CONF-8803129
BNL-41650

BROOKHAVEN NATIONAL LABORATORY

Associated Universities Inc.

Upton, New York 11973

BNL--41650

DE88 016766

INJECTION DYNAMICS*

A. U. Luccio, F. Z. Khiari

Lecture at the 4^{th} Workshop: *Pulse Power Techniques for Future Accelerators*, INFN Eloisatron Project. Erice–Trapany–Sicily: 4–9 March 1988.

*** Work performed under the auspices of the United States Department of Energy.**

INJECTION DYNAMICS

A. U. Luccio, F. Z. Khiari

Brookhaven National Laboratory

Accelerator Development Department

Upton, New York 11973, USA

1. TUTORIAL

To understand the injection process into a collider like Eloisa, we need some of the basic definitions of quantities for these machines. We will review them briefly here[1].

LUMINOSITY

When two equally and uniformly populated beams of particles (say protons) collide and a given process occurs, the rate R of occurrence of the process is given by

$$R = f\frac{N_b^2 \sigma_R}{A}, \tag{1}$$

where A is the geometrical transverse area of either beam (supposed the same); N_b is the number of protons per beam; f is the frequency of collisions and σ_R is the process cross section. The luminosity of the collider is defined as the process rate per unit cross section

$$\mathcal{L} = f\frac{N_b^2}{A}[cm^{-2}s^{-1}]. \tag{2}$$

Assuming that the beam has a transverse gaussian density of width 2σ expressed by

$$dN(x,y) = \frac{N}{2\pi\sigma^2}e^{[-\frac{x^2+y^2}{2\sigma^2}]}dxdy \tag{3}$$

and, since the frequency of collisions is given by

$$f = \frac{c}{S_b}, \tag{4}$$

where c is the speed of light (of the particles) and S_b is the spacing between bunches in the collider, an alternative expression for the luminosity is

$$\mathcal{L} = \frac{N_b^2}{4\pi\sigma^2}\frac{c}{S_b}. \tag{5}$$

The luminosity is determined by the detailed structure of the collider lattice. For Eloisa, a design value of the luminosity is 0.91 10^{33} $cm^{-2}s^{-1}$ with 2.7 10^9 protons per bunch.

TOTAL NUMBER OF PARTICLES

The circumference of the collider is given by

$$C = 2\pi\overline{R}, \tag{6}$$

where $\overline{R}$ is the average radius of the ring. The total number of particles is

$$N_T = N_b\frac{C}{S_b}, \tag{7}$$

with N_b number of particles per bunch and C/S_b number of bunches. The current in the beam is

$$I_b = \frac{ecN_T}{C}. \tag{8}$$

In Eloisa, the number of bunches is 39600 with a circumference of 210 Km. The beam current is 24 mA.

BETATRON AMPLITUDE FUNCTION

The particles perform (betatron) oscillations in the horizontal and vertical planes and the amplitude $\mathcal{A}$ of the beam envelope is proportional to the square root of the betatron amplitude function β

$$\mathcal{A} \propto \sqrt{\beta(s)}. \tag{9}$$

β is measured in meters. Its minimum value at the crossing points, i.e. at the points where collisions between counterstreaming particle beams collide is denoted by β^* which is determined by the detailed optics of the collider. In Eloisa $\beta^* = 1.25\ m$.

NORMALIZED BEAM EMITTANCE

The canonical conjugate quantity to the amplitude $\mathcal{A}$ is the momentum

$$\frac{\mathcal{A}}{\beta}\gamma mc, \tag{10}$$

with γ the normalized particle energy

$$\gamma = \frac{E}{mc^2}. \tag{11}$$

Liouville's theorem tells us that the particles are confined to phase space area which is the product of (10) and (11) times π

$$\pi\frac{\mathcal{A}^2\gamma mc}{\beta}. \tag{12}$$

With the transverse beam space charge density amplitude spread σ defined in Eq. (3), the definition of the normalized transverse emittance of the beam is the phase space area per unit mc

$$\varepsilon = \pi\frac{\sigma^2\gamma}{\beta}. \tag{13}$$

Since this quantity is an invariant (Liouville), it is at the crossings

$$\varepsilon = \pi\frac{\sigma^{*2}\gamma}{\beta^*}. \tag{14}$$

The emittance is measured in *m–rad* or *mm–mrad.* Inserting Eq. (14) into Eq. (5), a further expression for the luminosity is

$$\mathcal{L} = \frac{N_b^2 \gamma c}{4\pi \varepsilon \beta^* S_b}. \tag{15}$$

In Eloisa, the energy of protons per beam is 50 TeV ($\gamma = 5.3\ 10^7$) and the design emittance is $\varepsilon = 0.75\ 10^{-6}$ *m–rad.*

SYNCHROTRON RADIATION POWER

The power radiated, per particle, is

$$p = \frac{Z_o}{6\pi} \frac{e^2 c^2 \gamma^4}{\rho^2}, \tag{16}$$

with $Z_o = 377\ \Omega$ and ρ is the curvature radius of the orbit. The total power radiated by N_T particles is

$$p = \frac{Z_o}{6\pi} \frac{e^2 c^2 \gamma^4}{\rho^2} \frac{2\pi\rho}{C} N_T. \tag{17}$$

It is inversely proportional, for a given energy, to the fourth power of the particle mass ($\gamma = E/mc^2$). Hence, synchrotron radiation is the major factor limiting the energy of an electron ring but is not a problem for even very large proton colliders. The SR power of Eloisa is 385 KW per beam at 50 TeV.

BEAM–BEAM TUNE SHIFT

In head–on collisions, there is a defocusing effect on each beam due to the electromagnetic field of the other beam. This non–linear defocusing produces a tune shift (short range) per crossing

$$\Delta\nu_{SR} = \frac{N_b r_p}{4\pi\varepsilon}, \tag{18}$$

with $r_p = e^2/mc^2$ the classical proton radius = 1.535 10^{-18} cm.

Since the bunches in a beam are close to each other, there is also a long range detuning due to the interaction of a bunch in a beam with many bunches in the other beam expressed by

$$\Delta\nu_{LR} = \Delta\nu_{SR} 2n\left(\frac{\sigma}{\beta^* \alpha}\right)^2, \tag{19}$$

with n the number of long range encounters in the collision region and α the angle of crossing. It is safely assumed that an acceptable tune shift for stability is

$$\Delta\nu < 0.004. \tag{20}$$

In Eloisa, assuming 12 interaction regions, from Eqs. (19) and (20) a SR detuning of 0.0016 and a LR detuning of 3.2 10^{-5} are obtained.

BEAM LIFETIME

Because protons are lost in collisions, the beam lifetime depends on the number of collisions. The beam lifetime is

$$\tau_{pp} = \frac{N_T}{n_L} \frac{1}{\mathcal{L}\sigma_R}, \tag{21}$$

with n_L the number of crossings. From the above , we see that a small emittance of the beam will improve the luminosity of the collider, with upper luminosity limits set by detuning and beam lifetime.

The beam lifetime in Eloisa, due to collisions, assuming an inelastic pp section of 100 $mbarn$ at 50 TeV, is about 24 hours. The transverse emittance damping time, important to study injection, is about 1.2 hours.

AVERAGE NUMBER OF EVENTS PER BUNCH CROSSING

The average number of events in a collider per bunch crossing is proportional to the luminosity and bunch separation

$$< n >= \mathcal{L}\frac{S_b}{c}\sigma_R. \tag{22}$$

The probability distribution of collisions is Poisson's. Hence, the "effective" luminosity for k interactions is written as

$$\mathcal{L}_k = \mathcal{L}\frac{< n >^k}{k!}e^{-<n>}. \tag{23}$$

The complex operation of taking a large number of data from collider experiments calls for an expectation value of the order of one interaction per bunch crossing and, therefore, for a design luminosity

$$\mathcal{L}_1 = \mathcal{L} < n > e^{-<n>}, \tag{24}$$

which depends on bunch spacing. Fig. 1 shows $\mathcal{L}_1/\mathcal{L}$ as a function of $< n >$. A maximum is found for $< n >\approx 1$. From this value in Eq. (22), the optimum value for the bunch spacing is derived.

In Eloisa, at 50 TeV and for σ_R = 100 $mbarn$, we find $S_b \approx 3m$ or 10^{-8} sec. Design considerations have suggested the not too different value of 5.3 m.

Some of the main technical parameters (tentative) for Eloisa are given in Table 1 and a list of Hadron–Hadron colliders, existing or planned, is given in Table 2.

2. INJECTION INTO ELOISA

Eloisa consists of two rings, very close together, intersecting at a certain number of locations, say 12. A proposed scheme of injection is shown in Fig. 2. A first Linac brings the protons from rest to 500 MeV, a chain of Booster Synchrotrons, say 3, brings the protons from 500 MeV to 1 TeV. 1 TeV protons are continuously injected to fill one of the two rings to the required design current. While the protons coast in this ring, the injection line is switched to the other ring which is also filled with 1 TeV protons, in the opposite direction of circulation. The magnetic field in the dipoles of Eloisa is, at this stage, kept at 1,300 gauss.

When the filling of both rings is completed, and both beams have reached their equilibrium (damped) condition, the magnetic field is raised to 6.5 tesla and the energy of the protons reaches the design value of 50 TeV. The 50 TeV protons keep circulating in the collider for several hours before the next injection. The machine is designed to require one injection a day.

A second scheme, which is the main theme of the present study, is to substitute the synchrotron chain with a single synchrotron (500 MeV–1 GeV) followed by a Pulse Power Linac(1 GeV–1 TeV).

PROBLEMS OF INJECTION

The collider requires a train of proton pulses with a given intensity and spacings as discussed in Sect. 1; the beam should have a small emittance to achieve the desired luminosity; instabilities must be damped.

A first concern is how to achieve a high capture rate. The RF system of the collider produces phase space buckets which ought to be filled with protons from the injector as efficiently as possible to improve the capture time and to avoid unwanted radiation from stray particles.

A second problem is control of the emittance in the transverse phase space. Emittance may grow due to many factors; among them the coupling between betatron and synchrotron oscillations, i.e. between longitudinal and transverse phase space.

Third, we want to control the mode of bucket filling to achieve the required bunch spacing and to avoid instabilities.

In the following we will try to address some of these problems, limiting ourselves to an analysis of the longitudinal capture mechanism, for a conventional synchrotron and for a Pulse Power Linac.

RF CAPTURE IN THE LONGITUDINAL PHASE SPACE[2]

A basic RF bucket structure in the longitudinal phase space, referred to the coordinates E, proton energy, and θ, angular, or longitudinal position along the ring, is shown in Fig. 3. Particles inside the shaded areas perform stable synchrotron oscillations around an ideal synchronous particle, particles outside are unstable and soon lost from the beam.

We have studied the capture mechanism by computer simulation, using the code ESME from Fermilab[3], adapted and modified at BNL.

In this code, a number of representative particles (say 40,000) are tracked in phase space and the percentage inside the bucket is evaluated. Particle motion obeys the Courant–Snyder theory. The particles are accelerated by a prescribed external RF voltage $V(t)$ and their energy is controlled by an external dipole magnetic field $B(t)$.

In the case of a conventional Linac–synchrotron injection system, buckets are filled up with a continuous stream of protons, as shown in Fig. 4a, in the case of a pulse power Linac, delivering short proton pulses (tens of picoseconds), buckets are filled as shown in Fig. 4b. In the latter case, we can "paint" the bucket by injecting the protons at a different location at each pulse.

The first (A) series of plots (Fig. 5) shows the gradual filling and capture of particles in the RF bucket from a conventional injector delivering a continuous beam, at a constant proton energy. The RF voltage has been kept constant during injection and then raised. Since the θ–length of the bucket is proportional to the acceleration rate, as in the present case, the bucket occupies the entire available "horizontal" space. The vertical height of the bucket is proportional to the square root of the applied voltage. Therefore, increasing the voltage after injection provides more space to the beam to expand in phase space, while the beam still remains confined to the interior of the bucket.

Through the A–series the intensity of the beam was kept vanishingly low, providing a no–space charge reference case. 10,000 particles were tracked in the simulation. Energy and bucket length are referred to the 1 Tev and $h = 39{,}600$ of Eloisa, but can be scaled to any energy and harmonic number. The injected beam, at each revolution was simulated by a distribution of coordinates uniformly random in phase and gaussian in energy with a spread equal to 1/10 of the bucket half–height.

The A–series shows a small degree of losses of about 1%. With no space charge effects, the protons keep an orderly structure in the bucket "remembering" for while their starting pattern.

In a second series (B) of simulation (Figs. 6a and 6b), we still assumed a conventional injection with a continuous beam, but we assumed some space charge. The effect studied here was due to the interaction of each bunch with its own electromagnetic field. To enhance the effect, the number of protons was taken as $5\ 10^{12}$. This value is much higher than the design value for Eloisa. On the other hand, other effects would tend to blow up the beam, like beam–to–wall coupling, resonances of higher order e.m. modes of the RF cavities, vacuum chamber and so on, and finally beam–to–beam coupling, all unknowns at this time. Therefore, a high space charge impedance may represent a realistic lump impedance. For simulation with space charge, 40,000 representative particles were used in the computation.

Losses are about 3% more than in the A–series. The figures also show the current distribution inside the bunch at each time. We can see the rms length of each bunch is comparable to the separation between adjacent bunches, which is what we wanted.

A third series of simulations (C) (Fig. 7) was performed for the pulse power injection system with small current, or no space charge.

A fourth series (D) (Fig. 8) was done for the same conditions as in (C) but with a space charge of $5\ 10^{12}$ protons as in (B). For the (D) series, the calculated losses were of the order of 2%.

3. THE PULSE POWER LINAC AS AN ALTERNATIVE TO A BOOSTER CHAIN

A Pulse Power Linac can replace two or three Booster synchrotrons since it may be capable to give a very high acceleration ratio to the proton beam. For instance, three synchrotrons can be replaced by a linear structure operating between 2 GeV and 1 TeV, the injection energy to Eloisa. The protons will be brought to 2 GeV by a conventional Linac + Synchrotron chain.

The structure should be optimized, and here a problem is immediately apparent: namely, the 2 GeV proton output of the synchrotron would consist of pulses with a typical length of tens to hundreds of *nano*seconds, while the PP Linac is capable to accelerate pulses tens to hundreds *pico*seconds long. The repetition rate of the synchrotron is of the order of tens of protons per pulse, while the repetition rate of the Linac may be faster, depending on the switches employed.

A solution is to recirculate the 2 GeV proton beam, according to the scheme of Fig. 9, between successive pulses from the synchrotron. The PP Linac will then accelerate at each shot a successive slice of the pulse. By a suitable synchronization of the system, the Linac would sweep along a particle bunch in the recirculator and accelerate to 1 TeV a large fraction of it.

A further advantage of this scheme is more flexibility in the timing of the system. Therefore, the synchrotron could be a relatively slow structure, allowing injection from the low energy Linac at constant energy, with an overall higher capture efficiency.

4. CONCLUSIONS

The present study represents a first approach to the very complex problem of injection into Eloisa. Too many parameters of the collider are still unknown to allow for a truly realistic calculation. However, a main conclusion can be drawn, namely that injection with pulse power technique, when feasible, would not only be far less expensive than with conventional methods, but seemingly still results in a higher capture efficiency (losses are halved).

Beam current shows a more complicated structure with the described bucket painting depending strongly on the initial injection pattern. This suggests that the shape of the final current may be better controlled by injection of pulses rather than with a continuous beam.

REFERENCES

[1] *Conceptual Design of the Superconducting Super Collider*, SSC–SR–2020, March 1986.

[2] E. D. Courant & H. S. Snyder, Ann. Phys. 3, 1 (1958).

[3] J. A. Maclachlan, Fermilab TM–1274, 2041.000 (1984).

Proton energy	$50\ TeV + 50\ TeV$
Circumference	$210\ Km$
Average orbit radius	$33.42\ Km$
Dipole Magnetic field	6.5 Tesla (superconducting)
Radius of curvature	$25.64\ Km$
Period of revolution	$0.7\ msec$
Harmonic number	39600
RF frequency	$56.6\ MHz$
Bucket length	$5.3\ m$
Bucket spacings	$5.3\ m$
No. of crossings	12

Table 1. Technical Parameters for Eloisa

			Luminosity	Status
ISR (CERN)	30 GeV + 30 GeV	pp	10^{31}–10^{32}	completed 1971
		$p\bar{p}$	10^{28}	decomissioned 1984
$SppS$ (CERN)	310 GeV + 310 GeV	$p\bar{p}$	10^{29}	completed 1982
TEVATRON (FNAL)	1 TeV + 1 TeV	$p\bar{p}$	10^{30}	operation 1986
UNK (SERPUKHOV)	400 GeV + 3 TeV	pp	(10^{32})	$\rightarrow$ 1990's
LHC (CERN)	6 TeV + 6 TeV	$p\bar{p}$	(10^{32})	?$\rightarrow$ 1990's
SSC (USA)	20 TeV + 20 TeV	pp	(10^{33})	?$\rightarrow$ 1994
ELOISA	50 TeV + 50 TeV	pp	(10^{33})	?
RHIC	250 GeV + 250 GeV	pp	(10^{31})	?$\rightarrow$1991
	100 GeV/amu per beam	Au-Au	(10^{27})	?$\rightarrow$1991

Table 2. List of existing and (planned) Hadron–Hadron Colliders

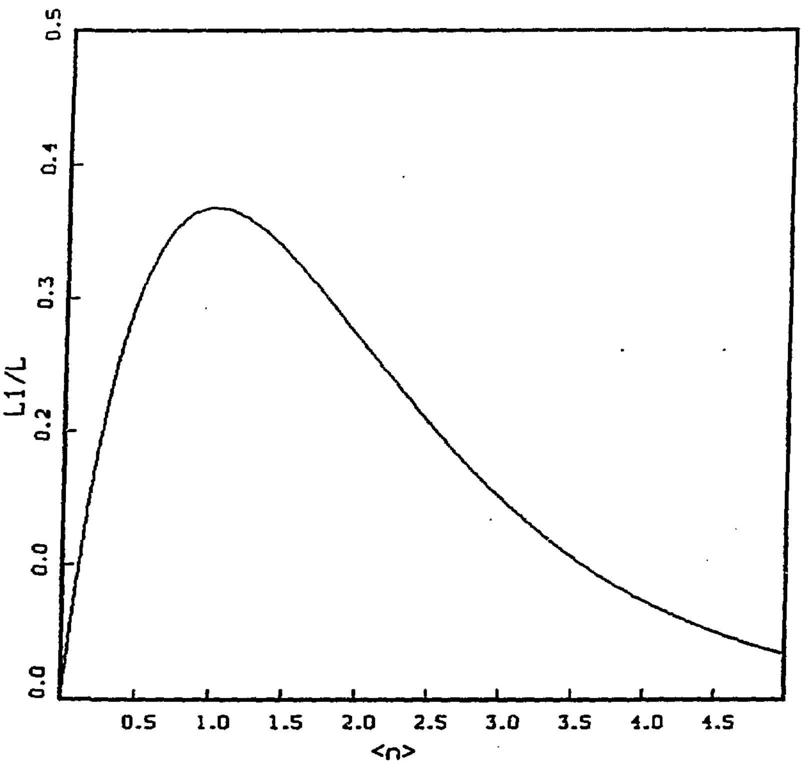

Fig.1. Luminosity ratio for one collision per crossing as a function of the average number of collisions.

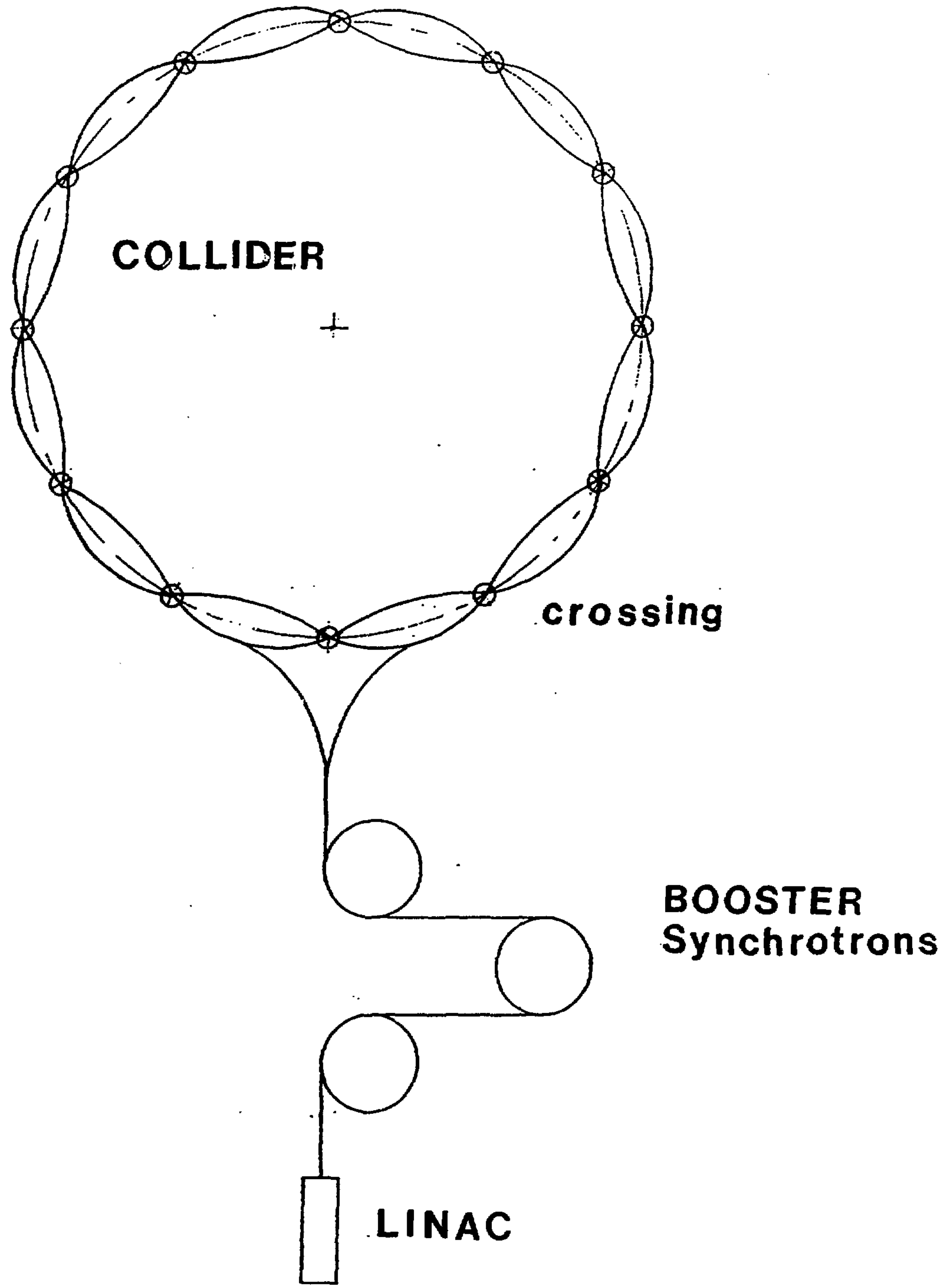

Fig.2. Injection scheme for ELOISA.

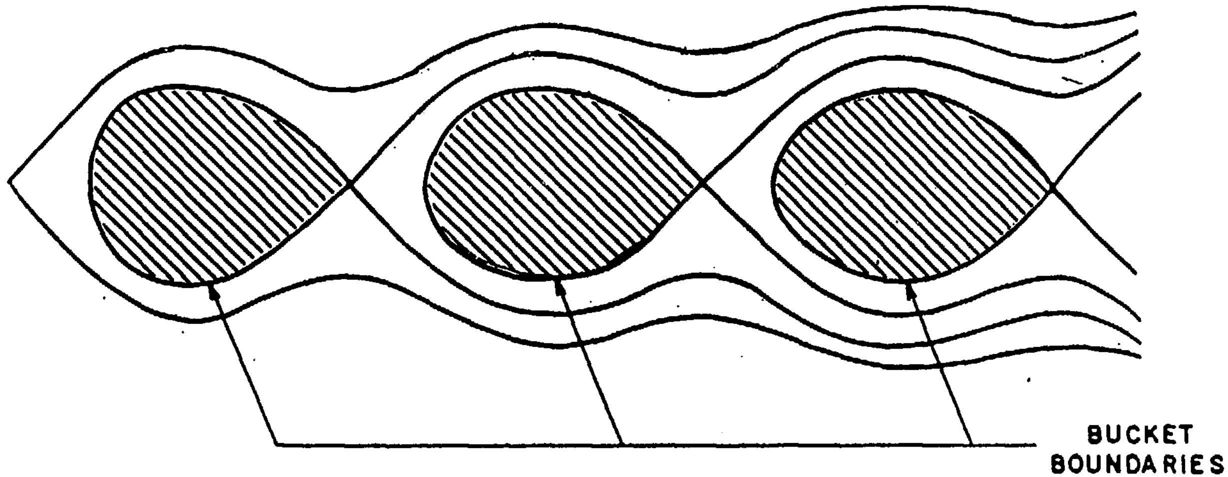

Fig.3. Structure of RF buckets. The shaded areas represent stable regions.

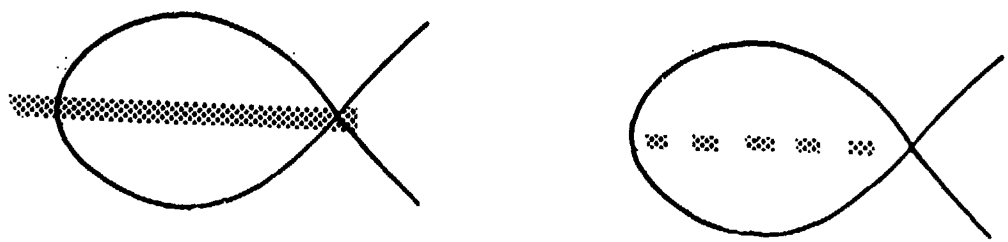

Fig.4. a) Injection with continuous beam; b) with pulsed beam.

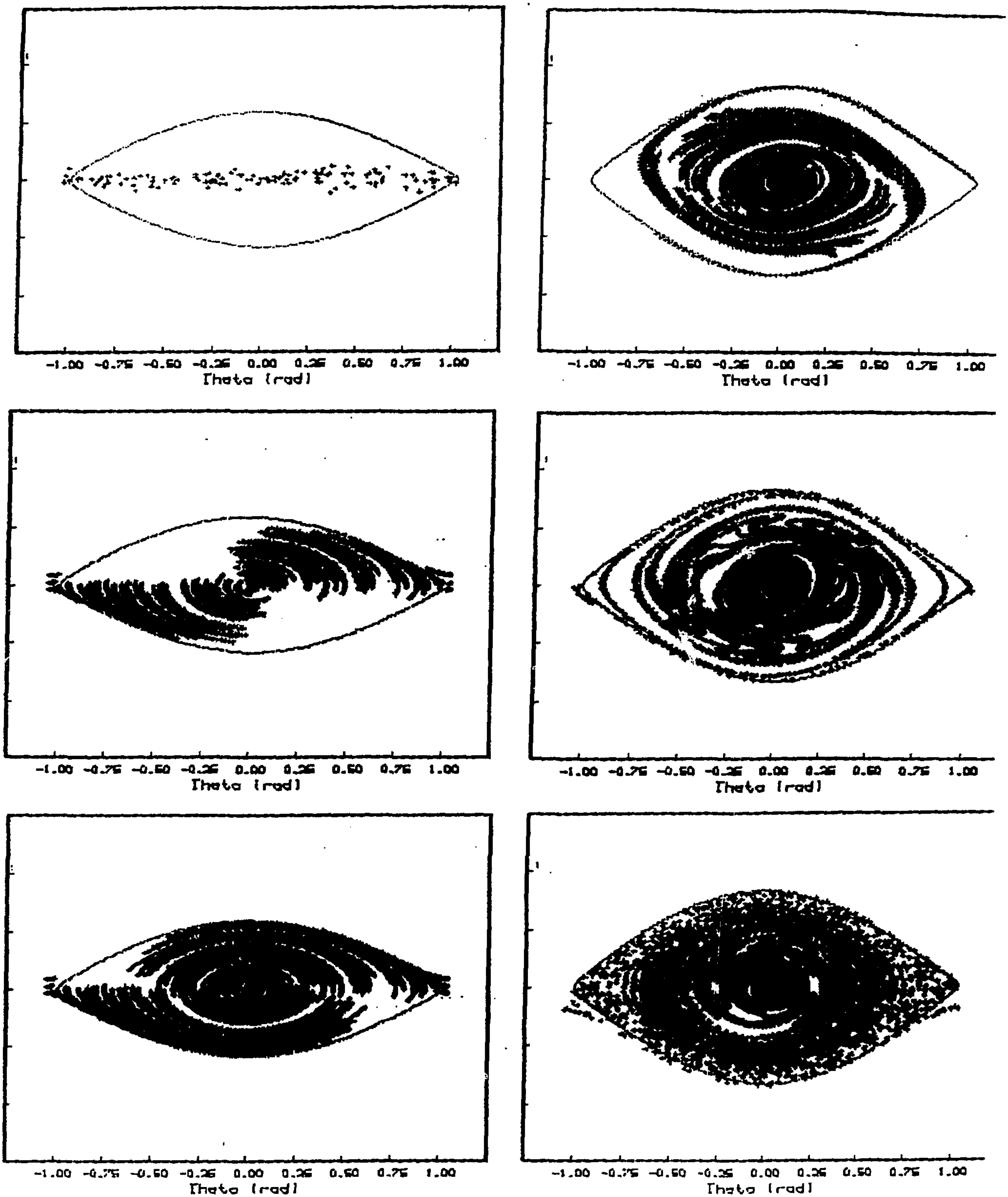

Fig.5. Time evolution of bucket. Continuous beam injected. Low current limit.

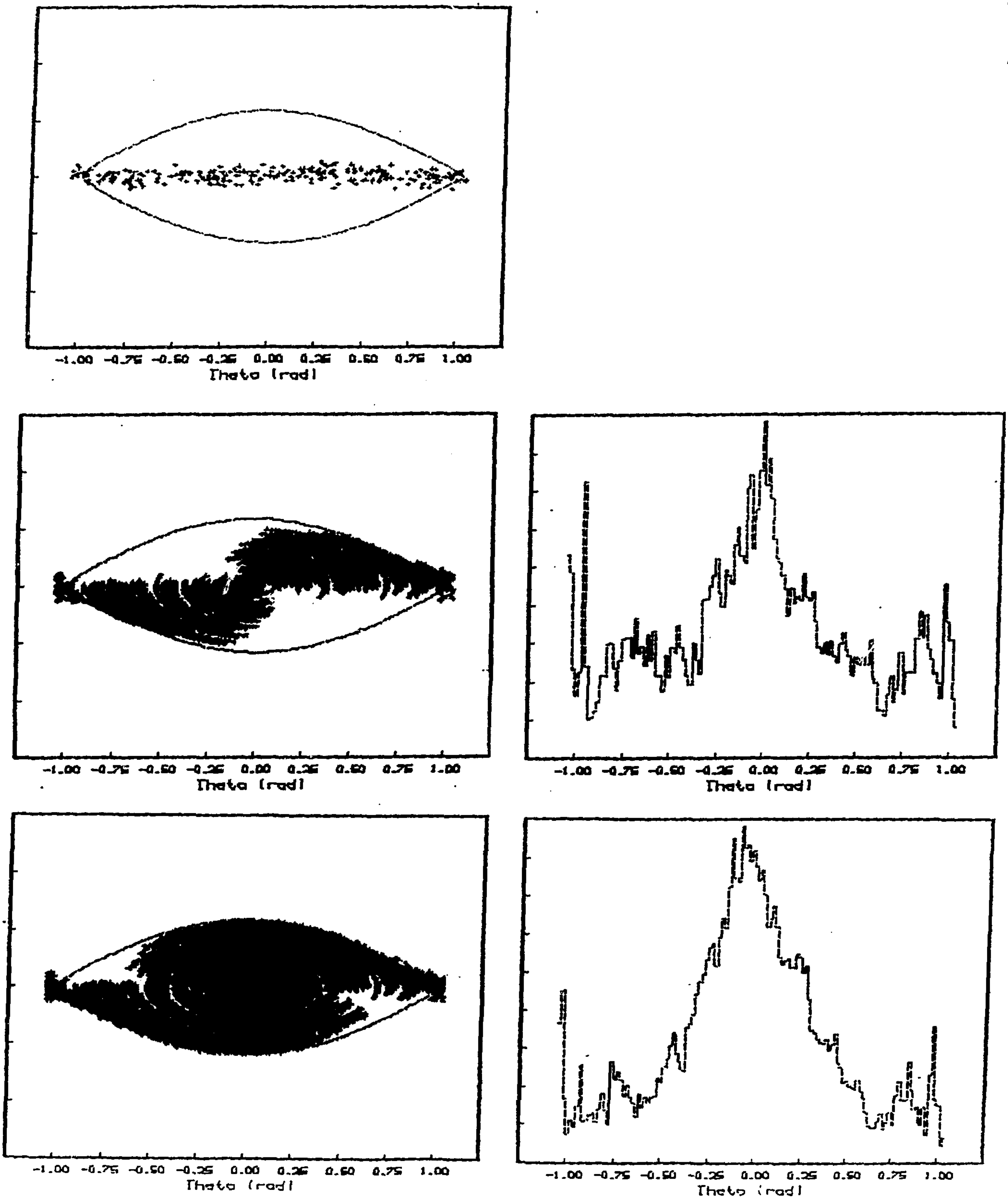

Fig.6a Time evolution of bucket and longitudinal profile of beam. Continuous beam injected. Substantial beam current.

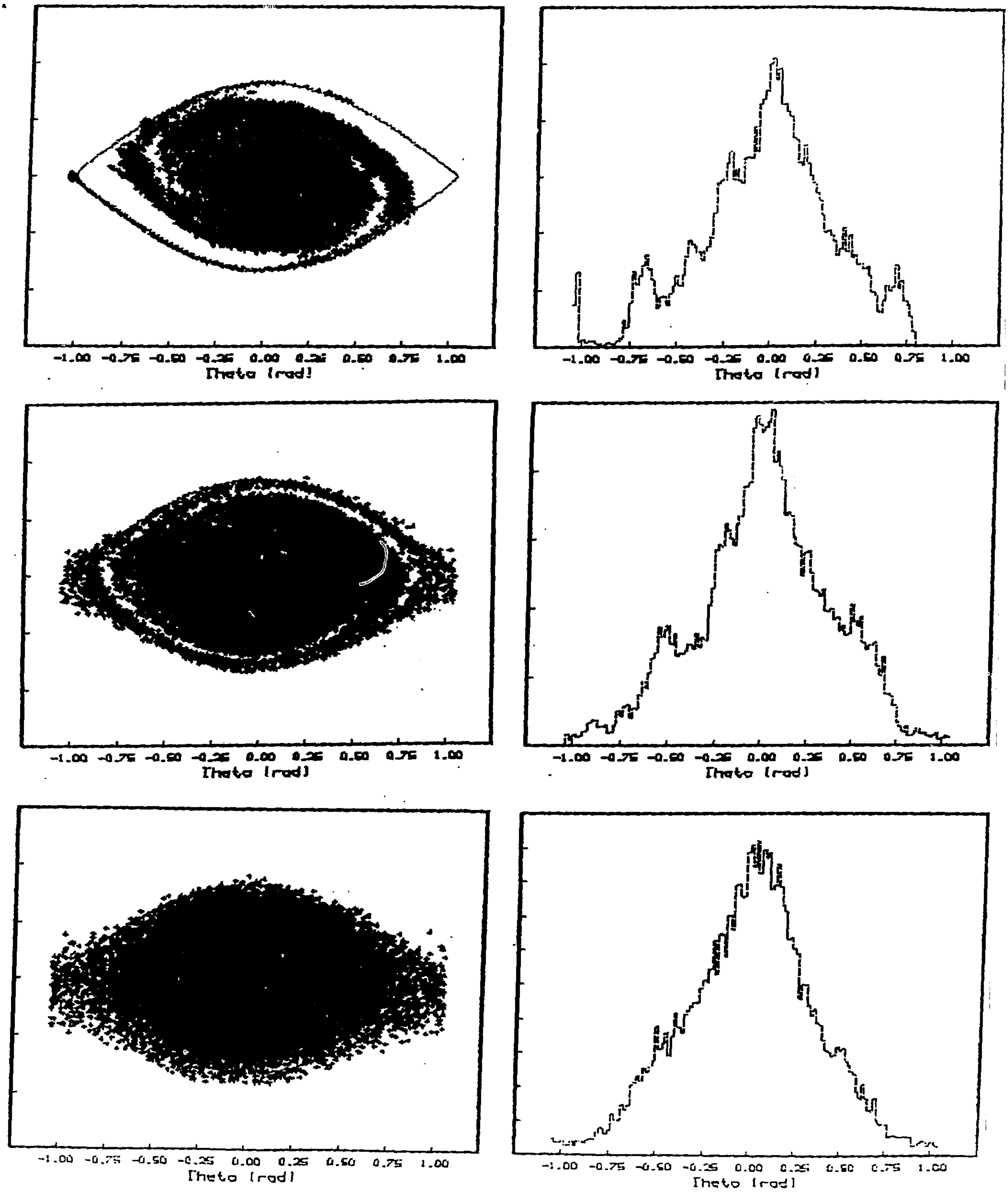
-1.00 -0.75 -0.50 -0.25 0.00 0.25 0.50 0.75 1.00
Theta [rad]
-1.00 -0.75 -0.50 -0.25 0.00 0.25 0.50 0.75 1.00
Theta [rad]
-1.00 -0.75 -0.50 -0.25 0.00 0.25 0.50 0.75 1.00
Theta [rad]
-1.00 -0.75 -0.50 -0.25 0.00 0.25 0.50 0.75 1.00
Theta [rad]
-1.00 -0.75 -0.50 -0.25 0.00 0.25 0.50 0.75 1.00
Theta [rad]
-1.00 -0.75 -0.50 -0.25 0.00 0.25 0.50 0.75 1.00
Theta [rad]

Fig. 6b

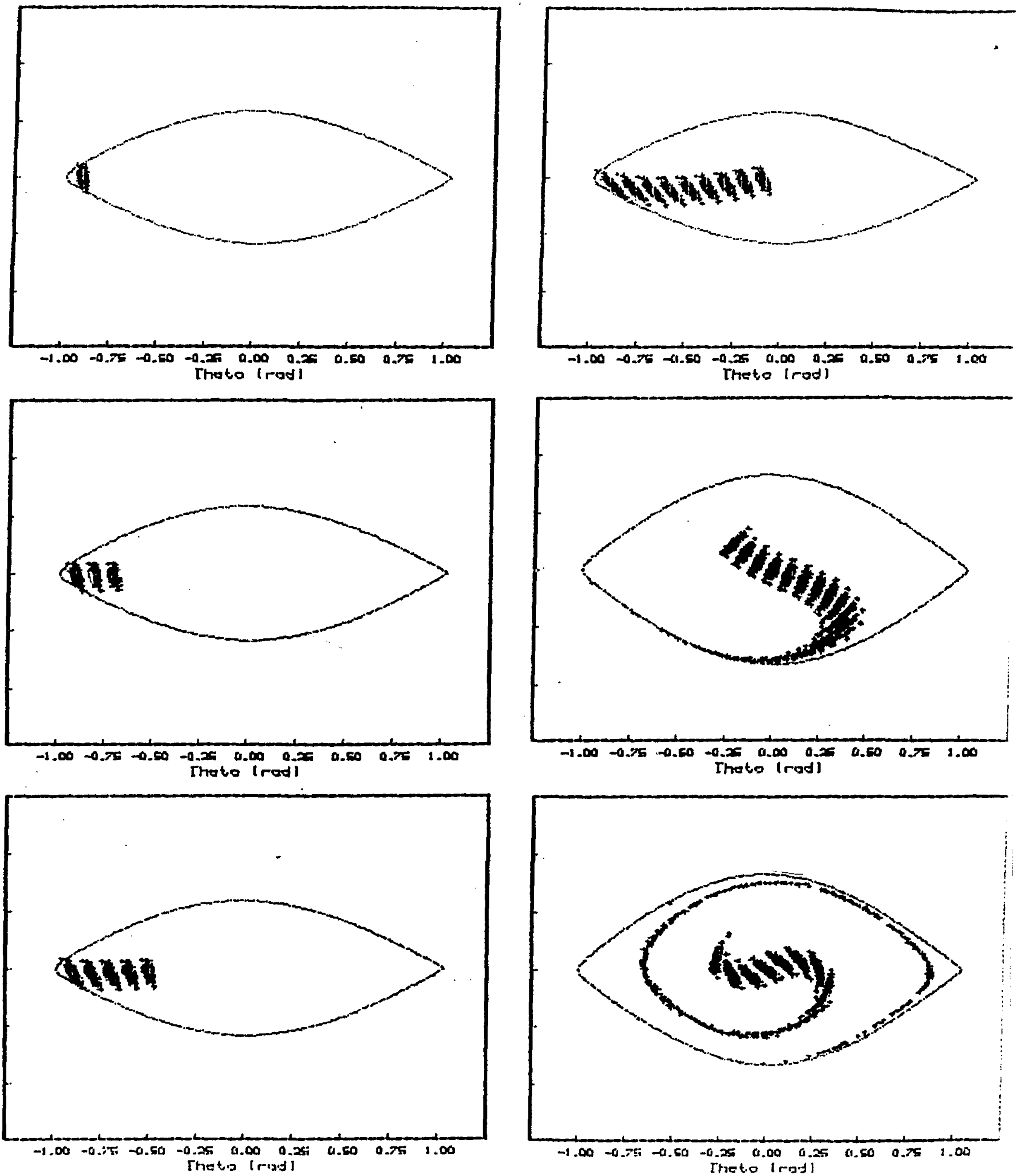

Fig.7. Time evolution of bucket. Pulsed beam injected. Low current limit.

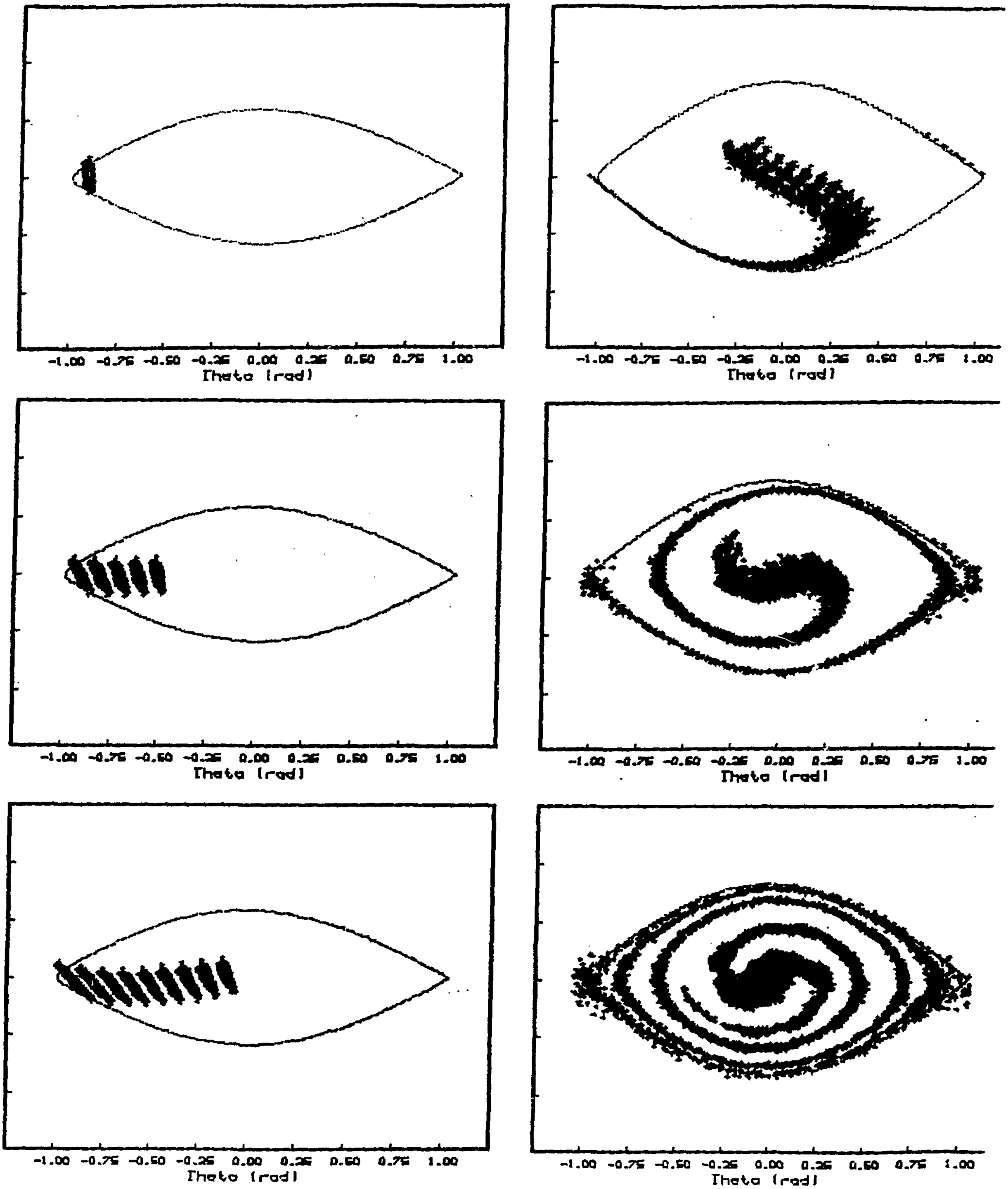

Fig. 8. Time evolution of bucket. Pulsed beam injected. High current.

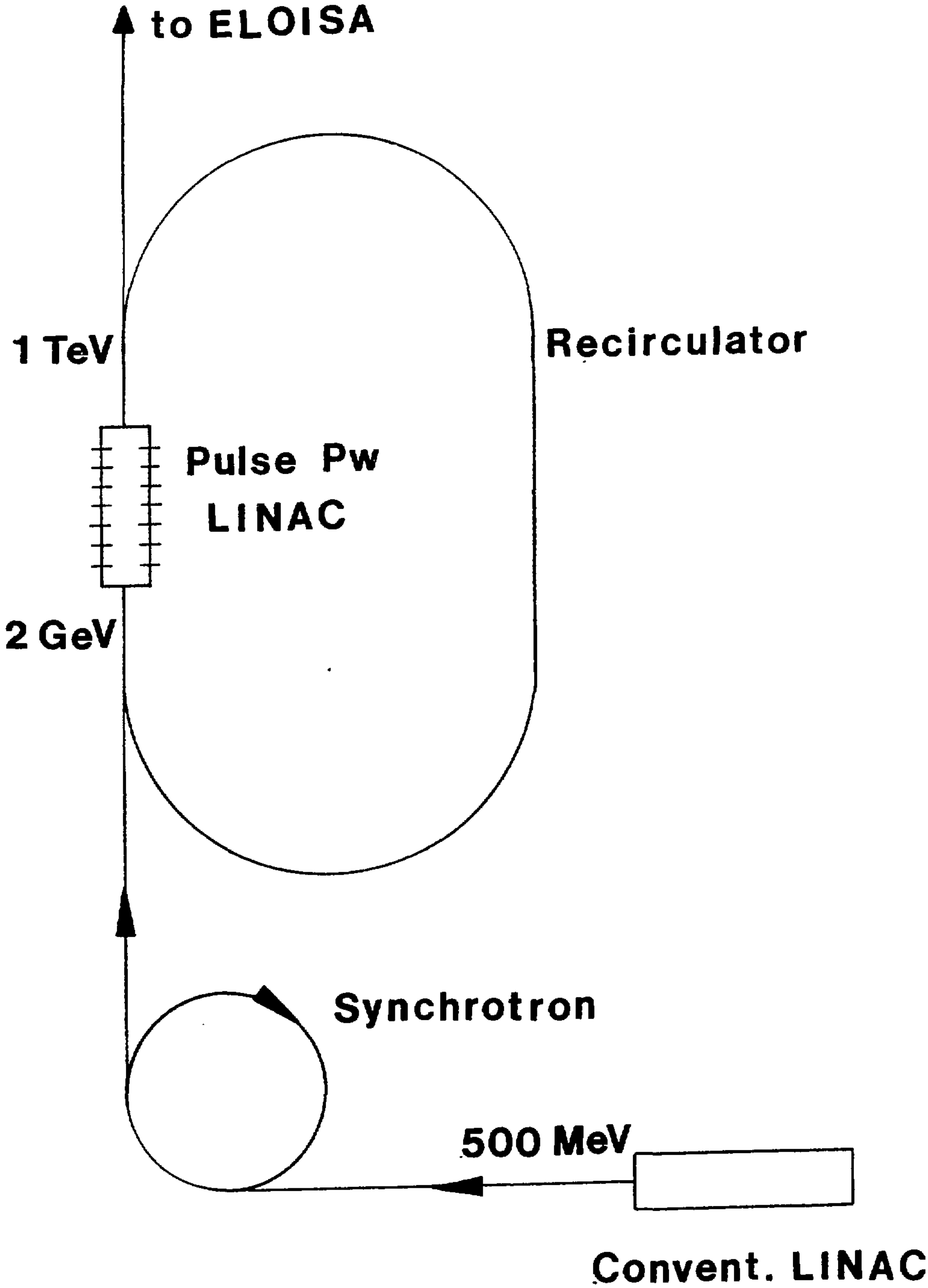

Fig.9. A recirculating proton storage ring will help to time match a Booster Synchrotron to a PP Linac to inject ELOISA.