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STABILITY AND DYNAMICS OF SPATIO-TEMPORAL STRUCTURES

Progress Report
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NOTICE

1 Abstract

The main goal of the project supported in this grant is to contribute to the understanding of *localized* spatial and spatio-temporal structures far from thermodynamic equilibrium. Here we report on our progress in the study of two classes of systems.
1) We have started to investigate localized wave-pulses in binary-mixture convection. This work is based on our recently derived extension of the conventionally used complex Ginzburg-Landau equations. We are considering three regimes: dispersionless supercritical waves; strongly dispersive subcritical waves; and localized waves as bound states of fronts between dispersionless subcritical waves and the motionless conductive state.
2) We have completed our investigation of steady domain structures in which domains of structures with different wave numbers alternate, separated by domain walls. In particular, we have studied their regimes of existence and stability within the framework of a Ginzburg-Landau equation and have compared it to previous results. Those were based on a long-wavelength approximation, which misses certain aspects which turn out to be important for the stability of the domain structures in realistic situations.
In addition, we give a description of our work on resonantly forced waves in two-dimensional anisotropic systems.

2 Description of Progress

2.1 Domain Structures

In collaboration with my Ph.D. student David Raitt we have completed the analysis of steady domain structures within the framework of a Ginzburg-Landau equation. Through the inclusion of a fourth-order derivative term it describes the bifurcation of a system to a steady structure with two competing wavelengths. Such a situation has been found very recently in the theoretical analysis of parametrically forced surface waves (Faraday experiment) [1]. It also describes convection in nematic liquid crystals. There, convection rolls are found which are oriented obliquely to the direction singled out by the dominant orientation of the rod-like liquid-crystal molecules. By reflection symmetry two orientations with opposite wave numbers in the y-direction are found, which can lead to 'zig-zag'-structures. A typical solution of these Ginzburg-Landau equations is shown in fig.1a. The solid line denotes the deviation of the local wave number from the critical wave number, or in the case of oblique rolls the wave number in the y-direction. In the center it clearly shows a stable domain of reduced wave number (or a 'zig'-domain between two 'zag'-domains of a 'zig-zag'). The thin line denotes the real part of the complex convective amplitude A. The work leads to two main results.

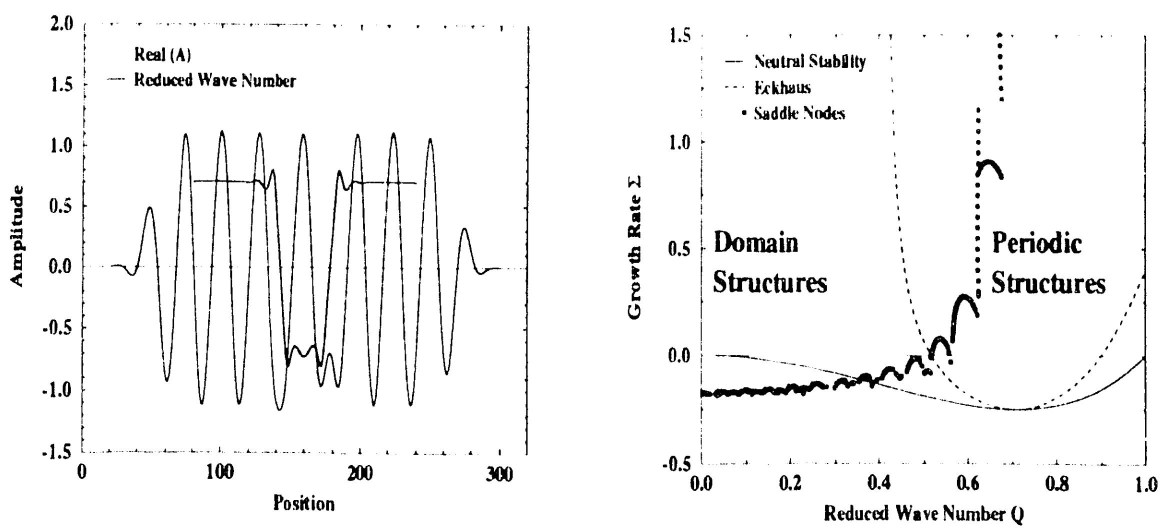

Figure 1:

a) Typical domain structure. Solid: local wave number, thin: $Re(A)$.

b) Phase diagram for domain structure. To the left of the solid line domain structures exist and are stable. Dashed: stability limit of periodic structures, thin: existence limit of periodic structure.

First, the existence region of these structures turnes out to be much more intricate than originally expected. The (numerical) result is shown in fig.1b. The domains exist and are stable to the left of the line marked by solid circles. When the Rayleigh number R with $\Sigma \propto R - R_c$ is reduced below this line the domain structure undergoes a saddle-node bifurcation and ceases to exist. The complicated shape of the bifurcation line is due to the interaction of the domain structure with additional modes emerging from the Eckhaus instability occuring along the dashed line.

Secondly, we have found that the domain structures can be stable under more general and more realistic conditions than expected previously. Within a phase-diffusion approach, which corresponds to a nonlinear WKB-analysis, the domains are stable only due to the conservaton of phase (total number of convection rolls). The corresponding phase equation is identical to the Cahn-Hilliard equation describing the phase-separation process (spinodal decomposition) of a binary mixture in the two-phase regime. Consequently, within this frame-work multiple domains always merge to form a *single* domain of one phase (with small wave number, say) embedded in the other. In the absence of phase conservation this state is also unstable and the system evolves eventually to a state with a single wave number. Going to the more complete description by the Ginzburg-Landau equation introduces an oscillatory character to the interaction between fronts (cf. oscillatory behavior of the local wave number in fig.1a) and fronts can lock into each other. Thus, domains can be stable even if the total phase is not conserved, as demonstrated in fig.1a where the amplitude of the structure vanishes smoothly at the boundary of the system and therefore allows convection rolls - and therefore phase - to enter and leave the system freely. A detailed account of our study is given in a preprint [2].

2.2 Extended Ginzburg-Landau Equations for Binary-Mixture Convection

We have continued the investigation of convection in binary mixtures emphasizing the effects which are due to the slow mass diffusion in liquids (small Lewis number $\mathcal{L}$). We have rederived the equations for the convective amplitude A and the new concentration mode C, which were presented earlier [3], in a way which allows to recover the conventional Ginzburg-Landau equations for the convective amplitude alone in the limit of large Lewis number. In this derivation certain terms in the expansion are effectively summed up to all orders in $\mathcal{L}$. This approach clarifies the relation between the new and the old equations.

We have started to investigate the new equations analytically. To do so we have subdivided the complex problem of studying dispersive waves arising from a subcritical bifurcation into various steps: a) dispersionless supercritical waves; b) soliton-related waves; c) localized waves as bound states of fronts.

a) We have started with the simplest case and studied the effect of the concentration mode on dispersionless waves arising from a supercritical bifurcation. We have performed a linear stability analysis of plane waves and found that at onset ($a = 0$) the stability of the waves is not affected by the new mode C. It becomes important, however, further above onset at a distance which scales with the Lewis number. This is shown in fig.2. In particuar, the waves can become unstable at **all** wave numbers (cf. fig.2a for $a > 0.03$). This resembles somewhat the Benjamin-Feir instability which is known to occur in strongly dispersive waves. It often leads to spatio-temporal chaos. For the dispersionless waves studied here this seems not to be the case. To investigate the nonlinear behavior arising from the instability we followed two paths.

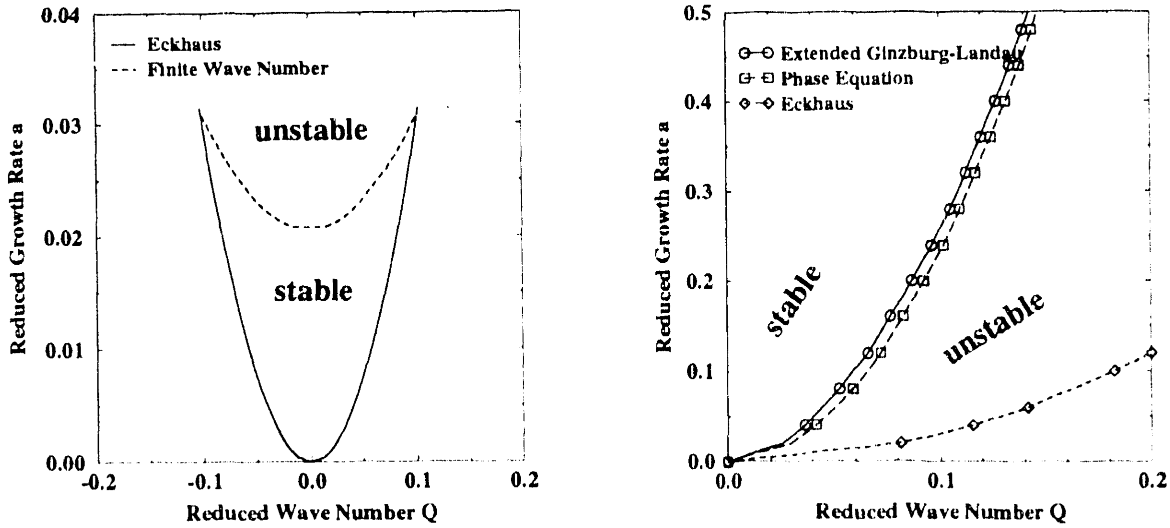

Figure 2:

a) Regime of stability of patially periodic, supercritical waves.

b) Compa*r*ison of stability limits of direct linear stability analysis (solid) with the long-wavelength theory (dashed).

First, we investigated the behavior of long-wave modulations of the plane waves. For finite Lewis number it is not affected by the concentration mode. Taking the gradients in the wave number q to be of the order of $\mathcal{L}^{1/2}$, however, one can derive coupled equations for q and C. These equations represent an extension of the usual phase equations to include a slow mean field. In fig.2b the stability limit obtained with these reduced equations is compared with the full linear stability analysis of the extended Ginzburg-Landau equations. Within the reduced equations we have studied the weakly nonlinear behavior of the instability, and find that, depending on parameters, the bifurcation can be subcritical as well as supercritical. In the latter case quasi-periodic waves are expected to arise.

Second, we solved the extended Ginzburg-Landau equations numerically guided by the previous analysis. This confirmed the existence and stability of quasi-periodic waves which arise from the instability of the plane waves to spatial modulations when it is supercritical. In the regime in which all plane waves become unstable and the bifurcation is subcritical (cf. fig.2a) perturbations were found to lead to stable **localized** waves. This establishes the existence of such pulses even if the primary bifurcation to convection is supercritical. This is similar to the results found in parity-breaking bifurcations [4].

b) In numerical simulations of the extended equations we have previously observed a slowing down of the pulses due to the concentration mode. We have now begun to investigate this analytically. We start from the soliton solutions of the nonlinear Schrödinger equation, which can be considered as a limiting case of the complex Ginzburg-Landau equation, and consider the dissipative terms as well as the additional equation for C as perturbations. Since the C-equation is linear in C to lowest order it can, in principle, be solved using Green's functions. So far, we could solve the resulting integral, which involves suitable derivatives of the pulse, only in an expansion in the inverse pulse velocity. But already in this restricted case the analysis suggests a jump-transition between slow and fast pulses when the group velocity parameter is changed. We could confirm this unexpected result in subsequent numerical simulations. We have made progress in applying soliton perturbation theory [5] to this relatively involved case. It is clear that we will be able to use it to determine the quantity of interest, the pulse velocity. Preliminary work shows that due to the continous Galilean symmetry of the soliton we will have to go to second order in that perturbation theory. This will be possible, albeit very involved, by using Maple.

c) We have started to investigate analytically the interaction between fronts connecting the conductive state with the convective state. This is done in collaboration with H. Herrero-Sanz from the University of Pamplona, Spain, who visited us for three months. Our goal is to understand how the C-mode can lead to bound states of fronts and 'backs', even in the absence of dispersion. Such bound states would constitute a stable localized wave. The method is based on an expansion in the distance L between fronts. For the real Ginzburg-Landau equation it is well known that the interaction is purely attractive and exponentially small in L. The coupling to the C-mode leads to additional contributions to the evolution equation for L. Within a first, simplified approach, in which the fronts are taken to be much steeper than any other length scale in the problem, the resulting equation suggests that such localized waves can be stable if they travel backwards, i.e. opposite to the advection of the concentration mode.

2.3 Temporal Forcing of Small-Amplitude Waves in Anisotropic Systems

In collaboration with M. Silber (California Institute of Technology) and L. Kramer (U. Bayreuth, Germany) we have investigated the influence of resonant temporal forcing on Hopf bifurcation in two-dimensional anisotropic systems. This work is motivated by experiments on convection in nematic liquid crystals [6]. As mentioned above, in the nematic phase the rod-like liquid crystal molecules are predominantly oriented along one axis and thus define an axis of anisotropy, the director. In the experiments it was found that convection can set in via a Hopf bifurcation to traveling waves which can be oblique to the director. Due to the remaining reflection symmetries of the system this leads to the interaction of waves in four different directions. Without forcing the resulting structures are very disordered. In the presence of forcing, however, ordered standing oblique rolls and standing rectangle patterns were observed as well as structures which alternate between the two different oblique orientations [6].

We have studied this problem within the frame-work of bifurcation theory with symmetry, which applies to small-amplitude waves and allows an efficient treatment of the resulting evolution equations for the 4 complex amplitudes. Building on our previous work on the case without forcing [7] we were able to classify all possible ways in which the full symmetry of the motionless basic state can be broken in this system. The main results concern waves which are phase-locked to the temporal forcing. In addition to convection patterns in the form of standing rolls and rectangles we found states in which rolls of different orientation alternate periodically in phase with the forcing. These states can be identified with those observed in experiments and the transitions between them are in qualitative agreement. To elucidate these transitions we considered the limiting cases of strong damping of the waves and of strong detuning of the temporal forcing, respectively. The latter leads to the consideration of a double-zero singularity of the Takens-Bogdanov kind. Finally, we considered the limit of vanishing angle of obliqueness. Further details are given in a preprint [8].

During this budget period the P.I. will have taken two months in salary during the summer 1993.

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Note: We will submit the preprints [2] and [8] in April. Therefore we decided not to enclose preliminary drafts at the present time.

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