

**HIERARCHICAL CONTROL OF A NUCLEAR REACTOR  
USING UNCERTAIN DYNAMICS TECHNIQUES\***

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# HIERARCHICAL CONTROL OF A NUCLEAR REACTOR USING UNCERTAIN DYNAMICS TECHNIQUES

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## Abstract

Recent advances in the nonlinear optimal control<sup>1,2,3</sup> area are opening new possibilities towards its implementation in process control. Algorithms for multivariate control, hierarchical decomposition, parameter tracking, model uncertainties, actuator saturation effects and physical limits to state variables can be implemented on the basis of a consistent mathematical formulation.

In this paper, good agreement is shown between a centralized and a hierarchical implementation of a controller for a hypothetical nuclear power plant subject to multiple demands.

The performance of the hierarchical distributed system in the presence of localized subsystem failures is analyzed.

## 1. Introduction

The computational burden for implementation of a nonlinear process controller for real time applications is considerably large and grows with the number of state variables. For a large scale system (LSS), decomposition into separate subsystems with their own computing resources may be necessary. Recent advances in the nonlinear optimal control<sup>1</sup> area are opening new possibilities towards the development of hierarchical decomposition algorithms based on uncertain dynamics formulations<sup>2,3</sup>. Algorithms for multivariate control with parameter tracking, actuator saturation effects and physical limits to state variables capabilities can be similarly developed on the basis of a consistent mathematical formulation.

Once system decomposition is accomplished, a hierarchical control structure can be established in which a coordinator module controls the operation of one or more subsystems<sup>4</sup>. The coordinator module can be controlled by other modules higher in the hierarchy.

Let a LSS be represented by the following vector state equation

$$dX/dt = F(X,U) \quad (1)$$

where  $X$  represents the state vector and  $U$  is the control vector. This system can be targeted for decomposition into  $n$  subsystems governed by a set of state equations of the form

$$dX_i/dt = F_i(X_i,U_i) + P_i \quad (2)$$

where  $X_i$  and  $U_i$  represent the state and control vectors of the  $i$ th subsystem, and  $P_i$  is an unknown term vector that accounts for both the effects of those state variables not explicitly modeled in the  $i$ th subsystem and the effects of its modeling inaccuracies.

Let us define, for each subsystem  $i$ , two Hamiltonian functions: one to be used to generate optimal controls for demand following, and the other to generate the unknown terms for optimal matching of the controller's state predictions and the measured signals. These Hamiltonians take the form:

$$H_{C_i} = 0.5(U_i - U_{0i})^T Q_i (U_i - U_{0i}) + 0.5(D_i - X_i)^T R_i (D_i - X_i) + W_i^T (F_i + P_i), \quad (3)$$

$$H_{S_i} = 0.5(P_i - P_{0i})^T M_i (P_i - P_{0i}) + 0.5(Y_i - S_i)^T N_i (Y_i - S_i) + Z_i^T (F_i + P_i) + E_i^T G_i, \quad (4)$$

where  $X_i$  and  $U_i$  are the state and control vector for the  $i$ th subsystem;  $W_i$ ,  $Z_i$ , and  $E_i$  are adjoint state vectors;  $Q_i$ ,  $R_i$ ,  $M_i$ , and  $N_i$  are weight matrices;  $D_i$  is

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the demand vector;  $Y_i$  and  $S_i$  are the estimated and actual sensor reading vectors respectively; and  $G_i$  is the sensor transfer function.

The application of Pontryagin's Maximum Principle (PMP) to the above Hamiltonian functions results in a set of differential and algebraic equations from which optimal control vectors and unknown terms can be computed.

## 2. Methodology

Detailed mathematical models of nuclear power plants may involve more than 500 state variables to represent the dynamics of mass, energy, momentum and neutron processes across the plant. For this study a simplified model consisting of 29 state equations and 4 controls has been implemented on a Vax 11/780 using ACSL simulation language.

Figure 1 is a schematic of a nuclear power plant showing the main energy flow loops, sensors, actuators and major components. The partitioning of the plant into four control subsystems is shown by shaded blocks. The decoupling points between subsystems were selected on the basis of physical boundaries that are present in the real system, such as heat exchangers, in which energy but no mass transfers are allowed, minimizing in this way the number of variables contributing to the coupling terms. This approach results in a set of unknown terms that have a physical meaning such as feedback reactivity and power transfer. Observation of the time dependency of these terms provides insight into the plant's behavior.

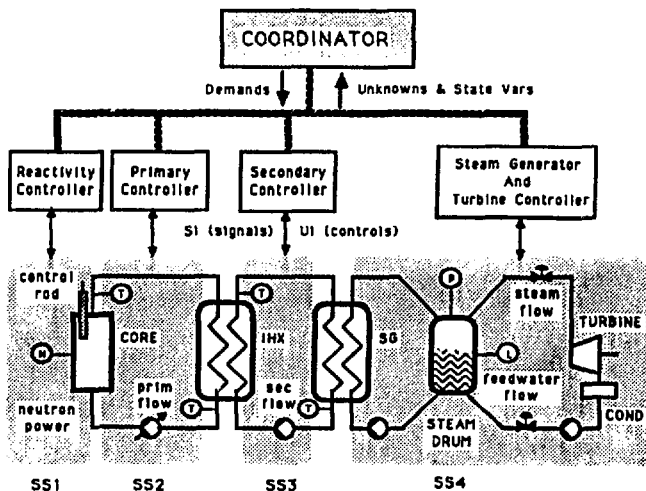


Figure 1 Subsystem decomposition of a LMR nuclear power plant

For illustration, let us consider the following system of equations representing energy balances for the 3-node model of the heat exchanger between subsystems SS2 and SS3:

$$\frac{dT_{op}}{dt} = \frac{W_p}{M_p} (T_{ip} - T_{op}) - \frac{(HA)_{pm}}{(MC)_p} (T_{op} - T_m) \quad (5)$$

$$\frac{dT_m}{dt} = \frac{(HA)_{pm}}{(MC)_m} (T_{op} - T_m) - \frac{(HA)_{ms}}{(MC)_m} (T_m - T_{os}) \quad (6)$$

$$\frac{dT_{os}}{dt} = \frac{W_s}{M_s} (T_{is} - T_{os}) + \frac{(HA)_{ms}}{(MC)_s} (T_m - T_{os}) \quad (7)$$

where  $T$  stands for temperature,  
 $M$  for mass,  
 $W$  for mass flow rate,  
 $HA$  for heat transfer coefficient,  
 $i, o$  for inlet and outlet,  
 $p, s, m$  for primary, secondary and metal.

Since the temperatures of the coolant at the outlet nodes are measurable it is possible to decouple the subsystems by replacing the metal equations and coupling terms with  $P_1$  and  $P_2$  as follows:

$$\frac{dT_{op}}{dt} = \frac{W_p}{M_p} (T_{ip} - T_{op}) + P_1 \quad (8)$$

$$\frac{dT_{os}}{dt} = \frac{W_s}{M_s} (T_{is} - T_{os}) + P_2 \quad (9)$$

Equations (8) and (9) are then made part of the controller's model for subsystems SS2 and SS3 respectively. By virtue of the Hamiltonian and the application of the PMP,  $P_1$  and  $P_2$  will be synthesized by their respective controllers while optimally matching their internal model's computations to the measured plant data.

## 3. Application

### 3.1. Demand Following

The performances of both a centralized optimal controller and of the corresponding uncertain-dynamics-based hierarchical controller

in response to a set of demands are shown in figs. 2. The transient presented corresponds to a ramp-down followed by a ramp-up on the demanded neutron power while all other demands are kept constant. Figures 2.1.a and 2.1.b show the perfect agreement between the demand, plant and controller's model for both the centralized and decomposed implementations respectively (all variables are scaled to reference values). Figures 2.2.a and 2.2.b show a similar comparison for the steam drum pressure in which the controller's model can track the plant exactly and the demand is maintained with an error of about 0.1% during the transient. It is clear from these figures that both centralized and decomposed controllers perform equally well.

Computationally, every time step the centralized controller requires the simultaneous solution of a set of 87 nonlinear differential equations (29 states and 58 adjoint states) and a set of 13 algebraic equations (4 controls and 9 unknown terms) which takes on the average 1.2 CPU sec per second of realtime simulation. The hierarchical controller requires the solution of four uncoupled sets of equations, one for each subsystem, with a total of 48 equations (16 states, 32 adjoint states) and a simpler set of algebraic equations (4 controls, and 9 unknown terms), which takes approximately 0.6 CPU sec per simulation second on the average. Note that the solution of each subsystem's equations can be performed at different time intervals and/or on different CPUs.

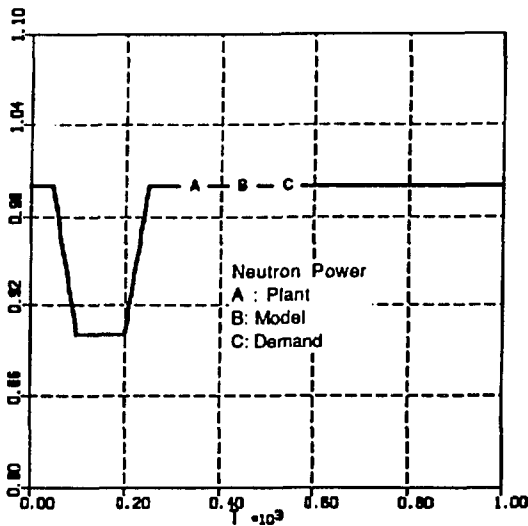


Figure 2.1.a Neutron Power (centr. controller)

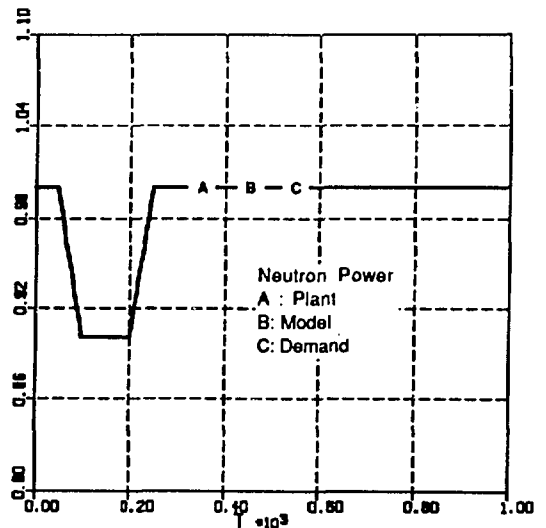


Figure 2.1.b Neutron Power (decomp. controller)

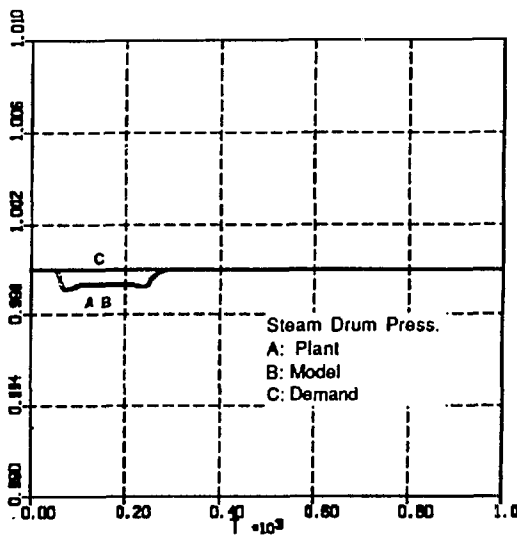


Figure 2.2.a Steam Pressure (centr. controller)

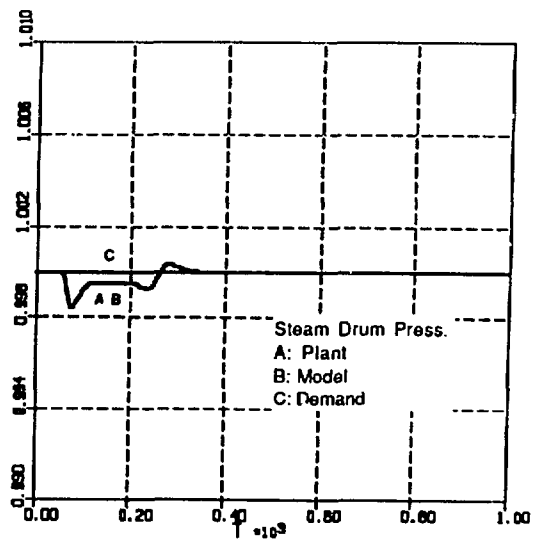


Figure 2.2.b Steam Pressure (decomp. controller)

### 3.2. Coordinator

The role of the coordinator module is to provide a set of consistent demands for each of the subsystems. Each subsystem's local controller generates the optimal control actions to be sent to the actuators to fulfill the supervisor's demands. In the process, the generated unknown terms and the computed state variables are sent up to the coordinator for plant performance evaluation, but this does not require any iterative computation with the subsystems.

By observing the time dependency of the unknown terms calculated by the different subsystems, the coordinator monitors the status of the plant and the local controllers. As new conditions arise, the coordinator recognizes the perturbation, identifies the failed subsystem, and changes the distribution of demands according to pre-established control strategies for the particular subsystem and for the entire plant.

A coordinator module with a small set of anomaly-detection rules was implemented to test the viability of the approach. One set of rules is based on analysis of the behavior of the coupling terms only, the other is based on the tracking of state variables only. Figure 3 is a schematic

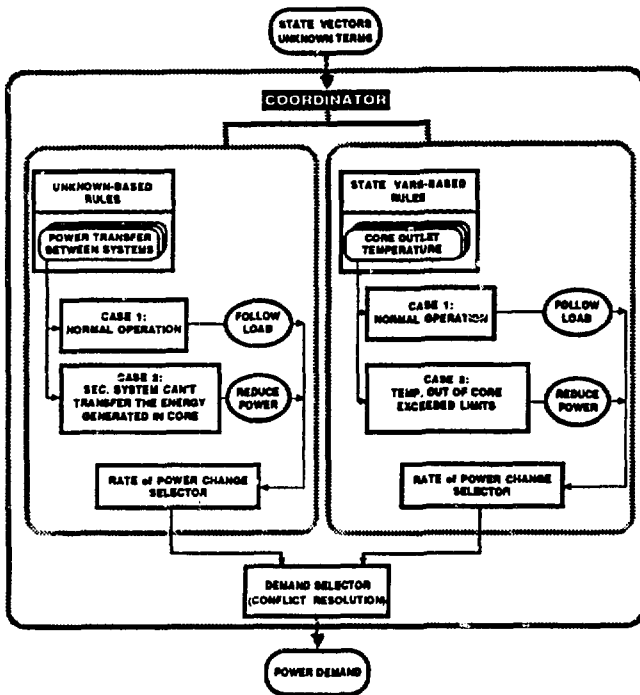


Figure 3 Coordinator Scheme

diagram of the coordinator with the two sets of rules used in the sample cases described here. These rules apply to the detection of specific anomalies in the intermediate heat transport loop (pump failure) and take corrective actions.

The set of rules #1 is shown in action in fig. 4, where the supervisor identifies the failure of subsystem SS3 by observation of the inconsistency of the unknown terms related to the energy transfer between SS2 and SS3 and between SS3 and SS4 (normalized to unity at nominal operating conditions in figs 4.1 and 4.2), and changes the neutron power demand until the balance is restored. State variables are shown in fig 4.3 and the corresponding control actions in fig. 4.4.

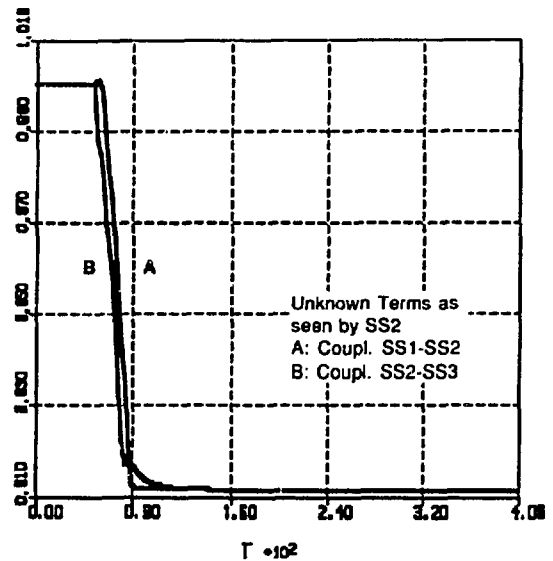


Figure 4.1 SS2 Unknown Terms (pump failure), rules # 1 triggered

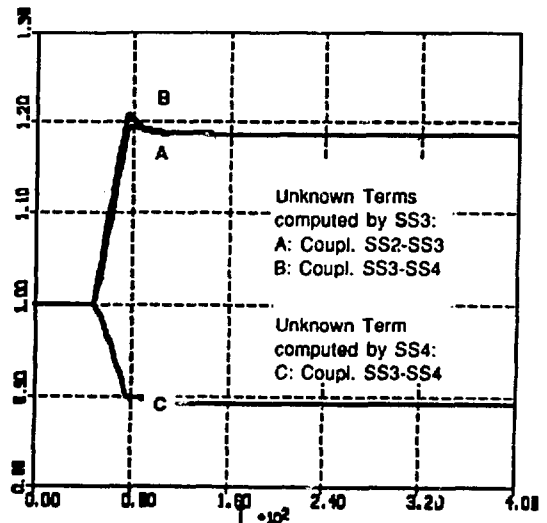


Figure 4.2 SS3 & SS4 Unknown Terms (pump failure), rules #1 triggered

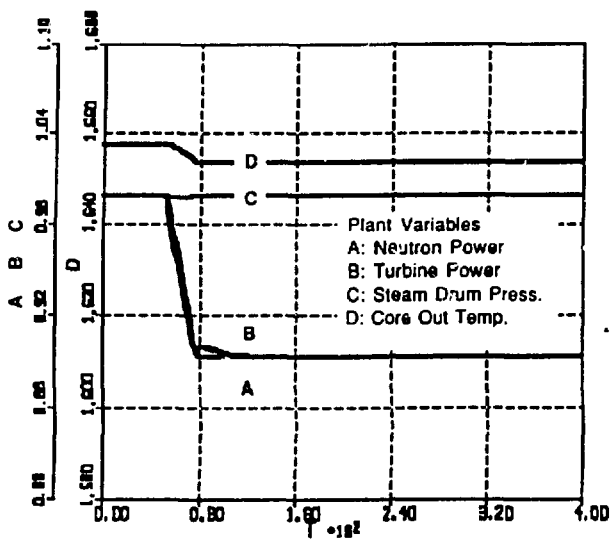


Figure 4.3 State Variables (pump failure), rules #1 triggered

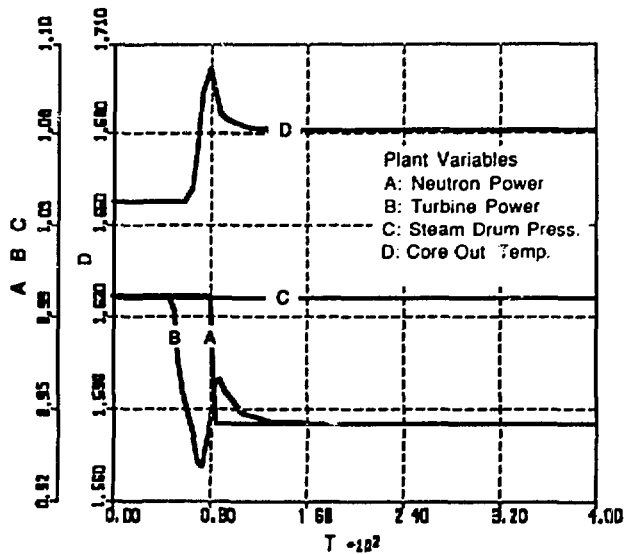


Figure 5.1 State Variables (rules #2 triggered)

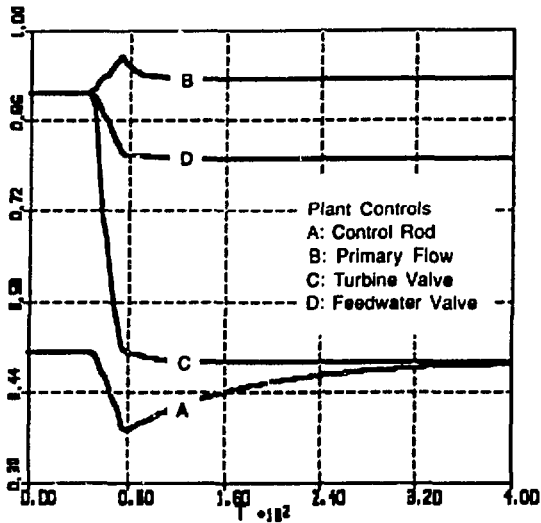


Figure 4.4 Plant Controls (pump failure), rules #1 triggered

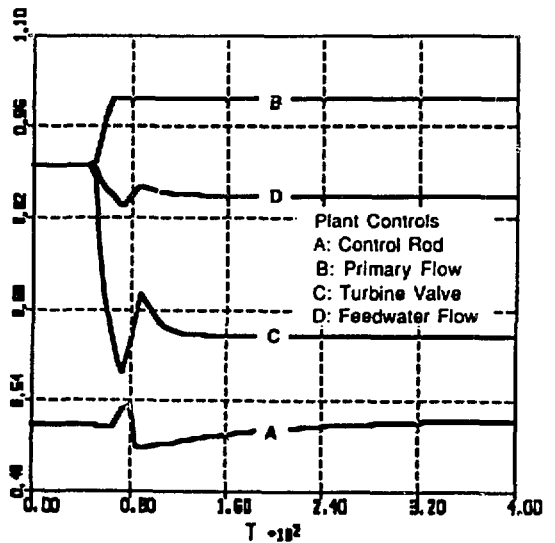


Figure 5.2 Plant Controls (rules #2 triggered)

The set of rules #2 are triggered in the simulation shown in figs. 5.1 and 5.2 where the supervisor detects that the core exit temperature has reached a high limit and imposes a runback demand on subsystem's SS1 neutron power. The core outlet temperature does not attain the demanded value since the capabilities of the flow controller are exceeded due to saturation. This kind of rule is of the type found in today's operating power plants.

Analysis of the dynamic behavior of the unknown terms can help identify the cause of plant-model mismatches due to sensor or component

failures. For instance, consider the case of a negative offset in the reading of the temperature sensor located at the outlet of the secondary side of the intermediate heat exchanger (IHX) in SS3. This erroneous signal generates a set of unknown terms which misrepresent the power being transferred into and out of SS3 (curves B and C respectively in fig. 6). Comparison between the unknown terms representing the power leaving the SS2 (curve A), and the power entering SS4 (curve D), rules out any component failures because no transient is being observed in neighboring subsystems. Since the subsystems are fed by independent sets of sensors the failure can be attributed to a sensor in subsystem SS3.

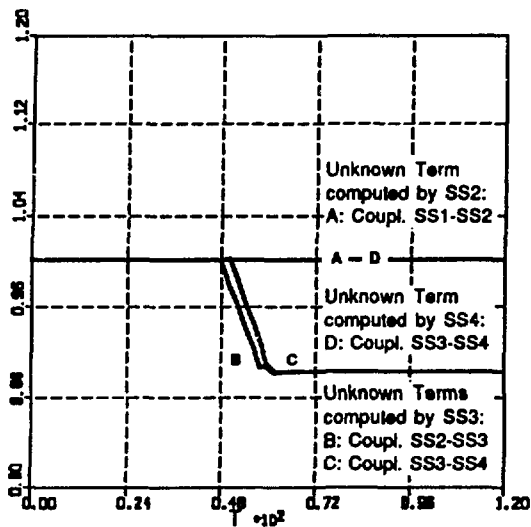


Figure 6 Unknown Terms (sensor failure)

#### 4. Conclusions

The approach to hierarchical control of large systems described here eliminates the need for the typical iterative computations to account for the coupling effects between subsystems. In addition, the computational independence between the unknown terms generated by each subsystem's controller facilitates both its implementation in a distributed network of CPUs, and the isolation and diagnoses of sensor and system component failures.

The role of the coordinator is transformed from that of an inflexible black-box-like numerical procedure to that of an intelligent supervisor of subsystem performance. By incorporating symbolic and numerical recipes, the supervisor issues the appropriate set of demands for each subsystem required for the specific goal. It is the task of the individual subsystem controllers to fulfil those demands in an optimal fashion.

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