.	25.	25.
0.024	25.	25.	25.	25.	25.
0.030	220.†	25.	25.	25.	25.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

† Denotes the start of an optimal range of values. (See section 2.6.7.)

$$\lim_{L \to \infty} \pi(L, Q) = \pi(Q) \tag{2.61}$$

for both the exponential and linear models. This follows from the fact that for both models, the revenues are monotone increasing functions of L and the terms containing L approach zero as $L \to \infty$. Furthermore the cost function $C(Q, T_i, L) \to C(Q)$ as $L \to \infty$. This last statement is a consequence of the fact that by eqn. 2.33 we can write

$$q(Q, T_i, L) = AC(Q)\left(\frac{1-e^{-iL}}{i}\right) + OP(Q)\left(\frac{1-e^{-\alpha L}}{\alpha}\right) + \sum_{j=1}^{3} C_j\left(\frac{e^{-iL} - e^{-iL_j}}{i}\right), \tag{2.62}$$

where AC indicates the annual costs (excluding pump operating costs), OP is the first year pump operating costs and the last term is the total termination costs. From eqn. 2.62,

$$\lim_{L \to \infty} q(Q, T_i, L) = \frac{AC(Q)}{i} + \frac{OP(Q)}{\alpha}$$

and therefore $C(Q, T_i, L)$ approaches $C(Q)$ as L gets large. On the other hand, the terms R_1 and R_2 in eqns. 2.53 and 2.54 depend only on Q as L approaches infinity and therefore for L large enough, π is dependent only on Q. The validity of eqn. 2.61 is thus established.

Let Q' be the maximand of $\pi(Q)$ which is defined by eqn. 2.61. It follows that

$$\lim_{L \to \infty} Q^*(L) = Q',$$

which implies that for L larger than some L', not only the pumping rate but the other quantities of interest, namely the injection temperature T_i^*, the breakthrough time τ^* and the cost function C remain effectively constant. Our computational experience indicates that for most cases, L' is around 100 years.

The asymptotic behavior of the profit function and the convergence of $\pi(L, Q)$ to $\pi(Q)$ and $Q^*(L)$ to Q' is illustrated in Tables 2. 12 and 2. 13. Here increasing values of L are given in the second column. For each L, the value of $\pi[L, Q^*(L)]$ and $Q^*(L)$ are shown as well as the corresponding values for optimal heat exchanger area A^*, injection temperature T_i^*, breakthrough time τ^*, total costs $C(Q, L, T_i)$ and pump capacity and operating costs (all optimal with respect to the given value of L).

Note also that in the base case presented in Tables 2. 12 and 2. 13 ($r = 0.024$, $i = 0.10$), the optimal life occurs at 25 years which is the assumed well life. Because a second well cost must be incurred if project life is greater than one well life, there is always a local maximum for the profit function $\pi[L, Q^*(L)]$ at L = well life. In the base case this local maximum exceeds $\pi(Q)$ and hence L = 25 is optimal. However, when the value of the energy is allowed to increase at a faster rate than $r = 0.024$ (or alternatively if the discount rates are small) $\pi(Q)$ is larger than peaks attained at L = WL and large lifetimes are optimal. This can be seen in Figure 2. 4 where for $r = 0.03$, $\pi(Q) \geq \pi(L, Q)$ for all L when $i \leq 0.12$. Note, however, that as i increases, $\pi(Q)$, the plateau of the profit function gets closer to zero, so that there is an i above which the local maximum at the well life dominates $\pi(Q)$. For $r = 0.03$, this interest rate is 0. 15 (as seen in Figure 2. 4, $\pi[25, Q^*(25)]$ dominates $\pi(Q)$). As we further increase i, the present worth of maximum profits decreases until at $i = 0.32$ when $\pi^* = 0$ (which means, incidentally, that the so-called internal rate of return is 32% for this particular value of r). For $i > 0.15$, the economic life remains at 25 years. Note that 32% is the maximum internal rate

Table 2.12

PROFIT AND DECISION VARIABLES RESPONSE TO LIFE
EXPONENTIAL GROWTH MODEL
(i = 0.10, r = 0.24)

$\Pi[L,Q^*(L)]$ ($10^3)	L (YRS)	$Q^*(L)$ (m^3/hr)	A^* (m^2)	T_1^* (°C)	τ^* (YRS)	$C(Q^*,T_1^*,L)$ ($10^3)	PUMPS ($10^3)	POC ($10^3)
47.51	5.	470.	447.	109.27	1.45	2021.8	96.0	655.4
345.50	10.	435.	432.	109.22	1.56	2488.7	98.0	950.1
443.58	15.	405.	396.	109.24	1.68	2740.4	113.1	1057.5
529.06	20.	395.	391.	109.22	1.72	2901.6	112.9	1157.6
560.89	25.	385.	379.	109.23	1.77	2993.9	117.0	1199.3
519.18	30.	380.	376.	109.22	1.79	3122.4	116.0	1234.6
532.02	35.	375.	371.	109.22	1.81	3151.7	116.6	1246.9
544.51	40.	370.	367.	109.22	1.84	3153.6	114.6	1244.0
550.38	45.	370.	367.	109.22	1.84	3181.2	115.7	1263.7
555.85	50.	370.	367.	109.22	1.84	3198.2	115.9	1277.3
552.71	55.	370.	367.	109.22	1.84	3216.3	116.4	1286.5
555.23	60.	365.	362	109.22	1.86	3185.9	113.8	1259.5
556.65	65.	365.	363.	109.22	1.86	3191.2	114.0	1263.7
557.83	70.	365.	363.	109.22	1.86	3194.5	114.0	1266.5
558.52	75.	365.	363.	109.22	1.86	3196.9	114.0	1268.5
558.59	80.	365.	363.	109.22	1.86	3198.9	114.1	1269.9
558.93	85.	365.	363.	109.22	1.86	3200.0	114.1	1270.8
559.19	90.	365.	363.	109.22	1.86	3200.7	114.1	1271.4
559.35	95.	365.	363.	109.22	1.86	3201.2	114.1	1271.8
559.47	100.	365.	363.	109.22	1.86	3201.5	114.1	1272.1
559.50	105.	365.	363.	109.22	1.86	3201.7	114.1	1272.3
559.56	110.	365.	363.	109.22	1.86	3201.9	114.1	1272.5
559.60	115.	365.	363.	109.22	1.86	3202.0	114.1	1272.6
559.62	120.	365.	363.	109.22	1.86	3202.1	114.1	1272.6
559.64	125.	365.	363.	109.22	1.86	3202.1	114.1	1272.7
559.65	130.	365.	363.	109.22	1.86	3202.1	114.1	1272.7
559.65	135.	365.	363.	109.22	1.86	3202.2	114.1	1272.7
559.66	140.	365.	363.	109.22	1.86	3202.2	114.1	1272.7
559.66	145.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.66	150.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.66	155.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	160.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	165.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	170.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	175.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	180.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	185.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	190.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	195.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	200.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	205.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	210.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	215.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	220.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	225.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	230.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	235.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	240.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	245.	365.	363.	109.22	1.86	3202.2	114.1	1272.8
559.67	250.	365.	363.	109.22	1.86	3202.2	114.1	1272.8

Table 2.13

PROFIT AND DECISION VARIABLES RESPONSE TO LIFE

LINEAR GROWTH MODEL

$(i = 0.10,\ r = 0.24)$

$\Pi[L,Q^*(L)]$ ($\$10^3$)	L (YRS)	$Q^*(L)$ (m^3/hr)	A^* (m^2)	T_i^* (°C)	τ^* (YRS)	$C(Q^*,T_i^*,L)$ ($\$10^3$)	PUMPS ($\$10^3$)	POC ($\$10^3$)
45.37	5.	470.	447.	109.27	1.45	2020.4	96.0	654.0
336.54	10.	435.	432.	109.22	1.56	2481.9	98.0	943.4
424.89	15.	405.	395.	109.24	1.68	2726.1	113.1	1043.4
500.12	20.	395.	389.	109.23	1.72	2877.8	112.9	1134.2
522.50	25.	385.	378.	109.23	1.77	2961.3	117.0	1167.1
472.82	30.	380.	374.	109.23	1.79	3081.3	116.0	1194.0
479.18	35.	375.	369.	109.23	1.81	3103.6	116.6	1199.4
486.68	40.	375.	369.	109.23	1.81	3135.8	117.2	1222.4
488.66	45.	370.	364.	109.23	1.84	3122.4	115.7	1205.6
491.36	50.	370.	365.	109.23	1.84	3135.1	115.9	1214.9
486.18	55.	370.	365.	109.23	1.84	3149.8	116.4	1220.8
487.18	60.	370.	365.	109.23	1.84	3154.9	116.4	1224.6
487.48	65.	370.	365.	109.23	1.84	3158.3	116.6	1227.1
487.86	70.	370.	365.	109.23	1.84	3160.3	116.6	1228.6
487.99	75.	370.	365.	109.23	1.84	3161.7	116.7	1229.6
487.65	80.	370.	365.	109.23	1.84	3163.0	116.7	1230.2
487.70	85.	370.	365.	109.23	1.84	3163.5	116.7	1230.6
487.76	90.	370.	365.	109.23	1.84	3163.8	116.7	1230.8
487.78	95.	370.	365.	109.23	1.84	3164.0	116.7	1231.0
487.80	100.	370.	365.	109.23	1.84	3164.2	116.7	1231.1
487.77	105.	370.	365.	109.23	1.84	3164.3	116.7	1231.1
487.78	110.	370.	365.	109.23	1.84	3164.3	116.7	1231.2
487.78	115.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	120.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	125.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	130.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	135.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	140.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	145.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	150.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	155.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	160.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	165.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	170.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	175.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	180.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	185.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	190.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	195.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	200.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	205.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	210.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	215.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	220.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	225.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	230.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	235.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	240.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	245.	370.	365.	109.23	1.84	3164.4	116.7	1231.2
487.78	250.	370.	365.	109.23	1.84	3164.4	116.7	1231.2

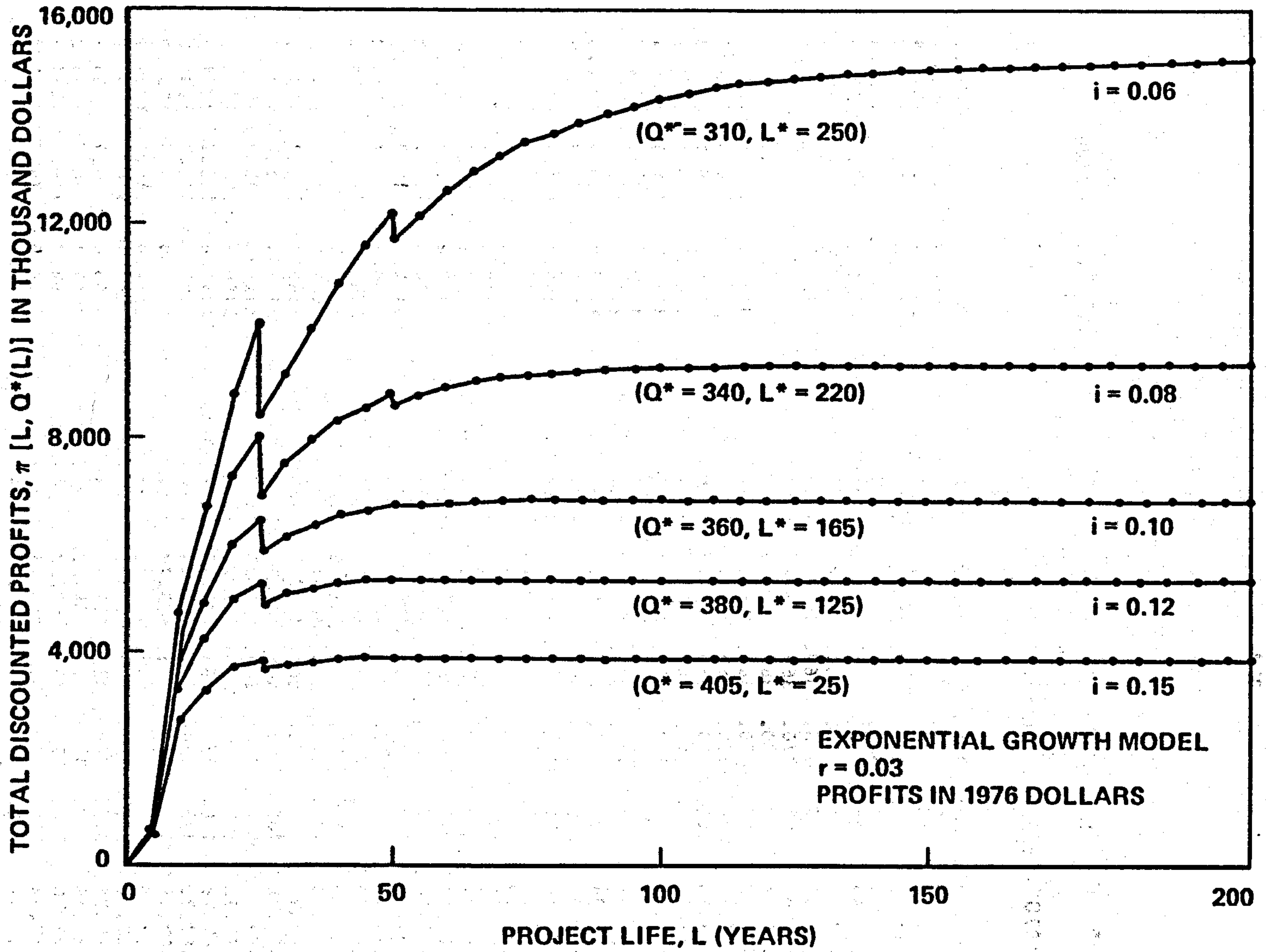

FIGURE 2.4 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN INTEREST RATES

of return in our range of growth rates. The minimum internal rate of return for the project is obtained when r = 0. This minimum rate is 25.6 %.

2.6.9 Practical Significance of the 'Profit Plateau'

In evaluating the significance of the 'profit plateau' one should bear in mind the assumption that the real value of energy increases indefinitely with time. In view of the great uncertainty surrounding future energy prices, it is difficult to support the position that energy value will rise forever. Perhaps its rising trajectory will slow and/or actually decline after a decade or two, in which case L^* would be much less than some of the values indicated in these results. We conclude that the 'profit plateau' and associated long reservoir lives should be interpreted as indicating optimal reservoir lives of at least five or six decades, as opposed to, say, two or three. Furthermore, it seems worth repeating that over a very wide range of L^*, the variation in Q^* is quite small, suggesting that decisions on whether to pump for 25 or 60 years are not necessary at the outset of production.

2.6.10 Effect of δ

In the discussion above, the effect of δ, the minimum allowable temperature difference between production and injection temperature, has been neglected. In Tables 2.12 and 2.13, the optimal pumping rates are small enough that even after 250 years the temperature difference is still greater than 6° C, and hence the pumping rates remain unaffected during the period under consideration (250 years). In fact, for every positive δ, there is an L_δ such that $Q^*(L)$ and hence the corresponding profits, $\pi[L, Q^*(L)]$, will decrease as L increases beyond L_δ. Thus L_δ signifies the end of the plateau region of the profit function. This phenomenon is observed in Figures 2.8 and 2.9 where for higher production temperatures (which imply higher flow rates), $Q^*(L)$ and π decrease after the corresponding L_δ.

Note that by eqns. 2.12 and 2.19, the temperature difference is a decreasing function of $L \cdot Q(L)$. If no restriction existed on the temperature difference, the pumping rate $Q^*(L)$ would remain constant for $L > L'$. Since $g(L_\delta/t_u) = \delta$, it follows that as L increases beyond L_δ, the value of $Q^*(L)$ decreases so that the δ constraint is not violated.

The quantity $Q^*(L_\delta) \cdot L_\delta$ can be easily computed. Noting that the first term in eqn. 2.19 dominates the other two ($\psi_1 = \frac{1}{47.5}\psi_2 = \frac{1}{580}\psi_3$), eqn. 2.17 can be written as

$$\gamma_1 e^{-\psi_1 Q^*(L_\delta) \cdot L_\delta} \approx \frac{\delta}{T_o - T_i}$$

yielding

$$Q^*(L_\delta) \cdot L_\delta = \frac{1}{\psi_1} \ln \frac{\gamma_1(T_o - T_i)}{\delta} \quad . \tag{2.63}$$

Unless the value for δ is chosen very large (and therefore not permitting $Q(L)$ to approach Q'), $Q^*(L_\delta) = Q'$. Note from eqn. 2.63 that for fixed Q, T_o increases the value for L_δ. However, as T_o increases, $Q^*(L)$ increases for all L (and hence for L_δ), reducing the value for L_δ. The total effect of increase in T_o is that

$$\frac{1}{\psi_1 Q^*(L_\delta)} \quad \ln \frac{\gamma_1(T_o - T_i)}{\delta}$$

and hence L_δ decreases as T_o increases. Figure 2.9 illustrates this point.

2.6.11 Optimal Injection Temperature

Tables 2.14 and 2.15 give the optimal injection temperature T_i^*. The optimal injection temperatures decrease slightly as r increases and increase slightly as i increases. This can be explained by the fact that at higher pumping rates, it becomes more expensive to attain lower injection temperatures.

Table 2.14

OPTIMAL INJECTION TEMPERATURE, T_i^* (°C)
FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	109.23	109.25	109.26	109.27	109.28
0.010	109.23	109.24	109.25	109.26	109.27
0.020	109.19	109.22	109.23	109.24	109.26
0.024	109.18	109.20	109.23	109.24	109.25
0.030	109.15	109.18	109.20	109.22	109.25
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.15

OPTIMAL INJECTION TEMPERATURE, T_i^* (°C)
FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	109.23	109.25	109.26	109.27	109.28
0.010	109.23	109.24	109.25	109.26	109.27
0.020	109.22	109.23	109.24	109.25	109.26
0.024	109.21	109.22	109.23	109.24	109.26
0.030	109.19	109.21	109.22	109.23	109.25
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

However, although there is some small variation of T_i^* with r and i, the most important information conveyed by this table is that: 1) the value of T_i^* is remarkably stable with respect to i and r, and 2) this value of T_i^* is very close to T_s (the temperature of 5 psi steam: 109 °C). The resulting high costs of heat exchanger equipment are evidently offset by the value of the extra energy extracted by having T_i^* close to T_s.

Note from the footnote on page 23 that although the production temperature T_o^t increases as T_i increases, the difference $T_o^t - T_i$ which determines the amount of recoverable heat at time t, decreases as T_i increases. This confirms eqns. 2.55 and 2.56 which show a decrease in total revenues as T_i increases. We conclude that, although injecting the brine at a high temperature prolongs the duration of time that the production temperature remains above a specified level, it has no effect on prolonging the economic lifetime of the project.

2.6.12 Optimal Heat Exchanger Area

Tables 2.16 and 2.17 give the optimal heat exchange areas. The optimal areas increase with i and decrease with r. As i increases and r decreases, the optimal flow rates increase, thus requiring larger heat exchanger areas, even though the fluid is injected at higher temperatures.

2.6.13 Optimal Breakthrough Time

Tables 2.18 and 2.19 give the optimal breakthrough times τ^*. These times are inversely proportional to Q^*. Note that the optimal breakthrough times occur very early in the project, namely during the first two years — long before L^*.

Table 2. 16

OPTIMAL HEAT EXCHANGER AREA, A^* (square meters)
FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	372.	378.	385.	387.	396.
0.010	358.	365.	372.	379.	389.
0.020	331.	371.	377.	384.	393.
0.024	331.	348.	379.	386.	395.
0.030	329.	349.	362.	376.	393.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2. 17

OPTIMAL HEAT EXCHANGER AREA, A^* (square meters)
FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	372.	378.	385.	387.	396.
0.010	357.	365.	372.	379.	395.
0.020	362.	369.	376.	383.	393.
0.024	364.	371.	378.	384.	394.
0.030	332.	373.	380.	387.	396.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.18

OPTIMAL BREAKTHROUGH TIME, τ^* (years) FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	1.79	1.75	1.70	1.68	1.62
0.010	1.86	1.81	1.77	1.72	1.66
0.020	2.09	1.81	1.77	1.72	1.66
0.024	2.13	1.97	1.77	1.72	1.66
0.030	2.20	2.00	1.89	1.79	1.68
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.19

OPTIMAL BREAKTHROUGH TIME, τ^* (years) FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	1.79	1.75	1.70	1.68	1.62
0.010	1.86	1.81	1.77	1.72	1.64
0.020	1.86	1.81	1.77	1.72	1.66
0.024	1.86	1.81	1.77	1.72	1.66
0.030	2.09	1.81	1.77	1.72	1.66
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

2.6.14 Costs

Tables 2.20 and 2.21 present total project costs and Tables 2.22 and 2.23 the total operating costs for pumps. Note that the pump operating costs, which are mainly the cost of the electricity used to operate the pumps, constitute a major portion of total costs. The electricity cost increases with time at the rate of e^{rt} or $1 + rt$ depending on the growth model. The pump operating costs constitute between 35 and 46% of total costs for the exponential model and between 35 and 41% for the linear model. Tables 2.20 through 2.23 show that as r increases and i decreases, not only do the pump operating costs and total costs increase, but the ratios of pump operating costs to total costs increase as well.

2.6.15 "Average Cost" per MBTU

Based on the optimal decision variables and costs, we can also compute a quantity which gives a measure of average costs of generating one MBTU of steam heat over the lifetime of the project. For example, let i = 0.10 and r = 0.024. Then for both models, $Q^* = 385\ m^3/hr$, $L^* = 25$ years, $T_i^* = 109.23°C$ and $\tau^* = 1.77$ years. Total heat produced, THP, is therefore

$$THP = 34.76 Q^* c_f \rho_f (T_o - T_i^*) \tau^* + \int_{\tau^*}^{L^*} 34.76 Q^* c_f \rho_f (T_o - T_i^*) g(t)\, dt$$

$$= 34.76 \times 385 \times 0.92\,(150-109.23) \left[1.77 + \int_{1.77}^{25} (0.338\, e^{-0.0013t} + 0.337\, e^{-0.06177t} + 1.368\, e^{-7.5386t})\, dt \right]$$

$$= 7{,}472{,}778 \text{ MBTU}\ .$$

Table 2.20

PRESENT WORTH OF ALL COSTS, $C(Q^*, T_i^*, L^*)$ ($1976, $1000) FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	3296.	2988.	2743.	2520.	2295.
0.010	3581.	3171.	2858.	2615.	2344.
0.020	4700.	3290.	2952.	2692.	2402.
0.024	4848.	3787.	2994.	2725.	2427.
0.030	5109.	3922.	3272.	2890.	2440.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.21

PRESENT WORTH OF ALL COSTS, $C(Q^*, T_i^*, L^*)$ ($1976, $1000) FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	3296.	2988.	2743.	2520.	2295.
0.010	3571.	3164.	2853.	2611.	2318.
0.020	3692.	3260.	2930.	2675.	2390.
0.024	3740.	3299.	2961.	2700.	2410.
0.030	4701.	3356.	3008.	2738.	2439.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.22

PRESENT WORTH OF PUMP OPERATING COSTS ($1976, $1000)
FOR EXPONENTIAL GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	1217.	1096.	998.	895.	803.
0.010	1380.	1203.	1065.	958.	838.
0.020	1936.	1319.	1158.	1034.	895.
0.024	2088.	1550.	1199.	1067.	919.
0.030	2357.	1688.	1346.	1161.	936.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Table 2.23

PRESENT WORTH OF PUMP OPERATING COSTS ($1976, $1000)
FOR LINEAR GROWTH MODEL

r \ i	0.06	0.08	0.10	0.12	0.15
0.000	1217.	1096.	998.	895.	803.
0.010	1371.	1196.	1060.	954.	829.
0.020	1490.	1290.	1137.	1017.	883.
0.024	1538.	1328.	1167.	1042.	902.
0.030	1936.	1385.	1213.	1080.	931.
P_o $/MBTU	1.104	1.107	1.110	1.113	1.118

Let C_a be an average yearly cost, so that the total discounted cost if annual payments of C_a are made for L^* years would be equal to $C(Q^*, T_i^*, L^*)$:

$$C(Q^*, T_i^*, L^*) = \int_0^{L^*} C_a e^{-it} dt .$$

With $C(Q^*, T_i^*, L^*) = \$2,993,900$ and $i = 0.10$, C_a equals \$326,163. The total undiscounted costs $= C_a \cdot L^* = \$8,154,075$, which makes the average cost for the exponential model equal to :

$$\$8,154,075/7,472,778 \text{ MBTU} = 1.09 \text{ \$/MBTU} .$$

Similarly, for the linear model, the total undiscounted costs would be \$8,070,993, yielding an average cost of 1.08 \$/MBTU. Comparing these two values, we see that the difference between results with each growth model is not great.

We wish to emphasize that these are <u>average</u> values over the life of the project when $T_o = 150\,^\circ C$. The unit cost of steam heat would be lower at first and higher toward the end of the project. These averages would be substantially lower if the initial temperature were higher than 150 °C.

2.6.16 Sensitivity Analysis

In this section we discuss the sensitivity of the optimal decision variables to changes in the parameters of the model. The rate of increase of the real value of energy has been assumed exponential (the exponential growth model) and the values for i and r are 0.10 and 0.024, respectively. Results for the linear model are generally similar to those of the exponential model, except that profits are lower and optimal pumping rates are higher for the linear model. The plots in Figures 2.5 through 2.8 and 2.10 through 2.14 represent profits, $\pi[L, Q^*(L)]$, as a function of L, for different values of the parameter under consideration. The maximum of $\pi[L, Q^*(L)]$ is of course π^*.

2.6.17 Sensitivity to Well Life

Figure 2.5 presents the behavior of the profit function with respect to changes in expected well life (WL). The well life has a great effect on both the profits and the optimal project life. The predominance of 25-year optimal lives in Tables 2.10 and 2.11 is due to the fact that the well life has been chosen as 25 years in the basic model. In fact, it seems that with the exception of the case where r = 0, the optimal lives are either equal to the well lives or at $L^\dagger$, the point where $\pi[L, Q^*(L)]$ reaches a plateau. As seen in Figure 2.5, the optimal project life when well life is below 40 years is equal to the well life. At WL = 40, the global maximum of π with respect to L is slightly higher than the local maximum at WL = 40. Had the interest rates been higher than 10, the plateau of the profit curves would have been lower so that even at WL = 40 years, L^* would be 40 years.

Note the discontinuity of the profit function at multiples of the well life in Figure 2.5. These discontinuities can be easily explained by the fact that new wells have to be installed at multiples of well life. For instance, if WL = 10, there is a sudden decrease in the amount of profits if L = 11, because the entire new well cost must be paid even though only one additional year of heat is produced. For longer lives, the discontinuities become less significant, because the discount rate reduces the incremental cost of each well, and because each cost increment becomes progressively smaller compared to total costs. After 50 years the profit function is relatively stable with respect to L, implying that the present worth of the well cost paid in full during the 51st year does not affect the present worth of profits significantly.

Note that well life has a significant effect on maximum profits. If expected well life increases from 10 years to 25 years, maximum profits increase from $420,570 to $560,890. If well life increases another 15 years

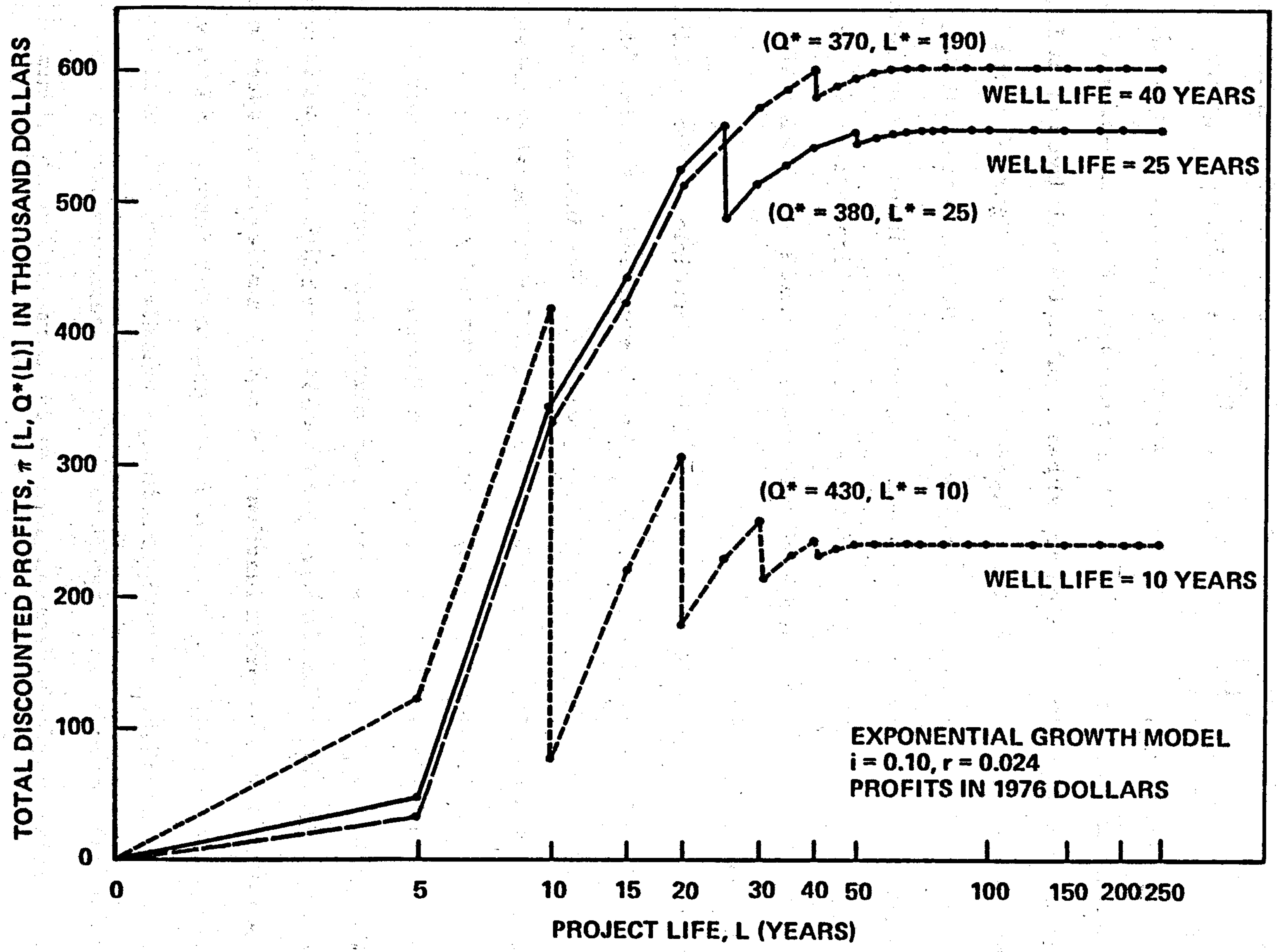

FIGURE 2. 5 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN WELL LIVES

from 25 to 40 years, maximum profits increase from $560, 890 to $606, 970. We conclude that the gains from prolonging well life (perhaps by extra maintenance expenditures) are substantial, but characterized by decreasing returns to additional "life-prolonging" efforts.

2.6.18 Sensitivity to Aquifer Porosity and Permeability

Figures 2.6 and 2.7 deal with significant geothermal parameters, namely porosity ϕ, and intrinsic permeability k. Figure 2.6 summarizes the effect of the uncertainty in the value of porosity. If porosity is 10% instead of 20% (base case), the optimal profit is only $493, 380 instead of $560, 890, a decrease of 12%. Note, however, that the optimal pumping rate is not at all sensitive to porosity in this range. If porosity is 30%, the maximum profit is $628, 190, an increase of 12%. This time, though, the optimal decision variable changes slightly: the pumping rate is reduced from 380 m^3/hr to 370 m^3/hr. Nevertheless, it is clear that the model is remarkably robust in determining the optimal pumping rate over a wide range of porosities. This is an important result, because it suggests major expenditures to accurately determine porosity in order to compute the "correct" pumping rate would probably not be warranted.

On the other hand, the effect of uncertainties in intrinsic permeability is greater than that for porosity. As seen in Figure 2.6, reduction in permeability from 200 millidarcies (base case) to 150 millidarcies reduces the optimal profits from $560, 890 to $241, 620, a reduction of 57%, and optimal pumping rate from 380 m^3/hr to 310 m^3/hr, a reduction of about 18%. An increase in permeability to 250 increases the profits to $859, 300, an increase of 53%, while Q^* increases 13% to 430 m^3/hr. The permeability governs pumping costs. As permeability increases, the drawdown in the production well decreases, thereby requiring smaller pumps and less energy for pumping the

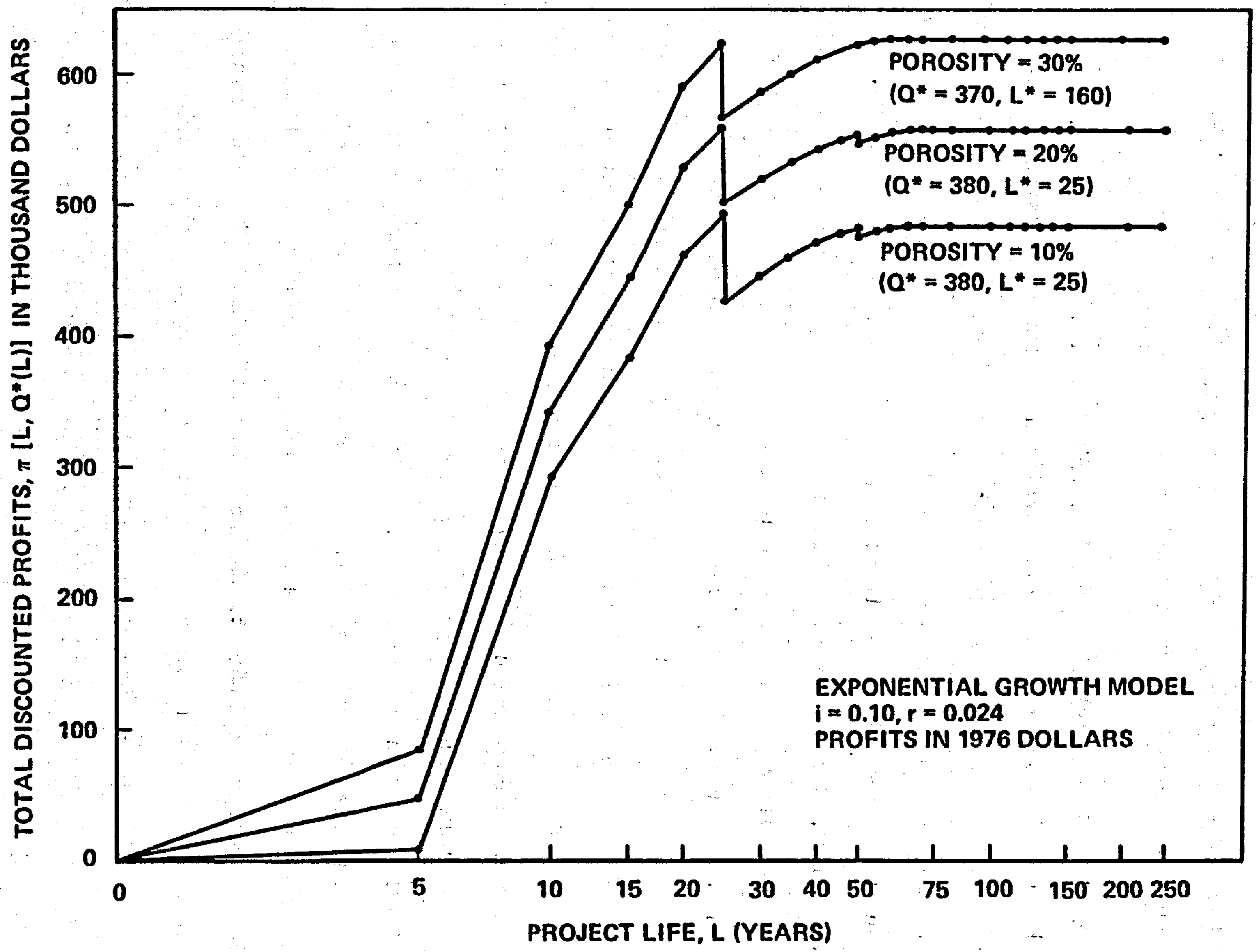

FIGURE 2.6 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN POROSITIES

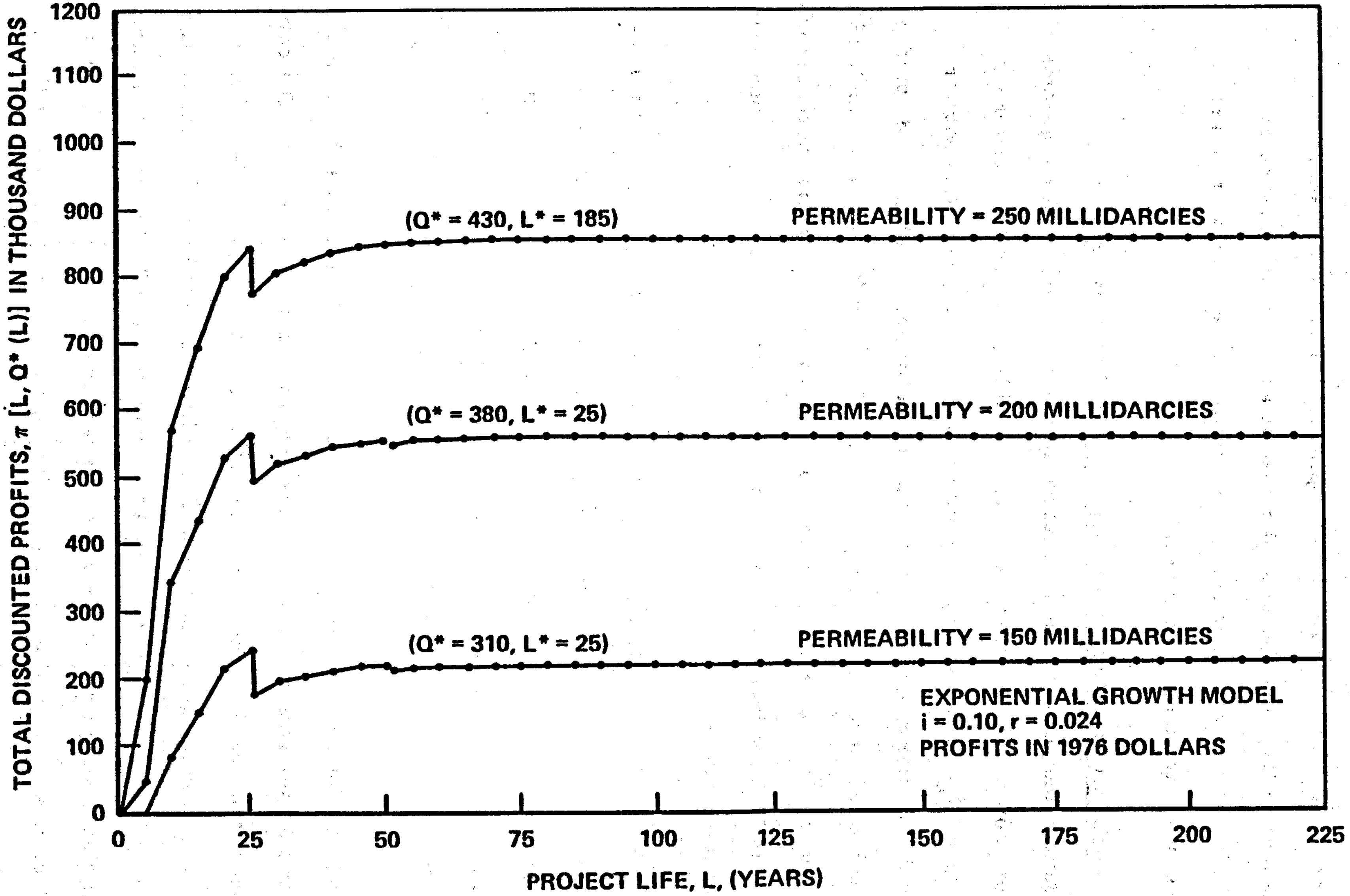

FIGURE 2. 7 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN PERMEABILITIES

same amount of flow. As a result, the optimal profits are increases. We conclude that unlike porosity, expenditures for accurate information on permeability may be very important, for profits and design flow rate are indeed sensitive to values of permeability.

2.6.19 Sensitivity to Initial Aquifer Equilibrium Temperature

Among the physical geothermal parameters, the one with the greatest impact on profits and the optimal decision variable is the initial equilibrium temperature, T_o. This is the temperature at which the geothermal aquifer fluid and the aquifer matrix are in thermal equilibrium. In Figure 2.8, profits have been plotted as function of project life for $T_o = 150\,^\circ C$ (our base case), $160\,^\circ C$, $170\,^\circ C$, and $180\,^\circ C$. As seen from this figure, optimal profits are increased with temperature in a nonlinear manner. Optimal profits and flow rates as functions of temperature can be tabulated as follows:

T_o (°C)	π^* ($)	Q^* (m^3/hr)	L^* (years)
150	560,890	380	50
160	1,982,890	875	150
170	5,007,890	2400	75
180	10,478,610	4900	50

In general, as the equilibrium temperature increases, it is optimal to extract energy at a higher rate and terminate the project in a shorter period. The only exception to this trend is when the temperature is 150 °C. Since the profit is rather low at this temperature, the well cost has a severe impact on the profits, thus making it imperative to terminate the project at L = 25. Even in this case, the total discounted profits if the project is terminated at L = 25 are not much higher than total profits had the project life been say, 200 years ($560,890 vs. $559,510).

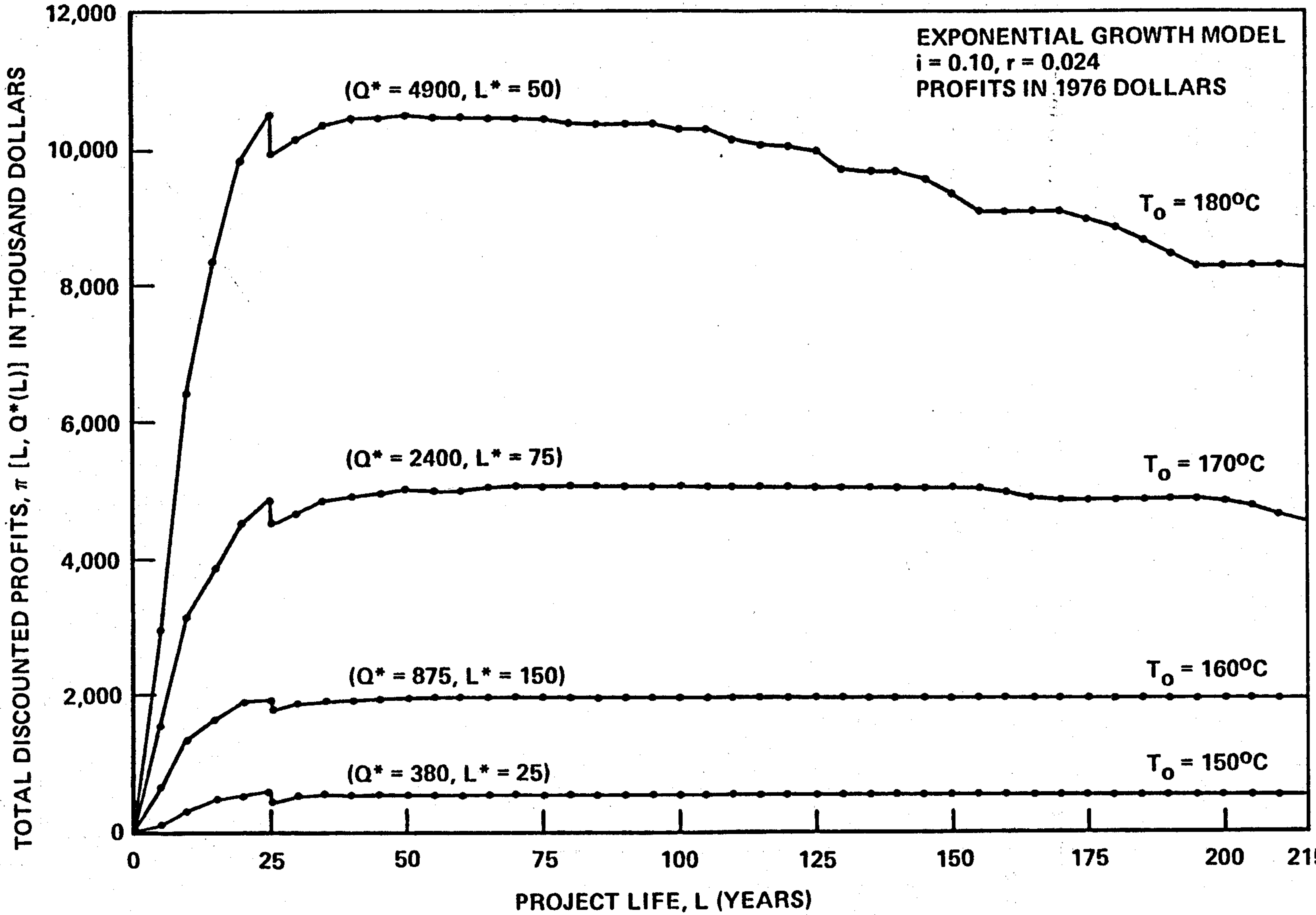

FIGURE 2. 8 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN INITIAL TEMPERATURES

Figure 2.9 gives a plot of the optimal flow rate as a function of project life. As seen from the plot, the δ constraint of 6 degrees does not have any effect on the pumping rate when T_o = 150 or 160 °C, because the optimal flow rates are so low that even at L = 250 years, the constraint is not violated when Q has reached its plateau level Q'. Note, however, that at the higher temperatures (when Q' is higher), the δ requirement forces $Q^*(L)$ to decrease as L increases. The "reduction point" occurs at 150 years when T_o = 170 °C and 55 years when T_o = 180 °C.

It is interesting that δ does not have any effect on Q^* or π^* (that is, we would have obtained the same Q^* and π^* even if δ had been zero. However, $Q^*(L)$ would have been larger for larger lifetimes). For both of these temperatures, the profits have peaked at lifetimes shorter than the "reduction point." The explanation for this is that when T_o = 170 °C for instance, the plateau flow rate is Q' = 2400 m^3/hr and at L = 75 years, the term containing L in the revenue function (eqn. 2.20) is negligible, so that the effect of increase in L is minimal. However, the cost function is still increasing in L (since L is not large enough) and therefore as L increases, the profit, which is the difference of these two terms decreases after L = 75 years.

2.6.20 Sensitivity to Economic Parameters

(i) Cost of Electricity

Figures 2.10 through 2.14 deal with economic parameters. The economic parameter that most influences profits and optimal flow rate is the present day cost of electricity for pumping.* Figure 2.10 shows that if electricity could be obtained at a lower cost, profits, optimal flow rate, and

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* Recall that electric power costs are assumed to rise with time at the same rate as the value of energy, namely, according to the particular growth model.

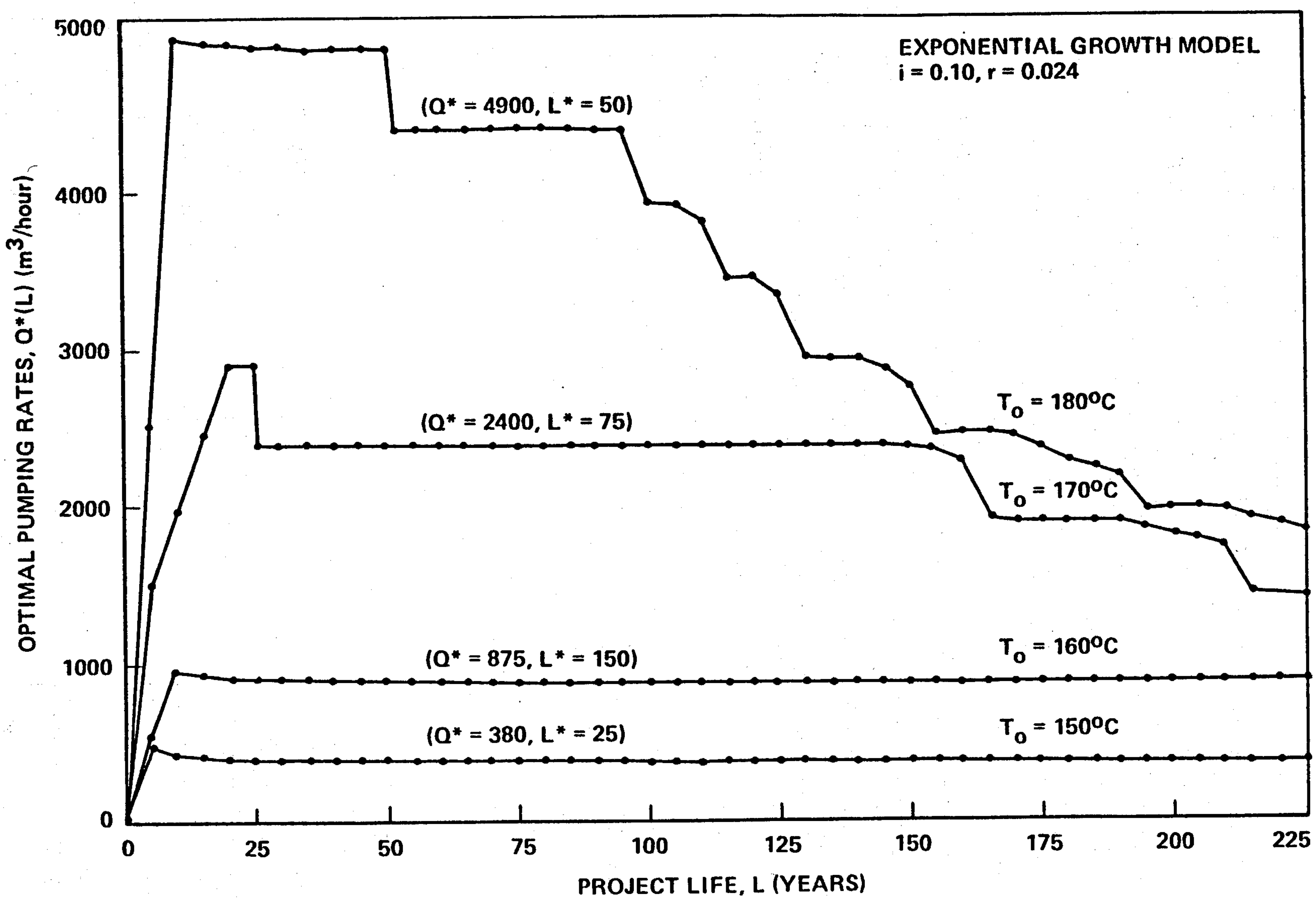

FIGURE 2. 9 OPTIMAL PUMPING RATES vs. PROJECT LIFE FOR GIVEN INITIAL TEMPERATURES

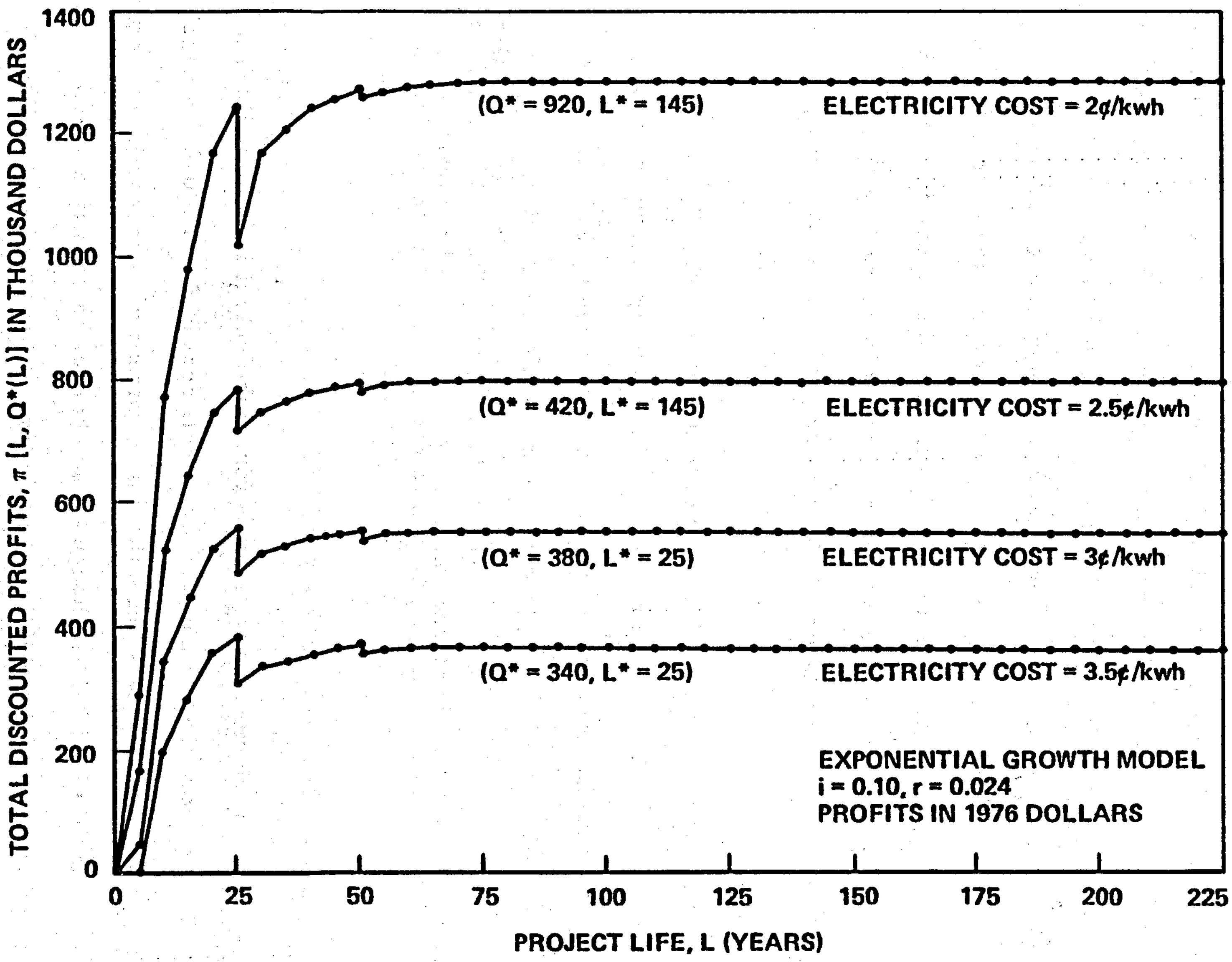

FIGURE 2.10 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN ELECTRICITY COSTS

economic lifetime would increase. Since pumping energy cost constitutes a major portion of the total costs, this result is hardly surprising. However, the magnitude of the increase in profits is interesting. The total discounted optimal profits increase by 43% when electricity cost decreases from 3¢ /kwh (base case) to 2.5¢ /kwh and by 130% when it decreases to 2¢ /kwh. It is interesting to observe that the optimal pumping rate increases by only 10.5% from 380 m^3/hr to 420 m^3/hr when electricity cost decreases from 3¢ /kwh to 2.5¢ /kwh, while it increases by 142% to 920 m^3/hr when electricity cost is decreased to 2¢/kwh. We conclude that when pumping is necessary, pumping energy plays a major role in the engineering-economics of geothermal energy production.

(ii) Royalty

The behavior of profits with respect to changes in royalty are more uniform. Figure 2.11 shows optimal profits decrease by 37% when royalty is increased from 10% (base case) to 15% while it increases by 38% when royalty is decreased to 5%. Although these changes in royalty cause significant changes in profits, they hardly affect Q^*, as Figure 2.11 shows. We conclude that the amount of royalty paid is of more concern to the investor than to the design engineer.

(iii) Well Cost

The effect of changes in well costs for each doublet is summarized in Figure 2.12. An increase of $200,000 over well cost of $600,000 (base case) reduces optimal profits by 52% while the same amount of decrease in well cost increases the profits by 57%. Although these changes in royalty and well cost do not significantly alter the optimal pumping rate, it is clear that well cost plays a major role in determining the economic viability of this type (nonelectric) of geothermal project.

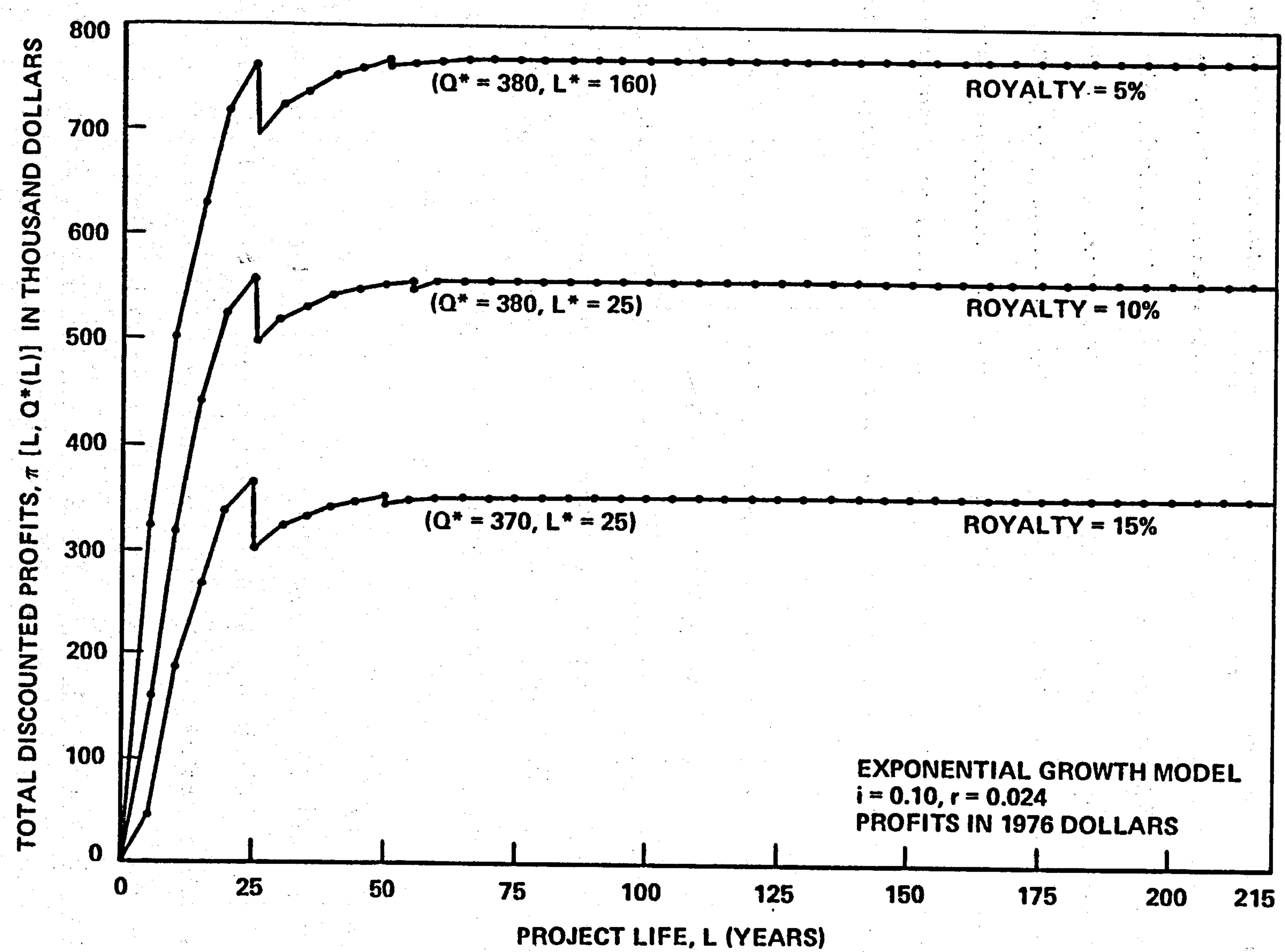

FIGURE 2.11 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN ROYALTIES

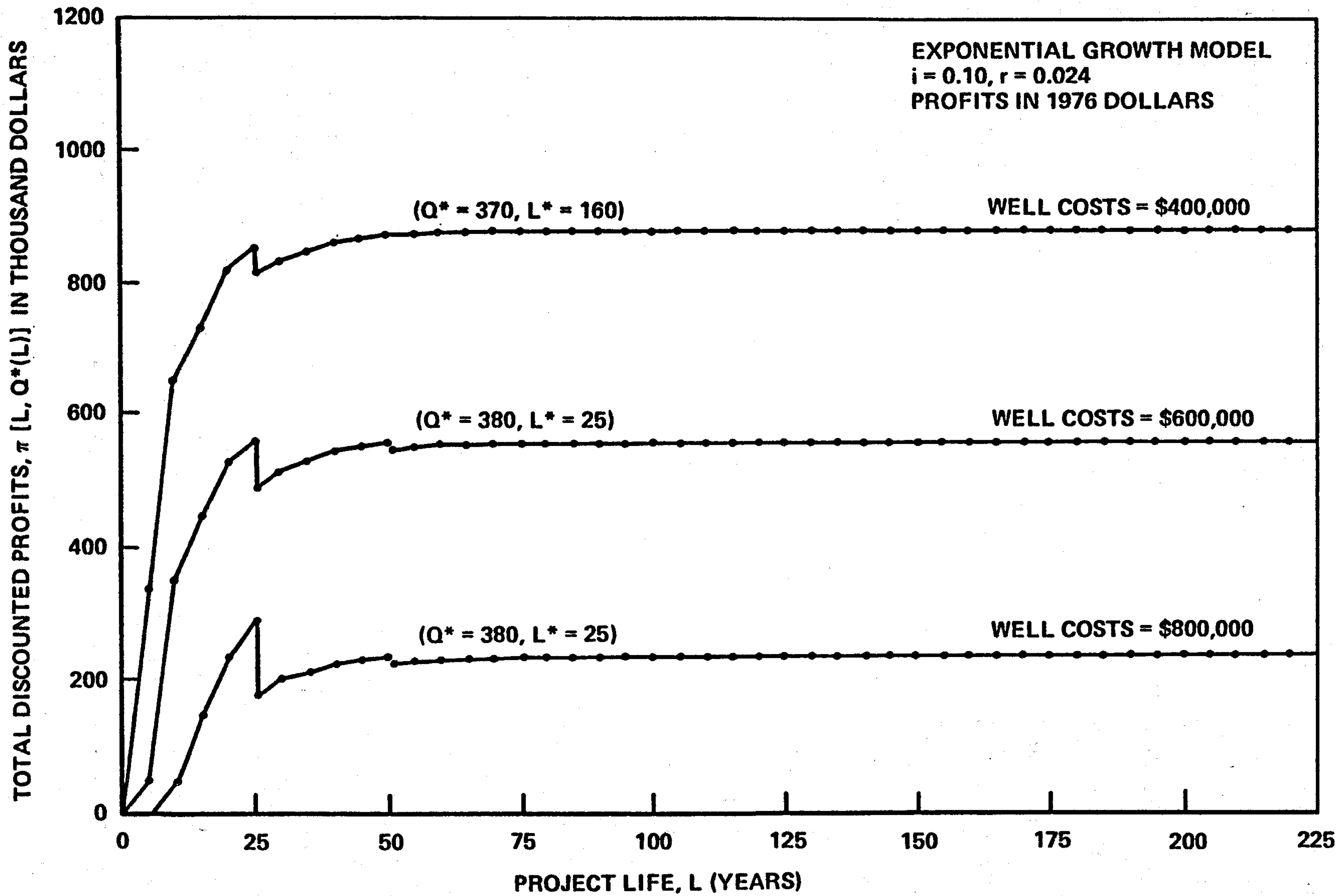

FIGURE 2.12 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN WELL COSTS

At this point it is appropriate to make a comment about the long economic lives in some of the cases presented in Figures 2.10, 2.11, and 2.12. In none of these cases is the optimal profit much higher than the profits at 25 years. For example, if WC = $400,000, the present worth of profits at 25 years is $854,840 while the present worth of profits at the optimal life of 160 years is $879,850.

(iv) Land Rent and Salaries

Figure 2.13 shows effects of changes in land rents and salaries. In our basic model this total amounts to $54,000/year. An increase of this total annual cost to $70,000 decreases the optimal profit by 26% while an increase to $100,000 decreases the profits by 75%. However, the optimal pumping rate and economic life remain constant at 380 m^3/hr and 25 years, respectively, in the face of these changes.

(v) Well Maintenance Costs

The effect of the annual well maintenance cost is the least among the main economic parameters. An increase of 50% in these costs reduces optimal profits by only 5% while doubling these costs reduces profits by less than 10%. Again the optimal pumping rate and economic lifetime remain unchanged. This is an interesting result when considered together with the effect of expected well life on profits. In section 2.6.16 we saw that there were significant gains in profits if well life could be prolonged. Here we see that this approach to well life prolongation — active well maintenance — is not very costly. Although we present no relationship between well maintenance expenditures and well life, it is reasonable to speculate that a good well maintenance program could be fairly cost beneficial. However, recalling that gains in profits were increasing at a decreasing rate as well life increased, we can also see that there would be some "optimal" maintenance expenditure, beyond which expenditure

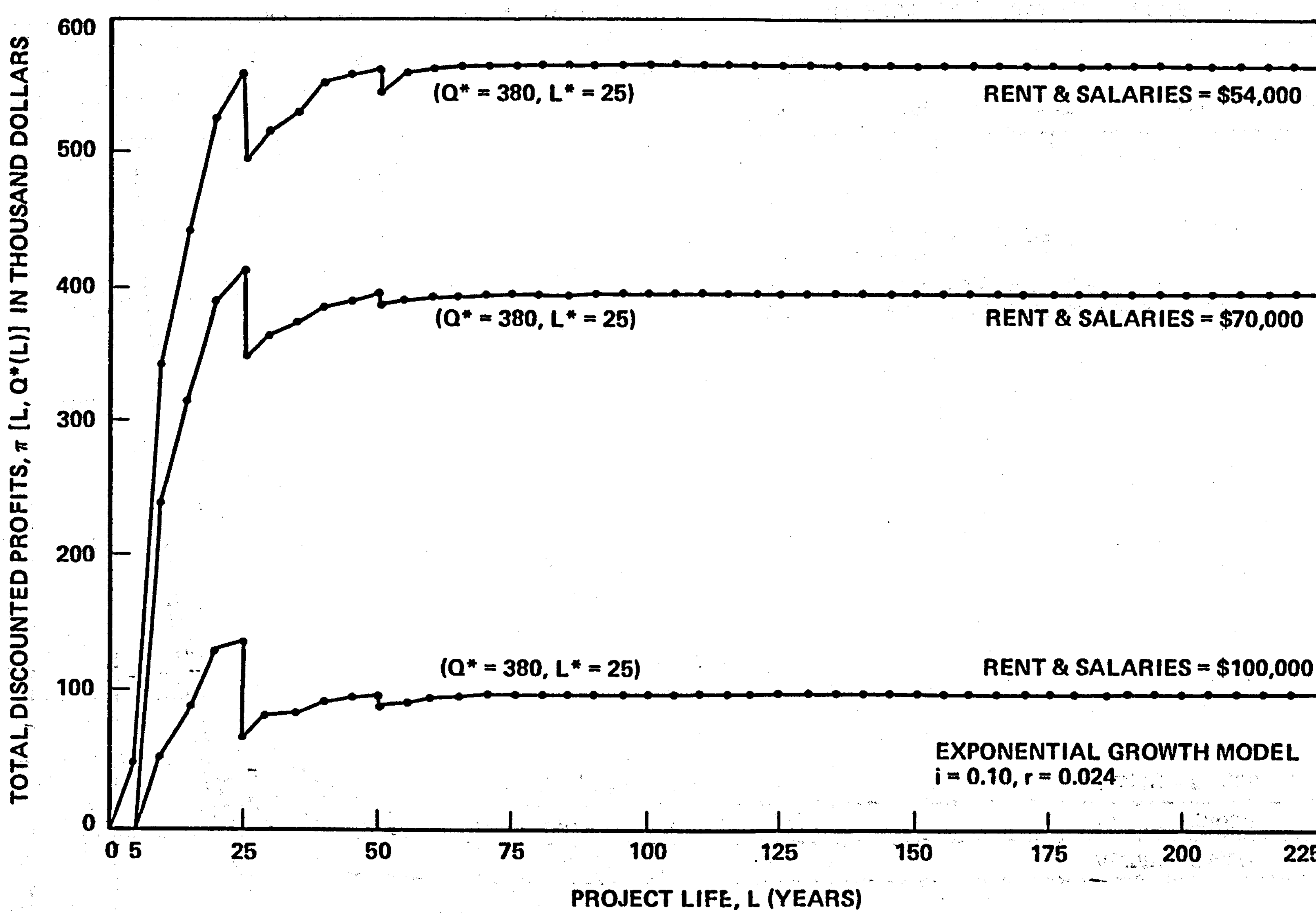

FIGURE 2.13 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN TOTALS OF RENT AND SALARIES

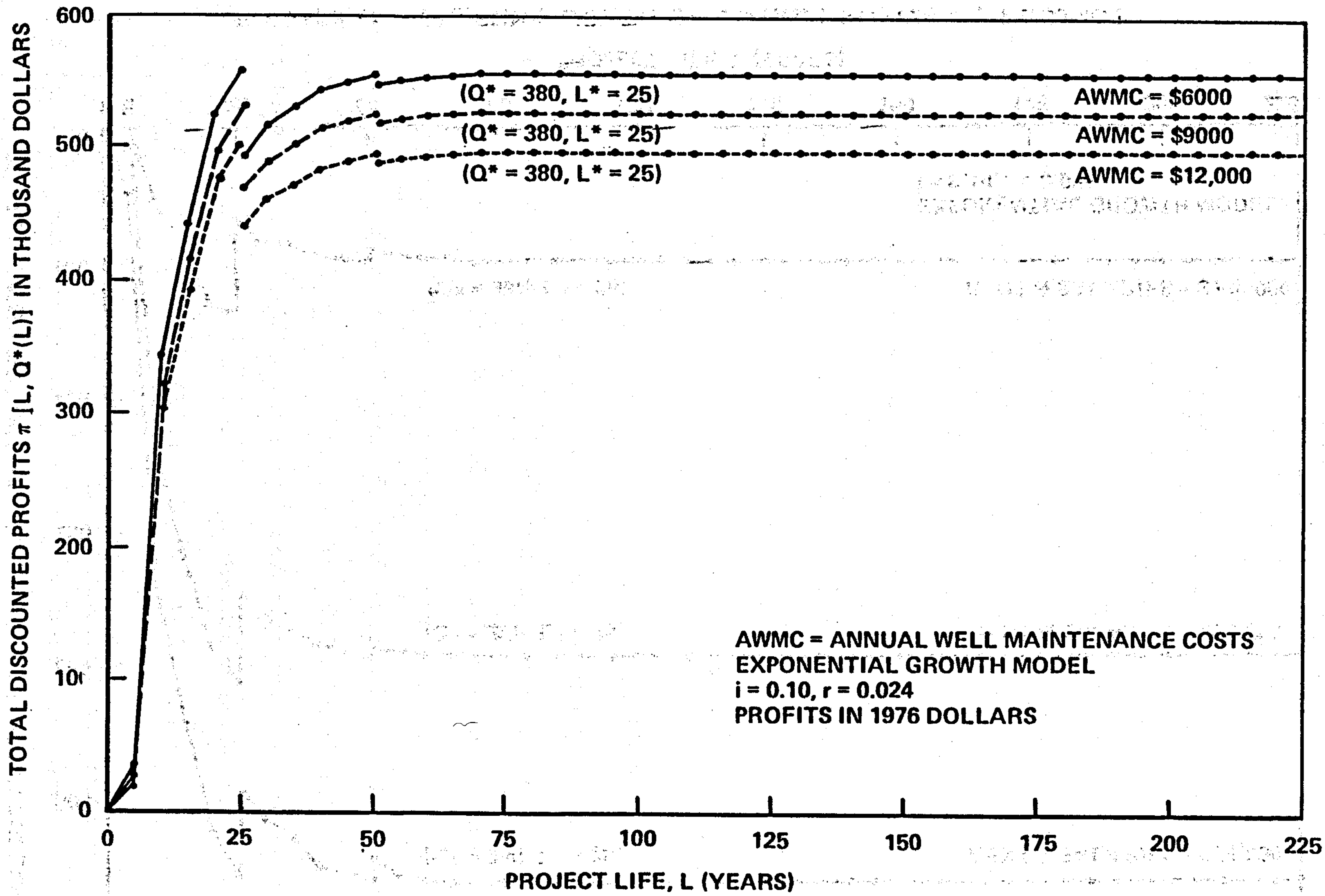

FIGURE 2.14 PRESENT WORTH OF TOTAL PROFITS vs. PROJECT LIFE FOR GIVEN ANNUAL WELL MAINTENANCE COSTS

would outweigh costs. We conclude that potential gains in profits may justify further investigation in this area of "optimal well maintenance."

2.6.21 Comment on the Shape of the Profit Function

We would like to make a final comment about the shape of the profit function $\pi(L, Q)$. As seen in Figure 2.3, for each L, the total cost function is a piecewise convex function of Q. It can be easily shown that the revenue function is concave in Q and therefore the profit function is a piecewise concave function of Q with discontinuities at multiples of $\bar{Q}$, the maximum flow rate from each production well. Since each segment of the profit function is concave each segment has a maximum. We can show that if Q_i is the maximand of the i^{th} segment, then

$$Q_1 \geq Q_2 - \bar{Q} \geq Q_3 - 2\bar{Q} \geq \ldots .$$

In other words, the distance between the maximands and the beginning of the segments becomes progressively smaller.

Using this result we can show that for a special case (when $\alpha = 0$) the line joining the maxima is also concave in Q. We think this result (which is domonstrated for our base case in Figure 2.15) is true for all values of α, but a formal proof has eluded us so far. The significance of this result is that an efficient algorithm can be designed to find the value of $Q^*(L)$ for each given L. For our base case for example (see Figure 2.15), after finding the values of $Q_1(L)$ and $Q_2(L)$ (which can be efficiently computed), the search for Q(L) terminates since $Q_1(L) > Q_2(L)$ implies that $Q^*(L) = Q_1(L)$.

2.6.22 Experiment on a Finite Aquifer

Recall that the hydrothermal model of Gringarten and Sauty, on which our economic model is based, assumes a horizontally infinite aquifer. Due to

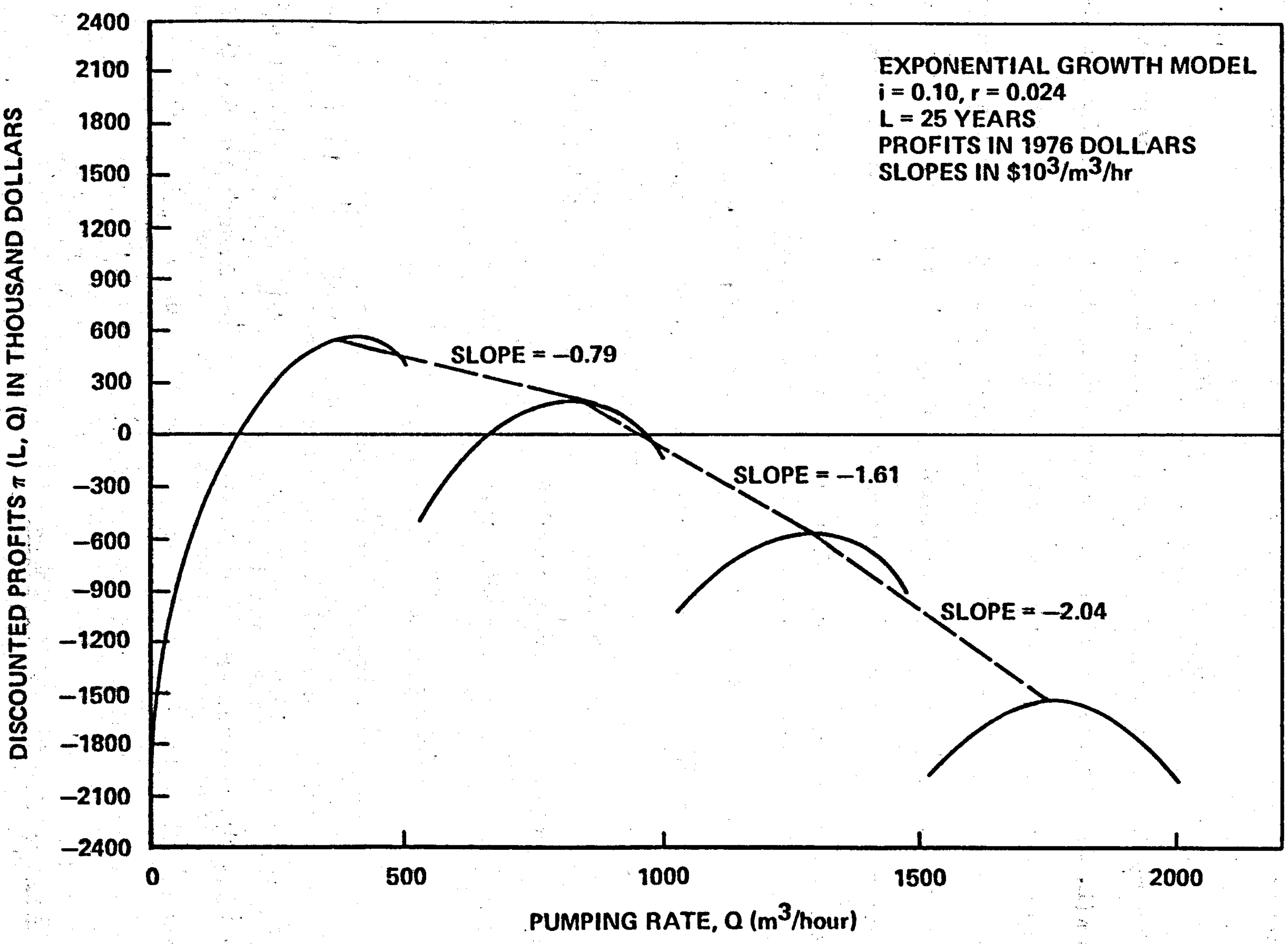

FIGURE 2.15 PRESENT WORTH OF PROFITS vs. PUMPING RATE FOR GIVEN PROJECT LIFE

interest in the effect of this assumption on our optimal decision variables, we investigated the case where the aquifer is finite (2 x 2 kilometers). For this purpose an equation showing the temperature decay vs. time for this aquifer was developed by Chin Fu Tsang of Lawrence Berkeley Laboratory and our computer program was accordingly modified to accommodate this case. Preliminary results indicate that the design decision variables (pumping rate and heat exchanger area) are not significantly different in the finite and infinite reservoir cases. However, an accurate determination of the relationship between reservoir size and decision variables requires further study.

2.7 SUMMARY

It is generally understood that a positive discount rate de-emphasizes the future in favor of the present, and we have found that to be the case in our results. Assuming shadow and actual market prices of energy are the same (as the effects of externalities have been ignored), we find that for each assumed rate of increase in the value of energy r, the present worth of maximum profits π^* (which is also the economic value of the reservoir) decreases while the optimal pumping rate Q^*, increases as the discount rate i, increases. While it can be easily shown that increasing the discount rate always reduces the present worth of maximum profits, the increase in Q^* as i increases is not necessarily inevitable. We conclude that, as theory predicts, a greater emphasis on the present "tilts" the design decisions on optimal pumping rate toward more rapid heat energy extraction rates. That is, we opt for extracting heat at a higher rate, leaving less for the future when profits are heavily discounted. Furthermore, for any two growth rates r_1 and r_2, with $r_1 < r_2$, the percentage change in both Q^* and π^* as i is increased is greater for r_2 than for r_1.

For each given interest rate, the maximum profit increases with r while optimal pumping rate Q^*, decreases. Since larger values of r mean energy is more valuable in the future, we conclude that as r is increases, we tend to extract heat more slowly, thus conserving a larger amount for production and sales in the future when its value is higher. Thus the interest rate and the growth rate work against each other in determining Q^*, the optimal pumping rate and π^*, the present worth of profits.

The economic life of a reservoir L^* (the planning horizon) is nonincreasing in i and nondecreasing in r. Thus, as i increases, future profits are discounted more heavily, and we extract a greater amount of heat per year over a period which tends to be shorter in comparison to when the discount rate is not as high. On the other hand, when energy is expected to rapidly increase in value with time, extraction of heat over a longer period of time tends to be more profitable.

Although the optimal extraction rate increases with discount rate when the value of r is small (r less than 1% for the exponential growth model and 2.4% for the linear growth model), we find that optimal economic life does not decrease in proportion as might be expected. Instead it remains constant for all values of i for these lower values of r. Accordingly, since Q^* increases with i, total energy extracted increases with i, another example of high discount rates that discourage "conservation." On the other hand when r is not small, the economic life tends to decrease as i increases.

In addition to the discount and growth rates, we find that well life is an important parameter in determining the economic life of the reservoir when r is positive. This is revealed in our results that show optimal project life is equal to expected well life, except when r is large and i is small. In this latter case, the relative importance of the future, when energy value is very high, is so great that much longer economic lives are chosen. However, here the effect of increasing the discount rate i is to reduce this economic life - since Q^* increases. We conclude that Q^* and L^* tend to move inversely as i increases.

In view of the above conclusions, we emphasize that the interpretation of L^* as the 'economic life of the reservoir' must be understood in the context of the deterministic nature of this model. Our results state that if all parameters (in particular i and r) are known with certainty, then it is indeed optimal to pump Q^* m^3/hr of brine over L^* years. However, as the values of these economic parameters tend to change with time according to some random process, it is more appropriate to consider L^* as an anticipated production period. Furthermore, since the same values of Q^* and other design variables are associated with a wide range of values of L, the interpretation of L^* is not especially important for the purpose of process design.

The amount of heat extracted per unit of time is not only a function of the extraction rate but the degree to which the extracted brine is cooled in the heat exchangers. Since the effect of heat losses in transmission is neglected, this heat exchanger outlet temperature is equal to the reservoir injection temperature T_i. Our results indicate that the optimal value of injection temperature T_i^*, is remarkably stable with respect to i and r and is very close to T_s, the generated steam temperature. We also find that in the context of the Gringarten-Sauty hydrothermal model, a higher T_i has no effect on prolonging the economic lifetime of the project. That is, although injecting the brine at a high temperature could prolong the duration of the (post breakthrough) time that the production temperature remains above a specified level, it is not actually profitable to do so.

In the examples we have studied in this report, the optimal breakthrough times occur very early in the project, namely during the first two years. The amount of heat that is extracted during this period is far less than that extracted after the breakthrough time. For example, in our base case the heat extracted before the breakthrough time is less than 12% of the total heat extracted during the life of the project. We conclude that for nonelectric uses of geothermal energy, termination of the extraction process at breakthrough time would be premature.

As mentioned earlier, well life has a substantial effect on both the economic life of the project and maximum profits. On the other hand, reduction in profits from increasing expenditures for well maintenance costs is minimal. Although we present no relationship between well maintenance expenditures and well life, it is reasonable to speculate that a good well maintenance program might be very cost beneficial. However, gains in profits increase at a decreasing rate as well life increases, so there will probably be some optimal maintenance expenditure beyond which marginal expenditures would outweight marginal costs.

Our results indicate that expenditures for well construction (well costs) do not have a significant effect on the optimal pumping rate but have a great impact on profits. Hence, even though engineering design is not highly sensitive to this parameter, it plays a major role in determining the economic viability of the project. As in the above paragraph, this also suggests an area of potentially productive investigation if a relationship can be generated between well capital and maintenance cost and well life.

A similar observation can be made for the importance of royalties and land rents. While both these items can significantly affect profits, their impact on optimal pumping rate is minor. Hence, we conclude that royalties and land rents are not the obvious economic incentives for control of production <u>rate</u> by public regulatory agencies, given that the decision has already been reached to produce a particular reservoir. We will see this same result in the next chapter where we find that rents and royalties are somewhat more important in influencing the profit maximizing entrepreneur's decision on the timing of production (when to commence).

The economic parameter that most influences profits and production rate is the present day cost of electricity for pumping, assuming it escalates at the same rate as other energy values. Pumping energy constitutes a major portion of the total costs and plays a major role in the economics and engineering design of geothermal energy production. This effect is demonstrated by the fact that if the electricity cost had been 2¢, kwh instead of 3¢/kwh, the maximum profits would have been higher by 130%, the optimal pumping rate by 142%, and the optimal heat exchanger area by 145%. Accordingly, these results represent a rather conservative estimate of the economic viability of geothermal (non-electric) energy production when the wells can be produced without extractive pumping.

Among the physical geothermal parameters, the one with the greatest impact on economic viability (profits) and the optimal decision variables, is the initial equilibrium temperature T_o. Both the profits and the optimal pumping rate greatly increase as the temperature increases. For instance, an increase of 10°C in the initial temperature of 150°C (which was chosen as the base case) increases profits by 253% and the optimal pumping rate by 130%. Since the economic worth of a reservoir is equal to the present worth of maximum profits, we confirm that the initial equilibrium temperature is a major consideration in assigning an economic value to a particular reservoir.

In contrast to the initial temperature, uncertainties in porosity of aquifer does not affect the optimal pumping rate or profits significantly. In fact, within the range of 10 to 30% porosity, the optimal pumping rate remains virtually constant. This suggests that major expenditures to accurately determine porosity may not be warranted for the purpose of computing the optimal pumping rate. On the other hand, intrinsic permeability has a relatively greater impact on the economics and design of a geothermal facility. When permeability is higher, the drawdown in the production well is not as great, so smaller pumps and less energy for pumping (the same amount of flow) is required. As a result, both the optimal profits and optimal pumping rate increase. We conclude that unlike porosity, expenditures for accurate information on permeability may be important, for profits and pumping rates are indeed sensitive to values of permeability.

Chapter 3

PRODUCTION TIMING AND ECONOMIC INCENTIVES

Kamal Golabi and Charles R. Scherer

3.1 INTRODUCTION

In the preceding chapter we developed an economic model for hot water geothermal energy production when the question was whether or not to produce a particular reservoir, and if so, at what pumping rate, etc. Although the results seemed to be sensitive to a number of parameters, the influence of land rents and royalty rate on production rate was seen to be minimal. We now consider a more general version of this model that contemplates not only how fast to pump the reservoir (and for how long, etc.), but also when to start. This is a useful extension for private sector producers. From the entrepreneurial point of view, it is a demonstration of how the profit maximizing geothermal company might determine how long to postpone the onset of production, assuming that the real value of energy is increasing with time. It is also valuable for public sector resource trustees and regulatory agencies who are responsible for lease timing and royalties, and who might consider using the latter to control production timing. For these parties, it contributes to development of a model of entrepreneurial exploitation activity that can be used to predict the effect of various incentives and penalties on private producers. These are the general goals of the model in this chapter. The more specific objectives have already been stated in section 1.3.

In the remainder of this chapter, we review the operation of the geothermal system to incorporate a waiting time variable. We then extend the economic model to include this delay. A procedure is presented for selecting the optimal time to start production, the best production rate and reinjection temperature, and the economic life of the project when the extracted energy is used for producing steam. Using the cost functions developed in the previous chapter and data for a typical aquifer, we present the results of our optimization and attempt to answer the questions raised in Chapter 1.

3.2 PRELIMINARIES

3.2.1 The Hot Water Geothermal System with Waiting Time

We begin the exposition by assuming the project starts at time u. As heat is transferred from the aquifer matrix to the fluid, the temperature of the matrix decreases. After τ years from the start of the project (i.e., at time $\tau + u$), the matrix can no longer heat the fluid to T_o by the time the fluid reaches point [1] in Figure 2.1. When this happens, the production well temperature at [1] (and hence at [2]) will begin to drop. If we denote the time-variable production well temperature as T_o^t, this process of temperature degradation over time can be plotted as shown previously in Figure 2.2. The time τ (after pumping starts) at which the temperature begins to decline below T_o is called breakthrough, referring to the time when the reduced fluid temperature breaks through to the production well.

The breakthrough time is inversely proportional to the production rate Q, Combining (2.1) and (2.2) we can write

$$\tau(Q) = \beta/Q \tag{3.1}$$

where

$$\beta = \pi h D^2 c_a \rho_a / 26,280\, c_f \rho_f \, . \tag{3.2}$$

Thus, the g function in (2.12), when production starts immediately, can be alternatively expressed as a function of Q and t. When waiting time is positive, g also depends on u and we can write:

$$\frac{T_o^t - T_i}{T_o - T_i} = g(t, u, Q) \quad . \tag{3.3}$$

Incorporating the waiting time, u, in the g function gives

$$g(t, u, Q) = \begin{cases} 1 & \text{if } t \leq u + \tau \\ \sum_{j=1}^{3} \gamma_j e^{-\psi_j Q(t-u)} & \text{if } t > u + \tau \end{cases} \tag{3.4}$$

where $\gamma_1 = 0.338$, $\gamma_2 = 0.337$, $\gamma_3 = 1.368$, $\psi_1 = 0.0023/\beta$, $\psi_2 = 0.1093/\beta$, and $\psi_3 = 1.3343/\beta$. Note that when u = 0, (3.4) and (2.11) give identical answers for each Q and t.

We note that for fixed Q, T_i is also a function of time, but as shown in Chapter 2, the variation in T_i is small and T_i can be assumed constant in our analysis. However, although it is reasonable to assume T_i constant with time, its value obviously affects the amount of heat removed per unit of time and hence affects discounted net revenues. That is, lower values of T_i yield greater heat removals per unit of time, but cause the field to cool more rapidly.

3.2.2 The Production Timing Problem

We can now state the problem of geothermal reservoir production timing. On one hand, the increase in the value of energy suggests that extraction be postponed to a time when the net profits (or social value) is greater. On the other hand, pumping energy cost also increases with time, and a positive discount rate discounts the greater future earnings. Furthermore, to hedge against the uncertainties in the availability of known geothermal reservoirs, the firm may wish to lease the land at the present time and incur annual rents, even though the actual extraction of energy is postponed to a later time. Conversely, the government, as part of its policy to encourage the early extraction of geothermal energy, may wish to levy an annual penalty (in addition to rents) on the firm during the time the land is under lease but the reservoir is not being exploited. Rent and penalties would be incentives for an early extraction time. Alternatively, an "early start" bonus could be provided, in the form of a tax break available during the first f years after the beginning of the lease. But this is just the reciprocal of the "penalty" proposal, and would probably have the same effect on starting date; the difference would be in the distribution of the "incentive" between the public (penalty) and the private sector (tax break).

To these factors we must add the effect of extraction rate on the quality of the unextracted resource, once the actual pumping of energy starts. The temperature-time profile for a particular pumping rate implies a trade-off between the quantity of extracted energy and the temperature of the unextracted resource. For example, if energy is extracted at a high rate, the temperature will decrease rapidly, seriously diminishing the quality of heat in the future. Another decision variable that must be considered in our analysis is the temperature at which the brine should be reinjected in the aquifer. More heat can be extracted by reinjecting at a lower temperature, but achieving lower reinjection temperatures is possible only by utilizing larger and hence more costly heat exchangers.

3.3 PRODUCTION TIMING MODELS

In the following pages we determine the best starting time u^*, project life L^*, extraction rate Q^*, and reinjection temperature T_i^*, and present an efficient method for computation of (u^*, L^*, Q^*, T_i^*). We investigate two related models. In the first, we assume the reservoir is either owned by the geothermal firm or it can be leased for production whenever the firm is ready for actual exploitation of the resource. In the second model, we assume the firm avoids the risk that an exploitable reservoir would not be available in the future by leasing the land immediately and paying rents and possibly penalties during the time the land is left unexploited. Once these two models are analyzed, based on the subjective probability that a reservoir of similar characteristics would be available at a later time, the firm can decide whether to lease the land now or start leasing at the onset of extraction. When the probability of availability is a function of time, making this decision is considerably more difficult.

In both of these models, a royalty is paid as a fraction of gross revenues once exploitation starts. According to the Geothermal Steam Act of 1970, the royalty is between 0.10 and 0.15 of gross revenues. The Geothermal Steam Act (Sections 6a and 6c) also specifies that the lessee should start exploitation of the field within ten years from the beginning of the lease. In order to determine the time when it is most profitable (from the entrepreneur's viewpoint) to extract geothermal energy, we waive this requirement in this study.

3.4 MODEL I

In this model we assume that no costs (including land rents) are incurred before the onset of extraction. We seek an extraction rate Q^*, starting time u^*, project life L^*, and reinjection temperature T_i^*, such that the present worth of profits is maximized. The amount of heat recovered per unit of time is the product of the flow rate, heat capacity of the fluid and the temperature drop experienced by the hot brine in the heat exchanger.* For the first τ years after the start of extraction, this temperature drop is $T_o - T_i$. From that time until the termination of the project at time L + u, the temperature drop is governed by eqn. 3.4. Since a certain amount of heat is lost in the heat exchange, we will require that the difference between the heat exchanger inlet and outlet temperatures remain above a certain degree, δ °C.

3.4.1 Revenue Function

Let $R(u, L, Q, T_i)$ denote the net revenues of the extraction process when the project starts at time u (years), the brine is extracted at the rate of Q (m^3/hr) for L years and is reinjected in the aquifer at temperature T_i (°C). We can write:

$$R(u, L, Q, T_i) = (1-\eta)\int_{u}^{\tau+u} 34.76\, P_o e^{rt} Q\, c_f \rho_f (T_o - T_i)\, e^{-it}\, dt$$

$$+ (1-\eta)\int_{\tau+u}^{L+u} 34.76\, P_o e^{rt} Q\, c_f \rho_f (T_o - T_i) g(t, u, Q)\, e^{-it}\, dt\,, \qquad (3.5)$$

where

η = royalty for geothermal lease paid as a fraction of the value of produced energy,

c_f = specific heat of the fluid (cal/g °C),

ρ_f = fluid density (g/cm^3),

P_o = assumed energy price at the present time ($/MBTU),

i = discount rate,

τ = breakthrough time (years)

and 34.76 is a conversion factor to yield revenues in dollars per year.

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* The maximum transferable heat, H_m, equals $Qc_f\rho_f(T_o^t - T_s)$ corresponding to an infinite exchange area. The heat actually transferred H_a, is the product of H_m and the effectiveness of heat exchanger defined as $(T_o^t - T_i)/(T_o^t - T_s)$ yielding $H_a = Qc_f\rho_f(T_o^t - T_i)$.

We will now evaluate eqn. 3.5 and show that once $R(0, L, Q, T_i)$ has been computed, $R(u, L, Q, T_i)$ can be readily obtained. Let

$$A = 34.76\,(1-\eta)\,P_o Q c_f \rho_f (T_o - T_i) \tag{3.6}$$

$$\alpha = i - r \tag{3.7}$$

Then eqn. 3.5 can be written as

$$R(u, L, Q, T_i) = A \int_u^{\tau+u} e^{-\alpha t} dt + A \int_\tau^{L+u} \sum_{j=1}^{3} e^{-\alpha t} \gamma_j e^{-(\psi_j Q)(t-u)} dt$$

which yields

$$R(u, L, Q, T_i) = \frac{A e^{-\alpha u}(1-e^{-\alpha\tau})}{\alpha} + A \sum_{j=1}^{3} \gamma_j e^{-\alpha u} \left[\frac{e^{-(\psi_j Q+\alpha)\tau} - e^{-(\psi_j Q+\alpha)L}}{\psi_j Q + \alpha} \right] \tag{3.8}$$

By letting u = 0 in eqn. 3.5, utilizing eqns. 3.6 and 3.7, and evaluating the integrals, we notice that the net revenues, when the extraction process starts immediately, may be written as:

$$R(0, L, Q, T_i) = A \left[\frac{1-e^{-\alpha\tau}}{\alpha} + \sum_{j=1}^{3} \gamma_j \frac{e^{-(\psi_j Q+\alpha)} - e^{-(\psi_j Q+\alpha)L}}{\psi_j Q + \alpha} \right] \tag{3.9}$$

which enables us to write:

$$R(u, L, Q, T_i) = e^{-\alpha u} R(0, L, Q, T_i) \;. \tag{3.10}$$

Note that in eqn. 3.5 the lifetime L is assumed to be greater than the breakthrough time τ. For $L \leq \tau$, the following relationship is used:

$$R(u, L, Q, T_i) = A \left[\frac{e^{-\alpha u}(1-e^{-\alpha L})}{\alpha} \right] .$$

3.4.2 Cost Function

In this section we develop the cost function $C(u, L, Q, T_i)$. The major costs associated with geothermal energy extraction are:

1)	Capital cost for wells and their casing	(WC)
2)	Annual well maintenance costs	(WM)
3)	Capital cost for well assemblies	(WA)
4)	Capital cost for pumps	(PM)
5)	Capital cost for heat exchangers	(HE)
6)	Capital cost for pipes	(PP)
7)	Annual pipe cleaning costs	(PC)
8)	Operating cost for pumps	(PO)
9)	Land rents and salaries	(S)
10)	Termination costs	(TC)

In Chapter 2 we developed detailed expressions describing the various components of costs as functions of the operating (design) decision variables, namely Q and T_i.* In this section we will categorize these costs and show the effect of postponement of extraction time on the cost function. We assume the total production flow is achieved by means of a cluster of production wells arranged close together so that the distance between them is small compared to D. A pair of production and injection wells is called a doublet. As explained in section 2.5.1, there are certain fixed costs that must be paid for each doublet, so the total cost function $C(u, L, Q, T_i)$ is a step function of Q, with jumps equal to the present value of well and overhead assembly costs plus fixed capital costs of pumps and heat exchangers. Denoting the total costs associated with a doublet (excluding rents and salaries which do not depend on the extraction rate) by $q(u, L, Q, T_i)$, and suppressing the dependence on u, L, and T_i we can write

$$C(Q) = nq(\bar{Q}) + q(Q - n\bar{Q}) + S \qquad \text{if } n\bar{Q} < Q \leq (n+1)\bar{Q} \tag{3.11}$$

for n = 1, 2, ... Here, $\bar{Q}$ is the maximum flow rate from each production well and is determined by the geology of the field and pump technology, as

* L is also a decision variable in our optimization, but ultimately not part of the equipment specification, as are Q and T_i.

explained in section 2.5.1. The term S denotes thepresent value of total salaries and land rents for the geothermal reservoir, i.e.,

$$S(u) = \int_u^{L+u} (\text{Annual Rents} + \text{Annual Salaries})\, e^{-it}\, dt \tag{3.12}$$

$$= e^{-iu} S(0) \ .$$

Therefore, to evaluate the cost function $C(u, L, Q, T_i)$, we only need to determine the function $q(u, L, Q, T_i)$. This function consists of capital costs, operating costs, maintenance costs, and termination costs.

To begin, we evaluate the present worth of total capital costs (KC). We take the useful life of pumps and heat exchangers as ten years and that of pipes and well assemblies as 25 years. Since the life of a geothermal well may be different for different fields, we let well life (WL) be an input parameter. We assume that payments for the cost of each type of equipment and accrued interests are distributed uniformly over the lifetime of the equipment, and we can therefore specify capital recovery factors for annualization of capital costs. The total capital cost is therefore

$$\begin{aligned} KC(u, L, Q, T_i) = \int_u^{L+u} & [(PM + HE)\, CRF(i, 10) \\ & + (WA + PP)\, CRF(i, 25) \\ & + (WC)\, CRF(i, WL)]\, e^{-it}\, dt \end{aligned} \tag{3.13}$$

$$= e^{-iu}\, KC(0, L, Q, T_i) \ ,$$

where $CRF(i, n)$ is the capital recovery factor for a piece of equipment when its useful life is n years and the interest rate is i.

To evaluate termination costs we note that each piece of equipment (with the exception of wells) has a salvage value equal to a percentage of its remaining unpaid costs, if the project terminates before the lifetime of the equipment is concluded. The termination cost is the present value of the extra costs associated with terminating the project prior to completion of lifetime cycles of various equipment components. Let s_1, s_2, s_3, s_4 denote the salvage value, as a fraction of the remaining payments, of pumps, heat exchangers, pipes, and well assemblies, respectively. Let L_1, L_2, and L_3 indicate the smallest multiples of 10, 25, and well life containing L. The termination costs are therefore

$$
\begin{aligned}
TC(u, L, Q, T_i) = {} & [(1 - s_1)PM + (1 - s_2)HE]\, CRF(i, 10) \int_{L+u}^{L_1+u} e^{-it}\,dt \\
& + [(1 - s_3)PP + (1 - s_4)WA]\, CRF(i, 25) \int_{L+u}^{L_2+u} e^{-it}\,dt \\
& + (WC)\, CRF(i, WL) \int_{L+u}^{L_3+u} e^{-it}\,dt \,, \qquad (3.14) \\
= {} & e^{-iu}\, TC(0, L, Q, T_i) \;.
\end{aligned}
$$

The operating cost of pumps consists of the cost of electricity to operate the production and injection pumps and the cost of maintaining the pumps and their motors. Note that the real cost of electricity increases at the same rate as the value of geothermal energy, so we can write

$$R_t = R_o e^{rt} \qquad (3.15)$$

where R_o and R_t are the prices of electricity ($/kwh) at the present and at time t. The present value of operating cost is therefore

$$PO(u, L, Q) = \int_u^{L+u} k_m E(Q) R_o e^{rt} e^{-it} dt$$

$$= e^{-\alpha u} \int_0^L k_m E(Q) R_o e^{-\alpha t} dt$$

$$= e^{-\alpha u} \quad PO(0, L, Q) \qquad (3.16)$$

where E(Q) is the energy requirement for the motors of the production and injection pumps and k_m is a multiplier indicating annual maintenance costs of pumps and their motors, and α is given in eqn. 3.7. The E(Q) function is developed in Chapter 2.

The maintenance cost consists of well maintenance costs and pipe cleaning costs.

Hence

$$MC(u, L, Q) = \int_u^{L+u} (WM + PC) e^{-it} dt$$

$$= e^{-iu} MC(0, L, Q) . \qquad (3.17)$$

We can now combine the different components of the cost function for one doublet and from eqns. 3.13, 3.14, 3.16, and 3.17 write

$$q(u, L, Q, T_i) = e^{-iu} [KC(0, L, Q, T_i) + TC(0, L, Q, T_i)$$

$$+ MC(0, L, Q)] + e^{-\alpha u}[PO(0, L, Q)] . \qquad (3.18)$$

Leq q_1 denote the terms inside the first bracket in eqn. 3.18. Substituting in eqn. 3.11 and suppressing the dependence of C and q on L and T_i, we can write

$$C(u,Q) = e^{-iu}[nq_1(0,\overline{Q}) + q_1(0,Q-n\overline{Q}) + S(0)]$$

$$+ e^{-\alpha u}[n \cdot PO(0,\overline{Q}) + PO(0,Q-n\overline{Q})]$$

$$= e^{-iu}C_1 + e^{-\alpha u}C_2 \ , \qquad (3.19)$$

where C_1 and C_2 represent respectively the total extraction costs (excluding pump operating costs), and pump operating costs for given Q and L when extraction starts immediately.

3.4.3 Optimization Problem

Having developed the revenue and cost functions, we are now in a position to present our optimization problem. We wish to maximize the total discounted net benefits subject to the constraints that the difference between the heat exchanger inlet and outlet temperatures remain above a certain degree $\delta^{\circ}C$, and the injection temperature be above the steam temperature. Our problem is

$$\underset{u,L,Q,T_i}{\text{Maximize}} \quad \pi(u,L,Q,T_i) = R(u,L,Q,T_i) - C(u,L,Q,T_i)$$

subject to

$$(T_o - T_i)\,g(t,u,Q) \geq \delta$$

$$T_i \geq T_s \qquad (3.20)$$

$$u, L, Q \geq 0 \ .$$

Let

$$B = R(0,L,Q,T_i) - C_2 \ . \qquad (3.21)$$

Then utilizing eqn. 3.10 and eqn. 3.19, we can write the objective function as

$$\pi(u, L, Q, T_i) = e^{-\alpha u} R(0, L, Q, T_i) - e^{-iu} C_1 - e^{-\alpha u} C_2$$

$$= e^{-\alpha u} B - e^{-iu} C_1 \quad . \tag{3.22}$$

In section 3.5.2 we present an algorithm that efficiently solves eqn. 3.22. Before doing so, we need some results enabling us to show that for solving eqn. 3.22 we need only to consider the problem when extraction is immediate, and then only the two decision variables Q and L. Once (Q^*, L^*) has been obtained when $u = 0$, (u^*, L^*, Q^*, T_i^*) and $\pi^*(u, L, Q, T_i)$ can be efficiently computed.

3.4.4 Some Results

Result 1: When $u = 0$, the optimal injection temperature can be expressed as a function of Q and L. Specifically

$$T_i^* = T_s + \frac{M}{a\sigma} \tag{3.23}$$

where

$$M = 30.96 \, CRF(i, 10)[(1 - e^{-iL_1}) - s_2(e^{-iL} - e^{-iL_1})]/i$$

and σ is the term inside the bracket of eqn. 3.9.

Proof: The optimal injection temperature is achieved at the point where the marginal revenue with respect to T_i equals the marginal cost of further reducing T_i. Eqn. 3.23 is obtained by setting

$$\frac{\partial R}{\partial T_i} = \frac{\partial C}{\partial T_i}\bigg|_{T_i^*} \quad .$$

This result reduces the number of decision variables to Q and L, as now T_i can be expressed as a function of these two variables.

<u>Result 2:</u> For each Q and L, the optimal starting time u^* is either equal to zero or is given by

$$u^* = -\frac{1}{r} \ln \left(\frac{\alpha B}{iC_1}\right) . \tag{3.24}$$

<u>Proof</u>: In our investigation we will confine ourselves to cases where Q and L are such that $B > 0$, that is $\frac{\alpha B}{iC_1} > 0$. For suppose $B \leq 0$. Then

$$\pi(u) = B e^{-\alpha u} - C_1 e^{-iu} \leq 0 \text{ for all } u ,$$

and at this (Q, L), the project is not profitable at any u, i.e., (u, L, Q) is dominated by (0, 0, 0).

When $B > 0$, we can distinguish two cases:

Case I: $\frac{\alpha B}{iC_1} < 1$.

This case includes the case where $0 < B < C_1$, that is, when revenues are greater than pump operating cost $(B = R - C_2 > 0)$ but not large enough for the venture to be profitable at $u = 0$, i.e., $R < C_1 + C_2$.

Setting the derivative of π (from eqn. 3.22) with respect to u equal to zero we obtain

$$\alpha e^{-\alpha u^*} B = i e^{-iu^*} C_1$$

yielding

$$\frac{\alpha B}{iC_1} = e^{-iu^*} e^{\alpha u^*} = e^{-ru^*}$$

as $\alpha = i - r$, which gives

$$u^* = -\frac{1}{r} \ln\left(\frac{\alpha B}{iC_1}\right) .$$

To check the second order condition:

$$\pi''(u) = \alpha^2 B e^{-\alpha u^*} - i^2 C_1 e^{-iu^*}$$

$$= \alpha^2 B e^{(\alpha/r)\ln(\alpha B/iC_1)} - i^2 C_1 e^{(i/r)\ln(\alpha B/iC_1)}$$

$$= \alpha^2 B e^{\ln(\alpha B/iC_1)^{\alpha/r}} - i^2 C_1 e^{\ln(\alpha B/iC_1)^{i/r}}$$

$$= \alpha^2 B(\alpha B/iC_1)^{\alpha/r} - i^2 C_1 (\alpha B/iC_1)^{i/r}$$

$$= (\alpha B/iC_1)^{\alpha/r} (\alpha^2 B - \alpha i B) < 0 .$$

Case II:

$$\frac{\alpha B}{iC_1} \geq 1$$

In this case $u^* = 0$. To show this, we first note that for any positive u,

$$\frac{i}{\alpha} > \frac{1 - e^{-iu}}{1 - e^{-\alpha u}} . \qquad (3.25)$$

This follows from the fact that $e^{-it} < e^{-\alpha t}$ as $\alpha < i$,

enabling us to write

$$\int_0^u e^{-it}\, dt < \int_0^u e^{-\alpha u}\, dt$$

or

$$\frac{1 - e^{-iu}}{i} < \frac{1 - e^{-\alpha u}}{\alpha} .$$

Since $\frac{\alpha B}{iC_1} \geq 1$, it follows that

$$\frac{B}{C_1} \geq \frac{i}{\alpha} > \frac{1 - e^{-iu}}{1 - e^{-\alpha u}}$$

which gives

$$B(1 - e^{-\alpha u}) > C_1(1 - e^{-iu})$$

or

$$B - C_1 > B e^{-\alpha u} - C_1 e^{-iu} \quad .$$

In other words,

$$\pi(0, Q, L) > \pi(u, Q, L) \quad ,$$

and hence u = 0 is optimal. Q. E. D.

The implication of the above result is that for every Q and L, the best starting time, u(Q, L) can be easily obtained. In our next result we show that in our search for u^*, we do not have to compute u(Q, L) for every Q and L. Rather, we can confine ourselves to a small subset, and thus compute (u^*, Q^*, L^*) efficiently. Before presenting our next result however, we will obtain an alternate expression for $\pi(u^*)$ when $\alpha B/iC_1 < 1$:

$$\pi(u^*) = B e^{-\alpha u^*} - C_1 e^{-iu^*}$$

$$= B e^{\frac{\alpha}{r} \ln (\frac{\alpha B}{iC_1})} - C_1 e^{\frac{i}{r} \ln (\frac{\alpha B}{iC_1})}$$

$$= B e^{\ln (\frac{\alpha B}{iC_1})^{\alpha/r}} - C_1 e^{\ln (\frac{\alpha B}{iC_1})^{i/r}}$$

$$= B(\frac{\alpha B}{iC_1})^{\alpha/r} - C_1 (\frac{\alpha B}{iC_1})^{i/r} \quad . \tag{3.26}$$

<u>Result 3:</u> Let $\Delta \equiv \{ (L, Q) \text{ s.t. } \frac{\alpha B}{iC_1} < 1 \}$,

Suppose $(\bar{L}, \bar{Q})$ maximizes B^i/C_1^α over Δ. Then either $(-\frac{1}{r}\ln(\alpha\bar{B}/i\bar{C}_1), \bar{L}, \bar{Q})$ is the optimal vector or $(0, \hat{L}, \hat{Q})$, where $(\hat{L}, \hat{Q})$ are the optimizing decision variables when extraction is immediate.

Proof:

Choose some arbitrary $(L, Q) \in \Delta$.

$$\frac{B^i}{C_1{}^\alpha} \leq \frac{\bar{B}^i}{\bar{C}_1^\alpha} \quad \text{implies that} \quad \frac{B^{i/r}}{\bar{C}_1{}^{\alpha/r}} - \frac{\bar{B}^{i/r}}{\bar{C}_1^{\alpha/r}} \leq 0 \ .$$

Since $(\frac{\alpha}{i})^{\alpha/r} \geq (\frac{\alpha}{i})^{i/r}$, we can write

$$(\frac{\alpha}{i})^{\alpha/r}\left(\frac{B^{i/r}}{C_1^{\alpha/r}} - \frac{\bar{B}^{i/r}}{\bar{C}_1^{\alpha/r}}\right) \leq (\frac{\alpha}{i})^{i/r}\left(\frac{B^{i/r}}{C^{\alpha/r}} - \frac{\bar{B}^{i/r}}{\bar{C}_1^{\alpha/r}}\right) \ .$$

Multiplying and rearranging the terms we obtain

$$B\left(\frac{\alpha B}{iC_1}\right)^{\alpha/r} - C_1\left(\frac{\alpha B}{iC_1}\right)^{i/r} \leq \bar{B}\left(\frac{\alpha\bar{B}}{i\bar{C}_1}\right)^{\alpha/r} - \bar{C}_1\left(\frac{\alpha\bar{B}}{i\bar{C}_1}\right)^{i/r} \ . \qquad (3.27)$$

Since both $\frac{\alpha B}{iC_1}$ and $\frac{\alpha\bar{B}}{i\bar{C}_1}$ are less than unity, their corresponding profit functions are given by eqn. 3.26, and hence eqn. 3.27 implies that we can do better with $(\bar{L}, \bar{Q})$ in comparison with (L, Q).

Now choose any (L, Q) which does not belong to Δ. Then by Case II of Result 2, $u^*(L, Q) = 0$. Therefore, if the maximum π is not achieved by $(\bar{L}, \bar{Q})$, then it must be obtained by some (L, Q) such that $u(L, Q) = 0$. Since $(\hat{L}, \hat{Q})$ is the optimizing decision when $u = 0$, our result is proven. Q. E. D.

We now discuss the second model which allows for the introduction of penalties and rents during the time the land is left unexploited.

3.5 MODEL II

In this section we investigate the problem of optimal timing and energy extraction when the KGRA is leased at the present time and land rents are paid not only during the active exploitation time L, but also during the period that energy is not being extracted. In addition, the government might levy an annual penalty on the firm during the time that the land is left unexploited. The other assumptions of the model are the same as Model I. In fact, Model I is a special case of Model II, in the sense that the two models are identical if the pre-exploitation rents and penalties in Model II are set to zero. As mentioned before, Model II can be used not only to compute the optimal extraction and timing of geothermal energy, but the extent and limitations of the influence that regulatory agencies can exert on extraction of geothermal energy through manipulations of rents, royalty, and penalties.

Let y be the annual pre-exploitation rents, and p the annual penalties that are imposed on the firm during the time the reservoir is left unexploited. The other symbols are those defined in section 3.4. Let

$$C_3 = p + y$$

$$C_3/i = G \qquad (3.28)$$

$$C_1 - G = H \qquad (3.29)$$

Then by eqn. 3.22 and the fact that the payment of C_3 is stopped after u years (the starting time of extraction), we can write

$$\pi(u, L, Q) = R(0, L, Q, T_i)\, e^{-\alpha u} - C_1 e^{-iu} - C_2 e^{-\alpha u} - \int_0^u C_3 e^{-it}\, dt$$

$$= B e^{-\alpha u} - C_1 e^{-iu} - C_3 (1 - e^{-iu})/i \ . \qquad (3.30)$$

which by eqns. 3.28 and 3.29 simplifies to

---

* Known geothermal resource area

$$\pi(u, L, Q) = B e^{-\alpha u} - C_1 e^{-iu} - G + G e^{-iu}$$

$$= B e^{-\alpha u} - H e^{-iu} - G . \qquad (3.31)$$

Note that land rents during the time period L that the reservoir is under exploitation is included in C_1. The dependence of π on T_i has been suppressed since T_i is a function of Q and L, as we showed earlier.

From eqn. 3.30 we observe that if $B < 0$ then $\pi(u, L, Q) < 0$, implying that extraction is not profitable at this Q and L. Accordingly, in our analysis we restrict ourselves to the cases where Q and L are such that $B > 0$. We now show this model has properties which are similar to those shown in Results 2 and 3 for Model 1.

3.5.1 Some Results

Result 4: For each Q and L, the optimal starting time u^* is either equal to zero or is given by

$$u^* = -\frac{1}{r} \ln\left(\frac{\alpha B}{iH}\right) . \qquad (3.32)$$

Proof: We distinguish three cases:

Case I: $0 < \frac{\alpha B}{iH} < 1$.

Setting the derivative of the profit function in eqn. 3.31 with respect to u equal to zero we obtain

$$\alpha B e^{-\alpha u^*} = iH e^{-iu^*}$$

yielding

$$\frac{\alpha B}{iH} = e^{-iu^*} e^{\alpha u^*} = e^{-ru^*}$$

as $\alpha = i - r$, which gives

$$u^* = -\frac{1}{r}\ln\left(\frac{\alpha B}{iH}\right)$$

Since $0 < \frac{\alpha B}{iH} < 1$, $0 < u^* < \infty$ and u^* is well defined. To check the second order condition:

$$\pi''(u^*) = \alpha^2 B e^{-\alpha u^*} - i^2 H e^{-iu^*}$$

$$= \alpha^2 B e^{(\alpha/r)\ln(\alpha B/iH)} - i^2 H e^{(i/r)\ln(\alpha B/iH)}$$

$$= \alpha^2 B e^{\ln(\alpha B/iH)^{\alpha/r}} - i^2 H e^{\ln(\alpha B/iH)^{i/r}}$$

$$= \alpha^2 B\left(\frac{\alpha B}{iH}\right)^{\alpha/r} - i^2 H\left(\frac{\alpha B}{iH}\right)^{i/r}$$

$$= \left(\frac{\alpha B}{iH}\right)^{\alpha/r}[\alpha^2 B - i\alpha B] < 0 \ .$$

Case II:

$$\frac{\alpha B}{iH} \geq 1 \ .$$

In this case $\frac{\alpha B}{iH} \geq 1$ implies by eqn. 3.25 that

$$\frac{B}{H} \geq \frac{i}{\alpha} \geq \frac{1 - e^{-iu}}{1 - e^{-\alpha u}}$$

which gives

$$B(1 - e^{-\alpha u}) \geq H(1 - e^{-iu}) \ . \qquad (3.33)$$

Subtracting G from both sides of eqn. 3.33 and utilizing eqn. 3.31, we can write

$$B - H - G \geq B e^{-\alpha u} - H e^{-iu} - G$$

or

$$\pi(0, L, Q) \geq \pi(u, L, Q) \qquad \text{for all } u,$$

and hence $u^*(L, Q) = 0$.

Case III:

$$\frac{\alpha B}{iH} \leq 0 .$$

Since B is positive, $H \leq 0$. From eqn. 3.31,

$$\pi(u, L, Q) = B e^{-\alpha u} - H e^{-iu} - G$$

$$\leq B e^{-\alpha u} - H e^{-\alpha u} - G e^{-\alpha u}$$

$$= e^{-\alpha u}(B - H - G) . \qquad (3.34)$$

Now if $B - H \geq G$,

$$e^{-\alpha u}(B - H - G) < B - H - G = \pi(0, L, Q)$$

and hence $u^{*}(L, Q) = 0$.

If $B - H < G$, then from eqn. 3.34, $\pi(u, L, Q) < 0$ for all u and (u, L, Q) is dominated by $(0, 0, 0)$. Hence, $u^{*} = 0$ or is given by eqn. 3.32.

Q. E. D.

Before presenting Result 5, we obtain an alternative expression for $\pi(u^{*})$:

$$\pi(u^{*}) = B e^{-\alpha u^{*}} - H e^{-iu^{*}} - G$$

$$= B e^{\ln(\alpha B/iH)^{\alpha/r}} - H e^{\ln(\alpha B/iH)^{i/r}} - G$$

$$= B\left[\frac{\alpha B}{iH}\right]^{\alpha/r} - H\left[\frac{\alpha B}{iH}\right]^{i/r} - G . \qquad (3.35)$$

<u>Result 5:</u> Let

$$\theta = \{(Q, L) \text{ s.t. } \quad 0 < \frac{\alpha B}{iH} < 1\} .$$

Suppose $(\overline{Q}, \overline{L})$ maximizes $\frac{B^{i}}{H^{\alpha}}$ over θ. Then either $(-\frac{1}{r}\ln(\frac{\alpha\overline{B}}{i\overline{H}}), \overline{L}, \overline{Q})$ is the optimal vector or $(0, \hat{L}, \hat{Q})$, where $(\hat{L}, \hat{Q})$ are the optimizing decision variables when extraction is immediate.

Proof: Choose an arbitrary (Q, L). If (Q, L) ε θ, then H and $\overline{H}$ (the value corresponding to $\overline{Q}$ and $\overline{L}$) are positive, as B > 0. Since

$$(\alpha/i)^{\alpha/r} > (\alpha/i)^{i/r} \text{ and } \frac{\overline{B}^i}{\overline{H}^\alpha} \geq \frac{B^i}{H^\alpha} ,$$

$$(\frac{\alpha}{i})^{\alpha/r}\left[\frac{B^{i/r}}{H^{i/r}} - \frac{\overline{B}^{i/r}}{\overline{H}^{\alpha/r}}\right] - G \leq (\frac{\alpha}{i})^{i/r}\left[\frac{B^{i/r}}{H^{\alpha/r}} - \frac{\overline{B}^{i/r}}{\overline{H}^{\alpha/r}}\right] - G$$

which gives

$$B\left[\frac{\alpha B}{iH}\right]^{\alpha/r} - H\left[\frac{\alpha B}{iH}\right]^{i/r} - G \leq \overline{B}\left[\frac{\alpha\overline{B}}{i\overline{H}}\right]^{\alpha/r} - \overline{H}\left[\frac{\alpha\overline{B}}{i\overline{H}}\right]^{i/r} - G ,$$

or by eqn. 3.35

$$\pi(u^*(Q, L), L, Q) \leq \pi(u^*(\overline{Q}, \overline{L}), \overline{L}, \overline{Q}) .$$

Now suppose (Q, L) is not a member of θ. Then if $\frac{\alpha B}{iH} \geq 1$, by Case II of Result 4, $u^* = 0$. If $\frac{\alpha B}{iH} \leq 0$, then either $u^* = 0$ or (u, L, Q) is dominated by (0, 0, 0) depending on whether $B > C_1$ or $B < C_1$. Thus, if the maximum of π is not achieved by (Q, L) ε θ, then $(0, \hat{L}, \hat{Q})$ must be optimal where $(\hat{L}, \hat{Q})$ is the optimal decision when there is no delay in extraction.

Q. E. D.

3.5.2 Optimization Algorithm

Since Model I is a special case of Model II, namely when pre-extraction rents and penalties are set to zero, it suffices to present an algorithm for Model II. The algorithm consists of a grid search routine over values of L and Q, combined with a procedure to compute the optimal starting time for each L.

The lifetime L is varied from L_{min} to L_{max} in increments of L_{inc}. For each L, the pumping rate Q is varied from Q_{min} to Q_{max} in increments of Q_{inc}.

These values are specified by the decision maker and are inputs to the program. For each Q and L, the δ constraint is checked so that the difference between the production and injection temperatures does not fall below δ degrees centigrade. In this regard, note that by eqn. 3.4, the temperature drop is a function of $Q \cdot L$. Therefore if, for a given L and some Q^1, $(Q^1 \leq Q_{max})$, $T_o^L(Q^1) - T_i \leq \delta$, it follows that $T_o^L(Q) - T_i < \delta$ for all $Q > Q^1$. For each L and feasible Q, $T_i^*(Q, L)$ and B, are computed. The program then computes π^*, the present worth of the maximum profits as follows:

If $B \leq 0$, since $\pi < 0$ for all u in this case, the program selects the next Q. If $B > 0$, then H is computed. For nonpositive H, the program selects the next Q if $B \leq C_1$ (as again the venture is not profitable) and computes $\pi(0) = B - C_1$, if $B > C_1$. The value of $\pi(0)$ is compared with the previous maximum which is stored say in S_1, and the maximum is retained. For positive H, the ratio $\frac{\alpha B}{iH}$ is computed. If this ratio is greater or equal to one, then $\pi(0)$ is computed and compared with the value in S_1. If $\frac{\alpha B}{iH} < 1$, then $\frac{B^i}{H^\alpha}$ is computed and stored in say S_2. When for the given L, all feasible values of Q are considered, S_2 contains the maximum B^i/H^α and the corresponding Q. The optimal starting time u and profit $\pi(u)$ are then computed for this value and compared with the value in S_1. The maximum of the two is the maximum profits for this particular L. When $L = L_{max}$, the program has computed $\pi(u^*, L^*, Q^*, T_i^*)$ as well as the optimal decision variables. Note that the values are computed more efficiently than Result 5 suggests, as we do not compute $\pi(0)$ for all Q's.

The flow chart for the algorithm is presented in Appendix B, along with the computer program for the models in this chapter. The computer program developed for this study can be readily utilized for decision making under a different set of conditions. Geohydrological and economic data are inputs to the program and the cost subroutine can be easily modified to accommodate the particular costs involved in the exploitation of each individual field.

The optimization is conducted with a particular set of data which to our best judgment reflects the current value of pertinent costs. The geo-hydrological data have generally been chosen in midrange of values associated with known hot water geothermal resources. Although most of these data are the same as those in Chapter 2, we list them all again for the readers' convenience.

3.6 THE DATA

The following set of data is common to all the results.

Minimum Allowable Project Life, L_{min}	0 years
Maximum Allowable Project Life, L_{max}	250 years
Project Life Increment, L_{inc}	5 years
Minimum Extraction Rate, Q_{min}	50 m^3/hr
Maximum Extraction Rate, Q_{max}	5000 m^3/hr
Extraction Rate Increment, Q_{inc}	10 m^3/hr
Thickness of Aquifer, h	100 m
Doublet Separation, D	300 m
Well Radius, r_w	0.15 m
Well Capacity, $\overline{Q}$	500 m^3/hr
Porosity of Aquifer, ϕ	0.20
Intrinsic Permeability, k	200 m.d.
Initial Equilibrium Temperature, T_o	150°C
Temperature of Generated Steam, T_s	109°C
Heat Capacity of Fluid, $\rho_f c_f$	0.92 cal/cc°C
Heat Capacity of Rock, $\rho_R c_R$	0.50 cal/cc°C

Specific Gravity of Fluid	0.9173
Overall Heat Transfer Coefficient of Fluid, U(0)	1000 BTU/hr ft^2°F
Friction Losses, b	20 m
Static Level of Fluid, z	0 m
Vertical Pump Efficiency, Eff_V	0.75
Horizontal Pump Efficiency, Eff_H	0.75
Pump Salvage Value as Fraction of Remaining Payments, s_1 .	0.40
Heat Exchanger Salvage Value as Fraction of Remaining Payments, s_2	0.40
Pipe Salvage Value as Fraction of Remaining Payments, s_3. .	0.40
Well Assembly Salvage Value as Fraction of Remaining Payments, s_4	0.40
Pipe Cleaning Cost, p_c	10 \$/m/year
Pipe Support Multiplier, k_p	1.25
Cost of 50 m^3/hr Bowl Unit, c_1	1250 dollars
Cost of 250 m^3/hr Bowl Unit, c_2	3941 dollars
Pump Maintenance Cost Coefficient, k_m	1.10
Well Cost per Doublet, WC	600,000 dollars
Well Maintenance Cost, WM	6000 \$/year/doublet
Useful Life of Wells, WL	25 years
Well Assembly Cost, WA	35,000 dollars
Electricity Cost in 1976, R_o	3 ¢ /kwh
Annual Salaries	50,000 \$/year
Annual Post-exploitation Land Rents	4,000 \$/year
Minimum Allowable Temperature Difference, δ	6°C

The absolute viscosity of the fluid is directly computed from the Bingham formula (eqn. 2.60a).

3.7 RESULTS

In this section we present the results obtained from exercising the economic model discussed in sections 3.4 and 3.5. We consider first the benefit (profit) maximizing levels of profits, starting time, production rate, injection temperature, and breakthrough time for several values of discount rate and rate of energy value growth. Since real prices and costs are used, an inflationless discount rate is also used. The purpose of this section is to show how these results are affected by these two important parameters. We consider a range of 4 to 15% for the discount rate and a range of 0.01 to 0.03 for r. In this section, the royalty η is assumed to be 10% of gross revenues, and the sum of annual pre-exploitation land rent and annual penalty, C_3, is 8,000 dollars. In the remaining sections we discuss the sensitivity of our result to variations in C_3 and η.

3.7.1 Profits

The present worth of maximum profits, π, is presented in Table 3.1. The values across the top row represent discount rates, and the left column denotes different values for r. The bottom row contains values of P_o, the 1976 value of one million BTU of 5 psi steam (which, by eqn. 2.25, also depends on the interest rate). For each i and r the maximum profit is given in the table.

As expected, profits decrease as i increases and increase as r increases. That this should always be so is shown in section 2.6.5.

3.7.2 Optimal Starting Time

In Table 3.2 we present the optimal starting times, u^*, for production. The optimal starting time increases with r and decreases with i, ranging from zero to 21 years. This is intuitive; when the rate of increase in value of energy is higher, the profit maximizing entrepreneur tends to postpone the

Table 3.1

PRESENT WORTH OF MAXIMUM PROFITS, π^* ($1976, $1000)

r \ i	0.04	0.06	0.08	0.10	0.12	0.15
0.010	677	554	455	375	308	226
0.015	1039	656	535	438	359	264
0.020	1778	860	619	505	412	303
0.024	2594	1116	731	561	457	337
0.030	4501	1598	952	688	537	389
P_o $/MBTU	1.101	1.104	1.107	1.110	1.113	1.118

Table 3.2

OPTIMAL STARTING TIME, u^* (years)

r \ i	0.04	0.06	0.08	0.10	0.12	0.15
0.010	0.0	0.0	0.0	0.0	0.0	0.0
0.015	14.15	0.0	0.0	0.0	0.0	0.0
0.020	15.28	4.72	0.0	0.0	0.0	0.0
0.024	16.70	5.63	1.24	0.0	0.0	0.0
0.030	20.98	6.97	2.45	0.30	0.0	0.0
P_o $/MBTU	1.101	1.104	1.107	1.110	1.113	1.118

onset of extraction. Conversely, the reservoir is produced sooner if the value of energy is not expected to rise as fast. However, the table also shows that even if the value of energy increases at a fast rate, it is not optimal to postpone extraction if future earnings are discounted heavily (for the range of parameters considered).

3.7.3 Optimal Extraction Rate

Table 3.3 presents the optimal production rate, Q^*. The optimal production rate increases with i and decreases with r. Thus, when r is increased, the optimal strategy is to start later and extract heat more slowly, leaving a larger amount for the future when value is higher. However, a high discount rate encourages the entrepreneur to extract energy at a faster rate (as well as start sooner).

It is interesting to note that even in cases where production is optimally postponed, the extraction rate is about the same as when production is necessarily immediate. This can be seen by comparing Tables 2.8 and 3.3. This means that even if the entrepreneur chooses to postpone the onset of production, he will nevertheless produce (whenever he starts) at about the same rate as when he starts immediately.

3.7.4 Optimal Project Life

Optimal reservoir life is non-increasing in i and non-decreasing in r. Thus, when future profits are discounted more heavily, the entrepreneur tends to start extraction sooner and pump the energy faster over a shorter period of time compared with when the discount rate is not as high. However, when the value of energy is expected to increase rapidly with time, he tends to postpone extraction and produce the energy over a longer period of time.

The optimal project life is 25 years in most cases, due to the fact that the useful well life has been assumed to be 25 years. Because a second well

Table 3.3

OPTIMAL PUMPING RATE, Q^* (cubic meters/hr)

r \ i	0.04	0.06	0.08	0.10	0.12	0.15
0.010	350	370	380	390	400	410
0.015	300	360	380	390	400	410
0.020	300	330	370	380	390	410
0.024	290	320	350	380	390	410
0.030	280	320	340	360	380	410
P_o \$/MBTU	1.101	1.104	1.107	1.110	1.113	1.118

cost must be incurred if project life is greater than one well life, there is always a local maximum for the profit function $\pi(\cdot, \cdot, L, \cdot)$ at L = well life (the dots represent the optimal decision variables for the L under consideration). When r is low or i is high this local maximum is the unique global maximum.

3.7.5 Optimal Injection Temperature

The optimal injection temperature is very stable with respect to i and r, and is close to T_s (within one degree centigrade). The resulting high costs of heat exchangers are evidently offset by the value of the large amount of energy that can be extracted when T_i^* is close to T_s.

3.7.6 Optimal Breakthrough Time

The optimal breakthrough times are inversely proportional to Q^*, and are therefore decreasing in i and increasing in r.

3.7.7 Sensitivity to Rent, Penalty, and Royalty

In the remaining sections we discuss the sensitivity of profits and our decision variables to changes in the total annual payments for the pre-exploitation rent and penalty C_3, as well as changes in royalty η. In the results presented in sections 3.7.1–3.7.6, C_3 had been assumed as 8000 \$/year and η as 10% of gross revenues. In the next sections we vary C_3 from 0 to 16,000 \$/year and η from 2.5% to 15%. Note that the post-exploitation rent is assumed to be 4,000 \$/year for all cases. Results are presented for three different interest rates, namely 0.04, 0.08, and 0.12, with r = 0.024 in all cases. This figure is based on the estimate of average rate of increase in the value of energy according to the 1977 National Energy Outlook (Federal Energy Administration, 1977).

3.7.8 Sensitivity of Profits

Table 3.4 shows the sensitivity of maximum profits to changes in royalty and C_3, the sum of annual rent and penalties before extraction starts. As expected, maximum profits decrease as royalty or C_3 is increased. This decrease is more prominent when the interest rate is low. The profit is more sensitive to changes in royalty. Moreover, as royalty is lowered, the effect of changes in C_3 on profits becomes less significant. In fact, when $\eta \leq 0.05$ and $i = 0.08$ or 0.12, profits remain unaffected by changes in C_3, because production is undertaken immediately even with no postponement penalty.

3.7.9 Sensitivity of Starting Times

Table 3.5 presents the effect of changes in C_3 and η on u^*. The optimal starting time decreases as C_3 is increased and increases as η is increased. Thus postponement penalties encourage early production while a high royalty effectively postpones production of the reservoir. Note that significant delays in production occur only when the discount rate is low. When future earnings are heavily discounted (i.e. $i = 0.12$), the entrepreneur prefers immediate production unless royalties are very high, in which case production is postponed, but for relatively short periods of time.

3.7.10 Sensitivity of Pumping Rates

Table 3.6 shows sensitivity of Q^* to changes in C_3 and η. As seen from this table, the optimal pumping rate is decreased as royalties are increased. Thus, when royalty payments are high, the entrepreneur tends to extract less energy, reserving more for the future when the value of energy is higher. We see, then, that η functions as an incentive for "conservation": as η increases, production is deferred, and undertaken at a slower rate, saving energy for the

Table 3.4

SENSITIVITY OF MAXIMUM PROFITS, π^*, TO RENTS, PENALTIES, AND ROYALTIES ($1976, $1000)

(i=0.04)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	3344	3118	2902	2696	2499	2311
4000	3303	3073	2853	2644	2443	2252
8000	3264	3030	2807	2594	2390	2195
12000	3228	2991	2762	2556	2347	2138
16000	3198	2951	2720	2510	2291	2086

(i=0.08)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	1115	984	859	747	647	557
4000	1115	984	855	738	632	538
8000	1115	984	855	731	620	521
12000	1115	984	853	726	610	505
16000	1115	983	850	722	602	491

(i=0.12)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	729	637	546	457	371	299
4000	729	637	546	457	370	291
8000	729	637	546	457	370	286
12000	729	637	546	457	368	282
16000	729	636	545	456	366	279

Table 3.5

SENSITIVITY OF OPTIMAL STARTING TIMES, u^*, TO RENTS, PENALTIES AND ROYALTIES (years)

(i=0.04)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	13.8	15.4	17.2	18.9	20.7	22.7
4000	12.7	14.3	16.1	17.8	19.6	21.6
8000	11.6	13.2	15.0	16.7	18.5	20.4
12000	10.4	12.0	13.9	15.6	17.4	19.2
16000	9.2	10.8	12.7	14.5	16.2	18.0

(i=0.08)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	0.0	0.0	1.5	3.1	4.9	6.7
4000	0.0	0.0	0.5	2.2	3.9	5.7
8000	0.0	0.0	0.0	1.2	3.0	4.8
12000	0.0	0.0	0.0	0.3	2.0	3.9
16000	0.0	0.0	0.0	0.0	1.1	2.9

(i=0.12)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	0.0	0.0	0.0	0.0	0.9	2.7
4000	0.0	0.0	0.0	0.0	0.0	1.8
8000	0.0	0.0	0.0	0.0	0.0	0.9
12000	0.0	0.0	0.0	0.0	0.0	0.2
16000	0.0	0.0	0.0	0.0	0.0	0.0

Table 3.6

SENSITIVITY OF OPTIMAL PUMPING RATES, Q^*, TO RENTS, PENALTIES, AND ROYALTIES (m^3/hr)

(i=0.04)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	310	300	300	290	280	280
4000	310	300	300	290	280	280
8000	310	300	300	290	280	280
12000	310	300	300	290	280	280
16000	310	300	300	290	280	280

(i=0.08)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	370	360	360	350	340	330
4000	370	360	350	350	340	330
8000	370	360	350	350	340	330
12000	370	360	350	350	340	330
16000	370	360	350	340	340	330

(i=0.12)

c_3 \ η	0.025	0.050	0.075	0.100	0.125	0.150
0	410	410	400	390	390	380
4000	410	410	400	390	380	380
8000	410	410	400	390	380	380
12000	410	410	400	390	380	380
16000	410	410	390	390	380	370

future. Pumping rate is practically unaffected by increase in rent and penalties, the latter being much more effective in influencing timing than rate of extraction.

3.8 SUMMARY

The primary focus of this chapter has been on when to commence production of a hot water geothermal reservoir, noting that this is appropriately a matter of interest for both the entrepreneur and pertinent government regulatory agencies. Our analysis has emphasized the likelihood of an increase in the real value of energy. However, our inquiry recognizes that the most desirable production rate Q^*, the best planning horizon L^*, the best injection temperature T_i^*, and the optimal starting time u^*, are interrelated, so we have analyzed their effect on the overall planning strategy simultaneously.

For the royalty of 10%, the results indicate that the best starting time, u^*, is quite sensitive to both the discount rate, i, and the rate of increase of the value of energy, r, ranging from 0 to about 21 years. Naturally this waiting time is longest when r is large and i is small. We conclude that if government perceives the appropriate social discount rate to be low (e.g. real discount rate of 4%, or a nominal value of 9%, assuming a 5% inflation), and anticipates annual real increases in costs of alternative forms of energy of at least 1%, then it may be best to postpone production for several years, either by not leasing a particular reservoir, or by the economic inducements discussed in this report.

Optimal production rate is increasing in i and decreasing in r and economic planning horizon is non-increasing in i and non-decreasing in r. Thus, when profits are discounted more heavily, the entrepreneur tends to start extraction sooner and produce the energy faster over a shorter period of time compared to when the discount rate is not as high. These results are consistent with the results of Chapter 2 when extraction was assumed to start immediately.

For fixed values of r and i ($r = 0.024$ and $i = 0.04$), we find u^* is quite sensitive to royalty and postponement penalty, ranging from 9 to 23 years as penalty decreases and royalty increases. For larger values of i, waiting time is less sensitive to penalty or royalty, and approaches zero as i is increased. On the other hand, pumping rates are remarkably insensitive to penalties and royalties.

The significant variation of waiting time with penalty and royalty suggests that these kinds of economic incentives could motivate a profit maximizing entrepreneur to accelerate (or postpone) production if his real alternative rate of return was less than 6% (about 11% nominal, assuming a 5% inflation). By the previous set of results, he would be less motivated by these incentives if his alternative rate of return were higher and if r were lower than 0.024 (in which case he tends to start immediately, regardless of penalty or royalty). Conversely, if r were higher than 0.024, this sensitivity would extend to higher values of rate of return.

Since decreasing royalty tends to accelerate starting time, it is important to note that the use of this incentive to accelerate production generally requires the government to forego some revenues from royalties (present worth of royalties would be smaller). Of course, this can be avoided - to a limited extent - by increasing the no-start penalty. This manipulation of incentives (penalties and royalties) can be done so that the entrepreneurial profits remain constant. As an example, note that the profit when $i = 0.04$, rent and penalty = 0 and royalty = 0.15 is about the same as when rent and penalty = 16,000 and royalty = 0.125, though u^* reduces from 22.7 to 16.2 years. This would suggest the possibility of identifying "iso-profit" curves along which the entrepreneur was indifferent, even though starting time varied significantly along this curve. Moving along one of these curves to decrease waiting time, the government would necessarily give up some royalty, though the entrepreneur would theoretically be indifferent.

On the other hand, if the government opts to delay production of a particular reservoir, these results suggest there are two ways of accomplishing postponement. The first is to withhold the subject reservoir from leasing proceedings, and the second is to lease now, but with a higher royalty rate (recall the Geothermal Steam Act specifies a range of royalties). Moreover, given the exogenous decision to postpone, these results indicate that the economically induced postponement would actually favor the government financially, since total present value of royalties would be greater, though at the expense of the entrepreneur's profits. As such this regulatory option would constitute a de facto transfer from the private sector to the public sector, with obvious political implications.

Finally, if the entrepreneur's alternative rate of return is large enough, the above incentives will be of little influence in forestalling production, in which case the only way for the government to achieve postponement (if this is suggested based on its own valuation of the discount rate), is to withhold the geothermal field from leasing.

We conclude that, in the context of our assumptions, the question of when to produce a hot water geothermal reservoir is non-trivial, but amenable to analysis — as demonstrated in this report. Secondly, we find that postponement of production (given appropriate values of i and r) could result in larger net present values of this energy resource. That is, it might be desirable from a social point of view to wait. Finally, under certain circumstances, government may be able to influence onset of production by manipulating royalty rates and nonstarting penalties.

Chapter 4

DISCUSSION

Charles R. Scherer and Kamal Golabi

4.1 PURPOSE OF THIS CHAPTER

In this chapter we offer some critical remarks on the models and results of chapters 2 and 3. Noting that research proceeds as a series of "closer looks," each providing a vantage point for the next "look," we present the following list of caveats and criticisms as a guide to others who may wish to extend the initial investigations. The list is long, for much remains to be done.

4.2 ASSUMPTIONS ON THE AQUIFER

4.2.1 Homogeneous Aquifer Medium

The technical "production function" used for this study assumes a homogeneous aquifer medium — as if the medium were some relatively uniform size sand or gravel. This may or may not be a good assumption, depending on a particular aquifer. If an aquifer is uniformly graded such that the flow through this porous medium actually is laminar, then the basic geohydraulic assumptions of the Gringarten-Sauty (1976) model are supported. If not, there may be some question about the valid application of this model and its results.

Suppose, for example, that the flow thought to occur through a particular sand aquifer is actually moving through large cracks in fractured but otherwise impermeable rock. Here the flow is probably turbulent and breakthrough is likely to occur much faster than might be expected with the effectively homogeneous aquifer, since the surface contact area is quite small in the case of the fractured rock. The point is that a production strategy obtained using an economic model based on an inappropriate geohydrologic assumption may be undesirable for guiding production decisions.

However, while the results of this study must be qualified by the assumption of a homogeneous aquifer, the concepts embodied in our economic model would be equally applicable for aquifers characterized by other than homogeneous media. One need "only" identify the analogous temperature-time decay function best suited to the particular non-homogeneous media under consideration. But the identification and specification of this function is not always straightforward. In fact, there may be great uncertainty about the actual composition of an aquifer, even after a production and reinjection well doublet is sunk. However, by drilling enough wells, one can obtain a fairly good idea. On the other hand, wells are very expensive, so the entrepreneur must often proceed to production with less than full information on the structure of the aquifer. Under these circumstances, it would be desirable to have an economic model that explicitly incorporates uncertainty and considers the value of additional information obtained by drilling more wells.

4.2.2 Infinite Aquifer

Throughout this report we have assumed the hot water geothermal aquifer was bounded top and bottom, but not horizontally. Of course this is a technical fiction, for no aquifer can be indefinitely unbounded. However, if the aquifer is large enough and the between-well distance small enough, the field flow streamlines will be effectively undisturbed and a finite aquifer will behave almost like an infinite reservoir. Then the question is, how small can a finite reservoir be (in diameter) before it ceases to function as an infinite reservoir. The answer to this question is beyond the scope of this report. However, when this minimum dimension is established, it will serve as a lower bound on the size of reservoir for which this model is valid.

4.3 MULTIPLE EXTRACTION RATES

Throughout this report we have assumed the production pumping ratio is held constant during the life of the reservoir. However, it would be possible to investigate a production strategy featuring more than one pumping rate during the life of the reservoir. Since the temperature histories for each pumping rate must be superimposed for the aggregate effect over time on the production hole temperature, optimization would be restricted to evaluation of a few different strategies. An obvious strategy for a multiple pumping scheme might be to increase pumping rate at or soon after breakthrough. As usual, the desirability of this management option would depend on the discount rate (i) and the rate of increase of energy value (r). If i is large and r small, then it would probably be profitable to step up pumping after breakthrough to capture the incremental energy sooner rather than later. On the other hand, if r itself is increasing with time, then this may not be the case.

4.4 HOT WATER GEOTHERMAL ENERGY AS A RENEWABLE RESOURCE

The possibility of multiple pumping rates leads naturally to a final remark on the "exhaustibility" of this natural resource. In section 1.1 we have assumed the aquifer is recharged so slowly (by interior earth heat) that it is non-renewable over a horizon of economic relevance. However, this is just an untested assumption and may be invalid if the aquifer is alternately pumped and then rested. If this on/off strategy is employed, the present economic worth of the reservoir might be enhanced. We have left further inquiry in this direction to another investigation. However, since one of the motives for this research was demonstration of economic extraction theory, we note that unlike fossil fuels, the distinction between renewable and non-renewable is not clear for hot water geothermal energy; perhaps geothermal energy should be considered "quasi-renewable."

4.5 OTHER DETERMINISTIC TRAJECTORIES FOR VALUE OF ENERGY

In the second chapter of this report we assumed the rate of increase of energy value was either exponential or linear. The latter option was dropped in Chapter 3. Both cases preclude the possibility of energy value increasing at a decreasing rate. Although the future of energy values is indeed uncertain, it is conceivable that the rate of increase of real value of energy might tend to zero over time, if not permanently, at least temporarily. If this were to happen, then optimal production rates would probably increase and reservoir life times would be correspondingly reduced. Delay times would be reduced. Although we carefully considered the significant sensitivity of production parameters to a range of values of the rate of increase in energy value, we did not investigate this latter case. Nevertheless, it should be considered in order to provide extraction plans for a fuller range of possible economic futures.

4.6 TREATING VALUE OF ENERGY AS STOCHASTIC PROCESS

A range of values was investigated for r in this study because the actual trajectory over time of energy value is unknown. Had there been no sensitivity of design variables and waiting time to r, there would be little need for a model that explicitly incorporates energy value as a random variable. But our results indicate this sensitivity is significant, suggesting that a stochastic treatment merits further attention. We emphasize the practical importance of this because entrepreneurial activity in the geothermal area is characterized by investments made in a physical and economic environment that is highly uncertain. The questions of how much capital to commit to production capacity and what the optimal management policy is are clearly related to the value of energy over time. How the producer makes these decisions in the face of uncertainty is a real and practical problem. Furthermore, although it is the question faced by the private entrepreneur, it is also of potential interest to the regulator responsible for accelerating or arresting the rate of geothermal energy development, Stochastic versions of the economic models contained in this study could reveal actions for reduction of risk that would increase the value of the resource for all.

Assuming the entrepreneur is a price taker, the value of energy is independent of pumping rate, so it would be straightforward to recast the economic models of this study in a probabilistic framework. We leave this to another investigation, anticipating that some formal mechanism can be developed to enable the entrepreneur to choose an appropriate level of capital to commit to reservoir development when future energy value is uncertain.

4.7 PRICE-SENSITIVE DEMAND

In section 2.4.2 we discussed the basis for evaluating the "social benefits" derived from production of a hot water geothermal reservoir. Our approach has been to impute to each BTU produced the unit value of the next more costly energy source in lieu of which geothermal energy is purchased. The tacit assumption is that a quantum of heat energy would be procured, regardless of the cost (within bounds), for space heating (or some other price-inelastic demand); the question simply is how much of this quantum should be of geothermal origin. It is also implicit that the energy demand local (within a hundred miles) to the reservoir is small compared to the market for the next most expensive energy source (oil-fired boiler), so that the energy customer and the seller of the alternative source are both price takers in this alternative market. The upshot of this is that price-sensitive demand is never actually considered. Benefits are linear in BTU output. If geothermal energy demand were treated as dependent on price (reflecting cost), then this would no longer be the case.

If geothermal energy demand is price dependent with a downward sloping demand curve, and if the area under the demand curve (between limits denoting the energy output) is taken as a surrogate for benefits — a standard approach — then geothermal energy benefits will increase at a decreasing rate as annual energy consumption increases. This situation might obtain if the geothermal entrepreneur were able to command a local monopoly on geothermal energy supply. Though even here it seems there would be some price above which an "imported" alternative would be cheaper.

In any event, if the demand curve shifts out with time, then benefits associated with given output would also grow with time, as was the case in the models studied in this project. The relative elasticity and rate of increase of benefits with time (for a given output) would probably affect the timing and rate of production, as well as reservoir life. For example, if elasticity and the rate at which demand curve shifts increased with time so that benefits per unit increased dramatically with time, production might be delayed and production rate might be slower than the results contained in this report indicate. Alternatively, these demand curves might result in benefits (per unit) that increased at a decreasing rate, resulting in higher production rates.

As implied in the above discussion, the authors are somewhat uncertain about the existence of geothermal entrepreneurs who face significantly downward sloping demand curves. However, to fully evaluate the welfare implications of this resource production problem, it may be necessary to investigate production policies using a "willingness to pay" objective.

4.8 SEPARATION OF PRODUCTION AND INJECTION WELLS

Throughout this report we have assumed a production-injection doublet separated by a distance D. In section 2.2 we follow Gringarten and Sauty in showing breakthrough time increasing with the square of D. Since brine pumped prior to breakthrough yields the greatest heat per unit of fluid, it follows that the capturable heat and the "value of the reservoir" will increase with D, at least up to a point. However, as D increases, the energy required to move the fluid from injection well to production well will increase, and so will insulation costs for preventing surface pipeline heat losses. Hence, there may come a point where the gains from retarded breakthrough are outweighed by the associated extra costs.

We need only add here that even before D approached the horizontal boundaries of the aquifer, it is possible that the production and injection wells would be so far apart that there would be a high likelihood of a geological fault between them. In this event the injected fluid might move off in the faulted plane, so distorting the field of flow as to render the Gringarten-Sauty assumptions totally invalid. The problems introduced by this eventuality have essentially already been discussed in section 4.2.1 on homogeneity and uncertainty. Although we did not investigate the "optimal" well separation, it clearly merits further investigation in the kind of economic framework developed herein.

4.9 WELL FIELD SPACING

The subject of well separation is not limited to one-dimensional optimization. Having investigated the best value of doublet separation, D, the next question is how should the entrepreneur position a second doublet with respect to the first, assuming he owns the entire reservoir. By arranging several doublets on one field, we may increase the amount of energy obtainable from the reservoir. However, it is intuitive that these well systems will interfere hydraulically and thermally, and a simple generalization of the Gringarten and Sauty results suggests the trade-off between energy now versus energy later. By drilling 10 doublets on 100-foot centers and pumping them at rate Q, we obtain ten times as much energy as from one doublet pumped at rate Q, but breakthrough might come much earlier, and the decline of temperature after breakthrough would certainly be more rapid. The best strategy for well spacing on an exclusively owned reservoir remains to be investigated.

4.10 JOINT PRODUCTION OF A COMMON RESERVOIR

As mentioned in the footnote to page 3 of Chapter 1, the complications arising from joint production of oil* wells are characteristic of the classic "commons" problem where the activities of each party impose external costs on the other and cause the resource to be exploited "too fast" for their own common good. The external costs are imposed because the pumping activities of each party increase the pumping head for the other (as well as for themselves). The "too rapid" exploitation is due to the tendency of the resource to migrate to the extraction point, thereby enabling one owner to extract all the fluid. Under these circumstances, each hastens to remove as much as possible (so long as marginal operating cost is less than market price), selling it at whatever (depressed) price results.

In the case of oil and water (for consumptive uses), the fluid is not replaced; its volume is consumptively used. In the case of geothermal energy, the water is only a carrier of the desired resource, and while the water is replaced, the energy is not. This means that the injected cool water from producer A is likely to migrate to the production well of producer B, prematurely

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* Or water wells

cooling his well. In this case, B pumping faster to extract the energy before A will only induce more of A's injected effluent to migrate to the producing well, thereby further reducing the economic value of the produced well.

The obvious solution to this "commons" problem is to unitize the field, in which case the administrative and allocative arrangements worked out for oil and gas unitization would probably be readily adapted to this case. The remaining problem would then be how to best exploit a set of wells with interference (including the option of eliminating the interference by using only one doublet). This is essentially the problem considered in 4. 9.

Although the unitization approach may not be especially elegant in concept, it is certainly tangible and readily comprehended. In contrast, it is hard to imagine how one might compute (let alone administer) a scheme of charges or fines to "optimally" manage two or more independent but interfering geothermal producers. We have left this challenging problem to another investigation, noting that without some prearranged solution to the joint operation problem, all parties are likely to lose "in common."

Chapter 5

CONCLUSIONS

Kamal Golabi and Charles R. Scherer

5.1 BEST MANAGEMENT OF A HOT WATER GEOTHERMAL RESERVOIR

A major objective of this study was to determine when, how fast, and how long a geothermal reservoir should be produced, and to what degree the brine should be cooled, in order to maximize the net value of the reservoir (defined as the present worth of revenues minus costs of production), when the real value of produced energy increases at a rate of r percent per year. We have developed and demonstrated an engineering-economic model for answering these questions using realistic data for a hypothetical hot water geothermal system. The results are consistent with predictions from the economic theory of non-renewable resource management, as indicated in the next section.

5.2 SENSITIVITY OF DECISION VARIABLES TO INPUT DATA

The influence of physical and economic input data on the decision variables is summarized qualitatively in Table 5.1. Production rate (m^3/hr) increases with permeability, initial aquifer temperature, and the discount rate. It decreases with increases in initial* pumping power costs and r, the the rate of increase in energy value (see 5.1 above). The economic life of the reservoir increases with initial aquifer temperature, well life, and r, but decreases with initial pumping costs and the discount rate. Production starting time is postponed as r increases and moved forward in time as the discount rate increases. When the discount rate is low, the production starting time is postponed as royalty

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* Pumping costs increase with time in all cases. Here we are concerned only with the initial cost of pumping energy, to which the present worth of all pumping energy costs are tied. That is, doubling the initial cost doubles the total discounted pumping energy cost for a particular pumping rate.

(percent of total revenues) increases, and moved forward as land, rents, salaries, and delay penalty increase. When the discount rate is higher, this particular sensitivity is notably diminished. Although production timing can be influenced by these economic parameters, production rate is generally insensitive to them. In all cases the brine is best reinjected at a temperature very close to the temperature of the low pressure steam produced on the other side of the heat exchanger from the hot brine. Likewise, based on data used in our example, breakthrough times are generally short compared with reservoir lives, and in all cases most of the extracted energy is removed after breakthrough.

Of these data inputs, only initial pumping costs are known in the present with certainty, although the discount rate, while somewhat subjective, is relatively determinable. Likewise, aquifer temperature is known before production, though not before the first hole is drilled. On the other hand, the rate of increase of the value of energy is completely unknown and "better information" can be purchased only from professional diviners.

One of the most important physical parameters - one that is not readily measureable — is aquifer permeability. In view of the sensitivity of reservoir engineering decisions to this physical parameter, it should be carefully investigated by prospective geothermal entrepreneurs. In addition, it might be worthwhile for the U. S. Department of Energy to sponsor research on quick and accurate methods for determining aquifer permeability.

The other important parameter is well life. Here there is really no substitute for experience, but it would appear that serious consideration should be given to the prediction of well lives by experienced reservoir engineers prior to production of a reservoir. In addition, perhaps the U. S. Department of Energy would find it worthwhile to fund literature and field investigations of hot water well lives.

Table 5.1

SENSITIVITY OF PRODUCTION PARAMETERS TO PHYSICAL AND ECONOMIC DATA*

	Profits	Production Delay	Production Rate	Reservoir Life	Reinjection Temperature	Breakthrough
PHYSICAL DATA						
Porosity		†				
Permeability	X	†	X			
Initial Aquifer Temperature	X	†	X	X		
Well Life	X	†				
ECONOMIC DATA						
Well Cost	X	†				
Well Maintenance Cost		†				
Initial Electric Power Pumping Cost	X	†	X	X		
Land Rent, Salaries, and Delay Penalty	X	X				
Royalties	X	X				
Discount Rate	X	X	X	X		
Rate of Increase of Value of Energy	X	X	X	X		

* The symbol X indicates significant sensitivity of production parameter to input data. Blank space indicates sensitivity is not great.

† Indicates sensitivity not evaluated.

We conclude this section by noting that an active well maintenance program, the cost of which seems relatively unimportant, might contribute significantly to prolonged well life. If so, this would appear to be a potentially cost-beneficial subprogram, both in terms of production management and in terms of U. S. DOE research and development.

5.3 ECONOMIC VALUE OF THE RESERVOIR

In this study we define the "value of the reservoir" as the maximum of a function which is the present worth of benefits (production-derived revenues) less the present worth of the costs incurred in producing the reservoir. As such, the value of the reservoir is essentially expected discounted profits.* Table 5.1 indicates discounted profits are sensitive to all major physical and economic input data except aquifer porosity and well maintenance cost. Profits increase dramatically as initial equilibrium temperature increases and/or as initial pumping power costs decrease.

5.4 USE OF ECONOMIC INCENTIVES TO INFLUENCE PRODUCTION RATE AND TIMING

These results are useful for two purposes. First, they may be used to direct and/or predict the actions of a profit maximizing entrepreneur. Secondly, to the extent that a regulatory agency's valuation of future energy value and social discount rate diverge from the private entrepreneur's valuation of these parameters, the agency can use this evaluative technique to determine production times and rates deemed more in the public interest.

State and federal regulators who determine when and how fast to produce a geothermal reservoir have two control options available. They could

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* Strictly speaking, the value of the reservoir to society is equal to profits plus royalties, since the latter represent a transfer payment rather than a true opportunity cost.

accelerate or postpone production and/or specify production rate by fiat. Or they could use delay penalties and royalties as economic incentives to achieve the same ends. Our results indicate that production timing is indeed amenable to adjustment by these incentives, although production rate is not. However, when these incentives are exercised, the entrepreneur's profits, as well as the government's total royalty revenues, may also vary.

In the current socio-political milieu, decisions on timing and rate of geothermal energy production are likely to be made amidst tugging and hauling by energy companies, conservationists, and other interested government agencies including the Office of Management and the Government Administrative Office. In this context it is interesting to note that production timing can, to some extent, be adjusted using both royalties and delay penalty without affecting the entrepreneur's profit picture.

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APPENDIX A

Fortran IV Computor Program Listing and User's Guide for the Basic Production Model of Chapter 2

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             C
             C
             C      OPTIMAL GEOTHERMAL EXTRACTION MODEL
             C
             C             JUNE 17, 1977
             C
             C
 0001               DIMENSION  SM(10,15),PPC(10)
 0002               DIMENSICN ATIME(10,10)
 0003               DIMENSICN G(10,10,10)
 0004               DIMENSION EA(10,10),QP(10,10),ALI(10),DLT(10),AENSI(20,20)
 0005               DIMENSION AINT(10),R(10),AQ(10,10),AA(10,10),ATI(10,10)
 0006               DIMENSION ALIF(10,10),TOW(10,10),PRO(10,10),ATHET(10,10)
 0007               DIMENSION AT1(10,10),AT2(10,10),ACOST(10,10),ACON(10,10)
 0008               DIMENSICN APUM(10,10)
 0009               COMMON QS,PK,D,A,B,DRCON,EFFV,EFFH,C1,C2,EXA,SPGRF,CST10
 0010               COMMON CST25,EXCOF,CRF10,CRF25,AKM,Z,AM,E,COST,PUCST,E1,E2
 0011               COMMON S1,S2,S3,S4,SLV10,SLV25,EE,ENCST,E5,CRFN,WCST,WMC
 0012               CCMMON CHCST
             C
             C             INPUT DATA FOR PROGRAM
             C
             C             TABLE COMMENTS
 0013               READ (5,609)   ((SM(I,J),J=1,15),I=1,8)
             C
             C             GEOLOGICAL AND AQUIFER DATA
 0014               READ(5,11)   HCAPR,PHI,SPGRF,TO,TS,SMK
 0015               READ(5,13)  H,D,RW,A,B
             C
             C             TEMPERATURE DEPLETION EQUATION COEFFICIENTS
 0016               READ(5,14)  PHIO,PHI1,PHI2,GAMAO,GAMA1,GAMA2
             C
             C             INTEREST RATES
 0017               READ  (5,15)  M,(AINT(I),I=1,M)
             C
             C             ENERGY GROWTH RATES
 0018               READ  (5,15)  N,(R(J),J= 1,N)
             C
             C             COST EQUATION CONSTANTS
 0019               READ(5,16)  AM,C1,C2,UU,PK,CM,PP,AKM,Z
             C
             C             EXOGENOUS PARAMETERS
 0020               READ(5,17)  QMIN,QMAX,QINC,ALMIN,ALMAX,ALINC,QBAR
             C
             C             SALVAGE VALUES, PUMP EFFICIENCY, TEMPERATURE LIMIT
 0021               READ(5,707)  S1,S2,S3,S4,DELTA,EFFV,EFFH
             C
             C             ECONOMIC PARAMETERS
 0022               READ(5,621)  WCST,WMC,OHCST,RENT,SALRY,ROYLT
             C
             C             WELL LIFE, GROWTH TYPE (LIN OR LOG)
 0023               READ(5,39)  WLIFE,K
             C
             C
```

```
                C          CALCULATE HEAT CAPACITY OF FLUID, DENSITY OF FLUID,
                C          AND VISCOSITY OF FLUID.
                C
 0024                 HCAPF=1.008*SPGRF
 0025                 GAMAF=62.427*SPGRF
 0026                 FMU=.209/100./(2.1482*(TO-8.435+(8078.4+(TO-8.435)**2)
                     *    **.5)-120.0)
                C
                C       "CALL" TO ROUTINE TO PRINT ALL INPUT DATA
                C
 0027                 GOTO 1001
 0028           1000  CONTINUE
                C
                C          DETERMINE LIMIT FOR LOOPING
                C
 0029                 DKM=(ALMAX-ALMIN)/ALINC
 0030                 KML=IFIX(DKM)+1
 0031                 IF(ALMIN.EQ.0.0)  KML=KML-1
 0032                 AK=4.88*UU/(1000.0*HCAPF)
                C
                C     COMPUTE HEAT CAPACITY OF AQUIFER
                C
 0033                 HCAPA=(1.0-PHI)*HCAPR+PHI*HCAPF
                C
                C     COMPUTE BETA
                C
 0034                 BETA=6.2832*H*D**2.0*HCAPA/(HCAPF*52560.0)
                C
                C     COMPUTE AQUIFER CONDUCTIVITY
                C
 0035                 CKK=0.000000000011653
 0036                 CK=CKK*SMK*GAMAF/FMU
                C
                C     COMPUTE CONSTANT IN DRAWDOWN
                C
 0037                 RATIO=D/RW
 0038                 DRCON=ALOG(RATIO)/(6.2832*CK*H)
                C
                C     COMPUTE CHI
                C
 0039                 CHIO=PHIO/(6.0*BETA)
 0040                 CHI1=PHI1/(6.0*BETA)
 0041                 CHI2=PHI2/(6.0*BETA)
                C
                C          BEGIN LOOPING FOR EACH DISCOUNT RATE
                C
 0042                 DO 22 I=1,M
                C
                C          DETERMINE CAPITAL RECOVERY FACTOR FOR 10-YEAR
                C          LIFETIME EQUIPMENT, 25-YEAR LIFETIME EQUIPMENT,
                C          AND THE WELL.
                C
 0043                 CON10=(1.0+AINT(I))**10.0
 0044                 CON25=(1.0+AINT(I))**25.0
```

```
                  CCNN=(1.0+AINT(I))**WLIFE
                  CRF10=AINT(I)*CON10/(CON10-1.0)+CM
                  CRFN=AINT(I)*CONN/(CONN-1.0)+CM
                  CRF25=AINT(I)*CON25/(CON25-1.0)+CM
                  PO=0.1876*CRF25+PP
                  PPO(I)=PO
      C
      C                BEGIN LOOP FOR EACH ENERGY RATE
      C
                  DO 33 J=1,N
                  DO 933  KK=1,8
      933         G(I,J,KK)=0.0
                  WRITE(6,89)  AINT(I),R(J)
                  WRITE(6,40)
                  ALFA=AINT(I)-R(J)
      C
      C                SET INITIAL PROJECT LIFE
      C
                  AL=ALMIN
                  IF(ALMIN.EQ.0.0)   AL=ALINC
                  QP(I,J)=0.0
      C
      C                BEGIN LOOP FOR EACH PROJECT LIFETIME
      C
                  DO 84 J1=1,KML
                  CSCCN=0.0
                  CSFU=0.0
                  CSENT=0.0
      C
      C                INITIALIZE PUMPING RATE TO MINIMUM
      C
                  Q=QMIN
      C
      C                COMPUTE SMALLEST MULTIPLES OF 10, 25, WL CONTAINING L
      C
                  ZZ=(AL-.001)/10.0
                  WW=(AL-.001)/25.0
                  YY=(AL-.001)/WLIFE
                  KK=IFIX(ZZ)
                  LL=IFIX(WW)
                  MM=IFIX(YY)
      C
      C                SET PROFITS TO MINIMUM VALUE ($0.00)
      C
                  PRO(I,J)=0.0
      C
      C                INITIALIZE ARRAY OF DATA FOR OUTPUT
      C
                  AQ(I,J)=0.0
                  AR(I,J)=0.0
                  ATI(I,J)=0.0
                  ALIF(I,J)=AL
                  TOW(I,J)=0.0
                  ACOST(I,J)=0.0
```

```
0078            ACON(I,J)=0.0
0079            APUM(I,J)=0.0
0080            AENST(I,J)=0.0
          C
          C          COMPUTE INTERMEDIATE VALUES FOR EQUATIONS DESCRIBING
          C          THE COST FUNCTION OF BOTH EXPONENTIAL AND LINEAR GROWTH
          C
0081            DIS1=-AINT(I)*AL
0082            DIS2=EXP(DIS1)
0083            DIS3=-AINT(I)*10.0*(KK+1)
0084            DIS4=EXP(DIS3)
0085            DIS5=-AINT(I)*25.0*(LL+1)
0086            DIS6=EXP(DIS5)
0087            DIS7=-AINT(I)*WLIFE*(MM+1)
0088            DIS8=EXP(DIS7)
0089            E=(1.0-DIS2)/AINT(I)
0090            E1=(DIS2-DIS4)/AINT(I)
0091            E2=(DIS2-DIS4)/AINT(I)
0092            E3=(1.0-DIS4)/AINT(I)-S2*E2
0093            E4=(1.0-EXP(-ALFA*AL))/ALFA
0094            E5=(DIS2-DIS8)/AINT(I)
0095            EE=E+(1.0-DIS2*(1.0+AINT(I)*AL))*R(J)/AINT(I)**2.0
0096            IF(K.EQ.1) EE=E4
0097      23    IF(K.EQ.2) GO TO 401
          C
          C          DETERMINATION OF PARAMATERS FOR EXPONENTIAL CASE
          C
          C
          C          BREAKTHROUGH HAS NOT OCCURED.
          C          FIND EXCHANGER AREA, INJECTION TEMPERATURE, AND REVENUES.
          C
0098            BKT=BETA/Q
0099            IF(BKT.GT.AL) GO TO 333
0100            GO TO 335
0101      333   TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*EE)+TS
0102            EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
0103            EXCOF=EXA/C
0104            THETA=34.76*PO*HCAPF*EE*(TO-TI)
0105            GO TO 207
          C
          C          BREAKTHROUGH HAS OCCURED.
          C          FIND INJECTION TEMPERATURE, HEAT EXCHANGER AREA, AND
          C          REVENUES.
          C
          C          EVALUATE TEMPERATURE DEPLETION EQUATION.
          C
0106      335   CONTINUE
          C
          C     COMPUTE SIGMA1
          C
0107            SIGMA=EXP(-ALFA*BETA/Q)
0108            SIG1=(1.0-SIGMA)/ALFA
          C
          C     COMPUTE SIGMA2
```

```
            C
0109              SIG01=CHI0*Q+ALFA
0110              SIG11=CHI1*Q+ALFA
0111              SIG21=CHI2*Q+ALFA
0112              SI011=-SIG01*BETA/Q
0113              SI012=-SIG01*AL
0114              SI111=-SIG11*BETA/Q
0115              SI112=-SIG11*AL
0116              SI211=-SIG21*BETA/Q
0117              SI212=-SIG21*AL
            C
            C           TEST FOR EXPONENTIAL UNDERFLOW
            C
0118              D01=EXP(SI011)
0119              IF(SI012.LT.-170.0) GO TO 61
0120              D02=EXP(SI012)
0121              GO TO 62
0122        61    D02=0.0
0123        62    D11=EXP(SI111)
0124              IF(SI112.LT.-170.0) GO TO 63
0125              D12=EXP(SI112)
0126              GO TO 64
0127        63    D12=0.0
0128        64    D21=EXP(SI211)
0129              IF(SI212.LT.-170.0) GO TO 65
0130              D22=EXP(SI212)
0131              GO TO 66
0132        65    D22=0.0
0133        66    SIG2=GAMA0*(D01-D02)/SIG01+GAMA1*(D11-D12)/SIG11
                 *   +GAMA2*(D21-D22)/SIG21
0134        205   TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*(SIG1+SIG2))+TS
0135              GO TO 204
            C
            C           DETERMINIATION OF PARAMETERS FOR THE LINEAR CASE.
            C
0136        401   BKT=BETA/Q
0137              IF(BKT.GT.AL) GO TO 334
0138              GO TO 336
            C
            C           BREAKTHROUGH HAS NOT OCCURED.
            C           FIND INJECTION TEMPERATURE, HEAT EXCHANGER AREA,
            C           AND REVENUES.
            C
0139        334   TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*EE)+TS
0140              EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
0141              EXCOF=EXA/Q
0142              THETA=34.76*PO*HCAPF*EE*(TO-TI)
0143              GO TO 207
0144        336   CONTINUE
            C
            C           BREAKTHROUGH HAS OCCURED. EVALUATE TEMPERATURE
            C           DEPLETION EQUATION.
            C
0145              SS1=-AINT(I)*BETA/Q
```

```
0146              SS2=EXP(SS1)
0147              SS3=(1.0-SS2)/AINT(I)
0148              SS4=(1.0-SS2*(1.0-SS1))/AINT(I)**2.0
0149              DB1=CHIO*Q+AINT(I)
0150              DB2=CHI1*Q+AINT(I)
0151              DB3=CHI2*Q+AINT(I)
0152              DB11=DB1*BETA/Q
0153              DB12=DB1*AL
0154              DB21=DB2*BETA/Q
0155              DB22=DB2*AL
0156              DB31=DB3*BETA/Q
0157              DB32=DB3*AL
           C
           C          TEST FOR EXPONENTIAL UNDERFLOW
           C
0158              DB111=EXP(-DB11)
0159              IF(DB12.GT.170.0) GO TO 301
0160              DB122=EXP(-DB12)
0161              GO TO 302
0162       301    DB122=0.0
0163       302    DB211=EXP(-DB21)
0164              IF(DB22.GT.170.0) GO TO 303
0165              DB222=EXP(-DB22)
0166              GO TO 304
0167       303    DB222=0.0
0168       304    DB311=EXP(-DB31)
0169              IF(DB32.GT.170.0) GO TO 305
0170              DB322=EXP(-DB32)
0171              GO TO 306
0172       305    DB322=0.0
0173       306    CONTINUE
0174              FR1=GAMAO*(DB111-DB122)/DB1+GAMA1*(DB211-DB222)/DB2+GAMA2
                *   *(DB311-DB322)/DB3
0175              FR2=GAMAO*(DB111*(1.0+DB11)-DB122*(1.0+DB12))/DB1**2.0
                *   +GAMA1*(DB211*(1.0+DB21)-DB222*(1.0+DB22))/DB2**2.0
                *   +GAMA2*(DB311*(1.0+DB31)-DB322*(1.0+DB32))/DB3**2.0
0176              FR3=PC*(SS3+FR1)+R(J)*(SS4+FR2)*PO
0177              TI=(150.7*E3*CRF10)/(AK*34.76*HCAPF*FR3)+TS
           C
           C      COMPUTE AREA OF THE HEAT EXANGER
           C
0178       204    EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
0179              EXCOF=EXA/Q
0180              IF(DELTA.EQ.0.0) GO TO 32
           C
           C
           C          COMPUTE CONDITION THAT TEMPERATURE DIFFERENCE IS
           C          SMALLER THAN DELTA.
           C
0181              CON1=CHIO*Q*AL
0182              CON2=CHI1*Q*AL
0183              CON3=CHI2*Q*AL
0184              IF(CON1.GT.170.0) GO TO 27
0185              CON11=GAMAO*EXP(-CON1)
```

```
              GO TO 117
      27      CON11=0.0
      117     IF(CON2.GT.170.0) GOTO 28
              CON22=GAMA1*EXP(-CON2)
              GO TO 18
      28      CON22=0.0
      18      IF(CON3.GT.170.0) GO TO 29
              CON33=GAMA2*EXP(-CON3)
              GO TO 19
      29      CON33=0.0
      19      CON=CON11+CON22+CON33
              DEL=DELTA/(TO-TI)
      C
      C            IF THE TEMPERATURE HAS FALLEN BELOW THE ALLOWABLE
      C            LIMIT, CONTINUE TO NEXT LIFE AND START SEARCHING.
      C
              IF(CON.GT.DEL) GO TO 32
              GO TO 34
      32      CONTINUE
      C
      C       COMPUTATION OF THETA1 AND THETA2
      C
              IF(K.EQ.2)  GO TO 402
      C
      C            EVALUATION FOR THE EXPONENTIAL CASE.
      C
      206     THET1=34.76*PO*HCAPF*SIG1*(TO-TI)*Q
              THET2=34.76*PO*HCAPF*SIG2*(TO-TI)*Q
              THETA=THET1+THET2
              GO TO 207
      C
      C            EVALUATION FOR THE LINEAR CASE.
      C
      402     THETA=Q*34.76*HCAPF*(TO-TI)*FR3
      207     X=Q/QBAR
              IF(X.EQ.1.0) GO TO 78
              GO TO 79
      C
      C            FIND COSTS FOR A SINGLE WELL OPERATING AT MAXIMUM RATE
      C
      78      QS=QBAR
              CALL KOST
              CSCON=COST+SLV10+SLV25
              CSPU=PUCST
              CSENT=ENCST*EE
              COSTS=CSCON
              PUMCS=CSPU
              EECST=CSENT
              GO TO 80
      79      CONTINUE
              IF(X.LT.1.0) GO TO 99
              II=IFIX(X)
              GO TO 101
      99      II=0
```

```
                C
                C          CCMPUTE TOTAL COSTS
                C
0224            101   QS=Q-II*QBAR
0225                  IF(CS.EQ.0.0) GO TO 81
0226                  GO TO 82
0227            81    COSTS=CSCON*II
0228                  PUMCS=CSPU*II
0229                  EECST=CSENT*II
0230                  GO TO 80
0231            82    CALL KOST
0232                  COSTS=CSCON*II+COST+SLV10+SLV25
0233                  PUMCS=CSPU*II+PUCST
0234                  EECST=ENCST*EE+CSENT*II
0235            80    ALLCS=COSTS+(RENT+SALRY)*E
                C
                C          COMPUTE NET REVENUES
                C
0236                  REV=THETA*(1.0-ROYLT)
                C
                C          CCMPUTE PROFITS IN THOUSANDS OF DOLLARS.
                C
0237                  REV1=(REV-ALLCS)/1000.0
                C
                C          SCALE DATA TO 1,000-$ RANGE FCR OUTPUT
                C
0238                  SCCON=CSCON/1000.0
0239                  COSTS=ALLCS/1000.0
0240                  PUMCS=PUMCS/1000.0
0241                  EECST=EECST/1000.0
                C
                C          ARE THE PROFITS FCR THIS PROJECT LIFE A NEW MAXIMUM?
                C          IF SO, UPDATE DATA ARRAY.
                C
0242                  IF(REV1.GT.PRO(I,J)) GC TO 31
0243                  GO TO 30
0244            31    PRO(I,J)=REV1
0245                  AQ(I,J)=Q
0246                  AA(I,J)=EXA
0247                  ATI(I,J)=TI
0248                  ALIF(I,J)=AL
0249                  BKT=BETA/Q
0250                  TOW(I,J)=BKT
0251                  ACOST(I,J)=COSTS
0252                  ACON(I,J)=SCCON
0253                  APUM(I,J)=PUMCS
0254                  AENST(I,J)=EECST
                C
                C          NEXT VALUE OF Q (PUMPING RATE)
                C
0255            30    Q=Q+QINC
0256                  IF (Q.GT.QMAX) GO TO 34
0257                  GO TO 23
0258            34    CONTINUE
```

```
              C
              C        ROUND PROFITS TO NEAREST $10
              C
 0259                  IPRO=PRO(I,J)*100.0+0.5
 0260                  PRO(I,J)=IPRO
 0261                  PRO(I,J)=PRO(I,J)/100.0
 0262                  WRITE(6,87)  PRO(I,J),ALIF(I,J),AQ(I,J),AA(I,J),AII(I,J),
                     *   TOW(I,J),ACON(I,J),ACOST(I,J),APUM(I,J),AENST(I,J)
 0263         36       CONTINUE
              C
              C            ARE THE PROFITS FOR THIS COMBINATION OF DISCOUNT AND
              C            ENERGY RATES AT A MAXIMUM?
              C            IF SO, PLACE THE DATA INTO THE ARRAY.
              C
 0264                  IF(PRO(I,J).GT.QP(I,J)) GO TO 708
 0265                  GO TO 709
 0266         708      QP(I,J)=PRO(I,J)
 0267                  G(I,J,1)=PRO(I,J)
 0268                  G(I,J,2)=ALIF(I,J)
 0269                  G(I,J,3)=AQ(I,J)
 0270                  G(I,J,4)=AA(I,J)
 0271                  G(I,J,5)=ACOST(I,J)
 0272                  G(I,J,6)=AENST(I,J)
 0273                  G(I,J,7)=ATI(I,J)
 0274                  G(I,J,8)=TOW(I,J)
 0275         709      AL=AL+ALINC
 0276         84       CONTINUE
 0277         33       CONTINUE
 0278         22       CONTINUE
              C
              C            OUTPUT TABLES OF RESULTS
              C
 0279                  DO 503 JZ=1,8
 0280                  WRITE(6,618)
 0281                  WRITE(6,607)(SM(JZ,J),J=1,15)
 0282                  WRITE(6,610)
 0283                  WRITE(6,604)
 0284                  WRITE(6,603)  (AINT(JY),JY=1,M)
 0285                  WRITE(6,604)
 0286                  WRITE(6,611)
 0287                  WRITE(6,604)
 0288                  WRITE(6,602)
 0289                  DO 501 JX=1,N
 0290                  WRITE(6,604)
 0291                  IF(JZ.GT.6) GO TO 500
 0292                  WRITE(6,617)  R(JX),(G(JY,JX,JZ),JY=1,M)
 0293                  GO TO 501
 0294         500      WRITE(6,619)  R(JX),(G(JY,JX,JZ),JY=1,M)
 0295         501      CONTINUE
 0296                  WRITE(6,604)
 0297                  WRITE(6,602)
 0298                  WRITE(6,604)
 0299                  WRITE(6,612)  (PRO(I),I=1,M)
 0300                  WRITE(6,620)
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```
0301                WRITE(6,604)
0302                WRITE(6,610)
0303                WRITE(6,613)
0304                IF(K.EQ.2) GO TO 505
0305                WRITE(6,605)
0306                GO TO 504
0307          505   WRITE(6,606)
0308          504   CONTINUE
0309                PH=PHI*100.0
0310          503   CONTINUE
0311          11    FORMAT(2F6.2,F6.4,3F6.2)
0312          13    FORMAT(5F6.2)
0313          14    FORMAT(6F6.4)
0314          15    FORMAT(I1,5X,8F8.4)
0315          16    FORMAT(8F7.2,F8.3)
0316          17    FORMAT(7F8.2)
0317          39    FORMAT (F6.2,3X,I1)
0318          40    FORMAT(8X,'NPV$',5X,'YRS',4X,'Q*',4X,'A*',4X,'TI',4X,'TAU',
                   *   4X,'CSTQB',4X,'TCST',4X,'PUCST',3X,'ENCST'//)
0319          89    FORMAT('1',///,17X,'INTEREST RATE=',F6.4,2X,
                   *   'RATE OF INCREASE OF PRICE=',F6.4///)
0320          87    FORMAT(6X,F9.2,2X,F5.0,1X,F5.0,1X,F5.0,1X,F6.2,1X,F5.2,2X,
                   *   F7.1,2X,F7.1,1X,F7.1,1X,F7.1)
0321          602   FORMAT(15X,'---------------------------------------------',
                   *   '-----------------')
0322          603   FORMAT(15X,'|     I=  |  ',F6.2,4X,F6.2,4X,F6.2,4X,F6.2,4X,
                   *   F6.2,5X,'|')
0323          604   FORMAT(15X,'|',8X,'|',53X,'|')
0324          605   FORMAT(26X,'P(T)=P(0)*EXP(+R*T)',/)
0325          606   FORMAT(26X,'P(T)=P(0)*(1+R*T)',/)
0326          607   FORMAT(15X,15A4,///////)
0327          609   FORMAT(15A4)
0328          610   FORMAT(15X,'_____________________________________________',
                   *   '__________________')
0329          611   FORMAT(15X,'|',2X,'R=',4X,'|',53X,'|')
0330          612   FORMAT(15X,'|  P(0)   |  ',F6.3,4X,F6.3,4X,F6.3,4X,F6.3,4X,
                   *   F6.3,5X,'|')
0331          613   FORMAT(////////)
0332          617   FORMAT(15X,'|  ',F5.3,' |  ',F6.0,4X,F6.0,4X,F6.0,4X,F6.0,
                   *   4X,F6.0,5X,'|')
0333          618   FORMAT('1',////,41X,'TABLE',////)
0334          619   FORMAT(15X,'|  ',F5.3,' |  ',F6.2,4X,F6.2,4X,F6.2,4X,F6.2,
                   *   4X,F6.2,5X,'|')
0335          620   FORMAT(15X,'|',1X,'$/MBTU',1X,'|',53X,'|')
0336          621   FORMAT(5F10.2,F6.4)
0337          707   FORMAT(4F5.2,3F8.2)
0338                WRITE (6,800)
0339          800   FORMAT ('1')
0340                STOP
              C
              C           SPECIAL SECTION FOR PRINTING OUT INPUT VALUES
              C
0341          1001  WRITE (6,1101)
0342                WRITE (6,1102) HCAPR,HCAPF,PHI
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0343              WRITE (6,1103) SPGRF,TC,TS
0344              WRITE (6,1104) SMK,GAMAF,FMU
0345              WRITE (6,1105) H,D,RW
0346              WRITE (6,1106) A,B
0347              WRITE (6,1107)  PHIO,PHI1,PHI2
0348              WRITE (6,1108)  GAMAO,GAMA1,GAMA2
0349              WRITE (6,1109) AM,CM,PE
0350              WRITE (6,1110) C1,C2,WCST
0351              WRITE (6,1111) UU,PK,OHCST,RENT,SALRY,ROYLT
0352              WRITE (6,1112) AKM,Z
0353              WRITE (6,1113) S1,S2,S3
0354              WRITE (6,1114) S4
0355              WRITE (6,1115) EFFV,EFFH
0356              WRITE (6,1116) QMAX,QMIN,QINC
0357              WRITE (6,1117) QBAR,ALMAX,ALINC
0358              WRITE (6,1118) WLIFE,WMC,DELTA
0359              IF (K.EQ.1)  WRITE (6,1119)
0360              IF (K.EQ.2)  WRITE (6,1120)
0361              WRITE (6,1121)  (AINT(I),I=1,M)
0362              WRITE (6,1122)  (R(I),I=1,N)
0363              GOTO 1000
0364       1101   FORMAT ('1',43X,'PROGRAM DATA'////)
0365       1102   FORMAT (' ','HEAT CAP. OF ROCK         ',F8.2,
                *         3X,'HEAT CAP. OF FLUID        ',F8.2,
                *         3X,'POROSITY OF AQUIFER       ',F8.2)
0366       1103   FORMAT (' ','SPEC. GRAVITY OF FLUID ',F8.4,
                *         3X,'AQUIFER INIT. TEMP.       ',F8.2,
                *         3X,'STEAM TEMPERATURE         ',F8.2/)
0367       1104   FORMAT (' ','INTRINSIC PERMEABILITY ',F8.2,
                *         3X,'FLUID UNIT WEIGHT         ',F8.2,
                *         3X,'FLUID VISCOSITY        ',F11.8/)
0368       1105   FORMAT (' ','AQUIFER HEIGHT            ',F8.2,
                *         3X,'WELL SEPERATION           ',F8.2,
                *         3X,'WELL RADIUS               ',F8.2)
0369       1106   FORMAT (' ','A - HEIGHT                ',F8.2,
                *         3X,'B - HEIGHT                ',F8.2/)
0370       1107   FORMAT (' ','PHI(1)                    ',F8.4,
                *         3X,'PHI(2)                    ',F8.4,
                *         3X,'PHI(3)                    ',F8.4)
0371       1108   FORMAT (' ','GAMMA(1)                  ',F8.4,
                *         3X,'GAMMA(2)                  ',F8.4,
                *         3X,'GAMMA(3)                  ',F8.4/)
0372       1109   FORMAT (' ','UNIT PIPE CLEANING COST',F8.2,
                *         3X,'INSURANCE/TAX % COST      ',F8.2,
                *         3X,'FUEL & OPERATING COSTS ',F8.2)
0373       1110   FORMAT (' ','BOWL UNIT COST 220 GPM ',F8.2,
                *         3X,'BOWL UNIT COST 1100 GPM',F8.2,
                *         3X,'WELL COST               ',F10.2)
0374       1111   FORMAT (' ','HEAT XFER. COEFF.         ',F8.2,
                *         3X,'PIPE INSTILLATION         ',F8.2,
                *         3X,'OVERHEAD COST             ',F8.2/
                *        ' ','RENT                      ',F8.2,
                *         3X,'SALARIES                 ' ,F9.2,
                *         3X,'ROYALTIES                 ',F8.2)
```

```
0375        1112  FORMAT (' ','PUMP SERVICE COST MULT.',F8.2,
                 *         3X,'ELECTRICITY COST        ' ,F9.3/)
0376        1113  FORMAT (' ','PUMP SALVAGE VALUE       ',F8.2,
                 *         3X,'HEAT XCHG. SALVAGE       ',F8.2,
                 *         3X,'PIPE SALVAGE VALUE       ',F8.2)
0377        1114  FORMAT (' ','WELL SALVAGE VALUE       ',F8.2/)
0378        1115  FORMAT (' ','VERT. PUMP EFFICIENCY    ',F8.2,
                 *         3X,'HORZ. PUMP EFFICIENCY    ',F8.2)
0379        1116  FORMAT (' ','PUMP RATE LIMIT (MAX)    ',F8.2,
                 *         3X,'PUMP RATE LIMIT (MIN)    ',F8.2,
                 *         3X,'PUMP RATE INCREMENT      ',F8.2)
0380        1117  FORMAT (' ','MAX. WELL FLOW RATE      ',F8.2,
                 *         3X,'PROJECT LIFE             ',F8.2,
                 *         3X,'TIME INCREMENT           ',F8.2)
0381        1118  FORMAT (' ','WELL LIFE                ',F8.2,
                 *         3X,'WELL MAINTENENCE         ',F8.2,
                 *         3X,'XCHG. TEMP. DELTA        ',F8.2)
0382        1119  FORMAT (///' ','GROWTH IS EXPONENTIAL')
0383        1120  FORMAT (///' ','GROWTH IS LINEAR')
0384        1121  FORMAT (///' ','DISCOUNT RATES      =  ',8F8.4)
0385        1122  FORMAT (  /' ','ENERGY COST RATES   =  ',8F8.4)
0386              END
```

```
0001          SUBROUTINE KOST
      C
      C          THIS SUBROUTINE COMPUTES THE DISCOUNTED COSTS FOR ONE
      C          DOUBLET (Q<=QMAX).
      C
      C          ALL PARAMETERS USED ARE LOCATED IN COMMON STORAGE.
      C
      C
0002          COMMON QS,PK,D,A,B,DRCON,EFFV,EFFH,C1,C2,EXA,SPGRF,CST10
0003          COMMON CST25,EXCOF,CRF10,CRF25,AKM,Z,AM,E,COST,PUCST,E1,E2
0004          COMMON S1,S2,S3,S4,SLV10,SLV25,E3,ENCST,E5,CRFN,WCST,WMC
0005          COMMON CHCST
      C
      C          PIPE COST
      C
0006          PCST=PK*D*(0.1313*QS+1.323*QS**0.5-4.36)
      C
      C          ANNUAL PIPE CLEANING COST
      C
0007          CLCST=AM*D
      C
      C          HEAT EXCHANGER COST
      C
0008          ECST=5000.0+150.7*EXCOF*QS
      C
      C          HORIZONTAL PUMP COST
      C
0009          PCSTH=24.0*QS
      C
      C          HORIZONTAL MOTOR COST
      C
0010          MCSTH=0.0546*SPGRF*QS**2.0*DRCON/EFFH+1907.1
      C
      C          BOWL UNIT COST
      C
0011          BUCST=C1+(C2-C1)*QS**2.0/40000.0
      C
      C          COLUMN ASSEMBLY COST
      C
0012          COCST=(A+4.0+QS*DRCON)*(0.1313*QS+1.323*QS**0.5-4.36)+3981.0
      C
      C          VERTICAL SHAFT COST
      C
0013          SHCST=(.001339*QS*SPGRF*(DRCON*QS+A+B)/EFFV+0.0768*(DRCON*
             *    QS+A)-10.132)*(DRCON*QS+A)
      C
      C          VERTICAL MOTOR COST
      C
0014          MCSTV=0.0546*SPGRF*QS*(DRCON*QS+A+B)/EFFV+1907.1+
             *    0.35*(DRCON*QS+A)
      C
      C          CAPITAL COSTS FOR PUMPS
      C
0015          PPCST=COCST+MCSTV+SHCST+MCSTH+BUCST+PCSTH
```

```
                C
                C         PRESENT WORTH OF PUMP COSTS
                C
0016                 PUCST=PPCST*CRF10*(E+E1*(1.0-S1))
                C
                C         ANNUAL PUMP OPERATING COSTS
                C
0017                 ENCST=AKM*Z*(23.80*QS*SPGRF*(DRCON*QS+A+B)/EFFV+152.86*
                    *   (DRCON*QS+A)+23.80*QS*SPGRF*DRCON*QS/EFFH)
                C
                C         TOTAL DISCOUNTED COST FOR ONE DOUBLET WITHOUT SALVAGE
                C
0018                 CST10=COCST+MCSTV+SHCST+MCSTH+BUCST+PCSTH+ECST
0019                 CST25=PCST+OHCST
0020                 COST=(CST10*CRF10+CST25*CRF25+CLCST+WMC+WCST*CRFN)*E+ENCST*EE
                C
                C         SALVAGE COSTS
                C
0021                 SLV10=PPCST*CRF10*E1*(1.0-S1)+ECST*CRF10*E1*(1.0-S2)
0022                 SLV25=PCST*CRF25*E2*(1.0-S3)+OHCST*CRF25*E2*(1.0-S4)
                    *   +WCST*CRFN*E5
0023                 RETURN
0024                 END
```

DATA CARD LAYOUT

Title Cards

Cards 1-8

1	61–80
TITLES FOR OUTPUT TABLES	(hatched)

Geological and Aquifer Data

Card 9

1	7	13	19	25	31	37–80
xxx.xx	xxx.xx	x.xxxx	xxx.xx	xxx.xx	xxx.xx	(hatched)
HCAPR	PHI	SPGRF	TO	TS	SMK	

Card 10

1	7	13	19	25	31–80
xxx.xx	xxx.xx	xxx.xx	xxx.xx	xxx.xx	(hatched)
H	D	RW	A	B	

Coefficients of the g Function

Card 11

1	7	13	19	25	31	37–80
x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	x.xxxx	(hatched)
PHIØ	PHI1	PHI2	GAMAØ	GAMA1	GAMA2	

Interest Rates

Card 12

1	2	7	15	23	31	39	47	55	63	71–80
x	(hatched)	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	(hatched)
M		AINT(1)	AINT(2)	AINT(3)	AINT(4)	AINT(5)	AINT(6)	AINT(7)	AINT(8)	

Growth Rates in Value of Energy

Card 13

1	2	7	15	23	31	39	47	55	63	71–80
x		xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	xxx.xxxx	
N		R(1)	R(2)	R(3)	R(4)	R(5)	R(6)	R(7)	R(8)	

Economic Parameters

Card 14

1	8	15	22	29	36	43	50	57	65–80
xxxx.xx	xxxx.xx	xxxx.xx	xxxx.xx	xxxx.xx	xxxx.xx	xxxx.xx	xxxx.xx	xxxx.xxx	
AM	C1	C2	UU	PK	CM	PP	AKM	Z	

Grid Search Parameters

Card 15

1	9	17	25	33	41	49	57–80
xxxxx.xx	xxxxx.xx	xxxxx.xx	xxxxx.xx	xxxxx.xx	xxxxx.xx	xxxxx.xx	
QMIN	QMAX	QINC	ALMIN	ALMAX	ALINC	QBAR	

Economic Parameters

Card 16

1	6	11	16	21	29	37	45–80
xx.xx	xx.xx	xx.xx	xx.xx	xxxxx.xx	xxxxx.xx	xxxxx.xx	
S1	S2	S3	S4	DELTA	EFFV	EFFH	

Card 17

1	11	21	31	41	51	57–80
xxxxxxx.xx	xxxxxxx.xx	xxxxxxx.xx	xxxxxxx.xx	xxxxxxx.xx	x.xxxx	
WCST	WMC	OHCST	RENT	SALRY	ROYLT	

Card 18

1	7	9	10	11–80
xxx.xx			x	
WLIFE			K	

GUIDE TO DATA CARD LAYOUT

CODES	DESCRIPTIONS	SYMBOLS	UNITS
HCAPR	Heat Capacity of Rock	$\rho_R c_R$	cal/cc° C
PHI	Porosity of Aquifer	ϕ	Fraction
SPGRF	Specific Gravity of Fluid	sp. gr.	———
TO	Initial Equilibrium Temperature	T_o	°C
TS	Steam Temperature	T_s	°C
SMK	Intrinsic Permeability	k	millidarcies
H	Thickness of Aquifer	h	m
D	Doublet Separation	D	m
RW	Well Radius	r_w	m
A	Static Level of Fluid	z	m
B	Friction Losses	b	m
PHIØ	Coefficient of g Function	ϕ_1	———
PHI1	Coefficient of g Function	ϕ_2	———
PHI2	Coefficient of g Function	ϕ_3	———
GAMAØ	Coefficient of g Function	γ_1	———
GAMA1	Coefficient of g Function	γ_2	———
GAMA2	Coefficient of g Function	γ_3	———
M	Number of Interest Rates	—	———
N	Number of Energy Growth Rates	—	———
AINT(I)	Interest Rates, $I \leq M$	i	Fraction
R(J)	Rates of Growth in Value of Energy, $J \leq N$	r	Fraction

CODES	DESCRIPTIONS	SYMBOLS	UNITS
AM	Pipe Cleaning Cost	P_c	\$/m/year
C1	Cost of 50 m^3/hr Bowl Unit	c_1	\$
C2	Cost of 250 m^3/hr Bowl Unit	c_2	\$
UU	Overall Heat Transfer Coefficient of Fluid	U(0)	BTU/hr-ft^2-^{o}F
PK	Pipe Support Multiplier	k_p	———
CM	Miscellaneous Costs of Capital in Equation (26)	m	Fraction
PP	Constant term in Equ. (27)	———	———
AKM	Pump Maintenance Coefficient	k_m	———
Z	Electricity Cost at Time Zero	R_o	\$/kwh
QMIN	Min. Pumping Rate to be Considered	Q_{min}	m^3/hr
QMAX	Max. Pumping Rate to be Considered	Q_{max}	m^3/hr
QINC	Incremental Increase in Pumping Rate	Q_{inc}	m^3/hr
ALMIN	Min. Project Life to be Considered	L_{min}	years
ALMAX	Max. Project Life to be Considered	L_{max}	years
ALINC	Incremental Increase in Project Life	L_{inc}	years
QBAR	Capacity of Each Production Well	$\bar{Q}$	m^3/hr
S1	Salvage Value of Pumps	s_1	Fraction of Remaining Payments

GUIDE TO DATA CARD LAYOUT (continued)

CODES	DESCRIPTIONS	SYMBOLS	UNITS
S2	Salvage Value of Heat Exchangers	s_2	Fraction of Remaining Payments
S3	Salvage Value of Pipes	s_3	Fraction of Remaining Payments
S4	Salvage Value of Well Assemblies	s_4	Fraction of Remaining Payments
DELTA	Min. Allowable Temperature Difference	δ	^oC
EFFV	Vertical Pump Efficiency	Eff_V	Fraction
EFFH	Horizontal Pump Efficiency	Eff_H	Fraction
WCST	Well Cost per Doublet	WC	$
WMC	Annual Well Maintenance Cost	WM	$/year/Doublet
OHCST	Well Assembly Cost per Doublet	WA	$
RENT	Annual Land Rents	Rent	$/year
SALRY	Annual Salaries	Salaries	$/year
ROYLT	Royalty	η	Fraction of Gross Revenues
WLIFE	Well Life	WL	Years
K	No. Denoting Model :	——	——
	1 = Exponential Growth Model	——	——
	2 = Linear Growth Model	——	——

APPENDIX B

Flow Chart and Computor Program for Optimal Timing Model (Chapter 3)

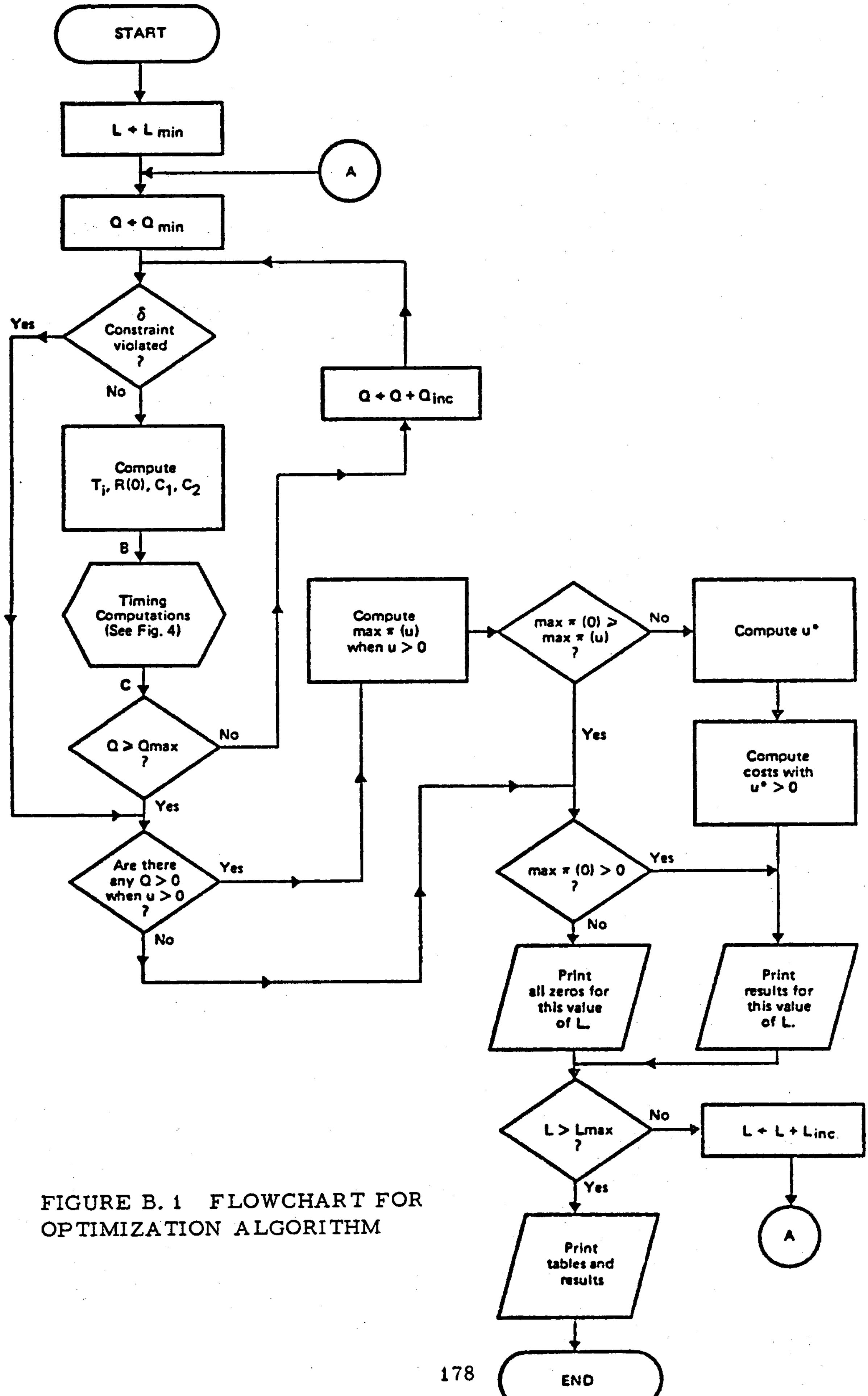

FIGURE B.1 FLOWCHART FOR OPTIMIZATION ALGORITHM

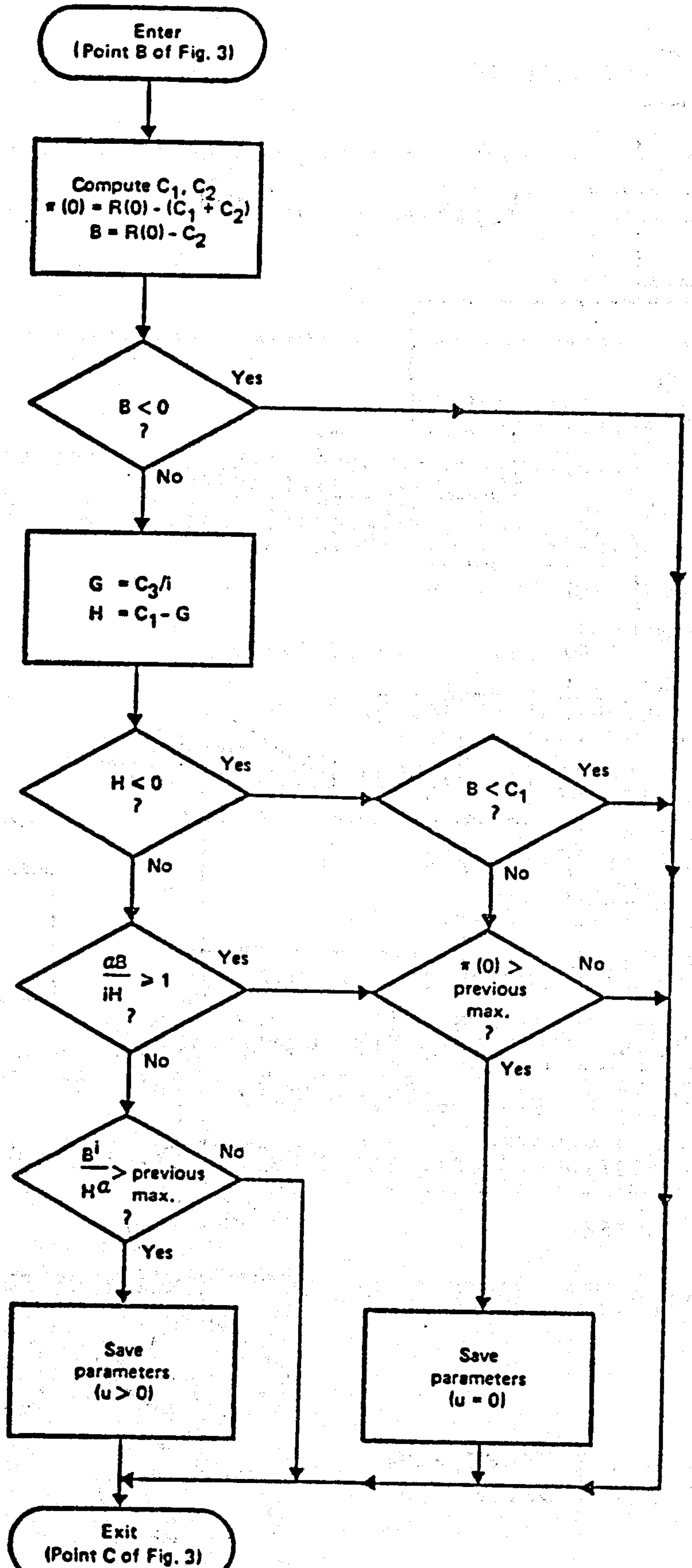

FIGURE B. 2 FLOWCHART FOR TIMING COMPUTATIONS

```
NO RUN CARD -- DECK NOT ACCEPTED.
*JOB
C
C
C      ******************************************************************
C      *                                                                *
C      *                                                                *
C      *    OPTIMAL TIMING AND EXTRACTION OF GEOTHERMAL ENERGY          *
C      *                                                                *
C      *                                                                *
C      *    OCTOBER 7, 1977                                             *
C      *                                                                *
C      *                                                                *
C      ******************************************************************
C
C
      DIMENSION  SM(11,15),PPO(10)
      DIMENSION ATIME(10,10)
      DIMENSION G(10,10,11)
      DIMENSION EA(10,10),ALI(10),DLT(10),AINT(10),R(10),ATHET(10,10),
     *   AT1(10,10),AT2(10,10),TSAV1(10),TSAV2(5)
      COMMON QS,PK,D,A,B,DRCON,EFFV,EFFH,C1,C2,EXA,SPGRF,CST10,CST
     %25,EXCOF,CRF10,CRF25,AKM,Z,AM,E,COST,PUCST,E1,E2,S1,S2,S3,S4,SLV10
     %,SLV25,EE,ENCST,ES,CRFN,WCST,WMC,OHCST
C
C           INPUT DATA FOR PROGRAM
C
C
C           TABLE COMMENTS
      READ (5,609)   ((SM(I,J),J=1,15),I=1,11)
C
C           GEOLOGICAL AND AQUIFER DATA
      READ(5,11)  HCAPR,PHI,SPGRF,TO,TS,SMK
      READ(5,13) H,D,RW,A,B
C
C           TEMPERATURE DEPLETION EQUATION COEFFICIENTS
      READ(5,14) PHI0,PHI1,PHI2,GAMA0,GAMA1,GAMA2
C
C           INTEREST RATES
      READ (5,15) M,(AINT(I),I=1,M)
C
C           ENERGY GROWTH RATES
      READ (5,15) N,(R(J),J=1,N)
C
C           COST EQUATION CONSTANTS
      READ(5,16) AM,C1,C2,UU,PK,CM,PP,AKM,Z
C
C           EXOGENOUS PARAMETERS
      READ(5,17) QMIN,QMAX,QINC,ALMIN,ALMAX,ALINC,QBAR
C
C           SALVAGE VALUES, PUMP EFFICIENCY, TEMPERATURE LIMIT
      READ(5,707) S1,S2,S3,S4,DELTA,EFFV,EFFH
C
C           ECONOMIC PARAMETERS
      READ(5,621) WCST,WMC,OHCST,RENT,SALRY,ROYLT,PENLTY,RENT2
C
C           WELL LIFE
      READ(5,39)  WLIFE
C
C
C           CALCULATE HEAT CAPACITY OF FLUID, DENSITY OF FLUID,
C           AND VISCOSITY OF FLUID.
```

```
C
      HCAPF=1.008*SPGRF
      GAMAF=62.427*SPGRF
      FMU=.209/100./(2.1482*(TO-8.435+(8078.4+(TO-8.435)**2)**.5)-120.)
C
C           @CALL@ TO ROUTINE TO PRINT ALL INPUT DATA
C
      GOTO 1001
1000  CONTINUE
C
C           DETERMINE LIMIT FOR PROJECT LIFE LOOP
C
      DKM=(ALMAX-ALMIN)/ALINC
      KML=IFIX(DKM)+1
      IF(ALMIN.EQ.0.0)  KML=KML-1
      AK=4.88*UU/(1000.0*HCAPF)
      CSTC3=RENT2+PENLTY
C
C           COMPUTE HEAT CAPACITY OF AQUIFER
C
      HCAPA=(1.0-PHI)*HCAPR+PHI*HCAPF
C
C           COMPUTE BETA
C
      BETA=6.2832*H*D**2.0*HCAPA/(HCAPF*52560.0)
C
C           COMPUTE AQUIFER CONDUCTIVITY
C
      CKK=0.000000000011653
      CK=CKK*SMK*GAMAF/FMU
C
C           COMPUTE CONSTANT IN DRAWDOWN
C
      RATIO=D/RW
      DRCON=ALOG(RATIO)/(6.2832*CK*H)
C
C           COMPUTE CHI
C
      CHI0=PHI0/(6.0*BETA)
      CHI1=PHI1/(6.0*BETA)
      CHI2=PHI2/(6.0*BETA)
C
C           BEGIN LOOPING FOR EACH DISCOUNT RATE
C
      DO 22 I=1,M
C
C           DETERMINE CAPITAL RECOVERY FACTORS FOR 10-YEAR LIFETIME
C           EQUIPMENT, 25-YEAR LIFETIME EQUIPMENT, AND THE WELL.
C
      CON10=(1.0+AINT(I))**10.0
      CON25=(1.0+AINT(I))**25.0
      CONN=(1.0+AINT(I))**WLIFE
      CRF10=AINT(I)*CON10/(CON10-1.0)+CM
      CRFN=AINT(I)*CONN/(CONN-1.0)+CM
      CRF25=AINT(I)*CON25/(CON25-1.0)+CM
      PO=0.1876*CRF25+PP
      PPO(I)=PO
```

```
C
C          BEGIN LOOP FOR EACH ENERGY RATE
C
      DO 33 J=1,M
      DO 933 KK=1,11
933   G(I,J,KK)=0.0
      WRITE(6,89) AINT(I),R(J)
      WRITE(6,40)
      ALFA=AINT(I)-R(J)
C
C          SET INITIAL PROJECT LIFE
C
      AL=ALMIN
      IF(ALMIN.EQ.0.0)  AL=ALINC
      QP=0.0
C
C          BEGIN LOOP FOR EACH PROJECT LIFETIME
C
      DO 84 J1=1,KML
      DO 986 LL=1,5
986   TSAV2(LL)=0.0
      REV3=0.0
      CSCON=0.0
      CSPU=0.0
      CSENT=0.0
C
C          INITIALIZE PUMPING RATE TO MINIMUM
C
      Q=QMIN
C
C          COMPUTE SMALLEST MULTIPLES OF 10, 25, WL CONTAINING L
C
      ZZ=(AL-.001)/10.0
      WW=(AL-.001)/25.0
      YY=(AL-.001)/WLIFE
      KK=IFIX(ZZ)
      LL=IFIX(WW)
      MM=IFIX(YY)
C
C          SET PROFITS TO MINIMUM VALUE ($0.00)
C
      REV0=0.0
      TRAT2=0.0
C
C          INITIALIZE ARRAY OF DATA FOR OUTPUT
C
C
C          COMPUTE INTERMEDIATE VALUES FOR EQUATIONS DESCRIBING
C          THE COST FUNCTION.
C
      DIS1=-AINT(I)*AL
      DIS2=EXP(DIS1)
      DIS3=-AINT(I)*10.0*(KK+1)
      DIS4=EXP(DIS3)
      DIS5=-AINT(I)*25.0*(LL+1)
      DIS6=EXP(DIS5)
      DIS7=-AINT(I)*WLIFE*(MM+1)
```

```
      DIS8=EXP(DIS7)
      E=(1.0-DIS2)/AINT(I)
      E1=(DIS2-DIS4)/AINT(I)
      E2=(DIS2-DIS4)/AINT(I)
      E3=(1.0-DIS4)/AINT(I)-S2*E2
      E4=(1.0-EXP(-ALFA*AL))/ALFA
      E5=(DIS2-DIS8)/AINT(I)
      EE=E4
      IQKEY=0
C
C          FOR EACH I AND R, IF QMIN > QBAR, BEFORE COMPUTING
C          USING Q = QMIN, COMPUTE Q = QBAR.
C
      IF(QMIN.GT.QBAR)  IQKEY=1
      IF(QMIN.GT.QBAR)  Q=QBAR
23    BKT=BETA/Q
      IF(BKT.GT.AL) GO TO 333
      GO TO 335
C
C          BREAKTHOUGH HAS NOT OCCURED.  FIND EXCHANGER AREA, INJECTION
C          TEMPERATURE, AND REVENUES.
C
C
333   TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*EE)+TS
      EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
      EXCOF=EXA/Q
      THETA=34.76*PO*HCAPF*EE*(TO-TI)
      GO TO 207
C
C          BREAKTHROUGH HAS OCCURED.  FIND INJECTION TEMPERATURE, HEAT
C          EXCHANGER AREA, AND REVENUES.
C          EVALUATE TEMPERATURE DEPLETION EQUATION.
C
335   CONTINUE
C
C          COMPUTE SIGMA1
C
      SIGMA=EXP(-ALFA*BETA/Q)
      SIG1=(1.0-SIGMA)/ALFA
C
C          COMPUTE SIGMA2
C
      SIG01=CHI0*Q+ALFA
      SIG11=CHI1*Q+ALFA
      SIG21=CHI2*Q+ALFA
      SI011=-SIG01*BETA/Q
      SI012=-SIG01*AL
      SI111=-SIG11*BETA/Q
      SI112=-SIG11*AL
      SI211=-SIG21*BETA/Q
      SI212=-SIG21*AL
C
C          TEST FOR EXPONENTIAL UNDERFLOW
C
      D01=EXP(SI011)
      IF(SI012.LT.-170.0) GO TO 61
      D02=EXP(SI012)
```

```
      GO TO 62
61    DO2=0.0
62    D11=EXP(SI111)
      IF(SI112.LT.-170.0) GO TO 63
      D12=EXP(SI112)
      GO TO 64
63    D12=0.0
64    D21=EXP(SI211)
      IF(SI212.LT.-170.0) GO TO 65
      D22=EXP(SI212)
      GO TO 66
65    D22=0.0
66    SIG2=GAMAO*(DO1-DO2)/SIGO1+GAMA1*(D11-D12)/SIG11+GAMA2*(D21-D22)/S
     %IG21
205   TI=(150.7*E3*CRF10)/(AK*34.76*PO*HCAPF*(SIG1+SIG2))+TS
      GO TO 204
C
C           COMPUTE THE AREA OF THE HEAT EXCHANGER
C
204   EXA=Q*(ALOG(TO-TS)-ALOG(TI-TS))/AK
      EXCOF=EXA/Q
      IF(DELTA.EQ.0.0) GO TO 32
C
C
C           COMPUTE CONDITION THAT TEMPERATURE DIFFERENCE IS SMALLER
C           THAN DELTA.
C
      CON1=CHIO*Q*AL
      CON2=CHI1*Q*AL
      CON3=CHI2*Q*AL
      IF(CON1.GT.170.0) GO TO 27
      CON11=GAMAO*EXP(-CON1)
      GO TO 117
27    CON11=0.0
117   IF(CON2.GT.170.0) GOTO 28
      CON22=GAMA1*EXP(-CON2)
      GO TO 18
28    CON22=0.0
18    IF(CON3.GT.170.0) GO TO 29
      CON33=GAMA2*EXP(-CON3)
      GO TO 19
29    CON33=0.0
19    CON=CON11+CON22+CON33
      DEL=DELTA/(TO-TI)
C
C           IF THE TEMPERATURE HAS FALLEN BELOW THE ALLOWABLE LIMIT,
C           CONTINUE TO NEXT LIFE AND START SEARCHING.
C
      IF(CON.GT.DEL) GO TO 32
      GO TO 34
32    CONTINUE
C
C           COMPUTATION OF THETA1 AND THETA2
C
206   THET1=34.76*PO*HCAPF*SIG1*(TO-TI)*Q
      THET2=34.76*PO*HCAPF*SIG2*(TO-TI)*Q
      THETA=THET1+THET2
```

```
207   X=Q/QBAR
      IF(X.EQ.1.0) GO TO 78
      GO TO 79
C
C         FIND COSTS FOR A SINGLE WELL OPERATING AT MAXIMUM RATE
C
78    QS=QBAR
      CALL KOST
      CSCON=COST+SLV10+SLV25
      CSPU=PUCST
      CSENT=ENCST*FE
      COSTS=CSCON
      PUMCS=CSPU
      EECST=CSENT
      GO TO 80
79    CONTINUE
      IF(X.LT.1.0) GO TO 99
      II=IFIX(X)
      GO TO 101
99    II=0
C
C         COMPUTE TOTAL COSTS WITH NO DELAY IN EXTRACTION
C
101   QS=Q-II*QBAR
      IF(QS.EQ.0.0) GO TO 81
      GO TO 82
81    COSTS=CSCON*II
      PUMCS=CSPU*II
      EECST=CSENT*II
      GO TO 80
82    CALL KOST
      COSTS=CSCON*II+COST+SLV10+SLV25
      PUMCS=CSPU*II+PUCST
      EECST=ENCST*FE+CSENT*II
80    ALLCS=COSTS+(RENT+SALRY)*E
C
C         COMPUTE NET REVENUES WITH NO DELAY IN EXTRACTION
C
      REV=THETA*(1.0-ROYLT)
C
C         COMPUTATION OF OPTIMAL DECISION VARIABLES AND
C         PROFITS WITH TIMING
C
C         BOX  1 OF FLOWCHART.
      REV1=REV-ALLCS
      IF(REV1.GT.REV3)  Q3=Q
      IF(REV1.GT.REV3)  REV3=REV1
      CSTC2=EECST
      CSTC1=ALLCS-CSTC2
      CONB=REV-CSTC2
C         BOX  2 OF FLOWCHART.
      IF(CONB.LT.0.0)  GO TO 30
C         BOX  3 OF FLOWCHART.
      CONG=CSTC3/AINT(I)
      CONH=CSTC1-CONG
C         BOX  4 OF FLOWCHART.
      IF(CONH.LE.0.0)  GO TO 90
```

```
C          BOX   5 OF FLOWCHART.
      CONTM=ALFA*CONB/(AINT(I)*CONH)
      IF(CONTM.GE.1.0)   GO TO 92
C          BOX   6 OF FLOWCHART.
      TSTRAT=CONB**AINT(I)/(CONH**ALFA)
      IF(TRAT2.GT.TSTRAT)   GO TO 30
C          BOX   7 OF FLOWCHART.
      TRAT2=TSTRAT
      TSAV1(1)=Q
      TSAV1(2)=EXA
      TSAV1(3)=TI
      TSAV1(4)=CSTC1
      TSAV1(5)=CSTC2
      TSAV1(6)=CSTC3
      TSAV1(7)=CONTM
      TSAV1(8)=CONB
      TSAV1(9)=CONG
      TSAV1(10)=CONH
      GO TO 30
C          BOX   8 OF FLOWCHART.
90    IF(CONB.LT.CSTC1)   GO TO 30
C          BOX   9 OF FLOWCHART.
92    IF(REV0.GE.REV1)   GO TO 30
C          BOX  10 OF FLOWCHART.
      REV0=REV1
      TSAV2(1)=Q
      TSAV2(2)=EXA
      TSAV2(3)=TI
      TSAV2(4)=CSTC1
      TSAV2(5)=CSTC2
C
C          NEXT VALUE OF Q (PUMPING RATE)
C
30    Q=Q+QINC
      IF (Q.GT.QMAX) GO TO 34
      IF(IQKEY.EQ.0)   GO TO 23
      Q=QMIN
      IQKEY=0
      GO TO 23
34    CONTINUE
      IF(TSAV1(1).EQ.0.0)   GO TO 31
323   Q=TSAV1(1)
      EXA=TSAV1(2)
      TI=TSAV1(3)
      CSTC1=TSAV1(4)
      CSTC2=TSAV1(5)
      CSTC3=TSAV1(6)
      CONTM=TSAV1(7)
      CONB=TSAV1(8)
      CONG=TSAV1(9)
      CONH=TSAV1(10)
      REV2=CONB*CONTM**(ALFA/R(J))-CONH*CONTM**(AINT(I)/R(J))-CONG
      IF(REV0.GE.REV2)   GO TO 31
      REV=REV2
      TIMST=-ALOG(TSAV1(7))/R(J)
      DCST=CSTC3*(1.0-EXP(-AINT(I)*TIMST))/AINT(I)
      EQCST=CSTC1*EXP(-AINT(I)*TIMST)
```

```
      DCST=CSTC2*EXP(-ALFA*TIMST)
      GO TO 331
31    Q=TSAV2(1)
      IF(Q.NE.0.0)  GOTO 378
C
C           NO ECONOMICALLY PROFITABLE PUMP RATE WAS FOUND.
C           OUTPUT IS SET TO ZERO TO INDICATE THIS (Q=0)
C
      WRITE (6,87) Q,AL,Q,Q,Q,Q,Q,Q,Q,Q,Q,Q,Q
      GOTO 709
378   EXA=TSAV2(2)
      TI=TSAV2(3)
      CSTC1=TSAV2(4)
      CSTC2=TSAV2(5)
      REV=REVO
      TIMST=0.0
      DCST=0.0
      EQCST=CSTC1
      OCST=CSTC2
331   IPRO=REV/1000.00*100.00+0.5
      REV=IPRO
      REV=REV/100.0
      DCST=DCST/1000.0
      EQCST=EQCST/1000.0
      OCST=OCST/1000.0
      BKT=BETA/Q
      TCST=DCST+EQCST+OCST
      REV3=REV3/1000.0
      WRITE(6,87) REV,AL,TIMST,Q,EXA,TI,BKT,DCST,EQCST,OCST,TCST,REV3,Q3
C
C           FIND THE MAXIMIZING PROJECT LIFE
C
      IF(REV.GT.QP) GO TO 708
      GO TO 709
708   QP=REV
      G(I,J,1)=REV
      G(I,J,2)=AL
      G(I,J,3)=Q
      G(I,J,4)=EXA
      G(I,J,5)=DCST
      G(I,J,6)=EQCST
      G(I,J,7)=OCST
      G(I,J,8)=TCST
      G(I,J,9)=TIMST
      G(I,J,10)=TI
      G(I,J,11)=BKT
709   AL=AL+ALINC
84    CONTINUE
33    CONTINUE
22    CONTINUE
C
C           OUTPUT TAPLES OF RESULTS
C
      DO 503 JZ=1,11
      WRITE(6,618)
      WRITE(6,607)(SM(JZ,J),J=1,15)
      WRITE(6,610)
```

```
      WRITE(6,604)
      WRITE(6,603)  (AINT(JY),JY=1,M)
      WRITE(6,604)
      WRITE(6,611)
      WRITE(6,604)
      WRITE(6,602)
      DO 501  JX=1,N
      WRITE(6,604)
      IF(JZ.GT.8)   GO TO 500
      WRITE(6,617) R(JX),(G(JY,JX,JZ),JY=1,M)
      GO TO 501
500   WRITE(6,619) R(JX),(G(JY,JX,JZ),JY=1,M)
501   CONTINUE
      WRITE(6,604)
      WRITE(6,602).
      WRITE(6,604)
      WRITE(6,612)  (PPO(I),I=1,M)
      WRITE(6,620)
      WRITE(6,604)
      WRITE(6,610)
      WRITE(6,613)
      WRITE(6,605)
      PH=PHI*100.0
503   CONTINUE
11    FORMAT(2F6.2,F6.4,3F6.2)
13    FORMAT(5F6.2)
14    FORMAT(6F6.4)
15    FORMAT(I1,5X,8F8.4)
16    FORMAT(8F7.2,F8.3)
17    FORMAT(7F8.2)
39    FORMAT(F6.2)
40    FORMAT(' ','          NPVS        L      U*      Q*      A*      TI     TAU',
     *  '   DELAY    EQUIP   OPER.    TOTAL                PI(0)    Q(0)'/
     *    48X,'COST     COST    COST     COST'/)
87    FORMAT(3X,F9.2,F5.0,F5.1,2F6.0,F7.2,F5.2,F7.2,2F8.2,F9.2,
     *    F15.2,F7.0)
89    FORMAT('1',///,17X,'INTEREST RATE=',F6.4,2X,
     *    'RATE OF INCREASE OF PRICE=',F6.4///)
602   FORMAT(15X,'-------------------------------------------
     %----------')
603   FORMAT(15X,'¬',4X,'I=',2X,'¬',2x,F6.2,4X,F6.2,4X,F6.2,4X,F6.2,4X,F
     %6.2,5X,'¬')
604   FORMAT(15X,'¬',8X,'¬',53X,'¬')
605   FORMAT(26X,'P(T)=P(0)*EXP(+R*T)',/)
607   FORMAT(15X,15A4,///////)
609   FORMAT(15A4)
610   FORMAT(15X,'@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
     %@@@@@@@@@@')
611   FORMAT(15X,'¬',2X,'R=',4X,'¬',53X,'¬')
612   FORMAT(15X,'¬',2X,'P(0)',2X,'¬',2X,F6.3,4X,F6.3,4X,F6.3,4X,F6.3,4X
     %,F6.3,5X,'¬')
613   FORMAT(////////)
617   FORMAT(15X,'¬',2X,F5.3,1X,'¬',2X,F6.0,4X,F6.0,4X,F6.0,4X,F6.0,4X,F
     %6.0,5X,'¬')
618   FORMAT('1',////,41X,'TABLE',////)
619   FORMAT(15X,'¬',2X,F5.3,1X,'¬',2X,F6.2,4X,F6.2,4X,F6.2,4X,F6.2,4X,F
     %6.2,5X,'¬')
```

```
620    FORMAT(15X,'¬',1X,'$/MBTU',1X,'¬',53X,'¬')
621    FORMAT(5F10.2,F6.4,2F10.2)
707    FORMAT(4F5.2,3F8.2)
       WRITE (6,800)
800    FORMAT ('1')
       STOP
C
C            SPECIAL SECTION FOR PRINTING OUT INPUT VALUES
C
1001   WRITE (6,1101)
       WRITE (6,1102) HCAPR,HCAPF,PHI
       WRITE (6,1103) SPGRF,TO,TS
       WRITE (6,1104) SMK,GAMAF,FMU
       WRITE (6,1105) H,D,RW
       WRITE (6,1106) A,B
       WRITE (6,1107)  PHIO,PHI1,PHI2
       WRITE (6,1108)  GAMAO,GAMA1,GAMA2
       WRITE (6,1109) AM,CM,PP
       WRITE (6,1110) C1,C2,WCST
       WRITE (6,1111) UU,PK,OHCST,RENT,SALRY,ROYLT
       WRITE (6,1112) RENT2,PENLTY,Z,AKM
       WRITE (6,1113) S1,S2,S3
       WRITE (6,1114) S4
       WRITE (6,1115) EFFV,EFFH
       WRITE (6,1116) QMAX,QMIN,QINC
       WRITE (6,1117) QBAR,ALMAX,ALINC
       WRITE (6,1118) WLIFE,AMC,DELTA
       WRITE (6,1119)
       WRITE (6,1121)  (AINT(I),I=1,M)
       WRITE (6,1122)  (R(I),I=1,M)
       GOTO 1000
1101   FORMAT ('1',43X,'PROGRAM DATA'////)
1102   FORMAT (' ','HEAT CAP. OF ROCK        ',F8.2,
     *         3X,'HEAT CAP. OF FLUID       ',F8.2,
     *         3X,'POROSITY OF AQUIFER      ',F8.2)
1103   FORMAT (' ','SPEC. GRAVITY OF FLUID ',F8.4,
     *         3X,'AQUIFER INIT. TEMP.      ',F8.2,
     *         3X,'STEAM TEMPERATURE        ',F8.2/)
1104   FORMAT (' ','INTRINSIC PERMEABILITY ',F8.2,
     *         3X,'FLUID UNIT WEIGHT        ',F8.2,
     *         3X,'FLUID VISCOSITY     ',F11.8/)
1105   FORMAT (' ','AQUIFER HEIGHT           ',F8.2,
     *         3X,'WELL SEPERATION          ',F8.2,
     *         3X,'WELL RADIUS              ',F8.2)
1106   FORMAT (' ','A - HEIGHT               ',F8.2,
     *         3X,'B - HEIGHT               ',F8.2/)
1107   FORMAT (' ','PHI(1)                   ',F8.4,
     *         3X,'PHI(2)                   ',F8.4,
     *         3X,'PHI(3)                   ',F8.4)
1108   FORMAT (' ','GAMMA(1)                 ',F8.4,
     *         3X,'GAMMA(2)                 ',F8.4,
     *         3X,'GAMMA(3)                 ',F8.4/)
1109   FORMAT (' ','UNIT PIPE CLEANING COST',F8.2,
     *         3X,'INSURANCE/TAX % COST    ',F8.2,
     *         3X,'FUEL & OPERATING COSTS  ',F8.2)
1110   FORMAT (' ','BOWL UNIT COST 220 GPM ',F8.2,
     *         3X,'BOWL UNIT COST 1100 GPM',F8.2,
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     *          3X,'WELL COST                 ',F10.2)
1111  FORMAT (' ','HEAT XFER. COEFF.          ',F8.2,
     *          3X,'PIPE INSTILLATION         ',F8.2,
     *          3X,'OVERHEAD COST             ',F8.2/
     *         ' ','RENT                      ',F8.2,
     *          3X,'SALARIES                  ',F9.2,
     *          3X,'ROYALTIES                 ',F8.2)
1112  FORMAT (' ','LAND NON-USE RENT          ',F8.2,
     *          3X,'LAND NON-USE PENTALY      ',F8.2,
     *          3X,'ELECTRICITY COST          ',F9.3/
     *         ' ','PUMP SERVICE COST MULT.',F8.2/)
1113  FORMAT (' ','PUMP SALVAGE VALUE         ',F8.2,
     *          3X,'HEAT XCHG. SALVAGE        ',F8.2,
     *          3X,'PIPE SALVAGE VALUE        ',F8.2)
1114  FORMAT (' ','WELL SALVAGE VALUE         ',F8.2/)
1115  FORMAT (' ','VERT. PUMP EFFICIENCY      ',F8.2,
     *          3X,'HORZ. PUMP EFFICIENCY     ',F8.2)
1116  FORMAT (' ','PUMP RATE LIMIT (MAX)      ',F8.2,
     *          3X,'PUMP RATE LIMIT (MIN)     ',F8.2,
     *          3X,'PUMP RATE INCREMENT       ',F8.2)
1117  FORMAT (' ','MAX. WELL FLOW RATE        ',F8.2,
     *          3X,'PROJECT LIFE              ',F8.2,
     *          3X,'TIME INCREMENT            ',F8.2)
1118  FORMAT (' ','WELL LIFE                  ',F8.2,
     *          3X,'WELL MAINTENENCE          ',F8.2,
     *          3X,'XCHG. TEMP. DELTA         ',F8.2)
1119  FORMAT (///' ','GROWTH IS EXPONENTIAL')
1121  FORMAT (///' ','DISCOUNT RATES      =  ',8F8.4)
1122  FORMAT (  /' ','ENERGY COST RATES   =  ',8F8.4)
      END
      SUBROUTINE KOST
C
C          THIS SUBROUTINE COMPUTES THE DISCOUNTED COSTS FOR ONE
C          DOUBLET (Q<=QMAX).
C
C          ALL PARAMETERS USED ARE LOCATED IN COMMON STORAGE.
C
C
      COMMON QS,PK,D,A,B,DRCON,EFFV,EFFH,C1,C2,EXA,SPGRF,CST10,CST
     %25,EXCOF,CRF10,CRF25,AKM,Z,AM,E,COST,PUCST,E1,E2,S1,S2,S3,S4,SLV10
     %,SLV25,EE,ENCST,E5,CRFN,WCST,WMC,OHCST
C
C          PIPE COST
C
      PCST=PK*D*(0.1313*QS+1.323*QS**0.5-4.36)
C
C          ANNUAL PIPE CLEANING COST
C
      CLCST=AM*D
C
C          HEAT EXCHANGER COST
C
      ECST=5000.0+150.7*EXCOF*QS
C
C          HORIZONTAL PUMP COST
C
      PCSTH=24.0*QS
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```
C
C         HORIZONTAL MOTOR COST
C
      MCSTH=0.0546*SPGRF*QS**2.0*DRCON/EFFH+1907.1
C
C         BOWL UNIT COST
C
      BUCST=C1+(C2-C1)*QS**2.0/40000.0
C
C         COLUMN ASSEMBLY COST
C
      COCST=(A+4.0+QS*DRCON)*(0.1313*QS+1.323*QS**0.5-4.36)+3981.0
C
C         VERTICAL SHAFT COST
C
      SHCST=(.001339*QS*SPGRF*(DRCON*QS+A+B)/EFFV+0.0768*(DRCON*QS+A)-10
     %.132)*(DRCON*QS+A)
C
C         VERTICAL MOTOR COST
C
      MCSTV=0.0546*SPGRF*QS*(DRCON*QS+A+B)/EFFV+1907.1+0.35*(DRCON*QS+A)
C
C         CAPITAL COSTS FOR PUMPS
C
      PPCST=COCST+MCSTV+SHCST+MCSTH+BUCST+PCSTH
C
C         PRESENT WORTH OF PUMP COSTS
C
      PUCST=PPCST*CRF10*(E+E1*(1.0-S1))
C
C         ANNUAL PUMP OPERATING COSTS
C
      ENCST=AKM*Z*(23.80*QS*SPGRF*(DRCON*QS+A+B)/EFFV+152.86*(DRCON*QS+A
     %)+23.80*QS*SPGRF*DRCON*QS/EFFH)
C
C         TOTAL DISCOUNTED COST FOR ONE DOUBLET WITHOUT SALVAGE
C
      CST10=COCST+MCSTV+SHCST+MCSTH+BUCST+PCSTH+ECST
      CST25=PCST+OHCST
      COST=(CST10*CRF10+CST25*CRF25+CLCST+WMC+WCST*CPFN)*E+ENCST*EE
C
C         SALVAGE COSTS
C
      SLV10=PPCST*CRF10*E1*(1.0-S1)+ECST*CRF10*E1*(1.0-S2)
      SLV25=PCST*CRF25*E2*(1.0-S3)+OHCST*CRF25*E2*(1.0-S4)+WCST*CRFN*E5
      RETURN
      END
*RUN
     PRESENT WORTH OF MAXIMUM PROFITS, PI*    ($1976,$1000)
               ECONOMIC RESERVOIR LIFE, L*    (YEARS)
          OPTIMAL PUMPING RATE, Q*    (CUBIC METERS/HR)
        OPTIMAL HEAT EXCHANGER AREA, A*    (SQUARE METERS)
         PRESENT WORTH OF DELAY COSTS,   ($1976,$1000)
       PRESENT WORTH OF EQUIPMENT COSTS,    ($1976,$1000)
     PRESENT WORTH OF PUMP OPERATING COSTS,     ($1976,$1000)
    PRESENT WORTH OF ALL COSTS, C(Q,L,TI,U)     ($1976,$1000)
                OPTIMAL STARTING TIME, U*   (YEARS)
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