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A THEORY FOR MIXED-LAYER-TOP LEVELNESS OVER IRREGULAR TOPOGRAPHY

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1. OBSERVATIONS

Observations show that the top of the mixed layer (ML) does not always follow the topography. Fiedler (1990) found the top of the afternoon mixed layer to be relatively level across the Rhine valley of Germany (Fig 1c). Earlier in the morning, however, the mixed-layer top (MLT) apparently followed the topography (Fig 1b). Lenschow et al (1979) observed a morning situation as sketched in Fig 1a, that was later modified by advection into a structure more like those sketched in Figs 1b & c.

As the altitude of the MLT changes relative to the topography, so does the local thickness of the ML. The local thickness affects pollutant dispersion, and controls the scaling of most turbulence variables within the ML. Levelness of the MLT might also affect appropriate flight plans for research aircraft.

2. LEVELNESS DEFINITION

Define a dimensionless "levelness number" by:

$$L = \Delta z_i / \Delta z_T \qquad (1)$$

where Δz_i is the altitude difference (relative to sea level) of the MLT, and Δz_T is the altitude difference of the topography. The local altitude of the top of the mixed layer (z_i) varies with horizontal distance, x, while h represents the local thickness of the ML above the local topography.

Factors that affect the levelness are listed in Table 1, where t is time, t_* is the convective time scale = h/w_*, w_* is the Deardorff convective velocity scale, C_{*D} is the convective drag coefficient (Stull, 1992a), U_A is the background mean horizontal wind speed,

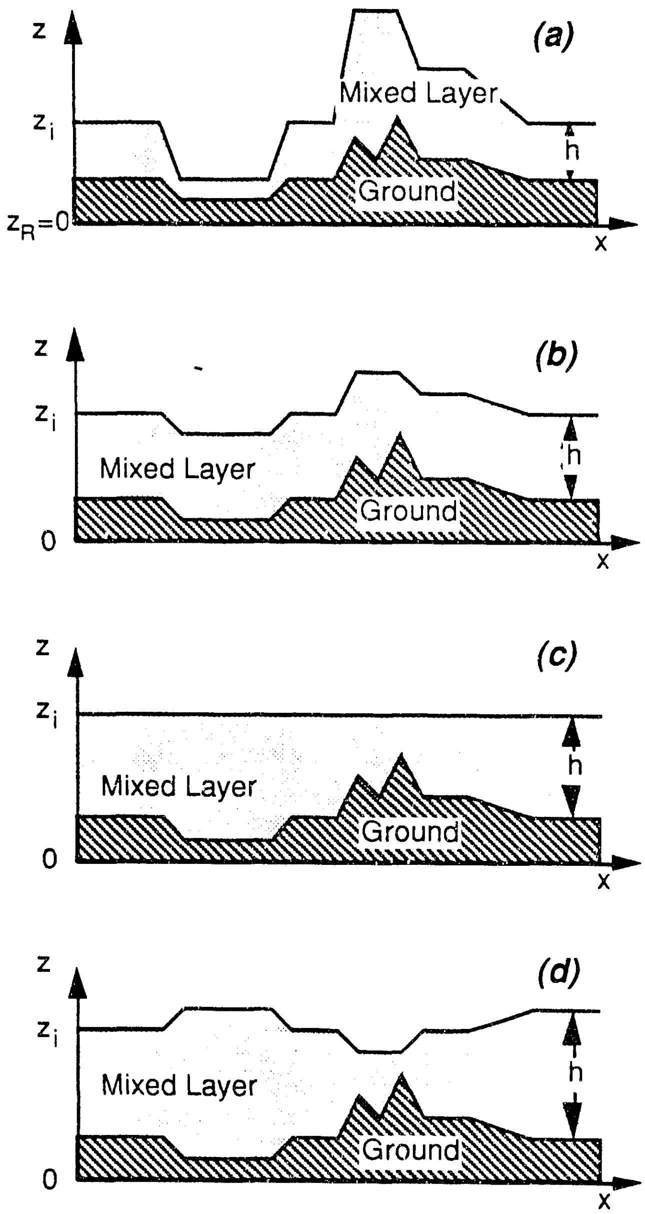

Fig 1. Four archetypical levelness situations:
(a) $L > 1$, for a MLT that amplifies topo. variations
(b) $L = 1$, for a MLT that follows the topography,
(c) $L = 0$, for a level MLT, and
(d) $L < 0$, for a MLT that varies opposite to topography.

Table 1. Dimensionless variables affecting levelness

Symbol	Definition	Name	Typical Range	Description
L	$\Delta z_i / \Delta z_T$	levelness	–0.5 to 1.5	relative amount of terrain following
T	$t / \overline{t_*}$	time	0 to 100	number of convective turnovers
R	$\Delta z_T / \overline{h}$	flatness	–1 to 1	relative depth of topography
C	C_{*D}	drag coef.	0.023	mixed-layer drag
π_1	$\lvert U_A \rvert \cdot \overline{t_*} / s$	advection	0 to 200	advective vs convective info propagation rate
π_2	$b_s \cdot \overline{t_*}$	divergence	0 to 0.05	synoptic and mesoscale relative spreading
π_3	$\overline{w_e} / \overline{w_*}$	entrainment	0 to 0.2	ML growth rate relative to convective vigor
π_4	$\overline{h} / s$	scale ratio	1 to 100	vertical ML depth to horizontal topo. dimen.
π_5	$\Delta q / \overline{\theta_*}$	cap strength	1 to 500	strength of capping stable layer

s is the horizontal scale of the topographic feature, b_s is the imposed synoptic and mesoscale divergence, w_e is the entrainment rate at the top of the mixed layer, Δq is the temperature difference across the ML top, and the overbar represents a horizontal average over the varied topography.

3. TENDENCY

When the mass conservation equation is applied to a ML over irregular topography, the levelness tendency is found to be a function of terrain-following terms, and leveling terms. Terrain following factors include entrainment at the top of the ML, advection, friction, and large-scale divergence. These oppose buoyant forces related to the cool mixed layer trapped under a warmer capping inversion, which tend to make the MLT more level.

Using the dimensionless groups from Table 1, Stull (1992b) derives the following equation for levelness tendency:

$$\frac{dL}{dT} = A\cdot(1-L) - \frac{B\cdot L}{C + (R\cdot B\cdot L)^{1/2}} \qquad (2)$$

where $A = \pi_1 + \pi_2 + \pi_3$, and $B = 4\cdot\pi_4^{\,2}\cdot\pi_5$.

Initial experiments with this equation indicate that the tendency is often a small difference between large forcing terms. This makes it very sensitive to errors. Numerical predictions made with this equation should either take very small time steps, or use a highly stable differencing scheme.

4. EQUILIBRIUM SOLUTIONS

By setting the time derivative in (2) to zero, one can solve for the equilibrium levelness as a function of the various forcings. Again, because of the sensitivity of the equation, care must be taken in its solution. We chose an iterative solution which is always stable and converges very rapidly, allowing easy computation and graphing in spreadsheet programs on desktop computers:

$$L = \left[1 + \frac{B/A}{C + (R\cdot B\cdot L)^{1/2}} \right]^{-1} \qquad (3)$$

The iterative equation (3) is a function of only four relevant variables: L, A, B, and R; plus the constant C.

As advection, divergence, and/or entrainment increase, these forcings tend to make the MLT follow the topography; that is, L Æ 1 as A ≠. This is shown in Fig 2, for constant buoyancy $B = 1$. However, as the height of the terrain feature decreases relative to the depth of the mixed layer, then the forcings are less effective at reducing levelness; that is, L Æ 0 as R Æ 0 .

As the strength of the capping inversion increases, or as the mixed-layer depth increases, then the buoyant leveling force is able to drive the excess air off of high terrain fast enough to maintain a nearly level MLT; that is, L Æ 0 as B ≠. This is shown in Fig 3, for constant $A = 1$. Both figures show that the MLT is level when the topography is flat; that is: $L = 0$ when $R = 0$.

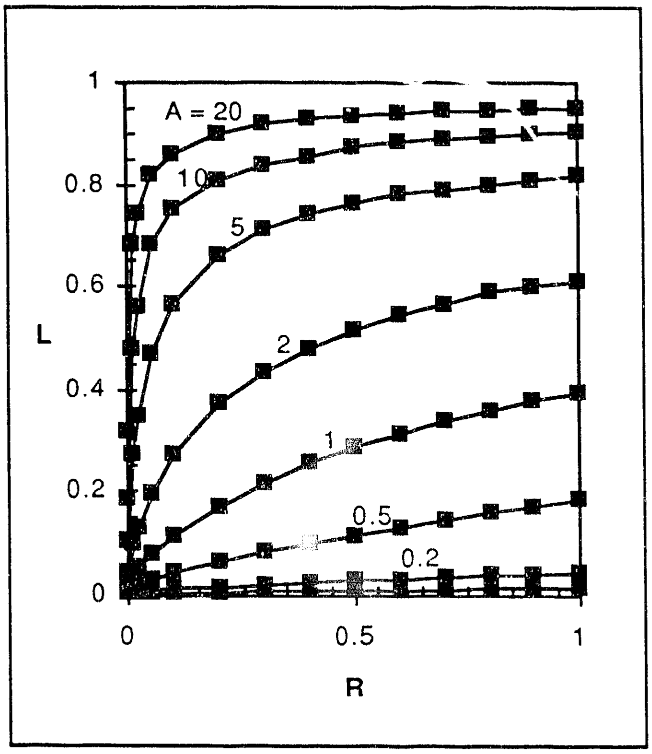

Fig 2. Variation of equilibrium levelness L as a function of topographic flatness R for a variety of forcing terms A that tend to reduce levelness. The whole graph is for B = 1, corresponding to a fixed amount of buoyancy leveling force. R = 0 is level topography, and L = 0 is level mixed-layer top.

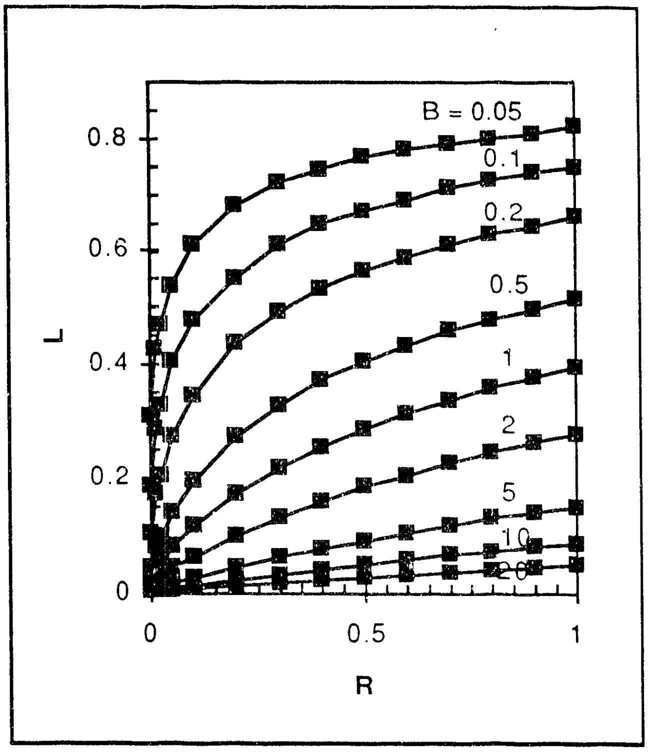

Fig 3. Variation of equilibrium levelness L as a function of topographic flatness R for a variety of intensities of buoyant forcing B that tends to maintain levelness. The whole graph is for A = 1, corresponding to a fixed amount of forcing tending to disturb the mixed-layer top away from being level.

5. CONCLUSION

Deeper mixed layers tend to be leveler than shallow ones. Thus, mixed layer tops tend to become more level during the day, because the depth usually increases during the day. The mixed-layer top is less level over topographic features that have larger horizontal extent.

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