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1

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TITLE BARYON NUMBER DISSIPATION AT FINITE TEMPERATRUE IN THE STANDARD MODEL

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Baryon Number Dissipation at Finite Temperature in the Standard Model

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ABSTRACT

We analyze the phenomenon of baryon number violation at finite temperature in the standard model, and derive the relaxation rate for the baryon density in the high temperature electroweak plasma. The relaxation rate, γ is given in terms of real time correlation functions of the operator $\mathbf{E} \cdot \mathbf{B}$, and is directly proportional to the sphaleron transition rate, Γ: $\gamma \propto n_f \Gamma / T^3$. Hence it is *not* instanton suppressed, as claimed by Cohen, Dugan and Manohar (CDM). We show explicitly how this result is consistent with the methods of CDM, once it is recognized that a new anomalous commutator is required in their approach.

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1 Introduction

Baryon (and lepton) number is not conserved in the standard $SU(2)_L \times U(1)$ electroweak theory. This derives from the fact that the pure $SU(2)$ vacuum is a periodic structure labelled by an integer Chern-Simons winding number,

$$N_{CS} = \frac{g^2}{16\pi^2}\int d^3\vec{x}\ \epsilon_{ijk}\left(A_i^a\partial_j A_k^a - \frac{g}{3}\epsilon_{abc}A_i^a A_j^b A_k^c\right). \tag{1}$$

In order to change from a vacuum configuration with one integer value of N_{CS} to that with another integer value, it is necessary to pass through non-vacuum, *i.e. finite energy* field configurations: Fig. 1. The height of the potential barrier between adjacent vacua is given by the energy of a certain static solution of the coupled Yang-Mills-Higgs classical field equations, called a sphaleron. In the Weinberg-Salam theory this energy barrier is of order M_W/α_W, or 7 to 10 Tev.[1]

Necessarily associated with the twisting of the gauge field from one vacuum state to another is the violation of chiral fermion number through the chiral anomaly. Because of (maximal) parity violation, the chiral anomaly becomes an anomaly in the lepton and baryon number currents as well:

$$\partial_\mu b^\mu = \partial_\mu \ell^\mu = \frac{n_f}{32\pi^2}\left\{-2g^2 F_{\mu\nu}^a \tilde{F}^{a\mu\nu} + g'^2 F'_{\mu\nu}\tilde{F}'^{\mu\nu}\right\}. \tag{2}$$

Here $F_{\mu\nu}^a$ and $F'_{\mu\nu}$ are the field strength tensors for the $SU(2)_L$ and $U(1)$ hypercharge gauge fields of the Weinberg-Salam theory, g and g' are the corresponding coupling constants, and n_f is the number of sequential generations (families) of quarks and leptons. Since $F_{\mu\nu}\tilde{F}^{\mu\nu}$ may be expressed as the total divergence of a four-vector whose time component is just that appearing in the definition of N_{CS}, the non-conservation of $B + L$ is related to the change in N_{CS} of the $SU(2)_L$ gauge vacuum. Because the anomalies in the baryon and lepton currents are identical, the difference $B - L$ is exactly conserved in the standard model.

At temperatures and energies low compared to 10 Tev, such transitions and concomitant $B + L$ violation are very severely suppressed.'t Hooft showed that instanton induced baryon number violating processes involving 12 fermions (for $n_f = 3$) are suppressed by a factor of

$$\exp(-4\pi\sin^2\theta_W/\alpha) \sim 10^{-170}, \tag{3}$$

and hence are entirely negligible at zero temperature.[2]

At high temperatures, the situation is quite different. Because the energy barrier represented by the classical sphaleron solution is finite, the rate of classical real time thermal transitions changing N_{CS} and therefore, $B + L$ has no such exponential suppression in electroweak theory.[3]-[7] The rate of such B and L violating processes has been computed in the Weinberg-Salam theory by semi-classical methods for the temperature range $M_W(T) << T << M_W(T)/\alpha_W$.[4],[5] At temperatures greater than $M_W(T)/\alpha_W$, the semi-classical analysis fails because perturbation theory around the zero temperature ground state is unreliable. The failure of the semi-classical approximation for the rate does not mean that the rate is small. Though this might seem paradoxical from the point of view of instanton methods,[8] it is borne out by calculations (both analytic and numerical) in two dimensional models. [5] [9] [10] It is also possible to argue from general properties of scaling in the high temperature phase that the rate of such transitions per unit volume is of order $\alpha_W^4 T^4$.[4] [11]

Another line of objection to this conclusion has been raised by Cohen, Dugan and Manohar,[12] hereafter referred to as CDM. These authors have tried to argue that the rate of *dissipation* for any $B + L$ asymmetry remains exponentially small, even though the rate of *fluctuations* of N_{CS} is large at high temperatures. Such a result would be contrary to quite general thermodynamic principles which relate fluctuation rates to relaxation processes,[13] but because of the critical role of a quantum anomaly in this case, it has nevertheless generated some degree of confusion and controversy.

The resolution of this controversy is important for cosmology. Since the seminal work of Sakharov[14] it has been recognized that the observed baryon number of the universe could be produced by out of equilibrium reactions which simultaneously violate baryon number, charge conjugation and CP. Moreover, the baryon number violating reactions must turn off (*i.e.* become insignificant) before the system returns to thermal equilibrium; otherwise any baryon density produced will relax to its equilibrium value, namely zero. A high rate of electroweak $B + L$ non-conservation at $T > M_W$ therefore carries with it the implication that any pre-existing $B + L$ asymmetry would be eliminated by the time of the electroweak phase transition.[3]-[4] [15] Thus, in order to obtain the observed baryon number either $B - L \neq 0$ at temperatures much greater than M_W, or baryogenesis must occur at the time of the electroweak phase transition.[16] This is a strong constraint on any theory of baryogenesis, and excludes some grand unified models (such as the minimal $SU(5)$ model) for generating the observed baryon excess in the universe, quite apart from the bounds provided by recent proton decay searches.

Khlebnikov and Shaposhnikov[11] (KS) used a well-defined formalism to evaluate the non-equilibrium dynamics of relaxation, and found a large relaxation rate at high temperatures. However, they did not explicitly evaluate *fermionic* quantities, which is at the heart of the CDM objection. In this contribution we redo the calculation of KS with fermions, and obtain a closed form relation between the baryon number relaxation rate and the transition rate. This relation is quite general, and *independent* of any sphaleron approximation, in accordance with general fluctuation-dissipation considerations. The expression (23) for the rate in terms of a certain spectral density function may provide for techniques of evaluation quite different from sphaleron methods.

Finally, we revisit the analysis of CDM, and show how the methods of those authors may be used to achieve the *same* result. The new ingredient in our reanalysis of CDM is an anomalous commutator between baryon number and $\mathbf{E} \cdot \mathbf{B}$, neglected in CDM, but required for consistency with the usual anomaly. Since these several different viewpoints all lead to the same conclusion, there ought to be no further controversy about unsuppressed electroweak B and L violation at high temperature and its implication(s) for early universe cosmology.

2 The Baryon and Lepton Number Relaxation Rate

Consider the standard electroweak theory at temperatures above M_W. In our discussion we neglect the contribution of the weak hypercharge to the baryon number anomaly. This is done for simplicity of notation. Inclusion of the hypercharge contribution would not change any of our conclusions. Let us assume that all of the dynamical variables of the system are in thermal equilibrium, except two: the baryon and lepton numbers $\mathbf{N_B}$ and $\mathbf{N_L}$, which have been driven out of equilibrium by a small amount due to some unspecified process. The initial condition for our problem then is $\langle \mathbf{N_B}(t=0)\rangle \neq 0$, $\langle \mathbf{N_L}(t=0)\rangle \neq 0$, and we wish to calculate the relaxation rate γ for B and L to return to their equilibrium value. In statistical mechanics, the time development of the dynamical variable $\dot{\mathbf{N}}_\mathbf{B} \equiv \frac{d\mathbf{N_B}}{dt}$ is given in terms of the statistical average $\langle \dot{\mathbf{N}}_\mathbf{B}\rangle \equiv Tr(\dot{\mathbf{N}}_\mathbf{B}\rho)/Z$ where $\rho(t)$ is the non-equilibrium statistical operator satisfying the quantum Liouville equation,

$$\dot{\rho} + i[\mathbf{H},\rho] = 0, \tag{4}$$

and $Z = Tr\rho$. Zubarev has shown that the operator

$$\begin{aligned} \rho &= \exp\left(-\beta\mathbf{H} + \varepsilon\int_{-\infty}^{t} e^{\varepsilon(t'-t)}\left(\mu_B(t')\mathbf{N_B}(t') + \mu_L(t')\mathbf{N_L}(t')\right)dt'\right), \qquad \varepsilon \to 0^+ \\ &\equiv \exp\left(-\beta(\mathbf{H} + \mathbf{h}(t))\right) \end{aligned} \tag{5}$$

satisfies the Liouville equation in the limit $\varepsilon \to 0^+$ and should be a good approximation in the case that only a few dynamical variables are out of equilibrium.[17]

Now, the number operators satisfy the anomalous equations of motion,

$$\dot{\mathbf{N}}_\mathbf{B} = \dot{\mathbf{N}}_\mathbf{L} = -n_f\int d^3\vec{x}\,\mathbf{q}(t,\vec{x}) \equiv -n_f\frac{\alpha_w}{2\pi}\int d^3\vec{x}\,\vec{\mathbf{E}}^a\cdot\vec{\mathbf{B}}^a, \tag{6}$$

where $\vec{\mathbf{E}}^a$ and $\vec{\mathbf{B}}^a$ are the $SU(2)_L$ electroweak electric and magnetic field strengths. In terms of the Chern-Simons charge $\mathbf{N}_{CS}$, we have:

$$\dot{\mathbf{N}}_\mathbf{B} = \dot{\mathbf{N}}_\mathbf{L} = +n_f\dot{\mathbf{N}}_{CS}. \tag{7}$$

Following KS we evaluate now ρ/Z to first order in $\mathbf{h}$:

$$\frac{\rho}{Z} = \left(1 + \beta\langle\int_0^1 d\lambda\, e^{-\beta\mathbf{H}\lambda}\mathbf{h}\, e^{\beta\lambda}\rangle_0 - \beta\int_0^1 d\lambda e^{-\beta\mathbf{H}\lambda}\mathbf{h}\, e^{\beta\mathbf{H}\lambda}\right)\frac{\rho_0}{Z_0}, \tag{8}$$

where

$$\mathbf{h} = -\mu_B(t)\,\mathbf{N_B}(t) + \int_{-\infty}^{t} e^{\varepsilon(t'-t)}[\dot{\mu}_B(t')\mathbf{N_B}(t') + \mu_B(t')\dot{\mathbf{N}}_\mathbf{B}(t')]dt' + (B \leftrightarrow L), \tag{9}$$

and the zero subscript denotes the equilibrium statistical operator with $\mathbf{h} = 0$.

Let us calculate first the average baryon number to this order. We find:

$$\langle \mathrm{N_B}(t) \rangle = -\beta \int_0^1 d\lambda \langle \mathrm{N_B}(t) e^{-\beta \mathrm{H} \lambda} \mathbf{h}(t) e^{\beta \mathrm{H} \lambda} \rangle_0, \tag{10}$$

where we have used $\langle \mathrm{N_B} \rangle_0 = 0$. Substituting the previous expression for $\mathbf{h}$, we find that the term involving $\dot{\mathrm{N}}_\mathrm{B}$ vanishes by the time reversal invariance of ρ in the limit $\varepsilon \to 0^+$. Ignoring the term involving $\dot{\mu}_B$, we obtain:

$$\begin{aligned} \langle \mathrm{N_B}(t) \rangle &= \beta \mu_B(t) \int_0^1 d\lambda \langle \mathrm{N_B}(0) e^{-\beta \mathrm{H} \lambda} \mathrm{N_B}(0) e^{\beta \mathrm{H} \lambda} \rangle_0 \\ &\to \beta \mu_B(t) \langle \mathrm{N_B}^2(0) \rangle_0, \end{aligned} \tag{11}$$

where the last expression is valid in the high temperature or weak coupling (classical) limit. An exactly analogous expression holds for $\langle \mathrm{N_L}(t) \rangle$. We may differentiate eq. (11) with respect to time, to find that $\dot{\mu}_B$ is of order ε, because of the Liouville equation (4), so that it is indeed legitimate to neglect the time variation of μ_B and μ_L in lowest order.

In a similar manner we may compute:

$$\langle \dot{\mathrm{N}}_\mathrm{B}(t) \rangle = -\beta \int_0^1 d\lambda \int_{-\infty}^t dt' e^{\varepsilon(t'-t)} \left(\mu_B(t') + \mu_L(t')\right) \langle \dot{\mathrm{N}}_\mathrm{B}(t) e^{-\beta \mathrm{H} \lambda} \dot{\mathrm{N}}_\mathrm{B}(t') e^{\beta \mathrm{H} \lambda} \rangle_0, \tag{12}$$

using (7), $\langle \dot{\mathrm{N}}_\mathrm{B} \rangle_0 = 0$, and $\langle \dot{\mathrm{N}}_\mathrm{B}(t) \int_0^1 d\lambda\, e^{-\beta \mathrm{H} \lambda} \mathrm{N_B}(t) e^{\beta \mathrm{H} \lambda} \rangle_0 = 0$ by the time reversal invariance of the equilibrium state. Since $\dot{\mu}_{B,L}$ are of order ε, we may replace $\mu_{B,L}(t')$ by $\mu_{B,L}(t)$ in the above expression and remove them from the integral. Then using the previous results for $\langle \mathrm{N_B}(t) \rangle$ and $\langle \mathrm{N_L}(t) \rangle$, we may eliminate the chemical potentials from (12) entirely, to arrive at:

$$\langle \dot{\mathrm{N}}_\mathrm{B} \rangle = \langle \dot{\mathrm{N}}_\mathrm{L} \rangle = -K \left(\frac{\langle \mathrm{N_B} \rangle}{\langle \mathrm{N_B}^2(0) \rangle_0} + \frac{\langle \mathrm{N_L} \rangle}{\langle \mathrm{N_L}^2(0) \rangle_0} \right), \tag{13}$$

where

$$K \equiv \int_{-\infty}^t dt' e^{\varepsilon(t'-t)} \int_0^1 d\lambda \langle \dot{\mathrm{N}}_\mathrm{B}(t) e^{-\beta \mathrm{H} \lambda} \dot{\mathrm{N}}_\mathrm{B}(t') e^{\beta \mathrm{H} \lambda} \rangle_0. \tag{14}$$

This derivation exactly parallels that of KS[11], who derive the equivalent result for N_{CS} instead of for the fermionic operator $\mathrm{N_B}$. We now depart from those authors by expressing K and the high temperature (sphaleron) transition rate Γ in terms of the *same* spectral function, thereby allowing us to find a direct relation between the two, independently of any specific approximation scheme.

To this end let us introduce the retarded response function,

$$G_R(t-t',\vec{x}-\vec{x}') \equiv -i\theta(t-t')\langle[\mathbf{q}(t,\vec{x}),\mathbf{q}(t',\vec{x}')]\rangle_0$$
$$= \int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\int\frac{d^3\vec{k}}{(2\pi)^3}e^{-i\omega(t-t')}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\tilde{G}_R(\omega,\vec{k}). \tag{15}$$

whose Fourier transform $\tilde{G}_R$ is analytic in the upper half complex ω plane:

$$\tilde{G}_R(\omega,\vec{k}) = \int_{-\infty}^{+\infty}d\omega'\frac{\rho(\omega',\vec{k})}{\omega-\omega'+i\varepsilon}. \tag{16}$$

The spectral density ρ (not to be confused with the density matrix of which we have no further use) is determined by the matrix elements of the topological charge density:

$$\rho(\omega,\vec{k}) = \frac{(2\pi)^3}{Z_0}\sum_{n,m}\left|\langle n\mid\mathbf{q}(0)\mid m\rangle_0\right|^2 e^{-E_n/T}\left(1-e^{-(E_m-E_n)/T}\right)$$
$$\times\delta(\omega-E_m+E_n)\delta^3(\vec{k}-\vec{p}_m+\vec{p}_n), \tag{17}$$

where the states $\mid n\rangle$ are a complete set of eigenstates of the full Hamiltonian with energy eigenvalues E_n.

By using the anomaly *operator* equation (7) and substituting the same complete set of intermediate eigenstates, it may be verified in a direct computation that the quantity K of eq. (14) is given by:

$$\begin{aligned}K &= iVn_f^2T\frac{d}{d\omega}\tilde{G}_R(\omega,\vec{k})\Big|_{\omega=\vec{k}=0}\\ &= -iVn_f^2T\int_{-\infty}^{\infty}\frac{d\omega}{\omega-i\varepsilon}\left[\frac{\rho(\omega,\vec{0})}{\omega}\right]\\ &= Vn_f^2T\pi\left[\frac{\rho(\omega,\vec{0})}{\omega}\right]\Big|_{\omega=0}\\ &= Vn_f^2T\pi\frac{d\rho}{d\omega}\Big|_{\omega=\vec{k}=0},\end{aligned} \tag{18}$$

where we have made use of the fact that

$$\rho(\omega) = \rho_+(\omega)-\rho_-(\omega) = \rho_+(\omega)-\rho_+(-\omega) \tag{19}$$

is explicitly an odd function of ω when $\vec{k}=0$.

The quantity, $\frac{d\rho}{d\omega}\Big|_{\omega=\vec{k}=0}$ occurs in a quite different context, as the rate for the (Brownian) diffusion of the toplological charge,

$$\mathbf{Q}(t) \equiv \int_0^t dt'\int d^3\vec{x}\,\mathbf{q}(t',\vec{x}) \tag{20}$$

in the periodic potential of Fig. 1. For we may calculate

$$\begin{aligned}
\langle \mathrm{Q}^2(t)\rangle_o &= 2V\int_0^\infty d\omega\rho_+(\omega,\vec{0})\frac{\sin^2(\omega t)}{\omega^2} \\
&\to 2\pi V t\rho_+(0,\vec{0}), \qquad t\to\infty \\
&= 2\pi V t T\frac{d\rho}{d\omega}\Big|_{\omega=\vec{k}=0}\,.
\end{aligned} \tag{21}$$

Since (in the absence of fermions) we define the diffusion coefficient of the random walk in Chern-Simons number by:

$$\begin{aligned}
\lim_{t\to\infty}\langle \mathrm{Q}^2(t)\rangle &= \lim_{t\to\infty}\langle(\mathrm{N}_{CS}(t)-\mathrm{N}_{CS}(0))^2\rangle \\
&= 2Vt\Gamma,
\end{aligned} \tag{22}$$

we have proven that

$$\begin{aligned}
\Gamma &= \pi T\frac{d\rho}{d\omega}\Big|_{\omega=\vec{k}=0} \\
&= \frac{8\pi^3}{Z_0}\sum_{n,m}\Big|\langle n\mid \mathrm{q}(0)\mid m\rangle\Big|^2 e^{-E_n/T}\delta(E_n-E_m)\delta^3(\vec{p}_n-\vec{p}_m),
\end{aligned} \tag{23}$$

and therefore,

$$K = Vn_f^2\Gamma. \tag{24}$$

which relates the baryon relaxation rate to the finite temperature diffusion rate in the absence of fermions. The last two expressions remain valid in the presence of fermions as well, provided only that the baryon number density is small compared to T^3, which is the same assumption necessary to derive the linear relations of eqs. (13).

In the previous literature[4],[11] Γ is evaluated in the semiclassical method of Langer[18], which relates it to the sphaleron energy in a semi-classical approximation. Expression (23) furnishes an *a priori* definition of Γ, which may (in principle) be evaluated from knowledge of the spectral density function near $\omega = 0$. In practice, this is quite difficult since it involves the long time behavior of the response function, which cannot be calculated in perturbation theory. Euclidean methods are also of little use since the long time limit is sensitive to any approximation(s) made in Euclidean time, and hence the continuation is generally unreliable. Nevertheless, we believe it is worthwhile to have a definition of the rate that is independent of any approximate method of evaluating it.

To complete the evaluation of the relaxation rate we must calculate the demominators of eq. (13). If we were dealing with a single species of left-handed fermion this would be straightforward in the regime where the temperature is much higher than fermion masses and chemical potential μ. In that case we would simply compute the partition function of a free fermion gas with a single helicity state:

$$\ln Z(\mu) = \frac{2VT^3}{\pi^2}\sum_{n=1}^{\infty}\frac{(-)^{n+1}}{n^4}\Big(1+\frac{(\mu\beta n)^2}{2}+O(\mu^4)\Big), \tag{25}$$

where μ is the chemical potential for this particle number, whose average is given by:

$$\langle \mathrm{N} \rangle = T\frac{\partial}{\partial\mu} \ln Z(\mu) = \frac{\mu V T^2}{6} \tag{26}$$

to linear order in μ. The mean square fluctuation of this number is:

$$\langle \mathrm{N}^2 \rangle = \frac{T^2}{Z}\frac{\partial^2}{\partial\mu^2} Z \Big|_{\mu=0} = T^2 \frac{\partial^2}{\partial\mu^2} \ln Z \Big|_{\mu=0} = \frac{VT^3}{6}, \tag{27}$$

which is consistent with eqs. (11) and (26).

In the standard model the accounting is a bit different. We must consider both baryon and lepton number together, since both are violated by the transition. Since

$$\begin{aligned} \mathrm{N_B} &= \frac{1}{3}\sum_f \sum_{c=1}^{3} (\mathrm{N}^c_{u_f} + \mathrm{N}^c_{d_f}), \\ \mathrm{N_L} &= \sum_f (\mathrm{N}_{\ell_f} + \mathrm{N}_{\nu_f}), \end{aligned} \tag{28}$$

where f labels the family or sequential generation, we have:

$$\begin{aligned} \langle \mathrm{N_B} \rangle &= n_f \times 3 \times 2 \times \frac{1}{3} \times \frac{\mu_B}{3} \times \frac{2VT^2}{6} = \frac{2}{9} n_f \mu_B V T^2, \\ \langle \mathrm{N_L} \rangle &= n_f \mu_L \left(\frac{2}{6} + \frac{1}{6}\right) VT^2 = \frac{1}{2} n_f \mu_L V T^2, \end{aligned} \tag{29}$$

The fluctuations in these quantities are likewise modified to become:

$$\begin{aligned} \langle \mathrm{N_B}^2(0) \rangle_0 &= \frac{2}{9} n_f V T^3, \\ \langle \mathrm{N_L}^2(0) \rangle_0 &= \frac{1}{2} n_f V T^3, \end{aligned} \tag{30}$$

in the high temperature or weak coupling limit. Substituting these last relations into the denominators of (13), and using the earlier result for K, eq. (24) yields the desired expression for the fermion number relaxation rate:

$$\langle \dot{\mathrm{N}}_\mathrm{B} \rangle = \langle \dot{\mathrm{N}}_\mathrm{L} \rangle = -\frac{n_f \Gamma}{T^3} \left(\frac{9}{2}\langle \mathrm{N_B} \rangle + 2\langle \mathrm{N_L} \rangle\right). \tag{31}$$

If we consider initial conditions with $\langle \mathrm{N_B} \rangle = \langle \mathrm{N_L} \rangle$, or simply consider the relaxation of the linear combination, $\frac{9}{2}\langle \mathrm{N_B} \rangle + 2\langle \mathrm{N_L} \rangle$, the fermion number relaxation rate becomes:

$$\gamma = \frac{13}{2} n_f \frac{\Gamma}{T^3}. \tag{32}$$

There is a simpler, heuristic way to derive this same result, based on detailed balance.[4] Suppose for $t < 0$ constant chemical potentials μ_B and μ_L are added to the Hamiltonian:

$$\mathbf{H} \rightarrow \mathbf{H} - \mu_B \mathrm{N_B} - \mu_L \mathrm{N_L}\,, \tag{33}$$

so that it becomes energetically favorable to create a net baryon and lepton number in the plasma. From the anomaly equation, this means that the periodic potential of Fig. 1 is replaced by a skewed potential near $\mathrm{N_B} = 0$: Fig. 2. Notice that the minima of Fig. 1 are forced to be degenerate, since all integer N_{CS} are equivalent to each other by a (topologically non-trivial) gauge transformation. Unlike the Chern-Simons number, $\mathrm{N_B}$ is *gauge-invariant*, so that states of different baryon number may have (and do have) different energies.

For large enough positive $\mathrm{N_B}$ the potential of Fig. 2 turns upward once more. This is because of Fermi-Dirac statistics: even if the fermions are treated as massless, it costs energy to create a fermion/anti-fermion pair with net chirality, since the pair must be created in an unoccupied momentum state. Since the spacing between states (and hence this energy cost) goes to zero in the infinite volume limit, the value of $\mathrm{N_B}$ at which the potential of Fig. 2 begins to turn upward is of order V. Indeed, to linear order in μ_B explicit evaluation of the thermal average in the Fermi-Dirac distribution just yields the results, (29) to linear order in μ_B and μ_L. The mean $\mathrm{N_B}$ is shifted to this positive value, so that the larger population of states with $\langle \mathrm{N_B} \rangle > 0$ diffusing to lower $\mathrm{N_B}$ can compensate for the energy bias to the right. Hence, there is detailed balance and

$$\langle \dot{\mathrm{N}}_\mathrm{B} \rangle = \langle \dot{\mathrm{N}}_\mathrm{L} \rangle = -n_f V \langle \mathbf{q}(t,\vec{0}) \rangle = 0\,, \qquad t < 0\,. \tag{34}$$

Suppose that the external chemical potentials are removed suddenly at $t = 0$. Now the large rate of diffusion to the left from the initial overpopulation with positive $\mathrm{N_B}$ is no longer balanced by an energy bias to the right. Hence there will be a net decrease of $\langle \mathrm{N_B} \rangle$ with time, *i.e.* the net baryon number will relax to zero. We may calculate the rate of relaxation if we assume that eqs. (29) continue to hold for $t > 0$ as well, effectively defining a *slowly varying* $\mu_B(t)$ and $\mu_L(t)$ in terms of the decreasing baryon and lepton numbers. That is, we assume that the relaxation is slow enough so that the system may be treated as approximately in equilibrium at all times during the relaxation, with an effective time dependent chemical potential. This adiabaticity assumption permits us to use detailed balance and equate $\langle \frac{d\mathrm{N_B}}{dt} \rangle$ for $t > 0$ to the *negative* of the transition rate to the right with the original skewed Hamiltonian that set up the distribution for $t < 0$ in the first place. Therefore,

$$\langle \mathbf{q}(t) \rangle = -\langle \frac{d\mathrm{N}_{CS}}{dt} \rangle = -(\Gamma_+ - \Gamma_-) = +n_f(\mu_B + \mu_L)\,\frac{\Gamma}{T}, \qquad t > 0 \tag{35}$$

since

$$\Gamma_\pm = \Gamma e^{\mp n_f(\mu_B + \mu_L)\beta/2} = \Gamma\left(1 \mp \frac{n_f(\mu_B + \mu_L)}{2T} + \mathcal{O}(\mu^2)\right) \tag{36}$$

to linear order in μ_B and μ_L in the skewed potential. Then we may eliminate $\mu_B + \mu_L$ from eq. (35) by using eqs. (6) and (29) to secure:

$$\langle \dot{\mathrm{N}}_\mathrm{B} \rangle = \langle \dot{\mathrm{N}}_\mathrm{L} \rangle = -n_f\,\frac{\Gamma}{T^3}\left(\frac{9}{2}\langle \mathrm{N_B} \rangle + 2\langle \mathrm{N_L} \rangle\right), \tag{37}$$

which is the same result for the fermion relaxation rate obtained by the more formal Zubarev approach.

3 CDM Analysis Revisited

CDM also calculate $\langle \dot{N}_B(t) \rangle$. However, they use a trick to obtain the thermal average in terms of the derivative of a generating function $F(\theta)$, defined in terms of the generalized electroweak Hamiltonian

$$\mathrm{H}(\theta) = \frac{1}{2}\int d^3\vec{x}\{\left(\vec{\Pi} + \alpha_w \frac{\theta}{2\pi}\vec{\mathrm{B}}\right)^2 + (\vec{\mathrm{B}})^2\} + \mathrm{H}_{fermion} \tag{38}$$

where

$$\vec{\Pi} = -\vec{\mathrm{E}} - \alpha_w \frac{\theta}{2\pi}\vec{\mathrm{B}} \tag{39}$$

is the momemtum conjugate to the gauge field. Define:

$$e^{-\beta F(\theta)} \equiv Tr(e^{-\beta \mathrm{H}(\theta)}) \tag{40}$$

such that

$$\frac{\partial F}{\partial \theta}\Big|_{\theta=0} = -\langle \int d^3\vec{x}\, \mathrm{q}\rangle_0 = 0 . \tag{41}$$

In fact, *all* derivatives of $F(\theta)$ vanish because F is independent of θ, as we now demonstrate. In order to do so it is sufficient to show that

$$e^{-i\mathrm{N_B}\phi}\mathrm{H}(\theta)e^{i\mathrm{N_B}\phi} = \exp\left(n_f\phi\frac{\partial}{\partial\theta}\right)\mathrm{H}(\theta) = \mathrm{H}(\theta + n_f\phi), \tag{42}$$

i.e. that a baryon number phase rotation can be used to rotate the angle θ to zero in the electroweak theory. Expanding (42) in a power series in ϕ gives:

$$\mathrm{H}(n_f\phi) = \mathrm{H}(0) - i\phi[\mathrm{N_B},\mathrm{H}(0)] - \frac{\phi^2}{2}[\mathrm{N_B},[\mathrm{N_B},\mathrm{H}(0)]] + \cdots . \tag{43}$$

The second term on the RHS is given by the anomaly eq. (6):

$$-i\phi[\mathrm{N_B},\mathrm{H}(0)] \equiv \phi\dot{\mathrm{N}}_\mathrm{B} = -n_f\phi\int d^3\vec{x}\,\mathrm{q} = +n_f\phi\frac{\partial \mathrm{H}}{\partial\theta}\Big|_{\theta=0} . \tag{44}$$

This verifies the first derivative term of the expansion. Integrating the anomaly relation $\dot{\mathrm{N}}_\mathrm{B} = n_f\dot{\mathrm{N}}_\mathrm{CS}$ and fixing the gauge by the condition that $\mathrm{N}_{CS} = 0$ when $\mathrm{N_B} = 0$ permits us to write the commutator in the third term on the RHS 0f (43) as:

$$\begin{aligned} -[\mathrm{N_B},[\mathrm{N_B},\mathrm{H}(0)]] &= -n_f[\mathrm{N}_CS,[\mathrm{N_B},\mathrm{H}(0)]] \\ &= -n_f[\mathrm{N}_{CS}, -in_f\int d^3\vec{x}\,\mathrm{q}] \\ &= +n_f^2\left(\frac{\alpha_w}{2\pi}\right)^2\int d^3\vec{x}\,\vec{\mathrm{B}}^a\cdot\vec{\mathrm{B}}^a \\ &= n_f^2\frac{\partial^2\mathrm{H}}{\partial\theta^2}\Big|_{\theta=0} . \end{aligned} \tag{45}$$

This verifies that terms quadratic in ϕ in eq. (43) are correct. Since $\vec{\mathbf{B}}^2$ no longer involves the electric field operator, its commutator with $\mathbf{N}_{CS}$ and $\mathbf{N}_\mathbf{B}$ vanishes, as do all the higher order commutators in the ellipsis, consistent with the fact

$$\frac{\partial^n \mathbf{H}}{\partial \theta^n}\Big|_{\theta=0} = 0\,, n > 2\,. \tag{46}$$

Thus, consistency requires a new anomalous commutator, *viz.*

$$[\mathbf{N}_\mathbf{B}, \int d^3\vec{x}\,\mathbf{q}] = -in_f\left(\frac{\alpha_W}{2\pi}\right)\int d^3\vec{x}\,\vec{\mathbf{B}}^2, \tag{47}$$

in addition to the original anomaly, eq. (6). If desired, one may verify this new anomalous commutator directly in terms of the canonical commutation relations of the theory, by defining the operator $\mathbf{N}_\mathbf{B}$ composed of fermion bilinears in terms of a gauge-invariant point splitting technique. Insertion of the path ordered exponential of $\int dx^i A_i$ between the fermion operators yields the anomalous commutator (47), which remains after the point splitting has been removed.

Hence eq. (42) is proven, and indeed we may rotate away the angle θ in eq. (40), proving that $F(\theta) = F(0)$ is independent of θ. Notice that this conclusion, verifiable also in a Lagrangian path integral approach requires the anomalous commutator (47). By taking the second derivative of F with respect to θ and using the fact that F is independent of θ, we find:

$$\langle\int d^3\vec{x}\,\mathbf{B}^2\rangle_0 = \beta\langle\int d^3\vec{x}\,\mathbf{E}\cdot\mathbf{B}\int d\lambda\, e^{-\beta\mathbf{H}\lambda}\int d^3\vec{x}'\,\mathbf{E}'\cdot\mathbf{B}' e^{\beta\mathbf{H}\lambda}\rangle_0. \tag{48}$$

Let us now consider non-equilibrium dynamics. As a trial non-equilibrium statistical operator, CDM consider the local operator,

$$\rho_\theta = N\exp\left(-\beta\left(\mathbf{H}(\theta) + \sum_k c_L \mathbf{O}_k(\theta)\right)\right), \tag{49}$$

where the $\mathbf{O}_k$ are arbitrary operators and the c_k are arbitrary coefficients. Define $F(\theta)$ as before with this new statistical operator $e^{-\beta F(\theta)} \equiv Tr(\rho_\theta)$ with $\mathbf{O}_k(\theta) = e^{-i\mathbf{N}_\mathbf{B}(0)\phi}\mathbf{O}_k\, e^{i\mathbf{N}_\mathbf{B}(0)\phi}$ and $\phi = \theta/n_f$. Then $F(\theta) = F(0)$ as before. Differentiating F with respect to θ we obtain:

$$n_f\langle\int d^3x\mathbf{q}\rangle = -i\sum_k c_k\,\langle[\mathbf{N}_\mathbf{B}, \mathbf{O}_k]\rangle_0, \tag{50}$$

to first order in the small parameters c_k.

CDM consider operators satisfying $[\mathbf{N}_\mathbf{B}, \mathbf{O}_k] = n_k\mathbf{O}_k$, but they do not consider operators such as $\mathbf{O} = \dot{\mathbf{N}}_\mathbf{B} = -n_f\int d^3\vec{x}\,\mathbf{q}$. Using the anomalous commutator, eq. (47), for this single operator, we obtain:

$$\langle\dot{\mathbf{N}}_\mathbf{B}\rangle \approx -c\left(\frac{n_f\alpha_W}{2\pi}\right)^2\langle\int d^3x\mathbf{B}^2\rangle_0. \tag{51}$$

Notice that this estimate for $\langle \dot{N}_B \rangle$ is *not* small or instanton suppressed. If we replace the non-equilibrium statistical operator of KS and the perturbing Hamiltonian of Zubarev with the *local* term,

$$h(t) \approx \frac{(\mu_B + \mu_L)}{T} \dot{N}_B \equiv c\, O\,, \tag{52}$$

which is valid in the limit that the autocorrelation function for $\dot{N}_B$ has support only when the time interval is of order T^{-1}, then eq. (51), obtained by the CDM local operator method, is identical to eq. (12) of the previous section, since:

$$\begin{aligned} K &= \int_{-\infty}^{t} dt' e^{\epsilon(t'-t)} \langle \dot{N}_B(t) \int_0^1 d\lambda\, e^{-\beta H \lambda} \dot{N}_B(t') e^{\beta H \lambda} \rangle_0 \\ &\approx \left(\frac{n_f \alpha_w}{2\pi}\right)^2 \beta \langle \int d^3x \mathbf{E} \cdot \mathbf{B} \int d\lambda\, e^{-\beta H \lambda} \int d^3y \mathbf{E} \cdot \mathbf{B} e^{\beta H \lambda} \rangle_0 \\ &= \left(\frac{n_f \alpha_w}{2\pi}\right)^2 \langle \int d^3x \mathbf{B}^2 \rangle_0 \end{aligned} \tag{53}$$

by eq. (48).

Thus, the main results of this paper, eqs. (23)-(24), and (31)-(32) relating the dissipation of fermion number at high temperature to the fluctuation or diffusion rate over the potential barrier are consistent with the methods of CDM, provided account is taken of the anomalous commutator (47). The relation (32) is a reflection of general fluctuation-diisipation theorems, and is a kind of analog to the relation found by Einstein for Brownian motion in a medium.[13] The local approximation of CDM leads to the estimate,

$$\Gamma \approx \left(\frac{\alpha_w}{2\pi}\right)^2 \langle \vec{\mathbf{B}}^2 \rangle_0 \tag{54}$$

by combining eqs (24) and (53). Actually, we might expect the time scale for the correlation function (14) to decay to be of order $(\alpha_w T)^{-1}$ rather than T^{-1}, since the former is the inverse dimensional coupling of the three dimensional gauge theory appropriate at high temperature. Then the above estimate for Γ would be enhanced by a factor of α_w^{-1} relative to (54). The same dimensional coupling enters the magnetic screening length,[9] so that we should expect:

$$\langle \vec{\mathbf{B}}^2 \rangle_0 \approx \alpha_w^3 T^4 \tag{55}$$

and

$$\Gamma \approx \frac{\alpha_w^4 T^4}{4\pi^2} \tag{56}$$

at high temperature. If the scaling relation $\Gamma \propto \alpha_w^4 T^4$ is correct, then the dissipation rate of baryon number in the hot electroweak plasma is of order $n_f \alpha_w^4 T$, which is much larger than the expansion rate of the universe at these temperatures. In the next contribution, Cline and Raby.[19] derive relations between the supposed high energy behavior of B violating inclusive cross sections that imply results for Γ different from this naive scaling behavior.

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Figure Captions

Fig. 1
The periodic vacuum structure of non-abelian gauge theory in the absence of fermions.

Fig. 2
The potential energy of gauge field plus massless fermion system as a function of fermion number. The potential is concave for large N_B in a finite volume, due to Fermi-Dirac statistics, as explained in the text.