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MAR 10 1990

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LA-UR--90-590

DE90 007544

TITLE NONOPTIMAL ENTROPY GENERATION BY HEAT TRANSFER  
IN CRYOGENIC DOMAIN

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SUBMITTED TO Florence World Energy Symposium  
Florence, Italy  
May 28-June 1, 1990

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NONOPTIMAL ENTROPY GENERATION  
BY HEAT TRANSFER  
IN CRYOGENIC DOMAIN

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ABSTRACT

The entropy generation caused by heat conduction has been analyzed for physical situations related to cryostats. The situations under consideration are characterized by heat conduction with presence or absence of heat generation and/or heat convection. The existing information regarding the thermodynamic optimal design is compared with more realistic nonoptimal situations. The difference between the optimal and nonoptimal entropy generation has been analyzed from the quantitative point of view. It has been shown that the nonoptimal temperature distribution through a cryostat component might be a realistic option from the thermal design point of view.

KEYWORDS

Cryogenics; cryostat; thermal design; entropy generation; heat transfer.

INTRODUCTION

Recent advances in new technologies in many direct applications, especially in the field of energy, are unavoidably involved with the development of cryogenic technology. The discovery of high temperature superconductivity emphasizes the above mentioned fact. The outstanding optimism of many scientists in the field of applied superconductivity research, and the modest expectation of cryogenic engineers, create a very stimulating atmosphere for new research efforts in the field of cryogenics.

One of the key components in a cryogenic system is the cryostat - a device for maintaining a constant and very low temperature. In a thermodynamic sense, the cryostat could be defined as an open nonadiabatic system in which heat flow (from the ambient high temperature level to cryogenic temperatures) must be reduced as much as possible. Therefore, the existence of large temperature differences along the paths of the heat flows is inherently connected with the basic function of a cryostat. The

direct consequence of this fact is a corresponding entropy generation in this, typically irreversible, process.

The rational approach to the efficient thermal design of a cryostat should be optimization based on an objective function which expresses the entropy generation. The optimization of different cryostat components has been studied relatively frequently: the design of mechanical support systems (Bejan and Smith, 1974, Hilal and Boom, 1977, and others); optimal distribution of the radiation shields (Martynovskij *et al.*, 1971, Eyssa and Okasha, 1978, Chato and Khodadadi, 1984); the analysis of the optimal regime of operation of the current leads (Agsten, 1973, Bejan, 1979); and the thermodynamic analysis of the superinsulation (Martynovskij and Snajd, 1964, Martynovskij *et al.*, 1966, Bisio, 1986, etc.). In addition to this, the analysis based on the second law of thermodynamics is a very attractive current research field of thermodynamic analysis (Bejan, 1988).

However, modern approaches to the thermal design of a cryostat should be applied under very rigid technological demands and constraints. Therefore, thermodynamically nonoptimal solutions during the design procedure at the component level are realistic option. Still the irreversibility of a heat transfer process (entropy generation) remains a key thermodynamic criteria for the evaluation of the solution. Despite of this, the analysis of the distribution of nonoptimal entropy generation within the cryostat components has not been adequately addressed in the existing literature.

#### ENTROPY GENERATION BY HEAT CONDUCTION

Usually, the most relevant heat transfer mechanism (regarding the contribution to the overall entropy generation balance in the cryostat) is the heat conduction through a nonisothermal cryostat component. This inherently dissipative process in general appears in a nonisotropic and nonhomogeneous conducting medium simultaneously with additional cooling and/or heating effects because of a possible heat convection (continuous or discrete cooling) and/or heat generation (Joule effect). Heat radiation effects are present very often, too.

The existing optimization procedures implicitly consider idealized operating conditions (i.e. ideal cooling effects, homogeneous distribution of idealized heat generation sources, simplified geometries and ideal thermal properties, etc.). However, the mechanical supports, current leads and other non-isothermal thermal communications are in many cases neither in idealized physical situations nor is the optimal design (from the thermodynamic point of view) possible because of unavoidable constraints. Therefore, the entropy generation in nonoptimal situations must be the subject of the quantitative analysis.

Let us consider the heat conduction through a cryostat component (see Fig.1). It consists of a conducting medium with cross section  $A$  and length  $L$ . One end is at hot ambient temperature ( $T_H$ ) while the other is at a cryogenic temperature ( $T_C$ ). The thermal communication is continuously cooled with a cooling medium and/or continuously heated because of the Joule effect. The heat radiation effects are considered negligible. In the first approximation, heating and cooling effects can be represented

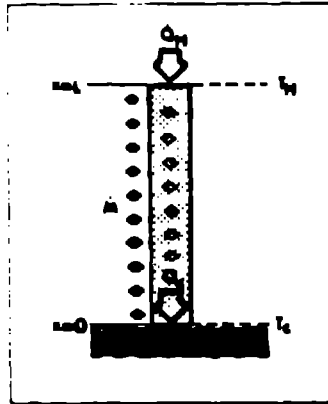


Fig. 1. One-dimensional cryogenic thermal communication

by homogeneously distributed, infinitesimally small heat sinks and/or heat sources within the isotropic homogeneous conducting material. Entropy generation under these conditions becomes:

$$S = \int_{T_C}^{T_H} \frac{\dot{Q}}{T^2} dT \quad (1)$$

or:

$$\dot{S} = \int_0^L \left( \frac{k(T)A(x) \left| \frac{dT(x)}{dx} \right|^2}{T(x)^2} + \frac{\rho(T) \cdot I^2}{T(x)A(x)} \right) dx \quad (2)$$

The temperature distribution  $T(x)$  under steady state conditions is described by:

$$\frac{d}{dx} \left( k(T)A(x) \frac{dT}{dx} \right) + \frac{I^2 \rho(T)}{A(x)} - \dot{M} c_p \frac{dT}{dx} = 0 \quad (3)$$

and by the above mentioned boundary conditions.

There are several attempts in literature to optimize the objective function  $\dot{S}$  (see equation (2)) in different simple physical situations ( $I=0$ ,  $k=\text{const.}$  or  $k=k(T)$ ,  $A=\text{fixed}$ , Martynovskij *et al.*, 1966,  $I \neq 0$ ,  $\rho=bT$ ,  $k=\text{const.}$ ,  $A=\text{fixed}$ , Bejan, 1979, etc.). However, no adequate efforts have been performed to analyze more closely the behaviour of the nonoptimal entropy generation (equation (2)) in these and more complicated situations. In Table 1. few characteristic situations are summarized. Let us consider the behaviour of the entropy generation in these physical situations as a function of relevant physical parameters.

In Figs. 2a and 2b, the entropy generation in some real cryogenic materials with no cooling and heating effects is presented. The situations with  $k=\text{const.}$  and  $k=aT$  are also included (cases  $a_1$ ,  $i=1, 2$  and 3, Table 1.). The entropy generation has been represented by the nondimensional entropy generation  $S$  given by:

$$\dot{S} = \frac{\dot{Q}}{k_{T=T_H} \left( \frac{A}{L} \right)} \quad (4)$$

Table 1. Physical situations under consideration

CASE a			CASE b	CASE c	CASE d
a <sub>1</sub>	a <sub>2</sub>	a <sub>3</sub>			
$k = \text{const.}$	$k = aT$	$k = k(T)$		$k = \text{const.}$	
$\dot{M} = 0$			$\dot{M} \neq 0$	$\dot{M} = 0$	$\dot{M} \neq 0$
$I = 0$			$I = 0$	$I \neq 0, \rho = \text{const.}$	$I \neq 0, \rho = \text{const.}$
$A = \text{fixed}$					

$k$ , thermal conductivity;  $\rho$ , electrical resistivity;  $I$ , electrical current;  $\dot{M}$ , mass flow rate of coolant ( $c_p, t$ );  $A$ , cross-sectional area

The results for case a (Fig. 2a and 2b) are obtained using equations (2) and (3). Temperature dependent thermal conductivity have been included according to Stewart and Johnson (1961).

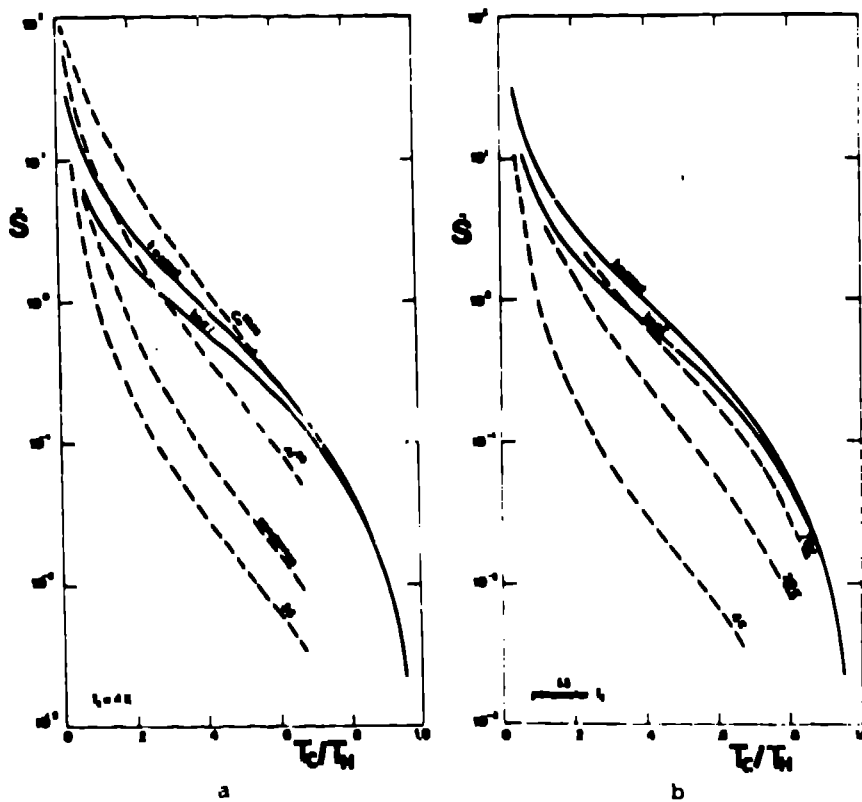


Fig. 2. a) Entropy generation within the cryogenic materials  
b) Entropy generation for stainless steel

The change of the temperature range under consideration influences dramatically the nondimensional entropy generation rate, as shown in Fig. 2b for stainless steel (SS). It is important to note that in this physical situation no optimal design regarding the irreversibility in the heat transfer process is possible.

The optimum design of cryogenic thermal communications (for example mechanical supports) should involve the use of a stream of cold gas (i.e. helium) flowing along with the thermal communication (Scott, 1959, Bejan, 1979). In other words, with the presence of continuous cooling, optimal temperature distribution exists which minimizes the objective function, Martynovskij and Snajd (1966). This temperature distribution assumes, however, idealized continuous cooling. In Fig. 3, the optimal nondimensional entropy generation has been compared with a few nonoptimal ideal cooling regimes. The curves representing nonoptimal

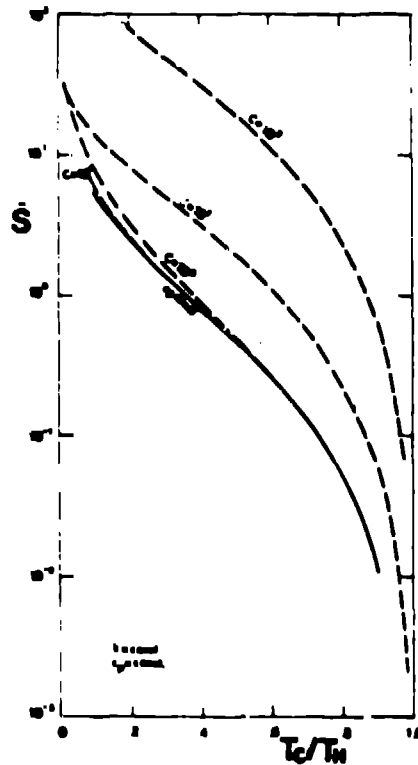


Fig. 3. Entropy generation and convective effects

regimes in Fig. 3 were constructed using the same procedure as for Fig. 2 but for the case b, Table 1. The characteristic parameter  $C$  represents the influence of the convective ideal cooling (the ratio of the heat capacity rate of the cooling medium and the thermal conductance of the conducting medium, i.e.

$C = (\dot{M} c_p) / k(A/L)$ . It is worthwhile to note that the "operating point" exists where the entropy generation is in the minimum (i.e. optimal thermal design), Fig. 4.

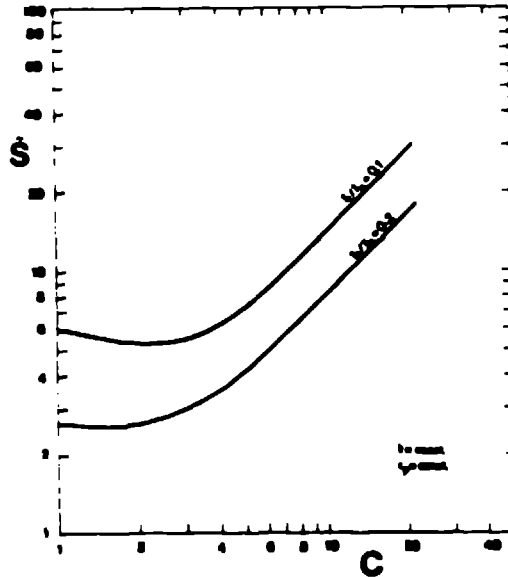


Fig. 4.  $\dot{S}$  vs.  $C$  for the continuous cooling case

The behaviour of the entropy generation within the conducting medium with the presence of Joule heating effects shows different characteristics. In Fig. 5a, the nondimensional entropy generation rate corresponding to the case c (Table 1) is shown. The main conclusions to be drawn from Fig. 5a are that: (i) the entropy generation rate dramatically increases as the ratio of the Joule heat release to the thermal conductance of the conducting medium increases (i.e. parameter  $B$  increases,  $B = (I^2 \rho L) / (2kA^2 T_H)$ ), and (ii) at "thermodynamic equilibrium" conditions ( $T_H = T_C$ ), a residual irreversibility exists (due to the Joule heat generation irreversibility). The difference between the optimum and nonoptimum design diminishes as the ratio between the cryogenic and ambient temperature decreases (as well as in the case b (Table 1), Fig. 3), i.e. nonoptimum thermal design is the real option in those regimes (for  $B = \text{ibidem}$ ).

Finally, let us consider simultaneous heat conduction, cooling the conducting medium and heating by heat generation within the thermal communication (case d, Table 1). The results of this analysis are shown in Fig. 5b.

The influence of the cooling and heating effects changes remarkably the behaviour of the nondimensional entropy generation rate in comparison to the previous physical situations.



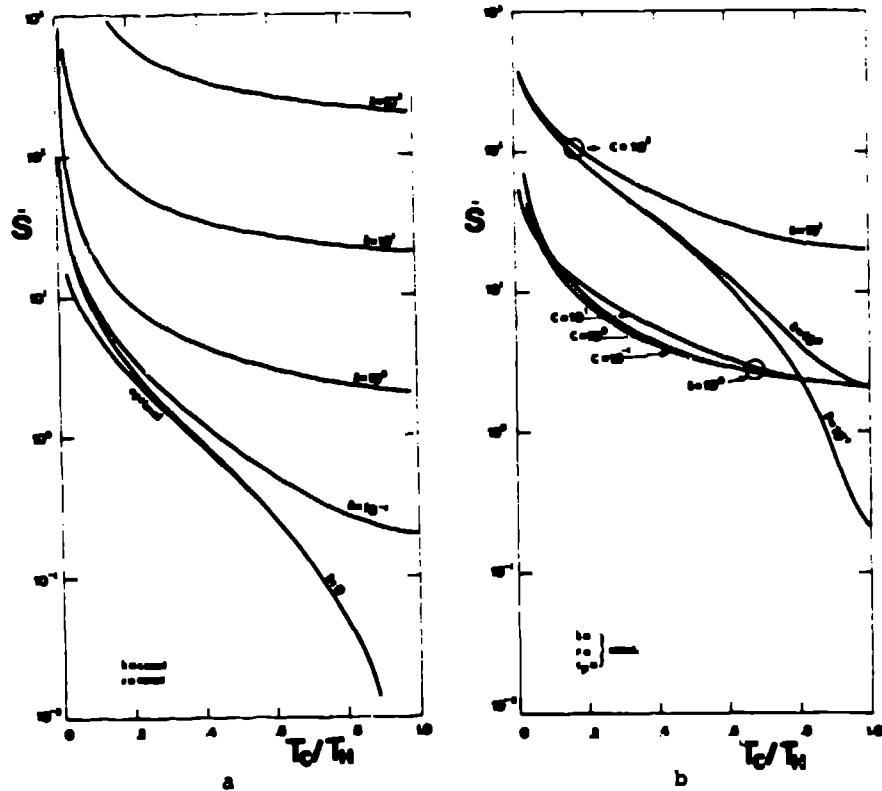


Fig. 5. a) Entropy generation and heat generation effects  
 b) Entropy generation and simultaneous cooling and heating

#### DISCUSSION AND CONCLUSIONS

The thermal design of a cryostat as a whole has been, until now, very often in the sphere of engineering judgement. One of the possible approaches to the thermal design has been the thermodynamic optimization of the cryostat components based on the second law of thermodynamics. The fact that the minimization of entropy generation in the cryostat means implicitly the minimization of the refrigeration power required to overcome heat leakage in through the thermal communications, promotes entropy generation methodology as a useful design tool in the thermal design. However, the soundness of the application of this methodology is strongly related to other pragmatical criteria and demands (dimension constraints, economical considerations, etc.). Therefore, the nonoptimal solutions from the thermodynamic point of view are also under consideration.

The analysis performed shows that nonoptimal solutions (expressed by entropy generation higher than for the optimal

temperature distribution within the conduction medium) are not a priori thermodynamically inconvenient.

The realistic temperature distribution is not always close to the optimal one, and in such situations the thermodynamic optimization might not be an adequate design approach. The results obtained for the two classes of heat transfer communications (mechanical supports and current leads) for cryostat applications clearly show that the analysis of entropy generation with the real temperature distributions needs to be addressed in order to obtain the relevant objective function for the optimization procedure.

#### ACKNOWLEDGEMENT

This paper is based on work supported by the U.S.-Yugoslav Joint Fund for Scientific and Technological Cooperation, in cooperation with the U.S. Department of Energy under Grant JF-DOE-818.

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