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# ON THE FLUCTUATION MAGNETOCONDUCTIVITY OF AN ANISOTROPIC DIRTY SUPERCONDUCTOR*

E.Z. Da Silva**† and B.L. Gyorffy**

Brookhaven National Laboratory, Upton, L.I., New York 11973



ABSTRACT--We calculate the fluctuation conductivity in presence of an external magnetic field H as function of the angle between H and the current for an anisotropic dirty superconductor.

RÉSUMÉ--On calcule la conductbilité due aux fluctuations en presence d'un champ magnétique applique H, en fonction de l'angle entre H et le courant, dans le cas d'un superconducteur anisotrope "dirty".

As it well known [1] the upper critical field $H_{c2}$ of a dirty type II superconductor depends on the lowest eignevalue of the differential operator $-\sum_{\alpha\beta} D_{\alpha\beta}\, \pi_\alpha\, \pi_\beta$ where $\alpha\beta$ denote cartesian coordinate axis x y z, $D_{\alpha\beta}$ is the diffusion tensor for the electrons and $\pi_\alpha$ is $\alpha$-component of the canonical momenta for a cooper pair in a magnetic field $\underset{\sim}{H}$ (if $\underset{\sim}{H}$ is represented by a vector potential $\underset{\sim}{A}$ then $\underset{\sim}{\pi} = \hbar\nabla + \frac{2ie}{c}\underset{\sim}{A}$). Also, it has been noted that superconducting fluctuations in presence of the field $\underset{\sim}{H}$ above $T_c(H)$ could provide information about the whole spectrum of $-\sum_{\alpha\beta} D_{\alpha\beta}\, \pi_\alpha\, \pi_\beta$. Consequently, when the interpretation of measurements on $H_{c2}$ are difficult, as in case of anisotropic superconductors like layered compounds [2] and $(SN)_x$ [3], it should be useful to study these fluctuations.

To facilitate such an investigation we present explicit formulae, valid for anisotropic dirty superconductors, for the fluctuation magneto-conductivity tensor $\sigma^f_{\alpha\beta}(T,H,\theta)$ where $\theta$ is the angle between the measuring current and $\underset{\sim}{H}$. Our discussion follows closely that of Mikeska and Schmidt [4] and, in the isotropic limit treated by them, our results for $\sigma^f_{\parallel} = \sigma^f_{zz}(T,H,0)$ and $\sigma^f_{\perp} = \sigma^f_{zz}(T,H,\frac{\pi}{2})$ agree with theirs. As a by-product of our calculation we obtain an expression for $H_{c2}(T,\theta)$ which agrees with that of Tilley [1].

Studying $\sigma^f$ along the chains in $(SN)_x$ without a magnetic field Citvak, et al. [5] found one-dimensional fluctuations. By contrast the measurement of other properties of $(SN)_x$ [3] suggests that it is a three-dimensional anisotropic semi-metal. If $\sigma^f_{\alpha\beta}(T,H,\theta)$ were measured and the results differed markedly from the theory presented here, the conclusion of Citvak, et al. [5] that superconducting fluctuations are dominated by reduced dimensionality due to the fibrous morphology of this metallic polymer would be considerably strengthened.

To second order in the order parameter $\Psi$ the generalized Ginsburg-Landau free energy for an anisotropic dirty superconductor in a magnetic field can be written as

$$\Delta F = \int d^3\underline{x} \left\{ \Psi^*(\underline{x}) \; f\; (T,\mathscr{D}) \;\; \Psi(\underline{x}) + \frac{H^2}{8\pi} \right\} \tag{1}$$

where f, from macroscopic theory [6] is given by

$$f(T,\mathscr{D}) = \frac{4\pi k_B T_c}{2\psi'(\frac{1}{2})} \left\{ \ln \frac{T}{T_{c0}} + \psi\left(\frac{1}{2} + \frac{\mathscr{D}}{4\pi k_B T}\right) - \psi(\tfrac{1}{2}) \right\}. \tag{2}$$

Here $\mathscr{D} = -\sum_{\alpha\beta} D_{\alpha\beta}\, \pi_\alpha\, \pi_\beta$, $\psi$ denotes the digamma function, $T_{c0}$ is the critical temperature in absence of the magnetic field. For a dirty superconductor $D_{\alpha\beta}$ is proportional to the normal state conductivity tensor [7].

In this theory, the $\alpha$-component of the current is

$$J_\alpha\;(x) = \frac{ie}{\psi'(\frac{1}{2})} \sum_\beta D_{\alpha\beta} \left\{ \left[ \psi'\left(\tfrac{1}{2} + \frac{\mathscr{D}}{4\pi k_B T}\right) \Psi\;(x) \right] \pi_\beta\, \Psi(\underline{x}) + \text{C.C.} \right\} \tag{3}$$

where $\psi'$ is the trigamma function.

For $T > T_c$ the order parameter $\Psi$ fluctuates about its mean value $\Psi=0$. To discuss these fluctuations it is useful to expand $\Psi$ in terms of the eigenfunctions $\phi_\mu$ corresponding to the eignevalues $\varepsilon_\mu$ of the operator $\mathscr{D}$:

$$\Psi(\underline{x},t) = \sum_{\mu} \Psi_{\mu}(t)\ \phi_{\mu}(\underline{x}) \tag{4}$$

and

$$-\sum_{\alpha\beta} D_{\alpha\beta}\ \pi_{\alpha}\ \pi_{\beta}\ \phi_{\mu}(\underline{x}) = \varepsilon_{\mu}\ \phi_{\mu}(\underline{x}). \tag{5}$$

The dynamics of the fluctuations can be obtained from microscopic theory [8], and for small frequencies the relax tion approximation gives

$$(i\omega + \Gamma_{\mu})\ \Psi_{\mu}(\omega) = 0 \tag{6}$$

where the relaxation rate $\Gamma_{\mu}$ near $T_c$ is given by

$$\Gamma_{\mu} = \frac{4\pi k_B T_c}{\psi'(\frac{1}{2}+\tilde{\varepsilon}_{\mu})} \left\{ \psi(\tfrac{1}{2}+\tilde{\varepsilon}_{\mu}) - \psi(\tfrac{1}{2}+\tilde{\varepsilon}_{0}) + \varepsilon(1-\tilde{\varepsilon}_{\mu}\psi'(\tfrac{1}{2}+\tilde{\varepsilon}_{\mu})) \right\} \tag{7}$$

with

$$\tilde{\varepsilon}_{\mu} = \varepsilon_{\mu}/4\pi k_B T \text{ and } \varepsilon = \frac{T-T_c(H)}{T_c(H)} .$$

Consider the case where the diffusion tensor is diagonal $D_{xx} = D_{yy} = D_{\perp}$, $D_{zz} = D_{\parallel}$ and $\underline{H}$ makes an angle $\theta$ with the axis of anisotropy. The eigenvalue problem of Eq. (5) may be solved for the gauge choice $\underline{A} = (Hz\sin\theta - Hy\cos\theta, 0, 0)$ if we make the following coordinate transformation

$$\begin{pmatrix} y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\left(\frac{D_{\parallel}}{D_{\perp}}\right)^{\frac{1}{2}} \sin\theta & \left(\frac{D_{\perp}}{D_{\parallel}}\right)^{\frac{1}{2}} \cos\theta \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix} \tag{8}$$

In the primed coordinate system Eq. (5) separates and the eigenvalues are

$$\varepsilon_{qn} = \hbar\omega_c(n+\tfrac{1}{2}) + \hbar^2 D_{\perp}\ g(\theta)q^2 \tag{9}$$

where $\omega_c = 2\left(\frac{2eH}{c}\right)D_{\perp}\ g(\theta)^{\frac{1}{2}}$ and $g(\theta) = \cos^2\theta + \frac{D_{\parallel}}{D_{\perp}}\sin^2\theta$ .

Furthermore the eigenfunctions are

$$\phi_{kqn} = \frac{1}{\sqrt{LL'}}\ e^{ikx}\ e^{iqz}\ u_n\left(y' - \left(\frac{\hbar c}{2eH}\right)k\right) \tag{10}$$

with $\mu_n$ being the normalized harmonic oscillator solutions.

We shall calculate $\sigma$ using Eq. (11) entirely in the primed frame of reference. Since we imposed the boundary conditions in this coordinate system we have changed both the shape and the volume of the sample. The former is irrelevant and we correct for the latter.

The fluctuation conductivity along the anisotropy direction is given by the "Kubo formula" [4].

$$\sigma^f_{zz}\ (T,H,\theta) = \frac{1}{\Omega k_B T} \sum_{\mu\mu'} I^z_{\mu\mu'} I^z_{\mu'\mu} \frac{<|\psi_\mu|^2> <|\psi_{\mu'}|^2>}{(\Gamma_\mu + \Gamma_{\mu'})} \tag{11}$$

where $\mu = (k,q,n)$, $I^z_{\mu\mu'}$ are the matrix elements of the current operator in Eq. (3) and from the equipartition theorem $<|\psi_\mu|^2> = \frac{2\psi'(\frac{1}{2})k_B T}{\psi'(\frac{1}{2}+\varepsilon_\mu)\Gamma_\mu}$. a full discussion of Eq. (11) will be published elsewhere. Here it is evaluated in three limits. Defining the pair breaking parameter $\rho(\theta) = \rho_0\ g(\theta)^{\frac{1}{2}}$ where $\rho_0 = \hbar\left(\frac{2cH}{c}\right) \frac{D_\perp}{4\ k_B T_c}$ we find:

(a) $\rho \ll \varepsilon \ll 1$ (vanishing magnetic fields)

$$\sigma^f_{zz}(T,0,\theta) = \frac{e^2}{16\hbar^3} \left(\frac{2k_B T_c}{\pi}\right)^{\frac{1}{2}} \left(\frac{D_\parallel}{D_\perp}\right)^{\frac{1}{2}} \frac{1}{D_\perp^{\frac{1}{2}} \varepsilon^{\frac{1}{2}}} \tag{12}$$

as one should expect for zero magnetic field there is no angular dependence.

(b) $\varepsilon \ll \rho \ll 1$ (small magnetic fields)

$$\sigma^f_{zz}(T,H,\theta) = \frac{e^2}{32\hbar^3} \left(\frac{2k_B T_c}{\pi}\right)^{\frac{1}{2}} \frac{D_\parallel^{\frac{1}{2}}}{D_\perp} \frac{\pi^2 \rho_0}{\varepsilon^{\frac{1}{2}}} \frac{\cos^2\theta}{g(\theta)^{\frac{1}{2}}} + \frac{e^2}{4\hbar^3} \left(\frac{2k_B T_c}{\pi}\right)^{\frac{1}{2}} \frac{D_\parallel^{3/2}}{D_\perp^2} \frac{1}{\varepsilon^{\frac{1}{2}}} \frac{\sin^2\theta}{g(\theta)} . \tag{13}$$

(c) $\rho \gg 1$ (large magnetic fields)

$$\sigma^{f}_{zz}(T,H,\theta) = \frac{3e^2}{\pi\hbar^3}\left(\frac{3k_BT_c}{\pi}\right)^{\frac{1}{2}} \frac{D_{\parallel}}{D_{\perp}}^{\frac{1}{2}} \frac{\rho_0^{5/2}}{\varepsilon^{3/2}} g(\theta)^{\frac{1}{2}} \cos^2\theta$$

$$+ \frac{e^2}{3\pi\hbar^2}\left(\frac{3k_BT_c}{\pi}\right)^{\frac{1}{2}} a^2 \frac{D_{\parallel}}{D_{\perp}^2}^{3/2} \frac{\rho_0^{\frac{1}{2}}}{\varepsilon^{\frac{1}{2}}} \frac{\sin^2\theta}{g(\theta)^{3/4}} \tag{14}$$

where $a = \frac{3}{4}\ \ln 3$.

Evidently for an arbitrary orientation of the magnetic field the conductivity has a component whose temperature dependence is characteristic of one-dimensional fluctuations ($\varepsilon^{-3/2}$) and another whose temperature dependence is three dimensional ($\varepsilon^{-\frac{1}{2}}$). Their relative contribution varies rapidly with $\theta$.

Measurements of $H_{c2}$ $(T,\theta=0)$ and $H_{c2}$ $(T,\theta=\pi/2)$ can be used to determine $D_{\perp}$ and $D_{\parallel}$. Thus the above theory is free of adjustable parameters.

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**Permanent Address: H.H. Wills Physics Laboratory, Univ. of Bristol, U.K.

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