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A ONE-BODY TRANSPORT MODEL OF FLUCTUATION PROCESSES IN NUCLEAR COLLISIONS

S. Ayik*
Tennessee Technological University, Cookeville, TN 38505, USA

and

C. Gregoire, E. Suraud, J. Stryjewski, M. Belkacem
GANIL, B.P. 5027, F14021, Caen Cedes, France

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1. INTRODUCTION

Many aspects of a many-body system can be described in terms of one-body transport models in which the system at any time is characterized by its single-particle density rather than by the full many-body information. In these one-body models, evaluation of the single-particle density is determined by a transport equation which contains the self-consistent mean-field potential and a collision term due to binary two-body collisions. Recently, this approach in a semi-classical limit with a Boltzmann-Uehling-Uhlenbeck (BUU) form of a collision term has been applied to nuclear collisions at intermediate energies [1]. Common to all one-body models, only the average effects of two-body collisions are retained in the equation of motion and higher order correlations are entirely neglected. This approximation corresponds to an ensemble averaging which is evident, for example, from the "molecular chaos assumption" introduced in derivation of Boltzmann equation. As a result, these one-body models determine the ensemble averaged single-particle density and cannot provide a description for the fluctuation processes in nuclear collisions. On the other hand, at low and intermediate energies dynamical fluctuations are substantial due to large available phase space for decay into many final states. Therefore, it is of great interest to improve one-body transport models by incorporating dynamical fluctuations due to high order correlations into the equation of motion.

2. STOCHASTIC BUU EQUATION

Recently, we proposed an extension of one-body transport theory by incorporating fluctuations into the equation of motion in a statistical approximation [2]. In a dilute system, dynamics is mainly determined by two-body collisions, which (i) produce dissipation by randomizing the single-particle momentum distribution and (ii) induce fluctuation by propagating correlations in phase-space. These two effects can be incorporated into the equation of motion for the single-particle density. This yields in semi-classical limit a stochastic BUU equation, or Langevin-Boltzmann equation, for the fluctuating single-particle density,

$$\left[\frac{\partial}{\partial t} + \frac{p}{m} \cdot \nabla_r - \nabla_r U(f) \cdot \nabla_p\right] f(r,p,t) = K(f) + \delta K(r,p,t). \tag{2.1}$$

Here, K(f) has the form of the usual collision term in terms of fluctuating density,

$$K(f_1) = \frac{g}{(2\pi\hbar)^3} \int dp_2 dp_3 dp_4 W(12;34)[f_3 f_4 \bar{f}_1 \bar{f}_2 - \bar{f}_3 \bar{f}_4 f_1 f_2] \tag{2.2}$$

where g is the degeneracy factor, $f_j = f(r_j,p_j,t)$, $\bar{f}_j = 1 - f_j$ and the spin-isospin averaged

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transition rates are given in terms of N-N cross-section by

$$W(12;34) = \frac{1}{2m^2}\frac{d\sigma}{d\Omega}\delta(p_1 + p_2 - p_3 - p_4)\delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4). \tag{2.3}$$

The additional term δK in eq. (2.1) arises from correlations not accounted by the collision term K(f), and it is referred to as a fluctuating collision term. The fluctuating collision term varies rapidly in time with a characteristic time in the order of duration time of a two-body collision, and it is nearly impossible to calculate it explicitly. Therefore, it is assumed that eq. (2.1) describes a stochastic process in which the entire single-particle density is a stochastic variable and δK acts like a random force. The fluctuating collision term is characterized by a correlation function,

$$\overline{\delta K(r,p,t)\delta K(r,p',t')} = C(p,p')\ \delta(r - r')\delta(t - t') \tag{2.4}$$

which is local in spatial coordinates due to localized two-body collisions. In a weak-coupling limit, the correlation function C(p,p′) is explicitly evaluated and given by

$$\begin{aligned} C(p,p') = \quad & \int dp_3 dp_4 W(11';34)[f_1 f_1' \bar{f}_3 \bar{f}_4 + \bar{f}_1 \bar{f}_1' f_3 f_4] \\ & -2\int dp_2 dp_4 W(12;1'4)[f_1 f_2 \bar{f}_1' \bar{f}_4 + \bar{f}_1 \bar{f}_2 f_1' f_4] \\ & +\delta(p - p')\int dp_2 dp_3 dp_4 W(12';34)[f_1 f_2 \bar{f}_3 \bar{f}_4 + \bar{f}_1 \bar{f}_2 f_3 f_4] \end{aligned} \tag{2.5}$$

where W is the same transition rate which enters into the collision term K(f).

The correlation function C(p,p′) is entirely determined by the average properties of the single-particle density. The parameters such as the mean-field potential and N–N cross-section determining the average properties also describe the fluctuations in the framework of the stochastic BUU model. This result can be regarded as a consequence of a "fluctuation-dissipation theorem" which relates the fluctuation and dissipation properties locally in phase-space. The stochastic BUU equation describes the dynamical evolution as a diffusion process for the trajectories of single-particle density in an abstract space of all single-particle densities. This description is equivalent to a generalized FP equation in infinite dimensions for the probability distribution function of the single-particle density. In some situations, instead of the full probability distribution of the single- particle density, we may consider its first moment and second moment, i.e., variance and co-variances of density. For small fluctuations, the equation for the first moment is just the BUU equation describing the mean trajectory, and the equation for the second moment can easily be deduced from the stochastic BUU eq. (2.1) [2]. A similar equation for the second moment of density is derived using a somewhat different approach in reference [3].

3. APPLICATION TO NUCLEAR COLLISIONS

The stochastic BUU model summarized in the previous section provides an extended one-body transport description of many-body dynamics by incorporating dynamical fluctuations in a theoretically sound basis. It opens up a possibility for a dynamical description of multifragmentation processes in nuclear collisions at intermediate energies. By employing standard methods for solving a typical Langevin equation, we can obtain the numerical solutions of eq. (2.1) iteratively over short time intervals [4]. Starting with a definite density f(t) at time t, eq. (2.1) generates a set of densities $\{f(t+\Delta t)\}$ at time $t+\Delta t$. For the next step, we choose one of such possible states as the initial state, and eq. (2.1) generates a new set of states at the next time step, and so on. At each step the generated states are randomly distributed around the initial state and their spread is determined by the correlation function $C(p,p')$. In order to make this simulation numerically tractable, we project the fluctuations on a collective property $Q(p)$ and determine the spread of trajectories in terms of the collective variable, $Q=\int dpQ(p)f(p,r,t)$. The reduced correlation function corresponding to the collective variable is given by

$$C_Q(t) = \int dp_1dp_2dp_3dp_4(\Delta Q)^2\, W(12;34)f_1f_2\tilde{f}_3\tilde{f}_4 \tag{3.1}$$

where $\Delta Q=Q(p_1)+Q(p_2)-Q(p_3)-Q(p_4)$. C_Q is proportional to the diffusion coefficient for the collective variable and it determines the rate of change of fluctuations in Q. Now, we can easily solve the Langevin process associated with Q by modifying it at each time step according to

$$Q \rightarrow Q + w\sqrt{\int_t^{t+\Delta t} dt'C_Q(t')} \tag{3.2}$$

where w is a normally distributed random variable. Once the fluctuations are inserted in Q, we can generate an event by renormalizing the momentum distribution to the new value of Q at each time step.

We performed numerical calculations based on the scheme presented here, in which the collective variable is chosen at the z-component of the quadrupole moment of the local momentum distribution along the beam axis, $Q(p)=2p_z^2-p_x^2-p_y^2$. Each event of eq. (2.1) is simulated with the help of the so-called Landau-Vlasov algorithm [5]. Fig. 1 shows the time evolution of the diffusion term together with the time evolution of the mean value of Q in a head-on collision of $^{12}C+^{12}C$ system at various energies. From Fig. 1, we can see that the fluctuations are large and peaked in time. There is a well–defined narrow peak just after touching. The peak in the fluctuations is an order of magnitude larger than the background which consists of numerical and thermal fluctuations. Consequently, large dynamical fluctuations are introduced during the early stages of the collision. As a first application, we study $^{40}Ca+^{40}Ca$ collision at bombarding energies E=20 and 60 MeV/nucleon. The collision at 20 MeV/nucleon is a typical low energy, incomplete fusion reaction. Fig. 2 shows the mass spectra obtained in both the BUU and the stochastic BUU approaches. Both calculations lead to a similar result, namely an incomplete fusion residue of mass $A\simeq45$ together with a large number of nucleons and a few very small fragments, $A\simeq2$–3. This result is very interesting for two reasons: (i) at low energies, the fluctuating theory does indeed lead to a fusion residue and not to an uncontrolled break-up of the nuclear system, (ii) as expected at low energies, the fluctuating theory gives essentially the same result as the average result of the BUU description. At higher energies, E/nucleon $\geq$ 50 MeV, multifragmentation is expected to occur. As can be seen from Fig. 3, the stochastic BUU calculations lead to a very reasonable mass spectrum of the produced fragments. The mass spectrum is an inclusive quantity which hardly discriminates between available theoretical descriptions. However, it provides a check on the reliability of our calculations. Investigation of more specific and sensitive observables is currently being performed.

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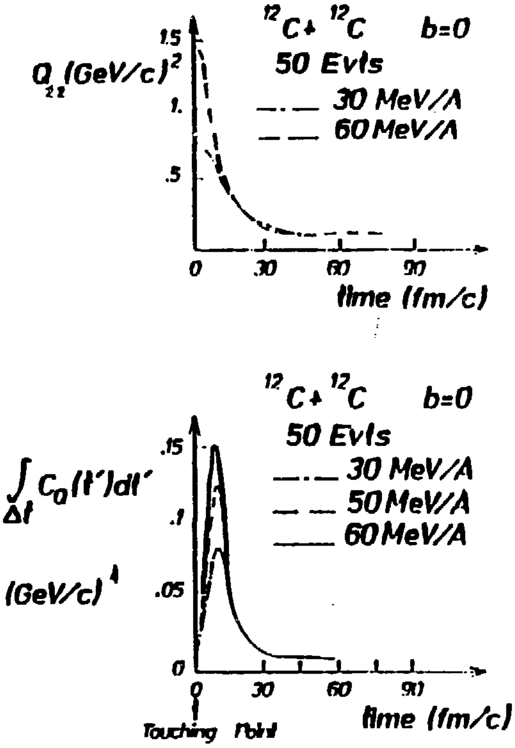

Figure 1

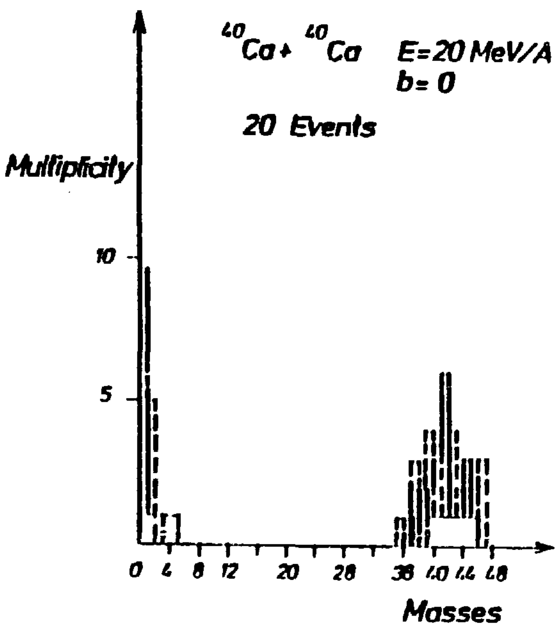

Figure 2

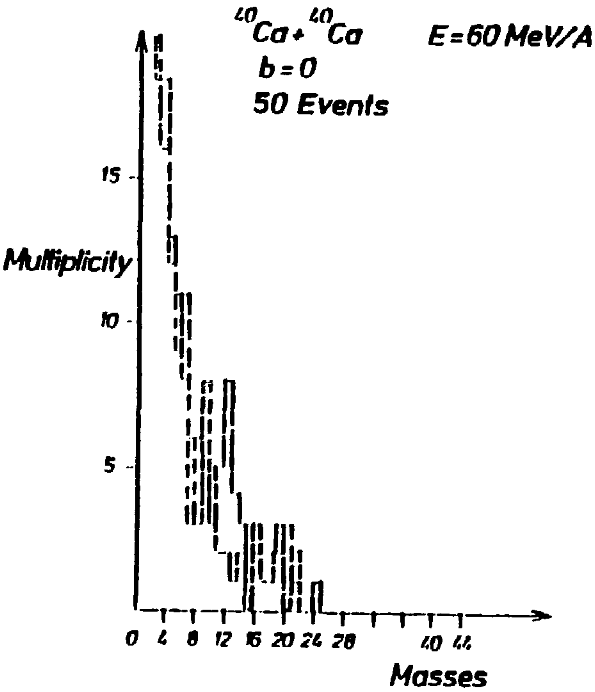

Figure 3

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