29
11-12-80
9¢L 24 to NTIS

SAND79-2212
Unlimited Release
UC-70

MASTER

MASTER

Creep Consolidation of Nuclear Depository Backfill Materials

Barry M. Butcher

Sandia National Laboratories

SF 2900-Q(3-80)

SAND79-2212
Unlimited Release

CREEP CONSOLIDATION OF NUCLEAR DEPOSITORY BACKFILL MATERIALS

B. M. Butcher
Geomechanics Division 5532
Sandia National Laboratories
Albuquerque, New Mexico 87185

ABSTRACT

Evaluation of the effects of backfilling nuclear waste repository rooms is an important aspect of waste repository design. Consolidation of the porous backfill takes place as the room closes with time, causing the supporting stress exerted by the backfill against the intact rock to increase. Estimation of the rate of backfill consolidation is required for closure rate predictions and should be possible if the creep law for the solid constituent is known. A simple theory describing consolidation with a spherical void model is derived to illustrate this relationship. Although the present form of the theory assumes a homogeneous isotropic incompressible material atypical of most rocks, it may be applicable to rock salt, which exhibits considerable plasticity under confined pressure. Application of the theory is illustrated assuming a simple steady-state creep law, to show that the consolidation rate depends on the externally applied stress, temperature, and porosity.

Table of Contents

List of Symbols

σ_{ij}	True stress
σ_{ij}'	True deviatoric stress
ϵ_{ij}	True strain
ϵ_{ij}^P	True plastic strain
$\sigma_1,\ \sigma_2 = \sigma_3$	True stresses in the triaxial loading configuration
$\epsilon_1,\ \epsilon_2 = \epsilon_3$ $d\epsilon_1,\ d\epsilon_2 = d\epsilon_3$ $d\epsilon_1^P,\ d\epsilon_2^P = d\epsilon_3^P$	True strains, strain increments, and plastic strain increments in the triaxial loading configuration
$\sigma_r,\ \sigma_\phi = \sigma_\theta$	True stresses in the spherical shell configuration
$\epsilon_r,\ \epsilon_\phi = \epsilon_\theta$ $d\epsilon_r,\ d\epsilon_\phi = d\epsilon_\theta$ $d\epsilon_r^P,\ d\epsilon_\phi^P = d\epsilon_\theta^P$	True strains, strain increments, and plastic strain increments in the spherical shell configuration
$\bar{\epsilon}^P$	Equivalent plastic strain
$\overline{d\epsilon}^P$	Equivalent plastic strain increment
$\bar{\sigma}$	Equivalent stress
$\dot{\epsilon}_1$	True axial strain rate in the triaxial loading configuration
t	Time
T	Temperature

List of Symbols (Cont.)

$\dot{r}$	$\frac{dr}{dt}$
a_o, a, b_o, b	See Figure 1
η	Porosity
$\dot{\eta}$	$\frac{d\eta}{dt}$
A, B, Q, R, μ, and n	Constants in the simple creep model given by Equation (3.1)

Introduction

Evaluation of the effects of backfilling nuclear waste disposal rooms is an important aspect of waste repository programs. Backfilling is proposed for a number of reasons, among which are greater isolation of the waste and reduction of subsidence. Barriers to radionuclide migration are also possible by proper selection of backfill material. All of these features have in common the requirement that the rate of consolidation of the backfill must be determined because the backfilled region, between the heat source (the waste container) and the structural components of the repository, influences heat flow, transmissivity of fluids, and the extent of deformation of the structure itself.

Complete elimination of porosity during backfilling operations is believed to be virtually impossible. Therefore, structural response and permeability calculations require a constitutive relationship for a porous material in a temperature field with an externally applied pressure (imposed by creep of the solid rock surrounding the repository). A considerable advantage will be gained in the development of these constitutive relations if the creep consolidation of the porous material can be described in terms of the creep characteristics of its solid constituent. Success in this approach minimizes the need for a large and costly material testing program to establish the creep properties of materials of varying porosity all composed of the same solid.

The relationship between dynamic inelastic compaction of a porous solid and deformation of its solid matrix has been considered in detail in shock mitigation studies.[1] The possibility exists, therefore, of addressing creep consolidation of porous materials in an identical manner.

The purpose of this report is to suggest a theoretical approach by which such a constitutive model can be developed.

The example discussed assumes voids in the material become approximately spherical in shape after a short transient period. Some justification for these assumptions can be made for rock salt, which is being considered for backfilling WIPP waste disposal rooms, because salt deforms in a manner similar to metals under certain conditions. The theory in a more advanced state can be extended to other void geometries and the incompressibility assumption, which in part satisfies the conditions of classical plasticity theory, can in principle also be removed. A more general theory of elastic materials with voids is also available.[2] These more complex questions are not addressed in this paper.

Model Description

The compaction model is based on the symmetric collapse of a hollow sphere of homogeneous isotropic incompressible elastic-plastic material. A time-dependent pressure P(t) is applied to the outer boundary while the inner boundary remains stress free as shown in Figure 1. The porous aggregate is imagined to be a three dimensional array of these spheres, with the outer boundaries overlapping so that the only void volume is that of the spherical voids. These voids are considered noninteracting so that the pressure supported by the spheres is assumed to be the pressure supported by the porous aggregate.

In the actual backfilling process, the aggregate is usually composed of sharp angular grains of rock which fit together in random orientations to cause a porosity in many cases of 60% or greater. Vibratory compaction can decrease the void volume to some extent, but clearly the voids are far from spherical. This does not completely discredit the model, however, because as further compaction occurs by creep the voids should become more spherical in nature. The progression to more spherical shaped voids is implied by the tendency of the voids to minimize their surface area. In practical application of the theory, therefore, it is necessary to obtain the initial compaction response experimentally up to the time when the voids have developed sufficient spheroidicity. This partially deformed state can then be used as an initial condition for the spherical void analysis. Such an approach is quite consistent with our objectives, because it is the long term response of the backfill that we seek to model. Very long duration experiments are usually not conducted in great numbers, and yet the later stages of such experiments are precisely the realm where the spherical void model is expected to give realistic predictions.

Theoretical Development

a) Equivalent Stress and Strain

The creep compaction equation derived in this section depends on the application of data from triaxial loading creep experiments to the radially symmetric spherical shell pore collapse model.[3] A similar analysis for cylindrical voids can be developed using a solution for creep of a tube under pressure given in Reference 4. In triaxial creep experiments, an axial stress σ_1 is applied to a right circular cylindrical sample surrounded by a lateral hydrostatic pressure. Stresses and strains for this configuration are therefore (see List of Symbols for symbol definitions):*

$$\left(\sigma_1,\ \sigma_2 = \sigma_3\right) \quad , \quad \left(d\epsilon_1,\ d\epsilon_2 = d\epsilon_3\right) \quad .$$

The spherical shell model, on the other hand, is described by the following stresses and strains:

$$\left(\sigma_r,\ \sigma_\phi = \sigma_\theta\right) \quad , \quad \left(d\epsilon_r,\ d\epsilon_\phi = d\epsilon_\theta\right)$$

A relationship between these two geometrical configurations can be obtained by the classical plasticity method of defining plastic deformation in terms of the plastic strain increment $\overline{d\epsilon}^P$ and the equivalent stress $\bar{\sigma}$. The equivalent plastic strain $\bar{\epsilon}^P$ is defined as:

$$\bar{\epsilon}^P = \int \overline{d\epsilon}^P \quad , \qquad (2.1a)$$

where

$$\overline{d\epsilon}^P = \sqrt{\frac{2}{3}} \left\{ d\epsilon_{ij}^P \ d\epsilon_{ij}^P \right\}^{1/2} \quad . \qquad (2.1b)$$

In terms of the principal strains,

$$\overline{d\epsilon}^P = \sqrt{\frac{2}{3}} \left\{ \left(d\epsilon_1^P\right)^2 + \left(d\epsilon_2^P\right)^2 + \left(d\epsilon_3^P\right)^2 \right\}^{1/2} \quad . \qquad (2.1c)$$

with all strains expressed as true (or logarithmic) strains. If the

*Subscripts 1, 2, and 3 are used instead of r, θ, and z because the published material data and constitutive equations have been presented in this form.

condition is added to this definition that no volume change occurs during creep,

$$d\epsilon_{ii}^{P} = 0 \quad , \tag{2.2a}$$

the plastic strain increments for the two configurations under consideration are given by:

$$d\epsilon_{1}^{P} = -2d\epsilon_{3}^{P} = \overline{d\epsilon}^{P} \tag{2.2b}$$

and

$$d\epsilon_{r}^{P} = -2d\epsilon_{\theta}^{P} = \overline{d\epsilon}^{P} \quad . \tag{2.2c}$$

The axial strain increment in the triaxial loading test is therefore equal in magnitude to the radial strain increment in the collapse of spherical shells and:

$$\bar{\epsilon}^{P} = \epsilon_{r}^{P} = \epsilon_{1}^{P} \quad . \tag{2.3}$$

The equivalent stress is defined by:

$$\bar{\sigma} = \sqrt{\frac{3}{2}} \left\{ \sigma_{ij}' \, \sigma_{ij}' \right\}^{1/2} \quad , \tag{2.4a}$$

or in terms of principal stresses

$$\bar{\sigma} = \sqrt{\frac{1}{2}} \left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \right\}^{1/2} \quad . \tag{2.4b}$$

This definition shows that the equivalent stress for spherical shell collapse is equal to the stress difference $(\sigma_r - \sigma_\theta)$, and for triaxial tests it is equal to the stress difference $(\sigma_1 - \sigma_3)$.

b) Compaction Equations

Three equations are required to define the collapse of a spherical void with time. Two of these equations, the equilibrium condition and the relationship between plastic strain and instantaneous porosity, define the extent of compaction and the externally applied stress at any given time. The third equation, the creep law for the solid material, defines the stress that can be supported by the solid as a function of temperature and time. For simplicity, these equations will be discussed starting with the creep law, then the equilibrium condition, and finally the plastic strain relationship.

Creep constitutive equations are generally of the form[5]

$$\dot{\varepsilon}_1 = \dot{\varepsilon}_1\left(\bar{\sigma}, \varepsilon_1, T\right) \quad , \qquad (2.5)$$

where T is the temperature. The effective stress $\bar{\sigma}$, strain ε_1, and temperature T are functions of t, the time, relative to some initial state of the material. The strain ε_1 in this equation is assumed to be completely nonelastic in nature in the present analysis: i.e., elastic strains are considered negligible. Furthermore, a yield point or a threshold value for $\bar{\sigma}$, below which no creep occurs, has yet to be established as physically realistic and so will not be considered.

Next, the equilibrium equation for a spherical shell,

$$r \frac{d\sigma_r}{dr} + 2\left(\sigma_r - \sigma_\theta\right) = 0 \quad , \qquad (2.6)$$

is written in terms of the equivalent stress to obtain

$$r \frac{d\sigma_r}{dr} + 2\bar{\sigma} = 0 \quad . \tag{2.7}$$

The creep law, Equation (2.5) must be solved for $\bar{\sigma}$, the result substituted into Equation (2.7), and then Equation (2.7) must be integrated to obtain the value of σ_r at $r = b$ (Figure 1) in order to find the externally applied stress that the porous solid can support. Before this step is possible, however, a relationship between ϵ_1, the plastic strain increment, and r is required.

In the absence of any yield point for the solid material, the true strain increment $d\epsilon_\theta$ of radially symmetric deformations is given by

$$d\epsilon_\theta = \frac{dr}{r} \quad , \tag{2.8a}$$

and, according to Equation (2.2c), is also related to the equivalent plastic strain $d\epsilon_\theta = - \frac{d\bar{\epsilon}^P}{2}$. Next, the equivalent strain is related to the strain ϵ_1 of the creep law, (2.2b) so that the relationship between the strain rate $\dot{\epsilon}_1$ and $\dot{r}$ is

$$\dot{\epsilon}_1 = \dot{\bar{\epsilon}}^P = -2 \frac{\dot{r}}{r} \tag{2.8b}$$

Equation (2.8b) can be integrated to give

$$\epsilon_1 = \bar{\epsilon}^P = -2 \ln \frac{r}{r_o} \quad , \tag{2.8c}$$

which in turn can be further simplified by evoking the constant volume condition in the form

$$r_o^3 - r^3 = a_o^3 - a^3 = b_o^3 - b^3 \quad . \tag{2.9}$$

so that

$$\varepsilon_1 = -2 \ln \left\{ \frac{r^3}{r^3 - \left[a^3 - a_o^3\right]} \right\} \quad . \tag{2.10}$$

One can also show that the porosity[‡] η is related to the dimensions of the spherical shell according to

$$\eta = \frac{a^3}{b^3} \quad , \quad \eta_o = \frac{a_o^3}{b_o^3} \quad , \tag{2.11a}$$

$$a^3 = \frac{\eta}{[1 - \eta]} \frac{\left[1 - \eta_o\right]}{\eta_o} a_o^3 \quad ,$$

and (2.11b)

$$b^3 = \frac{1}{[1 - \eta]} \frac{\left[1 - \eta_o\right]}{\eta_o} a_o^3 \quad ,$$

where η_o is the initial porosity. The strain rate $\dot{\varepsilon}_1$ from (2.8b) is related to the rate of change of porosity by

$$\dot{\varepsilon}_1 = -\frac{2}{3} \frac{\dot{\eta}}{\eta_o} \frac{\left[1 - \eta_o\right]}{[1 - \eta]^2} \frac{a_o^3}{r^3} \quad . \tag{2.12}$$

‡Porosity is defined as the fraction of the total volume occupied by the voids. This analysis is not sophisticated enough to consider packing factors and other microscopic void configuration details, all of which are embedded in the porosity parameters.

Equations (2.5), (2.7), and (2.8a) or its various forms define the compaction law. In applying these equations (2.7) is integrated assuming a stress free condition at the inner void surface to obtain the stress on the outer boundary,

$$\sigma_{r=b} = - \int_a^b 2 \frac{\bar{\sigma}}{r} \, dr \quad , \tag{2.13}$$

and the compaction stress is defined as

$$P(t) = -\sigma_{r=b} \quad . \tag{2.14}$$

Examples

Creep response of geological materials is often observed to exhibit all three primary, secondary and tertiary stages of creep. For simplicity, however, application of the compaction theory will be illustrated with a simple steady-state creep law:[5,6]

$$\dot{\epsilon}_1 = B \; e^{-\frac{Q}{RT}} \left(\frac{\bar{\sigma}}{\mu}\right)^{1/n} \tag{3.1}$$

where B, Q, R, μ and n are constants. This law states that the steady-state creep rate is dependent only on the instantaneous values of the temperature and the effective stress, both of which in the general case are time dependent. The next assumption is that (3.1) is a single valued function of the stress so that it can be inverted to

$$\bar{\sigma} = \mu \left(A e^{\frac{Q}{RT}} \dot{\epsilon}_1 \right)^n \tag{3.2}$$

where $A = B^{-1}$ is a constant. This equation and the strain-rate equation

(2.12) can now be substituted into (2.13) to compute the pressure applied to the outer boundary of the spherical shell:

$$\sigma_{r=b} = -2\mu \int_a^b \left[\frac{2A\left[-\dot{\eta}\right]\left[1 - \eta_o\right] e^{\frac{Q}{RT}}}{3\,\eta_o\left[1 - \eta\right]^2} \frac{a_o^{\,3}}{r^3} \right]^n \frac{dr}{r} \qquad (3.3)$$

Since $\dot{\eta}$ and η depend only on $\dot{a}$, $\dot{b}$, and the current shell dimensions, (3.3) can be integrated to give the compaction stress

$$P(t) = -\sigma_{r=b} = \frac{2\mu}{3n} \left[\frac{2Ae^{\frac{Q}{RT}}\left[-\dot{\eta}\right]}{3\left[1 - \eta\right]} \right]^n \left[\frac{1 - \eta^n}{\eta^n} \right] \quad . \qquad (3.4)$$

There are two simple ways of examining the implications of this result: (1) the rate of change of porosity at constant temperature can be assumed constant and various stress-porosity (or stress vs time) curves computed, (2) the external stress can be assumed constant and the porosity computed as a function of time. Computational results for both of these cases have been plotted in Figures 2 and 3 using reasonable values for the various material constants. Material constant values(5,6) for these computations are tabulated in Table 1. The fact that infinite stress is required to completely eliminate all porosity is a natural consequence of this type of analysis.

Summary and Critique

The main objective of this report is to suggest that the creep consolidation of a porous rock can be estimated from the creep properties of the solid constituent from which it is formed. The theoretical approach has been illustrated with a simple spherical void model of an homogeneous isotropic incompressible elastic-plastic material. While these assumptions are clearly oversimplifications of most granular materials, there may be some substance to the theory when rock salt is the backfilling material. Application of the theory is illustrated with a very simple steady-state creep law found to describe some parts of the creep of WIPP rock salt at elevated temperatures.

Application of the theory is also clouded by the fact that the initial state of the backfill is apt to contain highly irregular voids. Additional laboratory information is required under such circumstances, to experimentally define compaction response up to the point where the voids become more spherical in nature, if, in fact, they ever do.

Perhaps the biggest constraint on the current theory is imposed by the incompressibility restriction in connection with classical plasticity. Thus, this theory cannot offer any guidance in the consolidation of granular materials when fracture is an important mode of deformation. Dilation during inelastic deformation is a fracture related phenomenon which likewise does not fit within the framework of the present theory. If, on the other hand the rock exhibits some degree of plasticity, then the present theory may be of some assistance in predicting the extent of consolidation over a given time period.

References

1. "Shock-Wave Compaction of Porous Aluminum," B. M. Butcher, M. M. Carroll, and A. C. Holt, J. Appl. Phys., 45, No. 9, pp. 3864-3875, 1974.

2. "A Nonlinear Theory of Elastic Materials with Voids," J. W. Nunziato and Stephen C. Cowin, Arch. Rat. Mech., 72, pp. 175-201.

3. "Compression of a Spherical Shell of Work-Hardening Material," P. Chadwick, J. Mech. Sci., 5, pp. 165-182, 1963.

4. Strength of Materials, A. A. Ilyushin and V. S. Lensky, Pergamon Press, London, pp. 284-286, 1967.

5. "Preliminary Deformation-Mechanism Map for Salt (With Application to WIPP)," D. E. Munson, Sandia Laboratories Report SAND79-0076, January 1979.

6. "Constitutive Model for the Low Temperature Creep of Salt (With Application to WIPP)," D. E. Munson and P. R. Dawson, Sandia Laboratories Report SAND79-1853, October 1979.

Table 1: Rock Salt Material Constants

$B = 6.47 \times 10^{13}/s$

$Q = 0.042$ MJ/mol (10 kcal/mol)

$n = 0.2$

$\mu = 9.62$ GPa

$R = 8.317$ J/mol°C

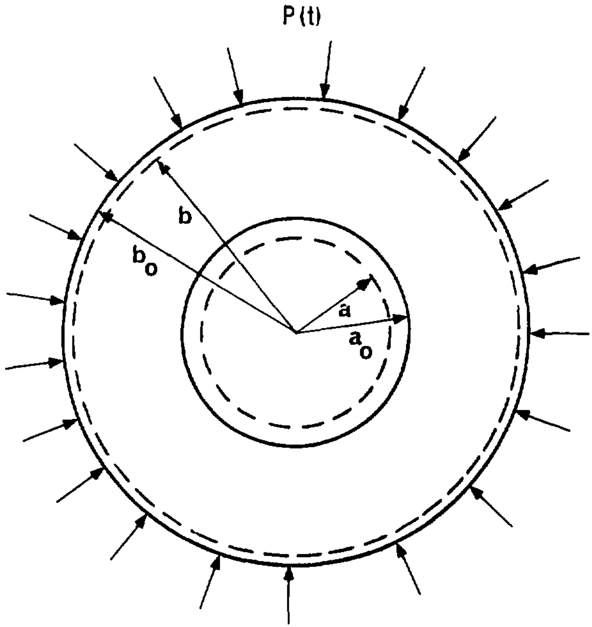

Figure 1: Spherical void model

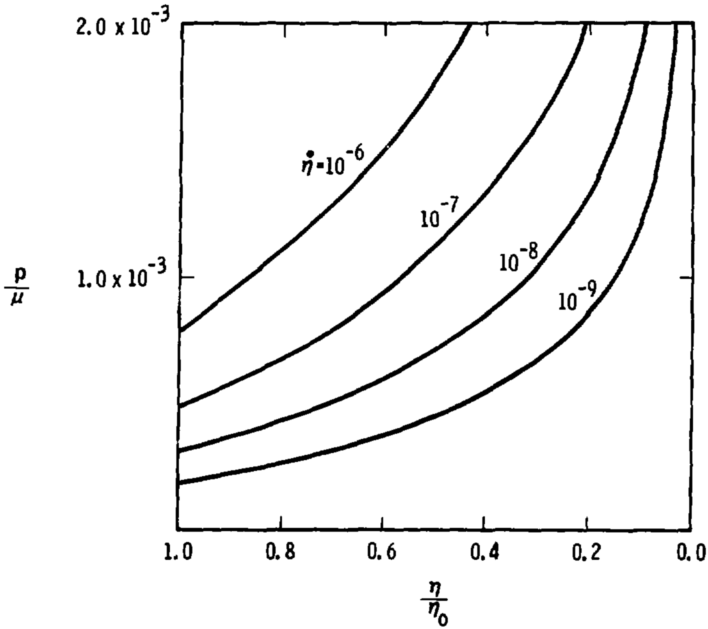

Figure 2: Reduced pressure-porosity curves for various values of constant compaction rate $\dot{\eta}$ assuming the initial porosity η_0 is 0.6. η is the porosity, μ is the shear modulus and P is the pressure supported by the backfill. The fact that the pressure becomes infinite as the porosity approaches zero is a common feature of spherical void models.

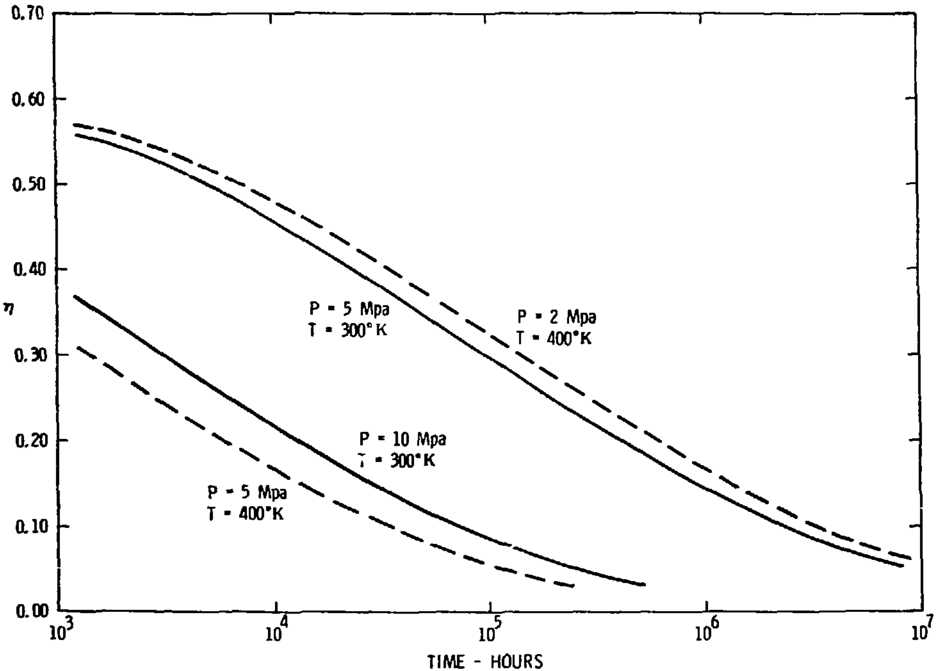

Figure 3: Consolidation time for various values of constant pressure P and temperature T. η is the porosity and an initial porosity η_o of 0.6 is assumed. The time to achieve complete compaction is infinite for spherical void models.