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EFFECT OF EMPTY BUCKETS ON COUPLED BUNCH INSTABILITY IN RHIC BOOSTER—LONGITUDINAL PHASE-SPACE SIMULATION

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ABSTRACT

Excitation of large amplitude coherent dipole bunch oscillations by beam induced voltages in spurious narrow resonances are simulated using a longitudinal phase-space tracking code (ESME). Simulation of the developing instability in a high intensity proton beam driven by a spurious parasitic resonance of the rf cavities allows one to estimate the final longitudinal emittance of the beam at the end of the cycle, which puts serious limitations on the machine performance. The growth of the coupled bunch modes is significantly enhanced if a gap of missing bunches is present, which is an inherent feature of the high intensity proton machines. A strong transient excitation of the parasitic resonance by the Fourier components of the beam spectrum resulting from the presence of the gap is suggested as a possible mechanism of this enhancement.

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INTRODUCTION

The environment of particles in a synchrotron is not completely described by the existing accelerating field. The physical presence of vacuum chamber and magnets can be expressed in terms of an impedance experienced by the beam, while the rf cavity itself may possess parasitic higher order modes. These, in particular, may have high enough Q's to allow consecutive bunches to interact through mutually induced fields, which is the case in the beam environment considered here. The cumulative effects of such fields as the particles pass through the cavity may be to induce a coherent build-up in synchrotron motion of the bunches, i.e. a longitudinal coupled bunch-instability[1].

The colliding mode operation of the present generation of high energy synchrotrons and the accompanying rf manipulations, make consideration of individual bunch area of paramount importance. Thus, a longitudinal instability in one of a chain of accelerators, while not leading to any immediate reduction in the intensity of the beam in that accelerator, may cause such a reduction of beam quality that later operations are inhibited (resulting in a degradation of performance).

In this paper we employ a longitudinal phase space tracking code (ESME)[2] as an effective tool to simulate specific coupled bunch modes arising in a circular accelerator. One of the obvious advantages of the simulation compared to existing analytic formalisms, e.g. based on a dispersion relation obtained from Vlasov's equation[3], is that it allows consideration of the instability in a self-consistent manner with respect to the changing accelerating conditions. Self-consistency with respect to the changing particle distribution in phase-space is an intrinsic feature of the simulation. Furthermore, this scheme allows one to model nonlinearities of the longitudinal beam dynamics, which are usually not tractable analytically.

Included in the simulation is the investigation of the effect of missing bunches on the development and growth of the coupled bunch modes. The machine-dependent parameters which are considered here are derived from the RHIC Booster, this study being motivated by a prospective instability problem for high intensity performance of this machine. Now, we will proceed with a detailed formulation of the above program.

1. LONGITUDINAL PHASE-SPACE TRACKING WITH WAKE FIELDS

Briefly summarized, the tracking procedure used in ESME consists of turn-by-turn iteration of a pair of Hamilton-like difference equations describing synchrotron oscillation in θ-ε phase-space ($0 \leq \theta \leq 2\pi$ for the whole ring and $\varepsilon = E - E_o$, where E_o is the synchronous particle energy). In order to include the effect of the beam environment one can consider the additional potential due to the wake field generated by the beam as it passes through the beam pipe. Knowing the particle distribution in the azimuthal direction, $\rho(\theta)$, and the revolution frequency, Ω_o, after each turn, one can construct a wake field induced voltage as follows[4,5]

$$V_i(\theta) = e\Omega_o \sum_{-\infty}^{\infty} \rho_n Z(n\Omega_o) e^{in\theta}, \tag{1}$$

where ρ_n represents the discrete Fourier spectrum of the beam and $Z(\omega)$ is a longitudinal coupling impedance. The numerical procedure involved in evaluating the above expression, Eq. (1), necessarily employs a discretization of the θ-direction. Some caution is required in this process of binning, due to the finite statistics inherent in such a simulation.

2. PARASITIC RESONANCES—MODE CROSSING

The longitudinal coupling impedance experienced by the beam in the RHIC Booster consists of a broad-band part associated with the magnet laminations and a number of sharp parasitic resonances of the rf cavities. These higher order modes of the proton rf cavity of the RHIC Booster were found numerically using the SUPERFISH[6] code. The results—the fundamental and 9 lowest parasitic modes are summarized in Table 1*. There are a total of 4 accelerating gaps and the shunt impedance listed in Table 1 refers to its net value. Considered here is a ferrite-loaded cavity. The SUPERFISH simulation includes the presence of the ferrites to determine the resonant frequencies, but no losses inside the ferrite fillings were considered. The only losses taken into account are those in the cavity copper walls. Each resonance can be modeled by the harmonic resonator of the impedance given by the standard expression:

$$Z(\omega) = \frac{R_{sh}}{1 + iQ(\omega/\omega_c - \omega_c/\omega)}. \tag{2}$$

Here R_{sh} is the shunt impedance, Q denotes the quality factor of the resonator and ω_c is its resonant frequency.

If the resonance is sufficiently strong (high Q and large R_{sh}) the wake field generated by one bunch has a range long enough to affect the motion of the neighboring bunches. Therefore, for the purpose of our simulation, only the relatively high-Q portion of the longitudinal impedance is relevant. For M equally spaced coupled bunches there are M possible dipole modes labeled by $m = 1, 2, \ldots, M$ (M coupled oscillators with periodic boundary conditions). To illustrate the m^{th} dipole mode one can look at the θ-position of the centroid of each bunch, $\theta_{\mathcal{L}}$, $\mathcal{L} = 1, 2, \ldots, M$. The signature of the simplest coupled bunch mode has the form of a discrete propagating plane wave illustrated schematically in Fig. 1.

$$\theta_{\mathcal{L}}(t) = \theta_o sin\left(\frac{2\pi m\mathcal{L}}{M} - \omega_s t\right), \tag{3}$$

where ω_s is the synchrotron frequency.

Fig. 1 also addresses the problem of the emittance degradation due to the presence of the coupled bunch oscillations. Although the longitudinal emittance of individual bunches is conserved, the total area in ϵ-θ phase space occupied by a sequence of consecutive bunches constituting one wavelength of the coupled bunch mode is significantly larger and scales like the square of the dipole mode amplitude. This in turn has an impact on any further bunch manipulation, e.g. bunch coalescing, and results in beam dilution and total effective emittance growth (in the previously described sense).

Based on an analytic model of coupled bunch modes proposed by Sacherer[1] one can formulate a simple resonance condition for the m^{th} dipole mode driven by the longitudinal impedance $Z(\omega)$ sharply peaked at ω_c. This condition is given by:

$$\omega_c = (nM + m)\Omega_o \pm \omega_s, \tag{4}$$

where n is an integer. Since Ω_o is time dependent (acceleration) and ω_c is fixed (geometry), and knowing that the width of the impedance peak is governed by ω_c/Q one can clearly see that the resonance condition, Eq. (4), is maintained over a finite time interval. This leads to the useful concept of a mode crossing the impedance resonance. Using the explicit time dependence of Ω_o (kinematics) and Eq. (4) one can easily calculate crossing intervals for various modes. This serves as a guide in the simulation since it allows us to select an appropriate time domain where the mode of interest crosses the resonance and will more likely become unstable. The kinematics of the RHIC Booster can be described as follows:

$$p(t) = \frac{1}{2}(p_o + p^*) - \frac{1}{2}(p^* - p_o)cos(2\pi f t), \tag{5}$$

where

$$f = 8.33 \ sec^{-1}$$

is the frequency of the RHIC Booster cycle and

$$p_o = 0.644 \ GeV/c$$

$$p^* = 2.251 \ GeV/c$$

are the values of momenta at injection ($t = 0$) and extraction ($t = 6 \times 10^{-2} sec$) respectively. Now the revolution frequency for the synchronous particle is given explicitly by

$$\Omega_o(t) = \frac{c}{R}\beta(t), \tag{6}$$

where

$$\beta(t) = \sqrt{\frac{(pc)^2}{(pc)^2 + (mc^2)^2}}.$$

Here $R = 32.114\ m$ is the average radius of the ring and $p(t)$ is expressed by Eq. (5). Both Eqs. (5) and (6) allow us to calculate the crossing intervals for different modes.

3. COUPLED BUNCH MODE—ESME SIMULATION

For the purpose of this model calculation we tentatively identified the parasitic resonance at $f_c = 20.68\ MHz$ (see Table 1) as the offending part of the impedance giving rise to a coupled bunch instability with harmonic number $m = 1$.

The rf system of the RHIC Booster provides 3 accelerating buckets. As a starting point for our simulation each bucket in θ–ϵ phase–space is populated with 300 macro–particles according to a bi–Gaussian distribution matched to the bucket so that 95% of the beam is confined within the contour of the longitudinal emittance of 0.3 eV–sec. Each macro–particle is assigned an effective charge to simulate a beam intensity of 1.5 10^{13} protons. The appropriate time interval to study mode $m = 1$ is chosen as 29–33$\times 10^{-3}$ sec.

In a real–life accelerator any coherent instability starts out of noise and gradually builds up to large amplitudes. In our model situation it proved necessary to create some intrinsic small amplitude—"seed" of a given mode in order to "start–up" the instability. The "seeding" procedure is basically prescribed by Eq. (3). Initially identical bunches are rigidly displaced from the center of each bucket (both in ϵ and θ) so that the positions of their centroids, θ_C, satisfy Eq. (3) for all the bunches around the ring (see Fig. 1). In practice a subroutine of ESME, which generates a closed contour in θ–ϵ space (given an appropriate starting point) under the action of a sinusoidally varying voltage, was used to establish the position of the bunch centroids. The intrinsic seed amplitude, θ_o, was assigned a value of 5×10^{-2} rad corresponding to an amplitude in energy of approximately 0.4 MeV.

The tracking results are illustrated in Fig. 2 by the snapshots of the longitudinal phase space taken at initial time, after 2000 revolutions and finally, after 3500 revolutions. Here the initial population was generated according to the above described seeding procedure. One can clearly see development of coherent coupled bunch motion and strong filamentation of the bunches due to the synchrotron tune spread inside a bucket (for large amplitudes of the dipole oscillations). To visualize the position and shape of individual bunches as they evolve in time one can compose a "mountain range" diagram by plotting θ–projections of the bunch density in equal increments of revolution number and then stacking the projections to imitate the time flow. The resulting mountain range plot for a coupled bunch mode $m = 1$ is given in Fig. 3a. One can clearly see development of the $m = 1$ coupled bunch mode corresponding to increasing amplitude of the dipole oscillations.

The impact of the initial beam spectrum, defined by the previously described seeding procedure, on the evolution of the simulated coupled bunch mode is quite essential; the same simulation carried out with the initial seed turned down to zero yields no buildup of coherent synchrotron motion in spite of the presence of the strong parasitic resonance. Obviously, the statistical noise spectrum does not contain a sufficiently large Fourier component at the coupled bunch mode frequency to bootstrap the instability and no development of the instability is observed.

Following the spirit of this section we carry out similar simulation for the same intensity–emittance conditions but now the protons are distributed between two buckets only leaving a gap of empty bucket in the beam. In spite of starting with no initial seed of the coupled bunch mode, one can see rapid development of the instability illustrated in Fig. 3b. Simple physical explanations of the above results will be given in the next section.

3. CONCLUSIONS

The results of these simulations clearly show evidence of large amplitude coherent dipole motion of bunches resulting from beam excitations of spurious high impedance narrow resonances in the ring. It is not, however, clearly established that the simulation program properly represents strictly coupled bunch motion. In order for this to be strictly correct the code would have to be able to generate excitation fields at frequencies ($n\Omega_o \pm \omega_s$), where ω_s is the coherent synchrotron frequency of a bunch prior to Landau smearing. The code, as is presently written, can only generate excitation fields at rotation frequencies, $n\Omega_o$, therefore the synchrotron harmonic oscillators are being excited near, but not on, resonance. The excitation currents resulting from the coherent bunch displacement are therefore not quite properly represented and extraction of coupled bunch growth times is suspect. Off-resonance excitation of a linear harmonic oscillator (see Fig. 4) would exhibit amplitude beating with the amplitude returning to zero periodically. In this, non-linear, case Landau smearing of the initial large amplitude of phase oscillation and the short duration of near resonance excitation give the appearance of coupled bunch instability growth.

This shortcoming of the representation is probably responsible for the unexpectedly large "seeding" requirement in the symmetric (full ring) case. In the case of a missing bunch, a large amplitude excitation current at rotation frequencies, $n\Omega_o$, is explicitely generated and seeding is unnecessary.

D. Kohaupt has shown[8] that the growth rate of an unsymmetrical bunch distribution (i.e. with missing or depleted bunches) is not greater than the growth rate for the symmetric distribution for a specified spurious impedance. This result is corroborated by R. Baartman[9] with the caveat that in the unsymmetrical case the growth need not start simply from the noise level and hence, large amplitude oscillations may appear more quickly.

The mentioned shortcoming in the ESME code, along with another difficulty having to do with treatment of transient response[10] are in the process of being corrected.

FOOTNOTE

* The values of R_{sh} and Q were scaled down by a factor of 1000 to qualitatively account for the losses in the Ferrite. The results of the simulation are still valid since R_{sh}/Q, which governs the integrated growth time of the instability remains the same.

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FIGURE CAPTIONS

Fig. 1 Schematic diagram of the longitudinal phase-space configuration corresponding to the coherent motion of coupled bunches. Below, illustration of the effective emittance of the beam modified by the presence of the coupled bunch instability.

Fig. 2 Time development of the m = 1 coupled bunch mode – θ–ε snapshots taken after various number of turns around the ring.

Fig. 3 Collection of mountain range plots illustrating the behavior of coupled bunch mode m = 1 with:

a) full ring (three bunches),

b) gap (one empty bucket)

Fig. 4 Schematic diagram of a synchrotron harmonic oscillator (two side-bands spaced by $\pm\omega_s$) driven off-resonance by a spurious impedance line at the revolution frequency harmonic $(h + m)\Omega_0$.

f_c [MHz]	R_{sh} [kΩ]	Q	R_{sh}/Q [Ω]
2.77	**225.12**	**145.**	**1,552.55**
20.68	2.304	1676.	1.37
25.99	1.104	1429.	0.77
31.51	0.096	1304.	0.07
32.82	0.912	2844.	0.32
36.96	0.432	876.	0.49
43.91	0.144	1508.	0.10
44.66	0.096	3244	0.04
45.02	0.528	1563.	0.34
54.44	0.192	977.	0.20

Table1 Selected resonant part of the longitudinal coupling impedance – the fundamental mode and several lowest parasitic spurious resonances. Result of the SUPERFISH calculation for a ferrite-loaded RHIC Booster cavity.

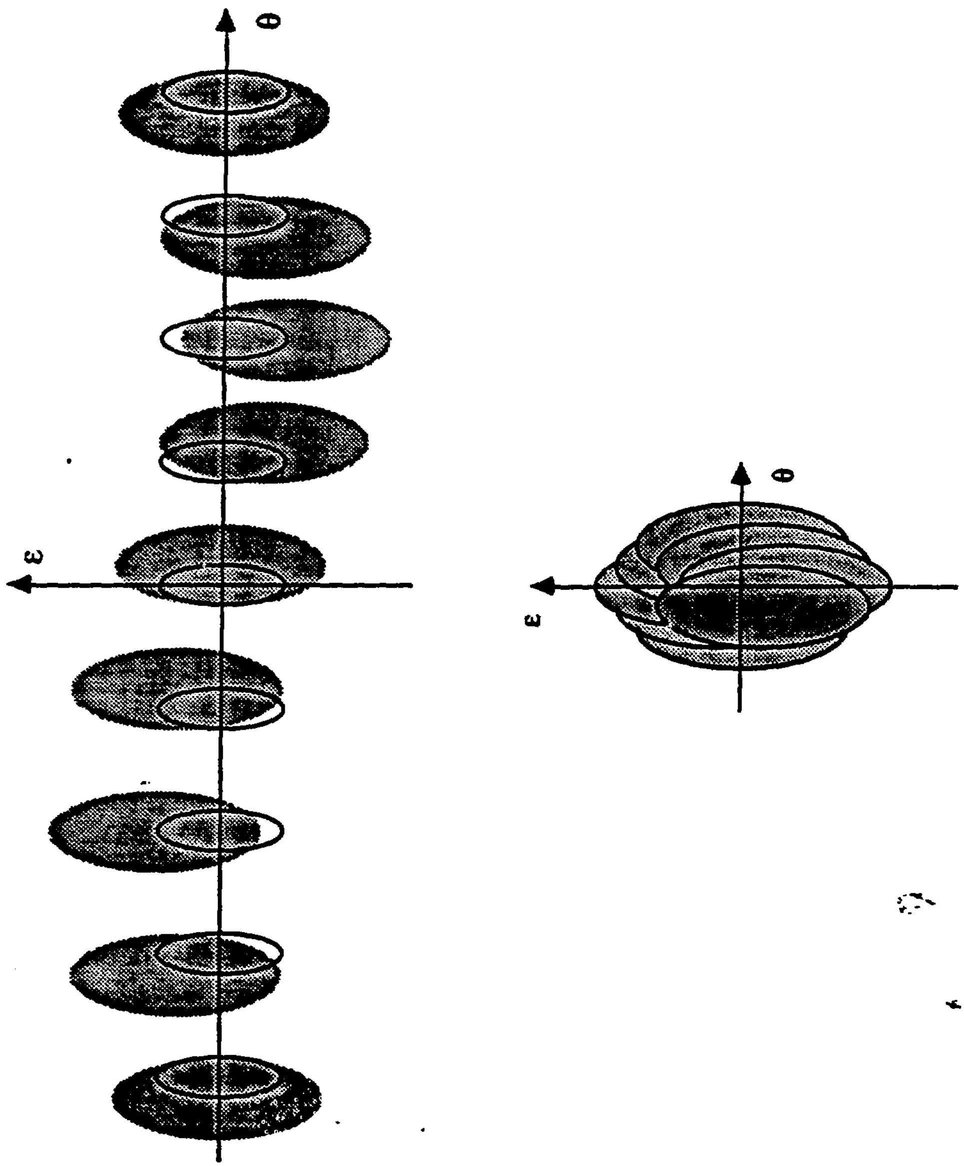

Fig. 1

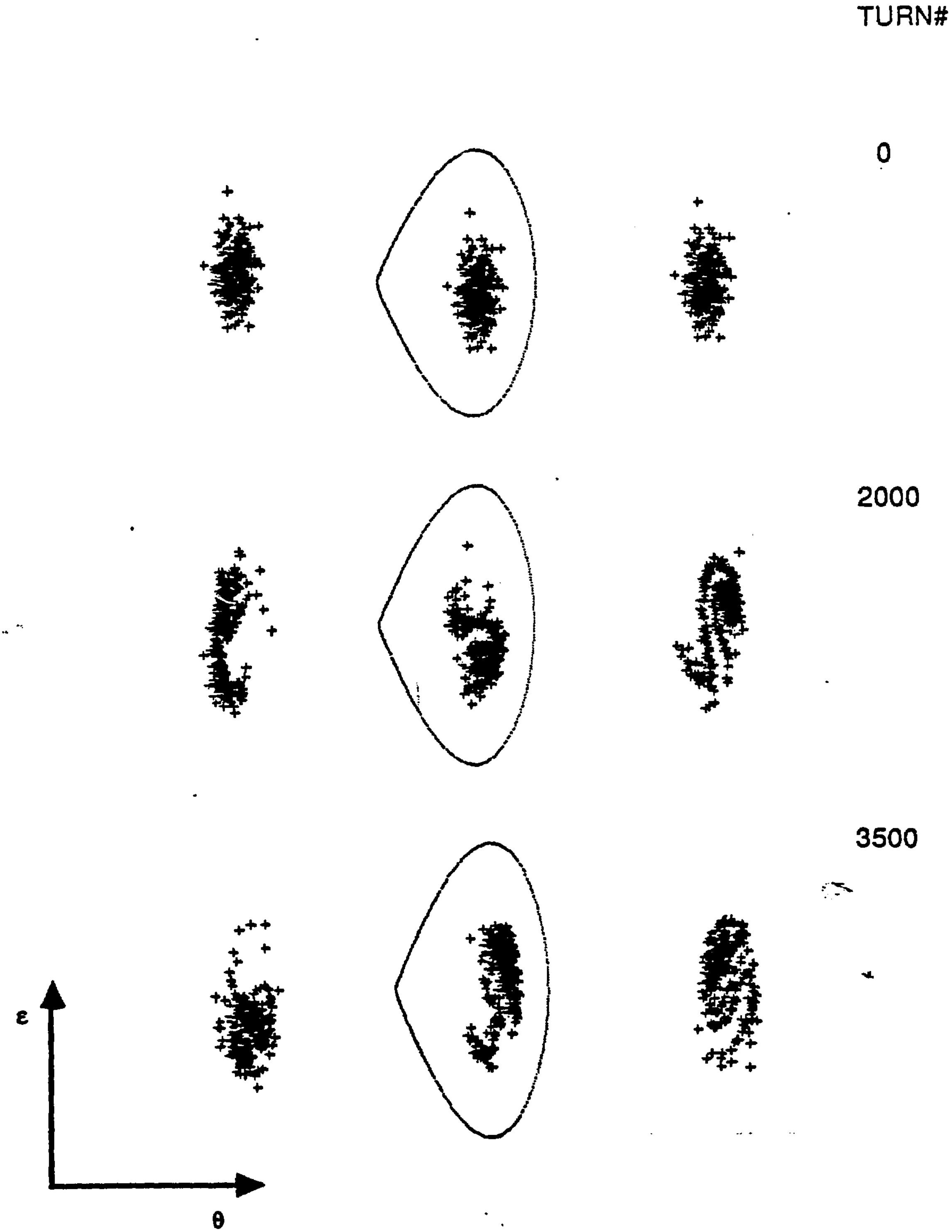

Fig. 2

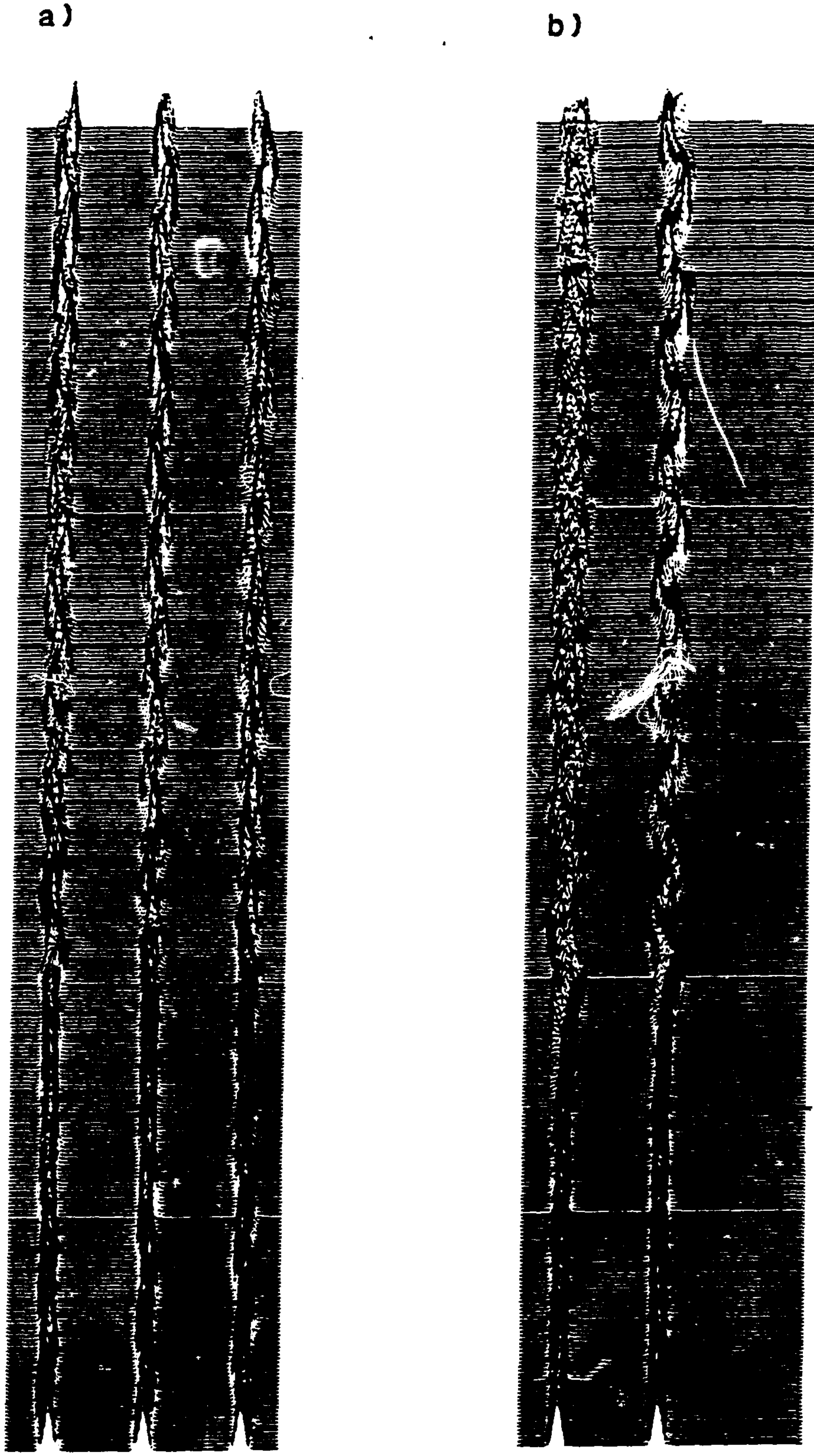

Fig. 3

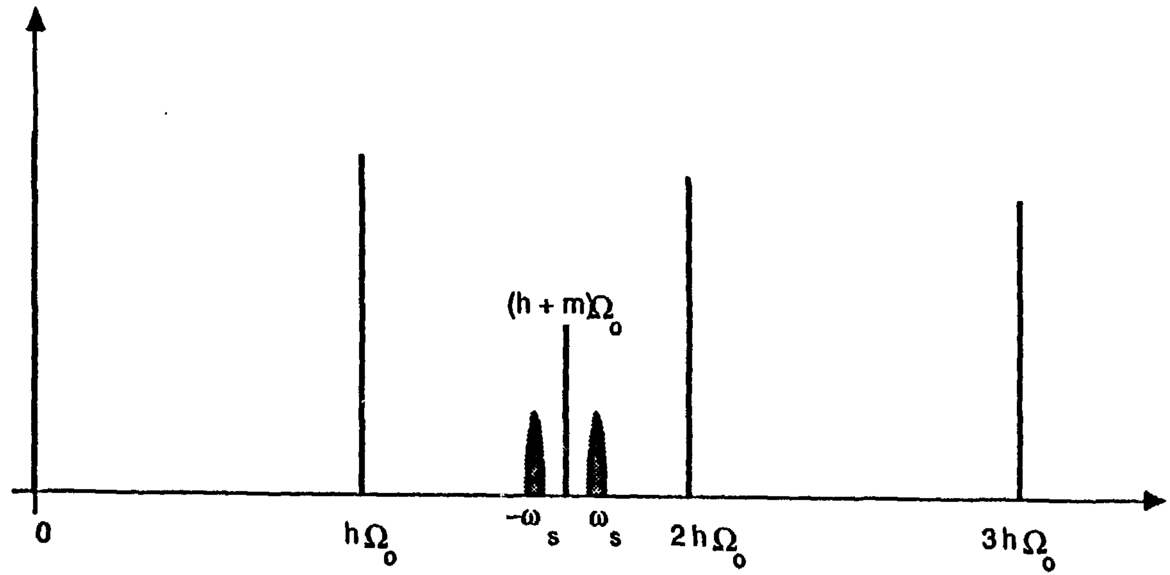

Fig.4