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The Gluon Propagator in Momentum Space

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We consider quenched QCD on a $16^3 \times 40$ lattice at $\beta = 6.0$. We give preliminary numerical results for the lattice gluon propagator evaluated both in coordinate and momentum space. Our findings are compared with earlier results in the literature at zero momentum. In addition, by considering nonzero momenta we attempt to extract the form of the propagator and compare it to continuum predictions formulated by Gribov and others.

1. INTRODUCTION

The possibility of studying nonperturbatively on the lattice gauge-dependent quantities provides in principle a unique tool to test QCD at the level of the basic fields entering the continuum Lagrangian. From this point of view, the gluon propagator in the quenched approximation is perhaps the simplest quantity. From its study one expects to obtain among other things a better understanding of the infrared behaviour of the theory and of the mechanism of gluon confinement.

The nonperturbative behaviour of the Euclidean gluon propagator has been investigated in the continuum by many authors with different methods and in different gauges[1–5]. In particular, a very peculiar momentum dependence has been predicted as a consequence of a modification of the standard path integral Faddeev-Popov formula in the Landau gauge by the introduction of a nonperturbatively correct gauge-fixing procedure[1, 4]. Such improved implementation of the Landau gauge is expressed by the equations

$$\partial \cdot A = 0 \qquad \text{and} \qquad FP[A] > 0 \qquad (1)$$

where $FP[A]$ is the Faddeev-Popov operator in the Landau gauge, which in general is not positive definite. The positivity requirement in (1) can be seen as a recipe to get rid (although not completely, see for example[6]) of Gribov copies[1]. In the gauge (1), the (transverse) gluon propagator in momentum space has been argued to be of the form[1, 4]

$$G_{Gribov}(k) \approx \frac{k^2}{k^4 + b^4} \qquad (2)$$

where b is a dynamically generated mass scale. The form (2) for the propagator in momentum space implies that in the continuum

$$G_{Gribov}(\vec{k} = 0, t) \approx e^{-\frac{b}{\sqrt{2}}t} \left(cos(\frac{b}{\sqrt{2}}t) - sin(\frac{b}{\sqrt{2}}t) \right) \qquad (3)$$

Remarkably, the same predictions were also obtained in the study of Schwinger-Dyson equations[3].

2. THE LATTICE PROPAGATOR

The lattice gluon field can be defined as[7]:

$$A_\mu(n) \equiv \frac{U_\mu(n) - U_\mu^\dagger(n)}{2ia} - \frac{1}{3} tr \left(\frac{U_\mu(n) - U_\mu^\dagger(n)}{2ia} \right) \qquad (4)$$

where a is the lattice spacing. Thus the lattice gluon propagator in x–space is the expectation value of:

$$G_{\mu\nu}(x, y) \equiv Tr \left(A_\mu(x) \; A_\nu(y) \right) \qquad (5)$$

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An important point is that on the lattice one can define and implement the analogue of the gauge condition (1) and reobtain from analytical arguments the predictions for the propagator mentioned in the above section[4].

In fact, given any link configuration $\{U\}$, one can define a function of the gauge transformations g on $\{U\}$

$$F_U[g] \equiv -\frac{1}{V}\sum_{n,\mu} Re\, Tr\, \left(U^g_\mu(n) + U^{g\dagger}_\mu(n-\hat{\mu})\right), \quad (6)$$

where V is the lattice volume and U^g indicates the gauge-transformed link $U^g_\mu(n) \equiv g(n)U_\mu(n)g^\dagger(n+\hat{\mu})$. An iterative minimization of $F_U[g]$ obtained by performing suitable gauge transformations generates a configuration $\{U^g\}$ that satisfies the lattice version of (1), defined in terms of a lattice Faddeev-Popov operator. Then it is natural to try and test numerically predictions like (2) and (3).

Numerical studies have been performed in the past years for the zero spatial momentum Fourier transform of (5), namely $G(\vec{k}=0,t) \equiv \sum_{i=1}^{3} G_{ii}(\vec{k}=0,t)$[7-9]. These studies reported some evidence of an effective gluon mass that increases with the time separation. This feature, which would be unacceptable for the propagator of a real physical particle since it violates the Kallen-Lehmann representation, is in qualitative agreement with the continuum prediction (3) and may be in principle acceptable for a confined particle[3, 7].

Our work aims to test at a more quantitative level continuum predictions and to extend the above results through the study of the gluon propagator at nonzero momenta. By requiring consistency between zero and nonzero momentum results, one has a better chance to determine the propagator's analytical form.

3. NUMERICAL RESULTS

3.1. Technical Remarks

It is perhaps worth remarking that, unlike simulations involving quenched quark propagators, evaluations of purely gluonic correlation functions can take full advantage of the translational symmetry of the theory in order to improve statistics. On the other hand, such quantities turn out to be very sensitive to the numerical accuracy of gauge fixing. Empirically, it turns out that when the minimization of $F_U[g]$ has reached an accuracy $\approx .05\%$ the signal for the propagators is sufficiently stable against additional gauge fixing.

3.2. Results

We report results for a set of 25 configurations on a $16^3 \times 40$ lattice at $\beta = 6.0$. As a first step we have evaluated $G(\vec{k}=0,t)$; our results confirm that the propagator exhibits a massive decay in time with an effective mass $a * m(t) \equiv ln(\frac{G(\vec{k}=0,t)}{G(\vec{k}=0,t+a)})$ that increases with t. In Fig.1) we plot $a * m(t)$ versus t in lattice units with jackknife errors. Assuming the value of the inverse

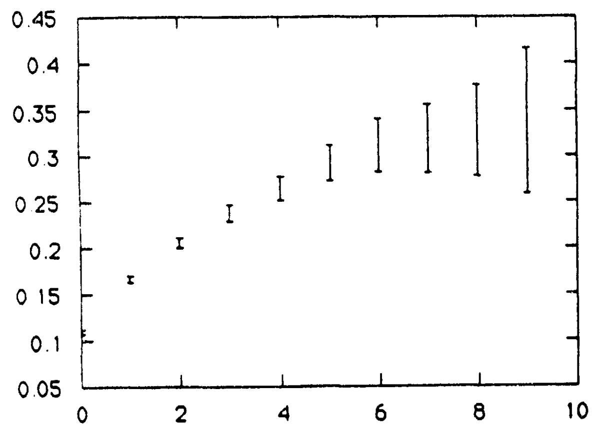

Figure 1. Effective gluon mass in lattice units

lattice spacing $a^{-1} \approx 2.0 GeV$ at $\beta = 6.0$, the effective gluon mass that we measure ranges approximately between 200 and 800 MeV.

We first attempt a 2-parameter least-squares fit of our data to the continuum form (3) without taking into account the correlations in the data. The parameters are an overall normalization factor and the mass scale b. The results are given in Fig. 2 and show a very good agreement between the data and the fitting points. For $a^{-1} = 2$ GeV one obtains $b = 225 \pm 5$ MeV, where the quoted error comes from the covariance matrix of the fitting parameters and does not include the systematic error on a^{-1}.

Good agreement is also obtained by using the form commonly referred to as particle + ghost, that is $G(\vec{k}=0,t) \approx C_1 exp(-M_1 t) + C_2 exp(-M_2 t)$, where C_2 is constrained to be negative.

On the other hand, one cannot get good agreement if one uses a conventional 4-parameter double exponential form, that is if one constrains C_2 in the above formula to take positive values. Indeed, in this case the effective mass would always decrease with t, in contrast to what is observed.

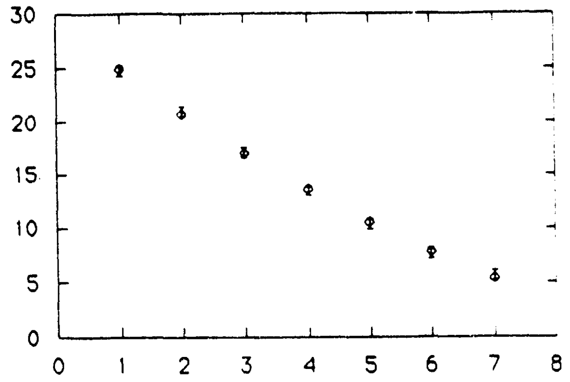

Figure 2. $G(\vec{k}=0,t)$ (error bars shown) and fit to the form G_{Gribov} (diamond points)

It is well known that the data points obtained from a Monte Carlo simulation are in general statistically correlated; in the present case, the correlated data are the values of the propagator $G(\vec{k}=0,t)$ at different timeslices.

The standard way to take into account this effect when performing χ^2 fits is to use the definition of χ^2 that involves the full covariance matrix[10]. Such a definition reduces to the standard one when the covariance matrix is diagonal, which would happen if the data points were uncorrelated.

By inspection of the covariance matrix for $G(\vec{k}=0,t)$ it turns out that the off-diagonal matrix elements are typically of the same size as the diagonal ones, i.e. our data points are highly correlated in t. Consequently, when we perform χ^2 fits taking into account the full covariance matrix, the fits are not well controlled because the correlation matrix is nearly singular. However, we still find that $G_{Gribov}(\vec{k}=0,t)$ fits the data better than other forms. There is also qualitative agreement between our results for $G(\vec{k}=0,t)$ and previous ones from other groups[7–9].

In spite of the difficulties in the statistical analysis, our interpretation of the results for $G(\vec{k}=0,t)$ receives a strong support from the analysis of the momentum space propagator $G(k) \equiv \sum_{\mu=1}^{4} G_{\mu\mu}(k)$. It turns out that such a quantity is very well determined in a significant interval of physical momenta, ranging from the lattice infrared cutoff $k_o = \frac{2\pi}{N_t a}$ to $k \approx 3k_o$ (see Fig. 3). In this range we fit the data to the continuum formula (2) and, for a comparison, to a standard massive propagator $G_{mass}(k) \approx \frac{A}{k^2+m^2}$.

An interesting point is that the covariance matrix associated with $G(k)$ turns out to be much more "diagonal" than the one for $G(\vec{k}=0,t)$; in other words, the data points are much less correlated in momentum space than they are in t. As a consequence we have been able to obtain good fits for $G(k)$ with or without taking into account correlations. We show in Fig. 4 a fit of $G(k)$ to the form $G_{Gribov}(k)$. With the full covariance

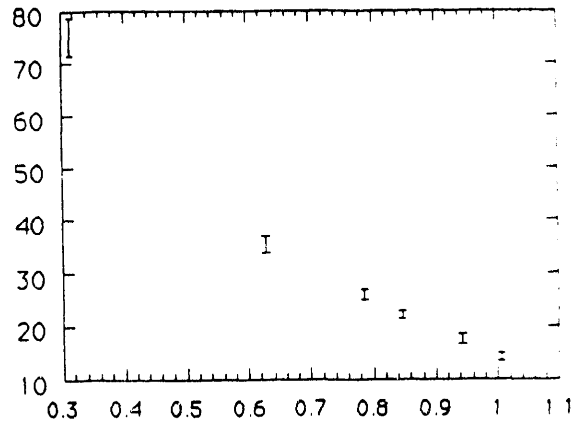

Figure 3. momentum space propagator vs. $|k|$ in GeV (assumes $1/a = 2.0$ GeV)

matrix, we get for the fit in Fig. 4 $\chi^2_{dgr} = 1.5$ and $b = 322 \pm 8$ MeV, assuming again $a^{-1} = 2$ GeV and neglecting systematic errors.

We compare this result to the best fit that one can obtain from the standard massive propagator, for which we obtain $\chi^2_{dgr} = 2.9$. On the other

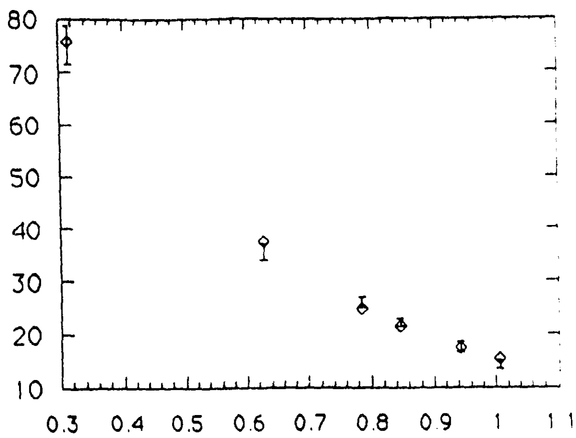

Figure 4. $G(k)$ (error bars shown) and fit to the form G_{Gribov} (diamond points)

hand, the b values that one obtains from the fits in coordinate and momentum space differ significantly, since they are respectively $b = 225 \pm 5$ and $b = 322 \pm 8$ MeV. Since an appreciable difference also occurs for the mass parameter m when we compare momentum and x−space fits to a simple massive propagator, we think that such discrepancies may be related to the different role that finite-size effects play in the two calculations. Further investigation of this issue is in progress.

4. CONCLUSIONS

We think that our results provide a significant (although preliminary) check of the continuum predictions (2) and (3). In particular, the study of the propagator in momentum space appears very promising since the data for such quantity turn out to be statistically rather clean.

Recalling (2) it is clear that a conclusive test requires the study of the propagator at very low momenta, in order to observe the suppression predicted by (2). Of course such a study calls for very big lattices.

The work in progress aims to obtain first a better understanding of systematic and statistical errors. After that, a study of the scaling properties of the mass scale associated to the gluon propagator is in order and, in a later stage, the issue of the gauge dependence of the propagator will be addressed.

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