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ABSTRACT

This paper presents a new formulation of a class of nonlinear optimal control problems in which the system's signals are noisy and some system parameters are changing arbitrarily with time. The methodology is validated with an application to a nonlinear nuclear reactor model. A variational technique based on Pontryagin's Maximum Principle is used to filter the noisy signals, estimate the time-varying parameters, and calculate the optimal controls. The reformulation of the variational technique as an initial value problem allows this microprocessor-based algorithm to perform on-line filtering, parameter tracking, and control.

INTRODUCTION

The availability of fast and reliable microprocessors has opened the possibility of performing on-line numerical calculations to improve and optimize the way nuclear reactors are controlled. In particular, there is general interest[1] in increasing the robustness of control algorithms when the system's behavior is affected by nonlinearities, unknown time-varying parameters, or noisy signals.

In this paper we present a new microprocessor-based algorithm that is able to perform on-line filtering, parameter tracking, and control of a nonlinear nuclear reactor. A variational technique based on Pontryagin's Maximum Principle (PMP)[2,3] is used to estimate the system's state and parameters and to calculate the optimal control.

Three simultaneous optimizations are performed: (a) estimates of the state variables are determined by optimal matching of the noisy plant signals to the model, (b) time-varying parameters are obtained by matching the filtered estimates to the control model results, and (c) optimal controls are obtained by matching a set of prescribed demands.

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To validate the algorithm, a nonlinear nuclear reactor was simulated in which the coolant feedback coefficient was an unknown time-varying parameter and the measured signals from the plant had additive noise. The plant was forced to follow demands in neutron density and outlet coolant temperature by adjusting two controls: (a) u1, reactivity, which was constrained to given set of maximum and minimum values, and (b) u2, inlet coolant temperature.

OPTIMAL CONTROL PROBLEM REFORMULATION

This section reviews the reformulation of the free terminal time (FTT) optimal control problem presented by the authors in ref. 2.

Let

$$dy/dt = F(y,u) \tag{1}$$

represent the plant model, where $y(t)$ is the vector of state variables and $u(t)$ is the control vector. Our goal is to find $u(t)$ such that the system will follow a given demand vector, d, in the sense that the cost function defined as

$$J = \int_0^{tf} \{[d - H(y)]^T Q[d - H(y)] + u^T Ru\} dt \tag{2}$$

is minimized. Q and R are weighting matrices, which are allowed to change with time. Note that the initial state vector $y(0)$ and final state $H(y(tf))$ are known but tf is free.

It has been shown[2] that, if the system is in equilibrium at $t=0$ and d is a well-behaved function, the minimization problem is equivalent to the solution of the following set of equations:

$$dy/dt = F(y,u) \quad , \tag{3}$$

$$dw/dt = [dF/dy)^T w - (dH/dy)Q(d - H(y)] \quad , \tag{4}$$

$$u = R^{-1} w^T (dF/du) \quad , \tag{5}$$

$$y(0) = y_0 \ ; \ w(0) = 0 \quad , \tag{6}$$

where w is a vector of Lagrange multipliers or adjoints. The above reformulation of PMP can be achieved by integrating backwards the cost function. Note that the initial conditions are known only if the initial state is in equilibrium.[2] Under this special condition, the calculation of the optimal control can be easily executed on-line on a microprocessor because the classical two-point boundary value problem is recast as an initial value problem.

STATE ESTIMATION: FILTERING

In this section the optimal control reformulation is applied to the problem of estimating the actual state of the plant from detector signals that are corrupted with additive noise.

Assuming that Eq. (1) represents a mathematical description of our system, let us define the vector of signals coming from the plant detectors as

$$s = y_d + n \quad , \tag{7}$$

where y_d represents the vector of detected state variables and n is an additive noise with zero mean.

Defining x as the estimated plant state, the dynamic estimation of x can be obtained from

$$dx/dt = G(x,u) + v \quad , \tag{8}$$

where G is an approximated model of the plant (predictor) and v is the dynamic correction (corrector) that can be obtained from the minimization of the cost function

$$J_F = \int_0^{tf} \{[s - x_d]^T Q_f [s - x_d] + v^T R_f v\} dt \quad . \tag{9}$$

PARAMETER TRACKING

This section shows how the reformulation of the optimal control problem can be used to update the time-varying parameters in the plant.

Let us assume that there is a set of parameters that can change arbitrarily with time. Our control model needs to be updated on-line, otherwise we cannot guarantee that the calculated control will be optimal. The updating of the control model can be achieved by applying once more the reformulation of the optimal control problem presented in the first section of this paper. In this case, we assume that the value of the unknown time-varying parameters, a, will be the one that minimizes the cost function

$$J_p = \int_0^{tf} \{[x - m]^T Q_p [x - m] + a^T R_p a\} dt \quad , \tag{10}$$

where m represents the control model state variables.

OPTIMAL CONTROL

At this point, our algorithm is able to estimate the actual state variables and the time-varying parameter magnitude; therefore, we have an adaptive model to which we can apply again the reformulated optimal control problem to obtain the controls necessary to follow the demands. Therefore, we only need to minimize the following cost function:

$$J_C = \int_0^{tf} \{[d - Hm]^T Q_c [d - Hm] + u^T R_c u\} dt \quad . \tag{11}$$

THE ALGORITHM

As stated before, our goal is to develop an algorithm that is able to update the control model by estimating the state variables and time-varying parameters while, at the same time, calculating the optimal control. This objective is achieved by minimizing the three cost functions J_F [Eq. (9)], J_P [Eq. (10)], and J_C [Eq. (11)] independently and simultaneously. The set of equations to be solved can be summarized as follows:

$$dx/dt = G(x,a,u) + v \quad , \tag{12}$$

$$dm/dt = G(m,a,u) \quad , \tag{13}$$

$$dw_f/dt = (dG/dx)^T w_f - Q_f(s - x_d) \quad , \tag{14}$$

$$dw_p/dt = (dG/dm)^T w_p - Q_p(x - m) \quad , \tag{15}$$

$$dw_c/dt = (dG/dx)^T w_c - (dH(m)/dm) Q_c (d - H(m)) \quad , \tag{16}$$

$$v = (R_f)^{-1} w_f{}^T (dG/dv) \quad , \tag{17}$$

$$a = (R_p)^{-1} w_p{}^T (dG/da) \quad , \tag{18}$$

$$u = (R_c)^{-1} w_c{}^T (dG/du) \quad , \tag{19}$$

$$m(0) = x(0) = y(0) = y_0 \quad , \tag{20}$$

$$w_f(0) = w_p(0) = w_c(0) = 0 \quad . \tag{21}$$

Equations (12) to (16) can be solved simultaneously with the initial conditions given by Eqs. (20) and (21) by using any standard differential equations solver. Equations (17) to (19) represent the filter corrector, v, unknown parameters, a, and optimal controls, u, respectively.

APPLICATION TO A NONLINEAR NUCLEAR REACTOR

The algorithm presented in this paper has been applied to a nonlinear reactor model. The plant was simulated with a four-dimensional model, and three detectors (details about this model can be found in ref. 2). During the transient, the feedback coefficient due to the coolant temperature was artificially changed sinusoidally with time to test the parameter-tracking capabilities of the present technique. In addition, the measured signals from the simulated plant were corrupted with additive noise. The plant was forced to follow demands in neutron density and outlet coolant temperature by adjusting two controls: (a) reactivity, which was constrained to have maximum and minimum values, and (b) inlet coolant temperature.

Figure 1 shows how both demands in power and outlet coolant temperature were followed by the plant despite the parameter changes and constraints in the controls. Figure 2 shows how the algorithm was able to update the coolant temperature feedback coefficient during the transient. Figure 3 shows the controls that were necessary to follow the transient. Finally, Fig. 4. shows how the variational filter eliminates the additive noise from the plant signals.

CONCLUSIONS

An optimal filtering, parameter-tracking, and control algorithm has been developed which allows one to find on-line solutions for demand-following problems in cases in which noisy signals, nonlinearities, and time-varying parameters invalidate other known algorithms. A crucial result has been the recasting of the Pontryagin's Maximum Principle technique for nonlinear systems as the solution of differential equations with initial conditions. The algorithm has been successfully applied to a nonlinear nuclear reactor model. Using the plant sensors signals, all the state variables were estimated, and the time-varying parameter was updated during the transient, allowing an adaptive optimal control calculation. We can conclude that this new formulation of the optimal control problem provides a suitable methodology for on-line, microprocessor-based applications.

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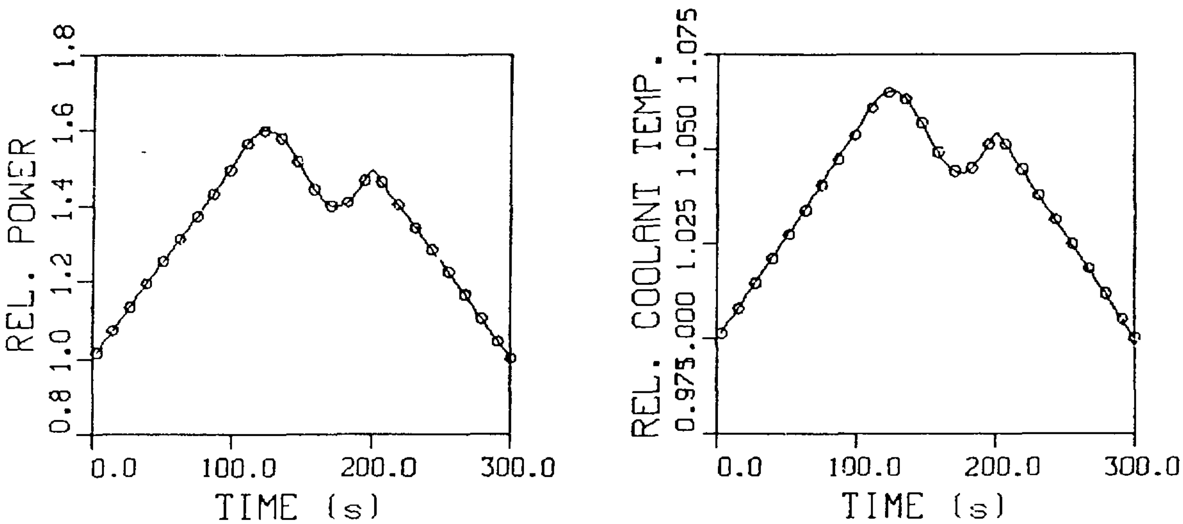

Fig. 1. Demand following: (a) demand on power (solid line) and plant power (circles) versus time, (b) demand on hot leg temperature (solid line) and plant hot leg temperature (circles) vs time.

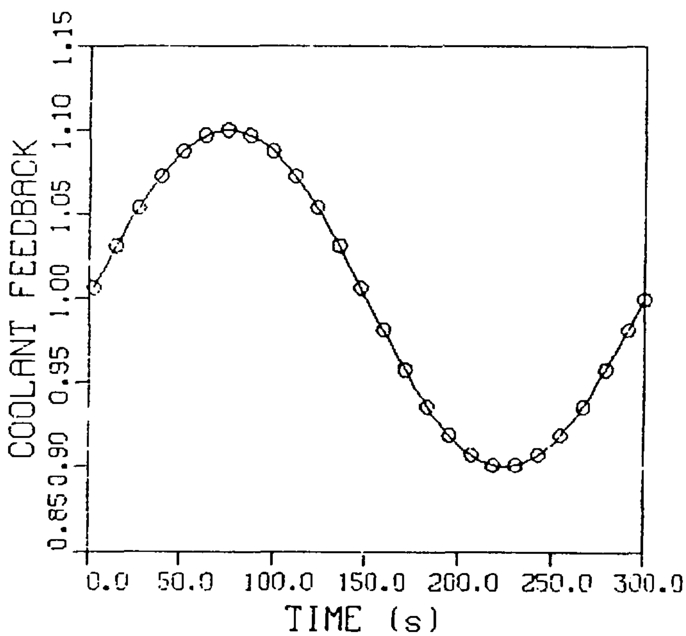

Fig. 2. Time-varying parameter update. Plant coolant feedback coefficient (solid line) and algorithm estimate (circles) vs time.

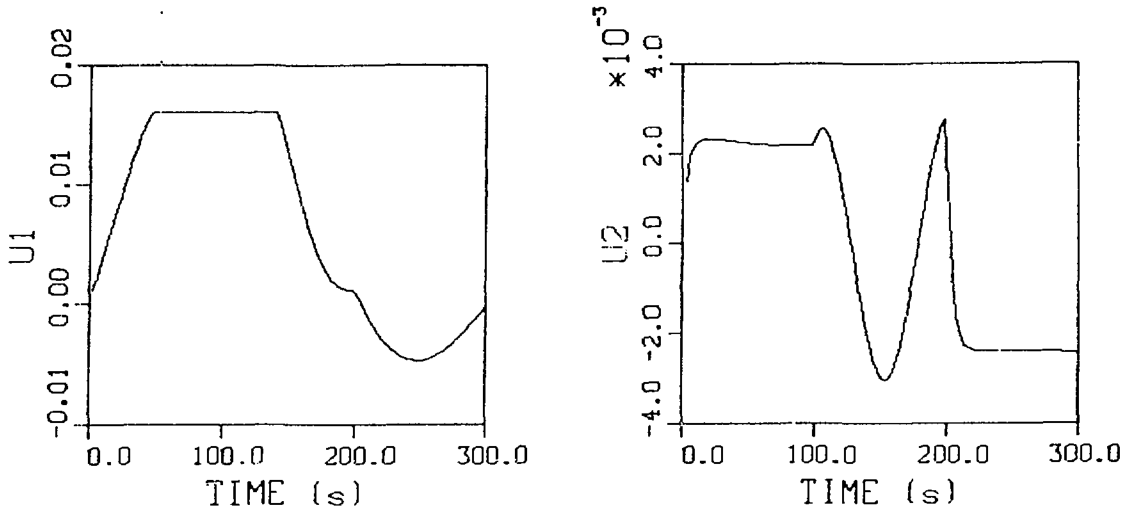

Fig. 3. Optimal controls: (a) u1, reactivity, which was constrained to be less than 0.016, and (b) *u2*, inlet coolant temperature change.

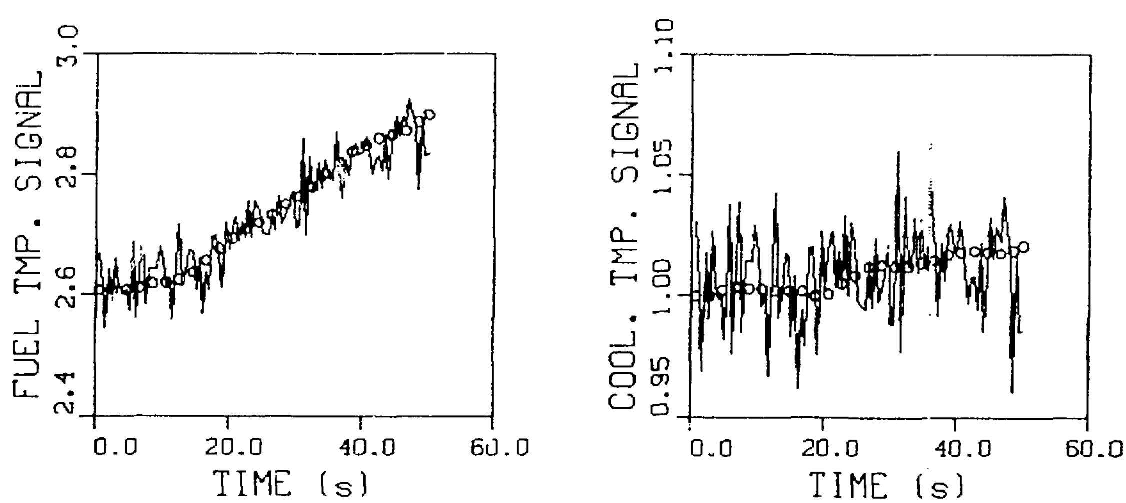

Fig. 4. Optimal filtering: (a) coolant temperature detector signal (solid line) and algorithm filtered estimate (circles) vs time, and (b) fuel temperature detector signal (solid line) and algorithm filtered estimate (circles) vs time.