Fermi National Accelerator Laboratory

FERMILAB-Conf-88/65

Coupled Bunch Instability in Fermilab Booster - Longitudinal Phase-Space Simulation*

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June 9, 1988

*To be presented at the European Particle Accelerator Conference, Rome, Italy, June 7-11, 1988

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

COUPLED BUNCH INSTABILITY IN FERMILAB BOOSTER - LONGITUDINAL PHASE-SPACE SIMULATION

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Introduction

The physical presence of vacuum structures can be expressed in terms of a coupling impedance experienced by the beam. The beam environment considered here consists of parasitic higher order modes of the r.f. cavities. These resonances may have high enough Q's to allow consecutive bunches to interact through mutually induced fields. The cumulative effect of such fields as the particles pass through the cavity may be to induce a coherent buildup in synchrotron motion of the bunches, i. e. a longitudinal coupled-bunch instability[1].

The colliding mode operation of the present generation of high energy synchrotrons and the accompanying r.f. manipulations, make considerations of individual bunch area of paramount importance. Thus, a longitudinal instability in one of a chain of accelerators, while not leading to any immediate reduction in the intensity of the beam in that accelerator, may cause such a reduction of beam quality that later operations are inhibited (resulting in a degradation in performance).

In this paper we employ a longitudinal phase-space tracking code (ESME)[2] as an effective tool to simulate specific coupled bunch modes arising in a circular accelerator. One of the obvious advantages of the simulation compared to existing analytic formalisms, e.g. based on the Vlasov equation[3], is that it allows consideration of the instability in a self-consistent manner with respect to the changing accelerating conditions. Furthermore this scheme allows to model nonlinearities of the longitudinal beam dynamics, which are usually not tractable analytically.

Included in the simulation is the investigation of possible cures aimed at eliminating or limiting growth of these instabilities. Most of the discussion is confined to three basic damping schemes:

1) Synchrotron tune spread induced inside an individual bunch due to a highly anharmonic (quartic) r.f. potential generated at the center of each bunch by a so-called Landau cavitiy[4],

2) Bunch-to-bunch synchrotron tune spread achieved through modulation of the fundamental r.f. voltage by a secondary voltage source of lower harmonic number so that consecutive bunches fill up slightly different buckets and obviously, their synchrotron tunes are no longer the same,

3) Damping through a radial position feedback loop where a longitudinal broadband kicker delivered an amplitude-limited correction voltage to each bunch on a turn-by-turn basis, thereby actively damping the coupled bunch modes.

The machine-dependent parameters, which are considered here, are derived from the Fermilab Booster; this study being motivated by an instability problem in that machine.

Longitudinal Phase-Space Tracking with Wake Fields

Briefly summarized, the tracking procedure used in ESME consists of turn-by-turn iteration of a pair of Hamilton-like difference equations describing synchrotron oscillation in θ–ε phase-space ($0 \le \theta \le 2\pi$ for the whole ring and $\varepsilon = E - E_o$, where E_o is the synchronous particle energy). Knowing the particle distribution in the azimuthal direction, $\rho(\theta)$, and the revolution frequency, ω_o, after each turn, one can construct a wake field induced voltage as follows[5]

$$V_i(\theta) = e\omega_o \sum_{n=-\infty}^{\infty} \rho_n Z(n\omega_o) e^{in\theta}, \qquad (1)$$

where ρ_n represents the discrete Fourier spectrum of the beam and $Z(\omega)$ is a longitudinal coupling impedance. The numerical procedure involved in evaluating the above expression, Eq. (1), necessarily employs a discretization of the θ-direction. Some caution is required in this process of binning, due to the finite statistics inherent in such a simulation.

For the purpose of our simulation, only the relatively high-Q portion of the longitudinal impedance is relevant. A single parasitic mode can be modelled by the harmonic resonator of the impedance given by

$$Z(\omega) = \frac{R}{1 + iQ(\omega/\omega_c - \omega_c/\omega)} \qquad (2)$$

Here R is the shunt impedance, Q denotes the quality factor of the resonator and ω_c is its resonant frequency. For M equally spaced coupled bunches there are M possible dipole modes labeled by m = 1, 2,..., M. To illustrate the m-th dipole mode one can look at the θ-position of the centroid of each bunch, θ_ℓ, $\ell = 1, 2,\ldots, M$. The signature of the simplest coupled bunch mode has the form of a discrete propagating plane wave:

$$\theta_\ell(t) = \theta_o \sin\left\{ \frac{2\pi m\ell}{M} - \omega_s t \right\}, \qquad (3)$$

where ω_s is the synchrotron frequency. Based on the analytic model of coupled bunch modes proposed by Sacherer[1] one can formulate a simple resonance condition for the m-th dipole mode driven by the longitudinal impedance $Z(\omega)$ sharply peaked at ω_c. This condition is given by:

$$\omega_c = (nM + m)\,\omega_o \pm \omega_s \;, \qquad (4)$$

where n is an integer. Since ω_o is time dependent (acceleration) and ω_c is fixed (geometry), and knowing that the width of the impedance peak is governed by ω_c/Q one can clearly see that the resonance condition, Eq. (4), is maintained over a finite time interval. This leads to the useful concept of a mode crossing the impedance resonance. Using the explicit time dependence of ω_o (kinematics) and Eq. (4) one can easily calculate crossing intervals for various modes. This serves as a guide in the simulation since it allows us to select an appropriate time domain where the mode of interest crosses the resonance and will more likely become unstable.

Undamped Coupled Bunch Mode

In the early stages of this study we tentatively identified the parasitic resonance at f_c = 85.5 MHz with Q = 3378 and R = 914x10^{-3} as the offending part of the impedance giving rise to a coupled bunch instability with harmonic number m = 53. This mode crosses the resonance earlier in the booster cycle, therefore the appropriate time interval to study the m = 53 mode is chosen as 19 – 26x10^{-3} sec. The r.f. system of the Fermilab Booster provides 84 accelerating buckets. As a starting point for our simulation each bucket in θ–ε phase-space is populated with 100 macro-particles according to a bi-Gaussian distribution matched to the bucket so that 95% of the beam is confined within the contour of the longitudinal emittance of 0.02 eV–sec. Each macro-particle is assigned an effective charge to simulate a beam intensity of 1.5x10^{12} protons.

In a real-life accelerator any coherent instability starts out of noise and gradually builds up to large amplitudes. In our model situation it proved necessary to create some intrinsic small amplitude - "seed" of a given mode in order to "start-up" the instability. The "seeding" procedure is basically prescribed by Eq. (3). Initially identical bunches are rigidly displaced from the center of each bucket (both in ε and θ) so that the position of their centroids, θ_ℓ, satisfy Eq. (3) for all the bunches around the ring. In practice, a subroutine of ESME, which generates a closed contour in θ–ε space under the action of a sinusoidally varying voltage, was used to establish the position of the bunch centroids. The intrinsic seed amplitude, θ_o, was assigned a value of 10^{-3} rad corresponding to an amplitude in energy of approximately 2 MeV.

To visualize the position and shape of individual bunches as they evolve in time one can compose a "mountain range" diagram by plotting θ-projections of the bunch density in equal increments of revolution number and then stacking the projections to imitate the time flow. The resulting mountain range plot for an undamped mode 53 is given in Fig. 1a.

In the next few sections we will proceed with the discussion of suggested damping mechanisms.

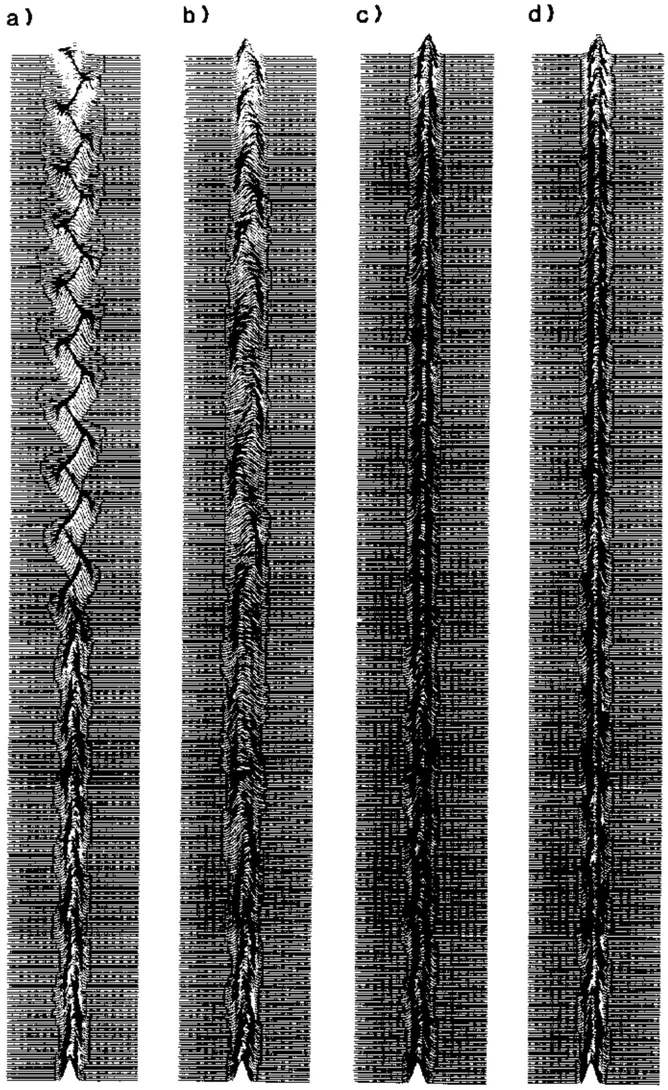

Fig. 1 Collection of mountain range plots illustrating the behavior of coupled bunch mode m = 53 with:
a) no damping,
b) passive damping via Landau cavity,
c) passive damping through the $h_2 = 77$ harmonic,
d) active damping via radial position feedback.

Fourth Harmonic Landau Cavity

Now let us consider a situation where, in addition to the fundamental r.f. voltage source, we have a secondary source of voltage whose frequency is equal to that of the fourth harmonic of the fundamental; the so-called Landau cavity. The phase and amplitude of the secondary voltage source are prescribed by the conditions that both the first and second derivatives of the net voltage vanish at the center of each bunch. The above condition can be formulated by introducing both voltages explicitly as follows

$$V_1(\phi) = V_{rf} \sin(\phi_s + \phi)$$
and $\qquad (7)$
$$V_4(\phi) = kV_{rf} \sin(\phi_4 + 4\phi).$$

Here ϕ_s is the synchronous phase relative to the fundamental r.f. waveform, ϕ_4 is the synchronous phase relative to the fourth harmonic waveform and f denotes the deviation of a particle from the synchronous phase, $\phi = h\theta - \phi_s$. Parameter k is the ratio of the secondary and primary voltage amplitudes. The combined net voltage is constrained by the condition that its first and second derivatives vanish at the center of each bunch. This fixes matching parameters k and ϕ_4 as follows

$$k = \frac{\sqrt{1+15\cos^2\phi_s}}{16}$$
and $\qquad (8)$
$$\phi_4 = \arccos\left\{\frac{\cos(\pi - \phi_s)}{4k}\right\}.$$

The resulting r.f. voltage is illustrated in Fig. 2. The purpose of imposing the above constraint, Eq. (8), is to provide a highly nonlinear bucket resulting in large synchrotron tune spread within each bunch. This in turn may eventually provide stability against coherent motion of coupled bunches (via a Landau damping mechanism). A family of closed orbits in θ–ε space corresponding to different amplitudes, was generated using a contour drawing subroutine of ESME. The result is depicted in Fig. 3. Each orbit is labeled with the respective synchrotron tune in frequency units ($\sec^{-1}$). The bounding curve, with tune 0, represents the separatrix (note the "squareness" of the bucket in this double r.f. voltage system).

The tracking was carried out for exactly the same initial condition as described in the previous section. In addition to the fundamental r.f. voltage the Landau cavity voltage, $V_4(\phi)$, is turned on linearly over the first 2×10^{-3} sec, matched to the fundamental voltage program according to Eq. (8) for a period of 3×10^{-3} sec and finally turned off linearly over the last 3×10^{-3} sec. The tracking results are illustrated by the mountain range plot collected in Fig. 1b. One can see by comparison with the corresponding plot for the undamped mode, Fig. 1a, that the Landau cavity provides substantial damping of an initially unstable coupled bunch mode.

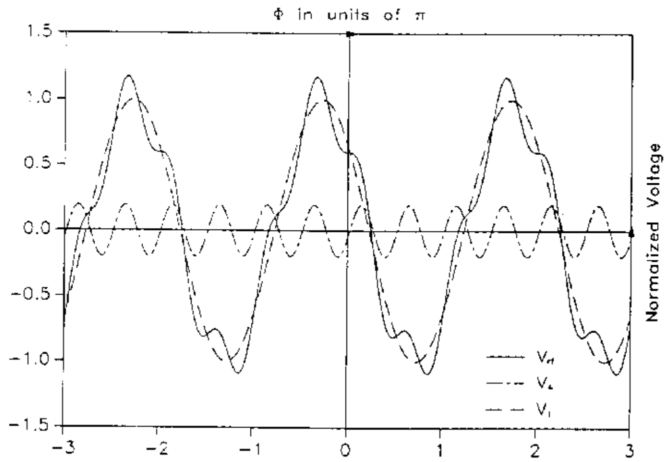

Fig. 2 Combined voltage of a double r.f. system with Landau cavity.

Damping Through Inter-Bunch Tune Spread

One can apply a secondary voltage source with lower than the fundamental harmonic number. We will consider a situation where 2 out of 18 r.f. cavities, modelled as a secondary source, provide voltage at harmonic number $h_2 = 77$ (the remaining 16 cavities, modelled as the fundamental r.f. source, will run at $h_1 = 84$). Now any seven ($h_1 - h_2 = 7$) consecutive buckets differ due to the voltage modulation provided by the secondary source. Therefore, the value of synchrotron frequency will vary from bunch to bunch (even for small amplitude oscillations in the linear region). For exactly the same initial conditions as in the simulation of the previous section the $h_2 = 77$ voltage source replaces the Landau cavity with the same linear turn on/off feature. As before, the phase-space evolution of a single bunch, given by the mountain range plot, Fig. 1c, illustrates effective damping of m = 53 coupled

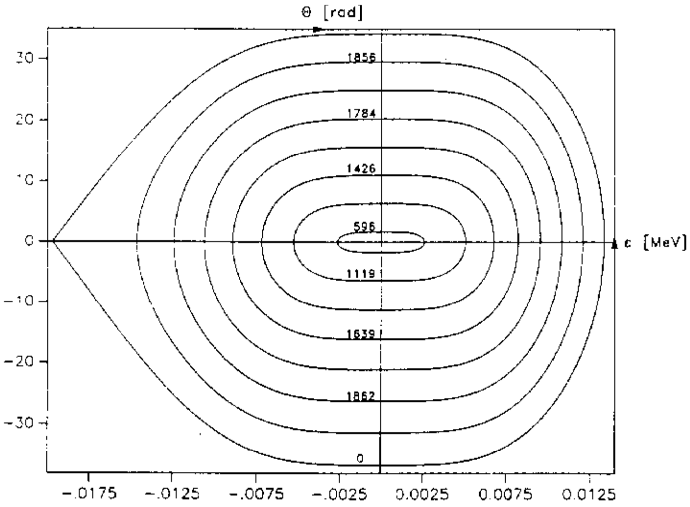

Fig. 3 Synchrotron tune spread inside a bucket corrected by Landau cavity as a function of relative amplitude,$(\varepsilon - E_s)/E_s$, where E_s is the height of the bucket.

bunch modes. In fact, in this case the damping is somewhat more evident.

Active Damping Through Radial Position Feedback

It was noted that the coupled bunch oscillations in the Fermilab Booster gave rise to a radial position signal in a circuit originally designed to damp horizontal betatron oscillations. It was suggested that this circuit be used to drive a longitudinal broadband kicker, thereby actively damping the coupled bunch modes. This scheme was also simulated. The "kicker" in our simulation delivered a maximum 1kV correction voltage to each bunch; the "seed" amplitude for the mode corresponded to a signal level safely above the noise level of the monitor in the damping circuit, as inferred from observations of the signal and knowledge of the dispersion in the region of the monitor.Here again the simulation results for m = 53 coupled bunch mode is illustrated by the mountain range profile given in Fig. 1d. Comparison with the other schemes indicates that such an active damper is very effective in both cases studied here.

Inter- and Intra- Bunch Tune Spread

At this point, some qualitative comments concerning the passive damping mechanisms and their relative efficacy are in order. We note that for mode 53, the cavities operating at harmonic 77 appear to be much more efficient than the Landau cavity (See Figs. 1b and 1c). This is not totally surprising, since the growth of the instability is dependent upon bunch-to-bunch "communication" via wake fields. The h_2 = 77 cavities disrupt this communication directly, via bunch-to-bunch tune spread . In addition, the distance between the bunches is modulated by the secondary r.f. voltage and therefore the components of the current at harmonics of the fundamental r.f. frequency are reduced. The Landau cavity, on the other hand, operates at an harmonic of the fundamental r.f. frequency, and therefore induces tune spread only within each bunch. The Landau cavity attempts to "discourage" the growth of the instability via suppression of the coherent motion inside each single bunch. The tune spread induced within a bunch, however, is a function of the range of amplitudes of the particles undergoing synchrotron motion. Thus, if a group of particles oscillate at amplitude "close" enough to each other, we might expect them to respond to a suitable driving force in a coherent fashion. Presumably, particles may be regarded as "close" if the tune spread among them is smaller than the frequency characterizing the growth of the instability. This "clustering" phenomenon did, in fact, occur in the simulation for mode 53, as illustrated by the mountain range plot in Fig. 1b. It is evident that a cluster of "almost coherent" particles (in the previously described sense) still participates in coupled bunch oscillation, while the remaining particles with synchrotron tune spread larger than some critical value do not respond coherently to the coupling wake field. This would suggest that there exists a threshold tune spread defining the extent of a "coherent blob" inside the bucket; that extent being a characteristic coherence length for a given driving frequency.

References

[1] F. Sacherer, IEEE Trans. Nucl. Sci., **24**, 1393 (1977)

[2] J.A. MacLachlan, FERMILAB TM-1274 (1984)

[3] S. Krinsky and J.M. Wang, Particle Accelerators, **17**, 109 (1985)

[4] A. Hofmann and S. Myers, CERN ISR-TH-RF/80-26 (1980)

[5] J.A. MacLachlan, FERMILAB FN-446 (1987)