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#### PRINCIPLES OF NEUTRON REFLECTION\*

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#### PRINCIPLES OF NEUTRON REFLECTION

#### G. P. Felcher

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#### Abstract

Neutron reflection is perhaps the most developed branch of slow neutrons optics, which in itself is a direct consequence of the undulatory nature of the neutron. After reviewing the basic types of interactions (nuclear and magnetic) between neutrons and matter, the formalism is introduced to calculate the reflectivity from a sample composed of stacked flat layers and, inversely, to calculate the stacking from reflectivity measurements. Finally, a brief survey of the applications of neutron reflection is given, both in technology and in fundamental research.

#### 1. Fresnel Reflectivity

When applied to the propagation of the neutron radiation and to its modification by material objects, the word "reflection" is by no means used figuratively. On the contrary, the close mathematical analogy in the field equations for the neutrons and the electromagnetic radiation gives rise to a set of parallel optical phenomena. More esplicitly, the propagation of the de Broglie waves associated with the neutron in a potential field V(z) is analogous to the propagation of light waves in a medium with variable refractive index n(z). The coordinate z is perpendicular to the surface of the material (which might be taken as the origin, z=0). The material has then graded optical properties along one direction only. Purpose of the present paper is to present the principles that enable to construct useful neutron-optical elements from graded materials; the detailed applications are the major topic of this conference. It will be shown also how, reversing the process, is possible to acquire a substantial knowledge on the composition and the magnetization of materials close to the surface just examining their neutron reflectivity.

Following the analogy with the electromagnetic radiation<sup>2</sup>, we can define a refractive index for neutrons as:

$$n(z) = [1 - V_0(z)/E]^{1/2}$$
 (1)

 $V_{\rm O}(z)$  is the potential due to the neutrons's interaction with the atomic nuclei of the matter and the magnetic fields encountered in its path. E is the kinetic energy of the neutron, which is expressed in terms of the de Broglie's wavelength  $\lambda$  is :

$$E = h^2/2m\lambda^2 \tag{2}$$

where h is Planck's constant and m the neutron mass.

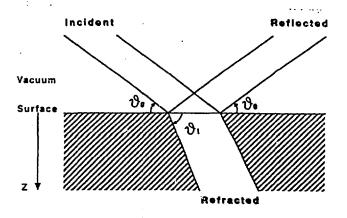


Figure 1. Scheme of the reflection and refraction of a neutron beam from a material surface. In the picture the materials is <u>less</u> reflecting than vacuum: this is true for only a handful of elements.

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Let us consider the optical process at the surface of an homogeneous material, i.e. with constant n. The angle of the neutron beam and the surface is  $\theta_0$  in the vacuum and  $\theta_1$  in the material. The two angles are correlated by Snell's law<sup>3</sup>:

$$\cos \theta_0 / \cos \theta_1 = n \tag{3}$$

which shows that, if n<1, there is a critical angle  $\theta_{\rm C}$  below which the neutrons do not penetrate the material; they are totally reflected. Total reflection can be achieved even keeping  $\theta_{\rm C}$  constant and changing n: by increasing the neutron wavelength up to a critical value  $\lambda_{\rm C}$ .

The refractive index for "thermal" neutrons differs from unity by only some  $10^{-4}$ , which means that total reflection takes place only at grazing incidence for the neutron wavelengths most readily available. Neutrons produced in nuclear reactor (or a spallation source) are cooled down (as a gas) in a moderating material until their temperature is in equilibrium. Even when the moderator is cooled to liquid hydrogen temperatures, the neutron spectrum covers a range between 2 and 16. A Angstroms, a range of wavelengths similar to that of soft X-rays. The difference between the two kinds of radiations lies in the origin of the interaction potential.

#### Interaction potential

The interaction of thermal neutrons with an individual atomic nucleus can be described by a scattering amplitude b which has a characteristic value for each type of nucleus. The general trend of b is to increase with the radius of the nucleus, but the presence of nuclear resonances introduces deep differences between the scattered amplitudes of adjacent elements or even between different isotopes of one element. In crossing a medium the aggregate effect of the local potentials on the neutrons is to give rise to a coherent, forward scattering wave resulting of the incident wave and the superposition of spherical wavelets emanating from each nuclear site. For this wave is operative an averaged, or optical potential of the incident wave is operative an averaged, or

$$V_{O}(z) = (2\pi h^{2}/m) N(z) b(z)$$
 (4)

where b is the average scattering at the depth z, and N(z) the corresponding atom density. The isotopic characterization of the scattering amplitude is a prerogative of the optics of neutrons, and points to the possibility of enhancing the optical contrast between two chemically similar species by selective isotopic substitution.

The magnetic fields present in the neutron path affect the neutron's motion by virtue of their interaction with the magnetic dipole moment of the neutron  $\mu_n$ . The magnetic potential is  $V_{mag}=\pm\mu_n B$ , if (for semplicity) is assumed that both B and  $\mu_n$  are aligned along a common axis. The + and - signs correspond to the cases, in which the neutron moment is respectively parallel and antiparallel to the field B. The refractive index is correspondingly two-valued: which means that for an unpolarized neutron beam a magnetized material is a birefringent medium. The more general and complex case where the magnetic fields and the neutron moments are not collinear will be dealt with in the appendix; it will be seen that the non-collinearity gives rise to new phenomena, such as a finite probabily that the neutron will flip its spin.

#### Calculating the reflectivities

With the interaction potential varying only as a function of the depth from the surface z, the neutron beams refracted and reflected have momenta mv which are affected only in their component perpendicular to the surface. This means that the calculation of the reflectivity reduces to the well-known problem of a particle in a one-dimensional potential box. Separating the coordinates, the Schroedinger equation along the z axis is 5:

$$f''_{\pm} + [k_0^2 - 4\pi(bN \pm cB)] f_{\pm} = 0$$
 (5)

where  $k_0=2\pi(\sin\theta/\lambda)$  is the component of the momentum of the incident neutron normal to the surface, and  $c=2\pi m\mu/h^2$ .  $f^+$ ,  $f^-$  are the time-indepent wavefunctions for neutrons polarized parallel and antiparallel to the magnetic field. In a region of z in which both b.N and B are constant (and for semplicity B=0) the solution of the Eq. (5) is:

$$f = A_1 \exp(ik_1z) + A_2 \exp(-ik_1z)$$
(6)

The wavefunction, which now is spin independent, is composed of two waves propogating at the left and the right of the z axis with momentum:

$$k_1 = \sqrt{k_0^2 - 4\pi bN} \tag{7}$$

The amplitudes  $A_1$  are determined by asking that the wavefunction and its derivative be continuous at the boundaries of the region in which the potential is constant. The most simple case to consider is that of a single interface between vacuum and an homogeneous material at z=0. For a unitary "incoming" neutron wave from the vacuum space at z<0 (fig.2a), the continuity conditions at z=0 become:

$$\exp(ik_0z) + r \exp(-ik_0z) = t \exp(ik_1z)$$

$$k_0 \exp(ik_0z) - k_0r \exp(-ik_0z) = k_1t \exp(ik_1z)$$
(8)

where  ${\bf r}$  is the reflectance and t the transmittance of the wave coming from the left. The reflectivity is then:

$$R = |r|^2 = \left| \frac{k_0 - k_1}{k_0 + k_1} \right|^2 \tag{9}$$

This relation can be verified experimentally, as it is shown at fig. 3.

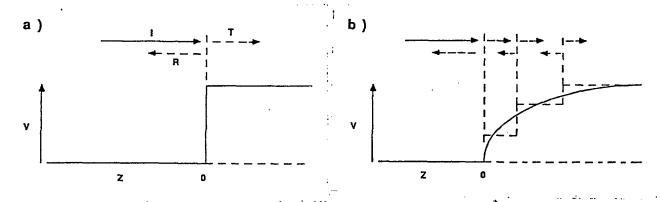


Figure 2. The reflection/transmission process, presented as the motion of a particle in a one-dimensional potential box. z is the coordinate perpendicular to the surface. a: case of a homogeneous material. b: reflection from a graded material. The reflectivity is calculated after substituting the continous profile with a histogram.

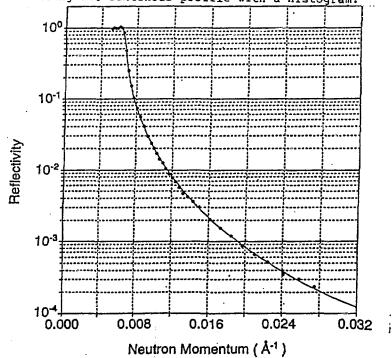


Figure 3. Measured and calculated reflectivity from the flat surface of a round of fused silica. The abscissa is the component of the neutron momentum perpendicular to the surface. The continuous line has been calculated without adjustable parameters.

The reflectivity from a material with a graded density (such as that shown in fig. 2b) is significantly more complex. The conventional way to calculate the reflectivity for such potential is to approximate it with a sufficiently fine histogram of layers of constant refractive indices  $^3$ . Imposing the conditions of continuity for the wavefunction and its derivative at the boundaries of each layer with boundaries  $z_i$ ,  $z_{i+1}$ , the wavefunction at the left of  $z_i$  ( $f_{i+1}$ ) is related to the wavefunction at the right of  $z_{i+1}$  ( $f_{i+1}$ ) by:

$$f_{1,i} = \cos(k_{i+1} \Delta z_{i+1}) f_{r,i+1} - \frac{1}{k_{i+1}} \sin(k_{i+1} \Delta z_{i+1}) f'_{r,i+1}$$

$$f'_{1,i} = k_{i+1} \sin(k_{i+1} \Delta z_{i+1}) f_{r,i+1} + \cos(k_{i+1} \Delta z_{i+1}) f'_{r,i+1}$$
(10)

where  $\Delta z_{i+1}$  is the thickness of the layer between  $z_i$  and  $z_{i+1}$ , and  $k_{i+1}$  is the relative neutron momentum (perpendicular component). Synthetically, eq. (10) can be written in matrix notation:

$$\mathbf{f}_{i} = \mathbf{M}_{i+1}\mathbf{f}_{i+1} \tag{11}$$

The wavefunction vector in the vacuum is related to that in the bulk by a matrix, which is simply the product of the matrices  $\mathbf{M}_i$ . While this numerical procedure (or variations of it') always allows the calculation of the reflectivity, its analytical form may be quite complicated. The general integral expression of the reflectance from a potential V(z) is  $^8$ :

$$r(0) = -\int_{0}^{\infty} \frac{k'(z)}{2k(z)} \left[1-r^{2}(z)\right] \exp\left\{-i\int_{0}^{z} k(z_{1}) dz_{1}\right\} dz$$

$$4. \text{ The inverse problem}$$
(12)

In the preceding paragraph it was shown how to calculate the reflectivity from the potential. The question is if the inverse process is possible: i.e. if one can calculate the potential V(z) from  $R(k_0)$  which constitutes the body of experimental information. In absence of a suitable backtransform, the procedure to follow is unpleasantly indirect. To start with, a reasonable model has to be proposed for the density profile (perhaps suggested from the sample preparation), and then its reflectivity should be calculated and compared with the experimental data. A good fit with the data is sought, by varying some of the parameters implanted in the model. Unfortunately such procedure does not assure, even in the case in which good fitting is achieved, that the mode we started with is correct, nor if the set of refined parameters is unique.

The "inverse problem" has been actively investigated even recently by numerous mathematical physicists Their findings have not yet been applied to solve the optical problem of determining  $V(z) = 4\pi b(z) N(z)$  from reflectivity data. A brief outline will be given of the treatment proposed by Hruslov to obtain V(z) once known the reflectance (in modulus and phase) over the entire range of  $k_0$ .

Suppose that the potential V(z) is real, locally summable and has different limits at  $\pm \infty$ 

$$\lim V_{z \to \infty}(z) = 0$$
  $\lim V_{z \to +\infty}(z) = c^2 (c>0)$  (13)

We impose on V(z) the following limitations: V(z) tends toward its asymptotic limits faster than  $\left|1/z\right|$  and represents a purely repulsive potential, without discrete eigenvalues. The wavefunction takes, close to the asymptotic limits, the simple form:

$$\frac{1}{2} \underbrace{im_{e}} I_{exp}(ik_{o}z) + 0_{exp}(-ik_{o}z) 
\underbrace{1}_{im_{e}} O_{exp}(ik_{1}z) + I_{exp}(-ik_{1}z)$$
where  $k_{1} = \sqrt{k_{0}^{2} - c^{2}}$ .

According to eq. (14), the free wave at  $-\infty$  is composed of a wave of amplitude I\_ coming into the field z, and one (of amplitude O\_) outgoing. Of similar terms is composed the free wave at  $z=+\infty$ . For the Schroedinger eq. (5), with asymptotic solutions (14), there is a matrix - the S or scattering matrix - which transforms the set of incoming elements into the set of outgoing elements:

$$O_{+} = I_{-}t(k) + I_{+}\rho(k)$$

$$O_{-} = I_{-}r(k) + I_{+}\tau(k)$$
(15)

r and t are respectively the reflectance and the transmittance for the wave moving from the left;  $\rho$  and  $\tau$  are the corresponding coefficients for the wave moving from the right. In matrix notation,  $S_{21}$  represents the reflectance as derived from the experiment. The knowledge of  $S_{21}$  is sufficient to determine the entire matrix, whose coefficients are releated by conditions of symmetry and unitarity.

The calculation of the potential proceeds along the following steps. First, the Fourier transform of  $S_{21}$  is introduced:

$$\Omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{21}(k_0) \exp(-2ik_0 z) dk_0$$
 (16)

In second place, the kernel K has to be calculated. The kernel is related to  $\Omega$  by a integral equation which has to be solved numerically 13:

$$K(z,y) + \Omega(z+y) + \int_{-\infty}^{\infty} \Omega(x+y) K(z,x) dx = 0$$
 (17)

with y<z

Finally, the potential is obtained simply as a derivative of the kernel:

$$V(z) = 2 \frac{d}{dz} K(z,z)$$
 (18)

The utility of these expressions might be assessed only after a significant amount of work, to determine how well the phase of the reflectance can be established or at least guessed; and how much error propagation results from utilizing sets of data numerically imprecise or incomplete. Finally, it might be worthwhile to inquire to what extent the solutions could be given in an analytical form rather than as the solution of an integral equation.

#### Applications

Historically, total reflection of neutrons was first observed by Fermi and Zinn in 1946 from polished mirrors of different metals 14. Their measurements of the critical angles for monochromatic neutrons established an important method for the determination of the scattering lengths. Note that in this way an absolute determination of the scattering length is obtained, provided that the neutron wavelength is known with sufficient accuracy. Later Koester 15 furthered measurements of this kind after characterizing the wavelength by the effect of gravity on the neutron (cfr. eq. (2)).

Neutron optics has found a number of practical applications. For instance, in a research reactor the source is small (ideally, pointform) and the number of experimental instruments which can be set up at a given distance is physically limited. The transport of neutron beams over relatively long distances, but with only a small loss of intensity, might be accomplished by making use of multiple internal reflections inside hollow tubes in a manner analogous to the use of light pipes. Put forward by Christ and Springer 16 and Maier-Leibnitz and Springer 17, guide tubes have been first developed and extensively applied at the high flux reactor of the Institute Laue-Langevin, and later at other reactors. Highly polished, nickel plated glass tubes of rectangular cross section allow a number of experiments to be accompodated at distances approaching 100 meters from the reactor core, in an environment of considerably reduced background. Further reduction of the background was obtained by giving the guide tube a gentle curvature (up to 2700 m radius) thereby eliminating from the beam fast neutrons and  $\gamma$ -rays.

In a neutron guide many reflections take place, hence the efficiency for the single reflection must be very close to one. If the guide wall is made of a homogeneous material the only reflection occurs at the vacuum/material interface, and the critical angle is entirely defined by the material. The best obtainable material, using conventional chemistry, is metallic nickel; but even in this case the critical angle is 0.1 degrees for a neutron wavelength of one Angstrom (and grows linearly with the wavelength). The only improvement that still can be obtained along this road is guite expensive, and consists in substituting natural nickel with one of its isotopes, Ni<sup>58</sup>, by which means a 20% increase is obtained for the critical angle.

It is still possible to go beyond this limit, but esploiting the optical properties of material composed of a suitable superposition of different layers. When these reflect a broad band of neutron wavelengths, they are called "supermirrors", and actually at this conference are presented the newest developments on their preparation, performance and utilization. These optical elements, which stretch the angular range of almost total reflection by a factor of three 18, are not used yet in extended guides but in short sections which are made integral part of various neutron scattering instruments. The most developed of these devices are the "polarizing supermirrors" which strongly reflect neutrons of one spin state only. They have been developed by Mezei 18 and Schaerpf 19 to polarize efficiently broad bands of cold neutrons. In a companion development, Majkrzak developed superlattices to polarize (but over a narrower range of wavelengths) neutrons of the "warmer" variety.

Aside from such applications, Fresnel reflection of neutrons has been used as a method to obtain in a systematic way the magnetic and composition depth profiles of thin laminar films. To enhance the contrast of different materials, extensive use has been made of

isotope substitution, in particular of hydrogen with deuterium in organic materials. After the first initial experiments 21,22 the interest of the scientific community (and in particular, of the polymer physicists and of the organic chemists) has grown almost explosively. This growth will be reflected in the literature in the near future 3. Examples of the subjects being studied are the concentration profile of a polymer in solution, close to the free surface 4 and the process of interdiffusion of two polymers 5: in both cases, a depth resolution approaching one nanometer has been achieved. Less abundant but equally interesting results have been obtained in the field of magnetism. The depth profile of a magnetic field into superconducting films has been probed 5 as well as in ferromagnetic films. For these it has been predicted 7 and demonstrated 8 that the technique has a sensitivity sufficient to detect a magnetic layer, of the thickness of a single atomic plane.

The first instrument dedicated to neutron reflection ("POSY") has been built at the Intense Pulsed Neutron Source at Argonne National Laboratory<sup>29</sup>. This was followed by a reflectometer ("CRISP") at the powerful pulsed neutron source ISIS at Rutherford Appleton Laboratory<sup>30</sup>. Every major neutron source (pulsed or continuous) has now an instrument of this type either commissioned or planned.

#### 6. Appendix: Non-uniaxial magnetization

We have seen in section 3 that, if the magnetization is parallel to the quantization axis of the neutron, the potential is proportional to:

$$\mathbf{v}^{\pm}(z) := 4\pi[\mathbf{b}(z)\mathbf{N}(z)\pm\mathbf{c}\mathbf{B}(z)] \tag{A1}$$

where the + and - sign correspond to a neutron polarization respectively parallel and antiparallel to the magnetic field. The equations for the two spin states being totally separated, their individual solution is obtained as for the non-magnetic case. In a few simple cases it is possible to find convenient relations between the two solutions. For instance if the reflecting materials are chemically and magnetically homogeneous, the two reflectivities have the identical form, and can be superimposed by rescaling the wavelengths:

$$\lambda_{+} \sqrt{bN+cB} = \lambda_{-} \sqrt{bN-cB}$$
 (A2)

this relations has been actually used to check the uniformity of magnetization in thin films of ferrite  $^{31}$ .

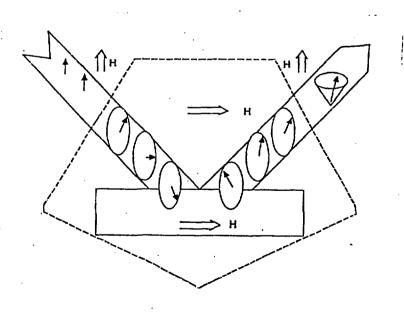


Figure 4 Precession of the neutron spin in a magnetic field perpendicular to its quantization axis. The neutron at the left is polarized parallel to the field and then encounters a magnetic 90° boundary (dotted line). This might lie in the sample, and be parallel to its surface.

When the magnetization is not parallel to the quantization axis of the neutrons the spinor equations cannot be solved separately  $^{29}$ ,  $^{32}$ . However, this separation is possible if the field is suddenly tilt away by 90° respect to the quantization axis, or, speaking in classical terms, the field is perpendicular to the neutron's magnetic moment. This is because the spinor equations are reduced to 29:

$$f_{+}^{*} + [k_{O}^{2} - 4\pi bN] f_{+}^{-} + 4\pi cB_{\perp} f_{-}^{-} = 0$$

$$f_{-}^{*} + [k_{O}^{2} - 4\pi bN] f_{-}^{-} + 4\pi cB_{\perp} f_{+}^{-} = 0$$
(A3)

The presence of both  $f_+$ ,  $f_-$  in each of the two equations (A3) shows that the spin state gradually changes as a function of z, or that "spin flips" are possible. It is easy to see that the variables can be separated by taking the linear combinations  $f_{\Sigma} = f^{+}+f^{-}$  and  $f_{\Delta} = f^{+}+f^{-}$ f<sup>+</sup>-f<sup>-</sup>. Their general solutions are:

$$f_{\Sigma} = A_{1} \exp(ik_{\Sigma}z) + A_{2} \exp(-ik_{\Sigma}z)$$

$$f_{\Lambda} = A_{3} \exp(ik_{\Lambda}z) + A_{4} \exp(-ik_{\Lambda}z)$$
(A4)

where

$$k_{\Sigma} = \sqrt{k_{O}^{2} - 4\pi(bN+B)}$$
  $k_{\Delta} = \sqrt{k_{O}^{2} - 4\pi(bN-B)}$  (A5)

and the coefficients  $A_i$  are determined by imposing the conditions of continuity for f, f' at the boundaries of the region of constant potential. We are interested in particular on the surface where the magnetic field tilts 90° away from the quantization axis. As seen in fig. 4, such boundaries might have a complex geometry (especially if the field is perpendicular to the sample's surface). If the magnetic boundary surface is not parallel to the surface of the material the Schroedinger equations have to be written in two dimensions, and as a result the neutron state is modified not only as a function of the depth from the surface but also of the length of the  $90^{\circ}$  region. If the magnetic boundary is parallel to the surface, the reflection of an initially polarized beam (parallel to the magnetic field) is described by:

$$r_{\pm\pm} = \frac{1}{2} \left( r_{\Sigma} \pm r_{\Delta} \right) \tag{A6}$$

where  $r_{++}$  indicates the reflectance for those neutrons, for which the final state of polarization is equal to the initial state, while  $r_{+-}$  is pertinent to the "flipped" neutrons. The reflectances  $r_{\Sigma}$ ,  $r_{\Lambda}$  are those relative to the momenta  $k_{\Sigma}$ ,  $K_{\Lambda}$  defined by (A5). As it can be seen, the unflipped neutrons give just an average reflectance, while for the flipped neutrons the reflectance, difference of those for the two wavefunctions, provides direct evidence of the presence of transversal fields. The separation the reflectivities with different final spin states (by polarization analysis) is entirely feasible but has not been yet systematically used. By this method it might be possible to obtain full maps of the magnetic fields in the sample, in direction and as well as in size, as a function of the depth from the surface.

#### Acknowledgments

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#### References

- 1. S.A. Werner and A.G. Klein, Neutron Scattering, Eds. D. Price and K. Skold, (Academic Press, New York 1984), Chapter IV
- 2. J. Als-Nielsen, in: " Structure and Dynamics of Surfaces II", W. Schommers and P. von Blanckenhahen Eds., In the serie "Topics in Current Physics", Springer Verlag 1987.
- M. Born and E. Wolf, Principles of optics, (Pergamon Press, Oxford, 1975)
   G.E. Bacon in "international Tables for X-Rays Crystallography", J.A. Ibers and W.C.
- Hamilton eds., vol. IV, Kynoch Press, Birmingham 1974
  5. G.L. Squires, Introduction to the Theory of Thermal Neutron Scattering, Cambridge University Press, Cambridge (1978)
- 6. L Schiff, Quantum Mechanics, McGraw-Hill, New York 1949

- O.S. Heavens, Optical Properties of Thin Solid Films, Dover, New York, 1965
   J. Jacobsson, in Progress in Optics, edited by E. Wolf (North Holland, Amsterdam 1966)
   K. Chadan and P.C. Sabatier, Inverse Problems in Quantum Scattering Theory, Springer Verlag, New York 1977
- 10. I. Kay, Comm. on Pure and Appl. Math. 13,371 (1960)
- 11. E. Hruslov, Math. USSR Sbornik 28,229 (1976)

- 12. S.N.M. Ruijsenaars and P.J.M. Bongaarts, Ann. Inst. Henri Poincare 26, 1 (1977) 13. H. Brunner and P.J. van der Houwen, The numerical solution of Volterra equations, North Holland, Amsterdam 1986 14. E. Fermi and W.H. Zinn, Phys. Rev. 70, 103 (1946) 15. L. Koester, in Neutron Physics. Springer Tracts in modern Physics, vol. 80, Springer Verlag (1977) 16. J. Christ and T. Springer, Nucleonik 4, 23(1962) 17. H. Maier-Leibnitz and T. Springer, J. Nucl. Energy A/3 17,217 (1963) 18. P.A. Degleish, J.B. Hayter and F. Mezei, in Neutron Spin Echo, Kecture Notes in Physics n. 128, edited by F. Mezei. Springer Verlag, Berlin 1980. 19. O. Schaerpf, Proc. Int. Conf. Neutron Scattering in the 90s, Juelich (1985) IAEA-Vienna. C.F. Majkrzak, Physica 136B,69 (1986)
   J.B. Hayter, R.R. Highfield, B.J. Pul J.B. Hayter, R.R. Highfield, B.J. Pullman, R.K. Thomas, A.I. McMullen and J. Penfold, J. Chem. Soc. Faraday Trans. 77,1437 (1981) R.M. Niklow, M. Pomerantz and A. Segmuller, Phys. Rev. B23, 1081 (1981). ISIS 1988 Annual Report. Rutherford-Appleton Laboratory Report RAL-88-050 23. (unpublished). 24. Y. Sun, E. Bouchaud, A. Lapp, B. Farnoux, M. Daoud and G. Jannink, Europhys. Lett. 6, 207 (1988). T. P. Russell, A. Karim, A. Mansour and G.P. Felcher, Macromolecules 18, 501 (1988) 25. G.P. Felcher, R.T. Kampwirth, K.E. Gray and R. Felici, Phys. Rev. Lett. 52, 1539 26. (1984)27. G. P. Felcher, K.E. Gray, R.T. Kampwirth and M.B. Brodsky, Physica 136B,59 (1986). 28. J.A.C. Bland, D. Pescia and R.F. Willis, Phys. Rev. Lett. 58, 1244, (1987). 29. G.P. Felcher, R.O. Hilleke, R.K. Crawford, J. Haumann, R. Kleb and G. Ostrowski, Rev. Sci. Intrum. 58, 609 (1987) 30. J. Penfold, R.C. Ward and W.G. Williams, J. Phys. E 20, 1411 (1987).
  31. S.S.P. Parkin, R. Sigsbee, R. Felici and G.P. Felcher, Appl. Phys. Lett. 48, 604
- (1986). 32. L.D. Landau and E.M. Lifshitz, Quantum Mechanics, Vol. 3 in: Course of Theoretical Physics, Pergamon Press 1958.

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	TUESDAY 16 AUGUST 1988	Monte Carlo simulation of convergin Copley, Univ. of Maryland and Natio	11. 11. 11. 11. 11. 12. 12. 12. 12. 12.
9	SESSION 1	Standards	
	Supermirrors	Angerson, Paul Scheifer Histitat (	
	Chair: F. Mezei, Hahn-Meitner-Institut (FRC)	Dixieland Band and Beer Fest,	6:00 to 7:30 pm Spe
	Principles of neutron reflection, G. P. Felcher, Argonne National Lab.		be
	Very high reflectivity supermirrors and their applications, F. Mezei, Hahn-Meitner-Institut (FRG)		A CONTRACTOR OF THE CONTRACTOR
	Pynn, Los Alamos National Lab.	SESSION 3Other Devices and	<b>企业,企业的企业,企业企业,企业企业</b>
	Coffee Break	Saving Office to Donner Dunn Los Alam	os manones entre ativos signaturas de la tratitua
	Design of high reflectivity supermirror structures, J. Hayter, Oakridge National Lab.	Use of mirrors and multilayers in co	Id and ultracoid frequency [983-14] Coffee
	Characterization of imperfections in multilayers for fabricating multiple d-spacing devices, A. M. Saxena, Brookhaven National Lab. [983-05]	Liltracold neutron facility of a supe	mirror turbine and a street it synchi
	Neutron reflectivity measurements on transition metal-based amorphous multilayers, J. E. Keem, Ovonic Synthetic Materials 374. Co., Inc.; B. Yelon, Univ. of Missouri; J. Wood, K. Hart, Ovonic 1983-061	Ebisawa, Kyoto Univ. (Japan)	particular, for small-angle Mon
	Synthetic Materials Co., Inc	neutron scattering, B. Alefeld, H. J	10:00 to 10:30 am 1.1 Chau
•	Keem, Ovonic Synthetic Materials Co., Inc.; S. Nutt, Brown Univ.; J. Wood, K. Hart, Ovonic Synthetic Materials Co., Inc. [983-07]	Coffee Break	ultilayers at the National
	Lunch Break Noon to 2:00 pm	WIND WIND CONTRACTOR OF THE PARTY OF THE PAR	Date of double Date
	SESSION 2 Tues. 2:00 pm		is the Linky (lanan); No Control of
`*		KVUSTU Olliv. (Japani) "	, [303-10] Kill

### Multilayer Monochromators for Xand Neutrons

Chair: Franz Schaefers, BESSY Gmb! (F

Note: This session is the same as Session 1 of C X-Ray Multilayers : Diffractometers, Monoch Spectrometers. Papers 983-20 through 983-25 984-01 through 984-06 in Conference 984. Th be published in both conference Proceeding vited Paper: Common aspects and basic differe evices for x-ray and neutron scattering, A. K. Fr Inchrotron Radiation Facility (France)......

vited Paper: Overview of the use or soft x-ray onochromators for synchrotrons, J. Feldhaus, F ıstitut (FRG) ...

offee Break . . . . . .

erformance of soft x-ray multilayer monochror nchrotron radiation, F. Schaefers, BESSY Gmb rioni, J. B. Goedkoop, J. C. Fuggle, Univ. of Ni letherlands); D. Duke, J. L. Wood, Ovonic Syr

lonochromator based on W-C multilayers of pacing, A. Smith, C. Riedel, B. Edwards, B. Lai, haudhuri, M. Lagally, F. Cerrina, Univ. of Wis H. Underwood, Lawrence Berkeley Labs. . . .

neory of multilayer neutron monochromator tomic Energy of Canada, Ltd. (Canada) ....

eutron reflectivity of NiSi multilayers, B. L. E abbagh, Univ. of Reading (UK)

Sangria Reception, Atlas Baliroom Foyer .... Anniversary Awards Banquet,