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THE ECONOMIC VALUE OF GEOTHERMAL RESOURCE ASSESSMENT INFORMATION

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A. The Potential of Decision Analysis for the Evaluation of Resource Assessment Expenditures

Decision theory is a powerful tool which can aid managers in making rational decisions about problems which involve uncertainty or trade-offs among incommensurable objectives. It is not a normative technique. Instead, decision theory structures our intuitive decisionmaking processes and exposes the assumptions and internal judgments that underlie our decisions.

Uncertainty about the consequences of a given action is a characteristic of social systems; human behavior is notoriously unpredictable. In contrast, physical systems exhibit remarkable predictability on a macroscopic scale: falling objects always obey the same law of motion. However, our knowledge of physical systems is frequently as limited and uncertain as our own behavior is unpredictable.

Throughout the course of man's intellectual development, we have created numerous analytical tools. Almost all of these deal exclusively with the world of physical systems and omit the problems associated with our uncertain knowledge about that world. Yet this uncertainty is at the core of the difficulties associated with rational decisionmaking about complex physical and social problems. Decision theory explicitly incorporates uncertainty into its analysis of the options available to us as decisionmakers.

A second consideration human beings must weigh in making decisions is the trade-off between conflicting objectives such as safety and economy or return on investment and risk. Most of the many analytical techniques developed to aid our internal decisionmaking processes either ignore the necessity of balancing conflicting objectives or artificially reduce all of the objectives to a common denominator. The explicit valuation of a human life in dollars and cents represents a particularly grotesque example of such methods. Decision theory, while analytically rigorous, permits a more flexible and, to some, a more sensitive technique for dealing with these questions.

Managers in industry and government who are responsible for the development of geothermal energy frequently face decisions involving the investment of substantial sums in geothermal facilities. These decisions entail considerable financial risk for the funding organization since the success of the facility depends upon the existence and upon the characteristics of the geothermal resource. Despite the fact that these characteristics are determined by physical laws, our knowledge of these characteristics is often highly uncertain.

For example, imagine a DOE official faced with a proposal for resource assessment funding. If he rejects the proposal, firms will decide whether to proceed with projects at that location on the basis of current geological information. If he accepts the proposal, firms will be able to decide better whether to proceed and how to design their facilities. Given that he has a limited budget, the official wishes to fund only those proposals which could yield potential benefits in terms of fossil fuel savings. If the technical quality of the proposal is not a consideration, how much should he be willing to spend on it?

It is unclear how much one should pay for additional information. Intuitively, we can say that the answer depends upon the value of the information to the decisionmaker. Setting aside the possibility of a discrepancy between the objectives of the decisionmaker and those of his organization, this value should be related to the usefulness of the information. If the information cannot affect the decision to be taken, it will only be valuable to the intellectually curious. If, however, the information can materially affect the decision to be made, the information may be extremely valuable.

This conclusion holds whether the decisionmaker is an executive in a firm about to embark upon an expensive drilling program or whether he is a program manager in the Department of Energy reviewing proposals for resource assessment funding. While the objectives of the decisionmakers may differ, the principle remains the same: one should not pay more for information than its value in light of the decision to be made.

Decision theory can be used to derive rigorously the dollar amounts that correspond to the value of geothermal resource information. The following sections describe the procedure in greater detail and provide a numerical example.

B. Value of Information

One of the most important factors involved in the evaluation of geothermal energy is the uncertainty that surrounds the geothermal resource itself. This uncertainty, which relates primarily to the temperature and production flow rate of the resource, is translated into a corresponding uncertainty in the anticipated cost of the project.

A potential user of geothermal energy might wish to reduce this uncertainty by commissioning additional geological studies or even by drilling an initial production well. In the present context, these activities will be termed experiments. However, the user should weigh the cost of such experiments against the potential value of the information to be gained from them.

The value of information derived from experiment to a potential user of geothermal energy depends upon the effect possession of the information will have upon his decisionmaking. At one extreme is the situation in which the decision to utilize geothermal energy and the choice of plant design are made in advance. In this case, possession of information from an experiment cannot alter the expected net present value of the project. At the other extreme lies the situation in which the decision to use geothermal energy and the choice of plant design can be postponed until the information has been obtained. If no experiment is performed, the user will design the plant on the basis of the expected resource characteristics. Thus if resource characteristics are found to differ in actuality from their expected values, the user will have built a suboptimal plant. However, if an experiment is performed, the user can design the plant to make optimum use of the discovered characteristics of the resource.

The value of the information to the user is represented by the difference between the expected value of the project without experimentation and the expected value of the project with experimentation.[1] Therefore, a rational decision-maker who has the opportunity to perform the experiment at a cost less than this value should choose to do so. If the cost of the experiment will exceed the value of the information to be obtained, the decision-maker should proceed without the experiment.

This concept has important implications for the practice of geothermal resource assessment as well. The most important of these implications is that location-specific assessment efforts can be rigorously justified on the basis of the value of the information to be procured. Furthermore, such a justification cannot be made without consideration of a particular application, since the economics of the specific application determine in part the value of the information to be gained from the experiment.

The procedure used to calculate the value of information for a given application is outlined in the next section. An example is then presented in Sections 2 and 3.

1. Calculation of the expected value of information

Among the simplifying assumptions made in the course of the analysis are the following. The net present value of the project is defined as the net present value of cash flows associated with the proposed system minus that associated with a conventional system. It is assumed that only a geothermal system can be used. Under certain circumstances, it is possible that the net present value of the project will be negative. If the user had the option of employing a conventional system, the net present value as defined here could never be below zero. It is also assumed that the user makes decisions solely upon the basis of their monetary consequences. It is further assumed that the user is not risk-averse. This condition is equivalent to the assumption that the value of a marginal dollar is constant regardless of the user's total financial wealth.

The second and third assumptions together imply that the net present value of the project is a sufficient measure of its attractiveness. In the present analysis, this figure is assumed to be a function of the geothermal fluid temperature and of the fluid flow rate used in the plant. [It is assumed that the plant design is optimized in terms of all other parameters.] The temperature is considered to be a random variable described by a probability distribution. The fluid flow rate is viewed as a decision variable under the control of the user since he is free to drill any number of wells into the geothermal reservoir. [The existence of this freedom depends upon the assumption that the reservoir is large compared to the range of possible flow rates.]

Thus

$$V = V(m,t) \quad , \qquad [1]$$

where

V = net present value of cash flows associated with the geothermal system minus that associated with a conventional system

m = mass flow rate of geothermal fluid purchased

t = temperature of the geothermal fluid.

To compute the value of information, one must first calculate the expected value of the project without experimentation and then ascertain the expected value with experimentation.

If no experiment is performed, the user can only attempt to maximize the expected project net present value. Applying the expectation operator (denoted by < >):

$$\langle V \rangle = \langle V(m,t) \rangle \quad . \qquad [2]$$

The maximum figure for this expected value will occur at a mass flow rate which can be found by differentiating with respect to m and setting the result equal to zero:

$$\frac{\partial \langle V \rangle}{\partial m} = 0 \quad . \qquad [3]$$

If the optimal value of m is denoted by $\hat{m}$ and the optimal expected net present value by $\langle \hat{V} \rangle$, the expected project net present value without experimentation can be written as

$$\langle \hat{V} \rangle = \langle V(\hat{m},t) \rangle \quad . \qquad [4]$$

The expected project net present value with experimentation depends upon the accuracy of the experiment. If it is assumed that the experiment yields perfect information (i.e. it determines with certainty the true value of t, or t_{actual}), then after the experiment is performed the user can compute a precise value for V which corresponds to any choice of m:

$$V = V(m, t_{actual}) \quad . \qquad [5]$$

The user would then maximize the net present value by setting the **derivative** of V equal to zero,

$$\frac{\partial V}{\partial m} = 0 \quad , \qquad [6]$$

which gives the optimal value of m, m', and thus the optimal value of V,

$$V' = V(m', t_{actual}) \quad . \qquad [7]$$

However, a priori the user does not know the results of the experiment and thus cannot compute either m' or V'. Nevertheless, he can calculate an expected value:

$$\langle V' \rangle = \langle V'(m', t) \rangle \qquad [8]$$

This figure represents the prior expected project net present value if a perfect experiment is performed.

The expected value of perfect information is defined to be the difference between the expected net present value if the best flow rate is chosen after perfect information is obtained and the expected net present value if the best flow rate is chosen without the benefit of further information. Thus

$$\text{EVPI} = \langle V' \rangle - \langle \hat{V} \rangle \quad , \tag{9}$$

where

EVPI = expected value of perfect information.

If it is assumed that the experiment yields imperfect information (i.e. it determines only a "narrower" probability distribution for t than that already established), then the computation of the expected value of information becomes somewhat more complicated.

The experiment gives a result τ^* for the resource temperature. However, this is only indicative of what the true value might be since the experiment is imperfect. After the experiment has been performed, the user is in an analogous situation to that if no experiment had been performed: he can only attempt to optimize the expected value of the project. Thus taking the expected value,

$$\langle V \rangle = \int V(m,\tau) P(\tau|\tau^*) \, d\tau \quad , \tag{10}$$

where

$P(\tau|\tau^*)$ = probability of the actual temperature being τ given that the experimental result was τ^*.

The user would then maximize the expected value by setting the derivative with respect to m equal to zero;

$$\frac{\partial \langle V \rangle}{\partial m} = 0 \quad , \tag{11}$$

which yields the optimum flow rate m" and the corresponding value V" in terms of the experimental result τ^*.

However, *a priori* the user does not know what τ^* will be. He can take an expected value, giving

$$\langle V'' \rangle = \int V''(\tau^*) P(\tau^*) \, d\tau^* , \qquad [12]$$

where

$\langle V'' \rangle$ = prior expected net present value of project if an imperfect experiment is performed.

$V''(\tau^*)$ = optimum project value as a function of the experimental result τ^*.

$P(\tau^*)$ = probability of the experimental result being τ^*.

The expected value of imperfect information is defined to be the difference between the expected net present value if the best flow rate is chosen on the basis of imperfect information and the expected net present value if the best flow rate is chosen without the benefit of further information. Thus

$$\text{EVIPI} = \langle V'' \rangle - \langle \hat{V} \rangle , \qquad [13]$$

where

EVIPI = expected value of imperfect information.

2. Example calculation of the expected value of perfect information

The net present value information required for the calculation of the EVPI (and the EVIPI) is determined by utilizing the computer model described in Reference 2.

For this example, the model is run using data from the second part of Chapter 6 of Reference 2. One modification is made in these data: it is assumed that the user pays for geothermal fluid on a "per unit mass" basis rather than investing directly in geothermal resource development. While the assumption of a constant price per unit mass may not be realistic over a wide range of flow rates, it is probably reasonable for the more limited range of flow rates of interest in this analysis.

The results for net present value as a function of mass flow rate and fluid temperature (as shown in Fig. 1) are represented by the following expression:

$$V = a_1 + a_2 m + a_3 m^2 + a_4 m^3 \quad , \qquad [14]$$

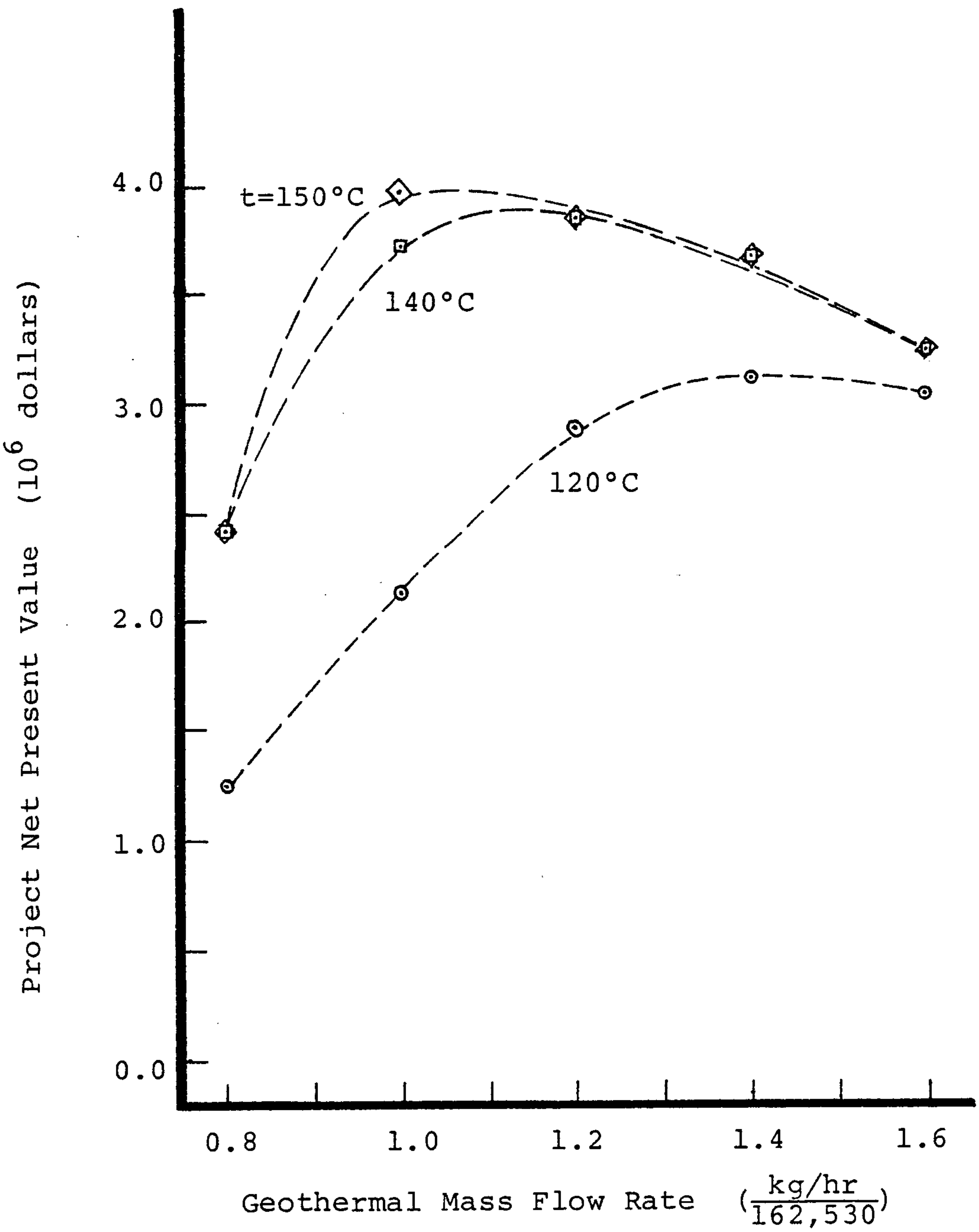

FIGURE 1 - NET PRESENT VALUE AS A FUNCTION OF GEOTHERMAL FLUID TEMPERATURE AND FLOW RATE

where for the sake of convenience V is in millions of dollars and m is normalized by division by 162,530 kg/hr, and where the coefficients a_i are functions of temperature. For each fluid temperature a least-squares analysis is used to determine the values of a a_2, a_3 and a_4 The coefficients are then cross-plotted against temperature. A linear least-squares analysis is then employed to yield expressions of the form

$$a_i = a_{i1} + a_{i2}\,\tau \quad , \qquad [15]$$

where

$$\tau = t-60 \quad .$$

The reason for using τ instead of t will become clear in the following discussion. The coefficients determined by the present analysis are given in Table 1.

The next step is to assume a probability density function for the geothermal fluid temperature. On the basis of the geological record for Ontario, Oregon (the site of the facility in this example), it is extremely unlikely that this temperature would be less than approximately 60°C. A two-tailed probability function is therefore inappropriate for modeling the distribution of possible

	$j=1$	$j=2$
a_{1j}	51.43	-0.9132
a_{2j}	-146.79	2.5378
a_{3j}	129.70	-2.1302
a_{4j}	-35.66	0.5651
d_{1j}	1.941	-8.961E-3

	b_i	c_i
$i = 1$	-17.06	-5.632
$i = 2$	43.54	0.535
$i = 3$	-30.07	-0.012
$i = 4$	6.72	1.189E-4
$i = 5$		-4.066E-7

Note: V = net present value in millions of dollars

$$m = \frac{\text{geothermal fluid flow rate, kg/hr}}{162{,}530}$$

τ = (geothermal fluid temperature, °C) - 60 .

TABLE 1 - COEFFICIENTS USED IN THE VALUE OF INFORMATION ANALYSIS

fluid temperatures. A more suitable distribution is the log-normal function,

$$P(\tau) = \alpha\tau^{\beta} \exp[-\gamma(\ell n\ \tau)^2] \quad \text{for } \tau>0 \quad , \qquad [16]$$

where

$P(\tau)$ = probability of the fluid temperature being equal to τ

α,β,γ = constants. Note that normalization of the probability function forces

$$\alpha = (\frac{\gamma}{\pi})^{1/2} \exp[-\frac{(\beta+1)^2}{4\gamma}] \quad .$$

Since the probability approaches zero as τ approaches zero and thus as t approaches 60°C, this function has the appropriate form. It can be shown by performing the following analysis with a different probability function that the ultimate results are relatively insensitive to the particular functional form assumed.

For the sake of completeness, it should be noted that use of the log-normal function implies that

$$\langle\tau\rangle = \exp\left(\frac{2\beta+3}{4\gamma}\right) \qquad [17]$$

$$\sigma^2(\tau) = \exp\left(\frac{2\beta+3}{2\gamma}\right) [\exp(\frac{1}{2\gamma}) - 1] \qquad [18]$$

and

$$\nu^2(\tau) = \exp(\frac{1}{2\gamma}) - 1 \quad , \qquad [19]$$

where

$\sigma(\tau)$ = standard deviation of τ

$\nu(\tau)$ = coefficient of variation of τ

$= \frac{\sigma(\tau)}{\langle\tau\rangle}$.

It is assumed for the purposes of this analysis that t equals 135°C ± 30%. However, it is ambiguous to state the uncertainty in terms of ±30% of the temperature. A more precise statement would be that three standard deviations equal 30% of the mean, or

$$\nu(\tau) = \frac{1}{3} \times 0.30 = 0.10 \quad . \qquad [20]$$

From Eqs. 17 and 19,

$\beta = 432.40$

$\gamma = 50.25$.

Now the expected net present value of the project in the absence of experimentation can be evaluated. By definition, the expected value of V is

$$\langle V\rangle = \int_0^\infty V(m,\tau)P(\tau)\, d\tau \quad . \qquad [21]$$

Replacing $V(m,\tau)$ by Eqs. 14 and 15, $P(\tau)$ by Eq. 16, and changing variables such that $x = \ln \tau$ yields

$$\langle V \rangle = \int_{-\infty}^{\infty} \alpha(C + De^{x}) \exp[(\beta+1)x - \gamma x^{2}]\, dx \quad , \qquad [22]$$

where

$$C = \sum_{i=1}^{4} a_{i1} m^{i-1} \qquad [23]$$

$$D = \sum_{i=1}^{4} a_{i2} m^{i-1} \quad . \qquad [24]$$

Separating the two terms and completing the squares in the exponents gives

$$\langle V \rangle = \int_{-\infty}^{\infty} \alpha C \exp\left[-\gamma\left(x - \frac{\beta+1}{2\gamma}\right)^{2} + \frac{(\beta+1)^{2}}{4\gamma}\right] dx \, +$$

$$+ \int_{-\infty}^{\infty} \alpha D \exp\left[-\gamma\left(x - \frac{\beta+2}{2\gamma}\right)^{2} + \frac{(\beta+2)^{2}}{4\gamma}\right] dx \quad . \qquad [25]$$

If the constant term in each integral is extracted and variables changed once again such that

$$r = \sqrt{2\gamma}\left(x - \frac{\beta+1}{2\gamma}\right) \qquad [26]$$

and

$$s \;=\; 2\gamma\ (x - \frac{\beta+2}{2\gamma}) \quad , \qquad [27]$$

then Eq. 25 can be reformulated as

$$\langle V\rangle \;=\; \frac{\alpha C}{\sqrt{2\gamma}} \exp\ [\frac{(\beta+1)^2}{4\gamma}] \int_{-\infty}^{\infty} \exp\ [-\ \frac{r^2}{2}]\ dr \ +$$

$$+\ \frac{\alpha D}{\sqrt{2\gamma}} \exp\ [\frac{(\beta+2)^2}{4\gamma}] \int_{-\infty}^{\infty} \exp\ [-\ \frac{s^2}{2}\ ]\ ds \quad . \qquad [28]$$

But these integrals are each $\sqrt{2\pi}$ times the integral of the normal probability distribution over the entire domain. Since the latter integral must equal unity, the given integrals both equal $\sqrt{2\pi}$. Performing this substitution and replacing α by its value in terms of β and γ (given below Eq. 16) yields

$$\langle V\rangle \;=\; C + D\ \exp[\frac{2\beta+3}{4\gamma}]$$

or

$$\langle V\rangle \;=\; C + D\langle\tau\rangle \quad . \qquad [29]$$

Thus it can be seen that

$$\langle V\rangle \;=\; V(m,\langle\tau\rangle) \quad . \qquad [30]$$

To evaluate Eq. 3, define

$$b_i = a_{i1} + a_{i2} \langle \tau \rangle \quad . \qquad [31]$$

The values of the b_i for this example are listed in Table 1. Thus

$$\langle V \rangle = \sum_{i=1}^{4} b_i m^{i-1} \quad , \qquad [32]$$

and

$$\frac{\partial \langle V \rangle}{\partial m} = 0 = b_2 + 2b_3 m + 3b_4 m^2 \quad . \qquad [33]$$

Finally, the optimal value for m in the absence of an experiment, $\hat{m}$, can be determined to be

$$\hat{m} = \frac{-b_3 - \sqrt{{}_3^{\,2} - 3\;{}_2b_4}}{3b_4} \qquad [34]$$

$$= 1.239 \quad ,$$

since only the smaller root is of significance in this example. Substituting this result into Eq. 32 then yields the expected project value without experimentation:

$$\langle \hat{V}(\hat{m},\tau) \rangle = 3.521 \text{ (dollars x } 10^{-6}) \quad . \qquad [35]$$

Next, the expected net present value of the project with experimentation must be evaluated. The optimal value of m after the experiment has been performed is found by differentiating V:

$$\frac{\partial V}{\partial m} = 0 = a_2 + 2a_3 m + 3a_4 m^2 \quad , \qquad [36]$$

or, solving,

$$m' = \frac{-a_3 - \sqrt{a_3^2 - 3a_2 a_4}}{3a_4} \quad , \qquad [37]$$

where it is again the smaller root which is of interest. This expression is awkward for the manipulations to follow. Since m' is a function of τ_{actual} through the coefficients a_i, the equation above may therefore be replaced by the linear relation

$$m' = d_{11} + d_{12}\,\tau \quad , \qquad [38]$$

which gives figures within 2.5% of those derived from Eq. 37 throughout the relevant domain. [The values of d_{11} and d_{12} for this example are given in Table 1.]

Then

$$V' = \sum_{i=1}^{4} (a_{i1} + a_{i2}\,\tau)(d_{11} + d_{12}\,\tau)^{i-1} \quad , \qquad [39]$$

or, rearranging,

$$V' = \sum_{i=1}^{5} c_i \tau^{i-1} \quad , \qquad [40]$$

where

$$c_1' = (a_{11} + a_{21}d_{11} + a_{31}d_{11}^2 + a_{41}d_{11}^3)$$

$$c_2 = (a_{12} + a_{21}d_{12} + a_{22}d_{11} + 2a_{31}d_{11}d_{12} + a_{32}d_{11}^2 + 3a_{41}d_{11}^2 d_{12} + a_{42}d_{11}^3)$$

$$c_3 = (a_{22}d_{12} + a_{31}d_{12}^2 + 2a_{32}d_{11}d_{12} + 3a_{41}d_{11}d_{12}^2 + 3a_{42}d_{11}^2 d_{12})$$

$$c_4 = (a_{32}d_{12}^2 + a_{41}d_{12}^3 + 3a_{42}d_{11}d_{12}^2)$$

$$c_5 = (a_{42}d_{12}^3) \quad .$$

Values for the c_i are given in Table 1. The expected value of V' is

$$\langle V' \rangle = \int_0^\infty (\sum_{i=1}^{5} c_i \tau^{i-1}) \alpha\, \tau^\beta \exp[-\gamma(\ell n\, \tau)^2]\, d\tau \quad . \qquad [41]$$

Following the same procedure as was used above, change variables such that $x = \ell n\ \tau$, separate terms, complete the squares in the exponents, and extract the constants from the integrals. Next, change variables once again such that

$$r = \sqrt{2\gamma}\,(x - \frac{\beta+1}{2\gamma})$$

$$s = \sqrt{2\gamma}\,(x - \frac{\beta+2}{2\gamma})$$

and analogous expressions are used for the remaining integrals. Each integral now equals $\sqrt{2\pi}$. Finally, substituting for α yields

$$\begin{aligned} \langle V' \rangle = \; & c_1 + c_2 \exp[\frac{2\beta+3}{4\gamma}] + c_3 \exp[\frac{\beta+2}{\gamma}] \\ & + c_4 \exp[\frac{6\beta+15}{4\gamma}] + c_5 \exp[\frac{2\beta+6}{\gamma}] \quad . \end{aligned} \tag{42}$$

It can be shown by use of Eqs. 17 and 19 that this is identical to

$$\begin{aligned} \langle V' \rangle = \; & c_1 + c_2\langle\tau\rangle + c_3\langle\tau\rangle^2\,(\nu^2+1) \\ & + c_4\langle\tau\rangle^3(\nu^2+1)^3 + c_5\langle\tau\rangle^4(\nu^2+1)^6 \quad . \end{aligned} \tag{43}$$

Evaluation of this expression reveals that for this example,

$$\langle V' \rangle = 3.561 \text{ (dollars x } 10^{-6}). \tag{44}$$

One can now compute the expected value of perfect information using Eqs. 35 and 44. Substitution of these equations into Eq. 9 results in

$$\begin{aligned} \text{EVPI} &= \$3.561 \times 10^6 - \$3.521 \times 10^6 \\ &= \$40{,}700 \quad . \end{aligned} \qquad [45]$$

3. Example calculation of the expected value of imperfect information

The underlying data and the equations for project net present value assumed above will also be adopted in this section. The development below utilizes several elements of the analysis presented in Reference 3.

The knowledge possessed by the user about the experiment to be performed is encoded in the following log-normal probability distribution:

$$P(\tau^*|\tau) = \alpha^* \tau^{*\beta^*} \exp[-\gamma^* (\ln \tau^*)^2] \quad , \qquad [46]$$

where

$P(\tau^*|\tau)$ = probability of an experimental result τ^* given that the true value is τ.

$\alpha^*, \beta^*, \gamma^*$ = constants.

This distribution represents a "calibration" of the precision of the experiment. If this cannot be specified, the experiment is probably poorly designed.

Two additional assumptions about this distribution can be made. First, it is assumed that no systematic bias is present, or

$$\langle \tau^* | \tau \rangle = \tau \quad , \qquad [47]$$

and second, it is assumed that the relative accuracy of the experiment is a constant, or

$$\nu(\tau^* | \tau) = \text{constant} \quad . \qquad [48]$$

Using the properties of the log-normal distribution given in Eqs. 17 and 19, one can show that for these two assumptions to hold, the following relations must be true:

$$\begin{aligned} \beta^* &= 2\gamma^* \ln \tau - 3/2 \\ \gamma^* &= \text{constant} \, . \end{aligned} \qquad [49]$$

In order to evaluate the probability distribution $P(\tau|\tau^*)$ in Eq. 10, note that by Bayes' Theorem

$$P(\tau | \tau^*) = \frac{P(\tau) P(\tau^* | \tau)}{P(\tau^*)} \quad . \qquad [50]$$

However, $P(\tau^*)$ must first be found to evaluate this expression. By definition,

$$P(\tau^*) = \int_0^\infty P(\tau^* | \tau) P(\tau) \, d\tau \quad . \qquad [51]$$

Substitution of Eqs. 16 and 46 yields (upon expansion of the expressions for α and α^*)

$$P(\tau^*) = \left(\frac{\gamma\gamma^*}{\pi^2}\right)^{\frac{1}{2}} \int_0^\infty \exp\left[-\frac{(\beta+1)^2}{4\gamma} - \frac{(\beta^*+1)^2}{4\gamma^*}\right] \tau^\beta \tau^{*\beta^*} \exp\left[-\gamma(\ln\tau)^2 - \gamma^*(\ln\tau^*)^2\right] d\tau \quad . \qquad [52]$$

Then substitution of $x = \ln\tau$, $x^* = \ln\tau^*$, Eq. 49, and finally rearrangement gives

$$P(\tau^*) = \left(\frac{\gamma\gamma^*}{\pi^2}\right)^{\frac{1}{2}} \int_{-\infty}^\infty \exp\left\{-(\gamma+\gamma^*)x^2 + (\beta+\frac{3}{2}+2\gamma^*x^*)x - \left(\frac{1}{16\gamma^*} + \frac{(\beta+1)^2}{4\gamma} + \frac{3}{2}x^* + \gamma^*x^{*2}\right)\right\} dx \quad . \qquad [53]$$

Following the same technique of integration used previously, this expression becomes

$$P(\tau^*) = \left(\frac{\gamma\gamma^*}{\pi(\gamma+\gamma^*)}\right)^{\frac{1}{2}} \exp\left[-\frac{1}{16\gamma^*} + \frac{(\beta+3/2)^2}{4(\gamma+\gamma^*)} - \frac{(\beta+1)^2}{4\gamma}\right] \tau^{*\left[\frac{\gamma^*(\beta+3/2)}{(\gamma+\gamma^*)} - \frac{3}{2}\right]} \exp\left[-\frac{\gamma\gamma^*}{(\gamma+\gamma^*)}(\ln\tau^*)^2\right] \quad . \qquad [54]$$

Note that this is a log-normal distribution with effective parameters

$$\beta_{eff} = \frac{\gamma^*(\beta+3/2)}{(\gamma+\gamma^*)} - 3/2$$
$$\gamma_{eff} = \frac{\gamma\gamma^*}{(\gamma+\gamma^*)} \quad , \tag{55}$$

and that as expected

$$<\tau^*> = <\tau> \quad . \tag{56}$$

The next step is to evaluate $P(\tau|\tau^*)$ using Bayes' Theorem (Eq. 50). Substitution of Eqs. 16, 46, 49, and 54 into Eq. 50 yields after some algebra

$$P(\tau|\tau^*) = \left(\frac{\gamma+\gamma^*}{\pi}\right)^{\frac{1}{2}} \tau^{\beta}\, \tau^{*\left[2\gamma^*\ell n\ \tau - \frac{\gamma^*(\beta+3/2)}{(\gamma+\gamma^*)}\right]}$$
$$\exp\left\{\tfrac{1}{2}\ell n\ \tau - (\gamma+\gamma^*)\ (\ell n\ \tau)^2 - \frac{\gamma^{*2}}{(\gamma+\gamma^*)}\ (\ell n\ \tau^*)^2 - \frac{(\beta+3/2)^2}{4(\gamma+\gamma^*)}\right\} \quad . \tag{57}$$

With these results in hand one can evaluate Eq. 10 for the posterior expected value of the project given an experimental result τ^*. Since from Eqs. 23 and 24

$$V(m,\tau) = C + D\tau \quad , \tag{58}$$

where C and D are functions of m, Eq. 10 can be written as

$$\langle V\rangle = \int (C+D\tau)P(\tau|\tau^*)\,d\tau \quad . \qquad [59]$$

Substitution of Eq. 57 into this equation, transformation of variables such that $x = \ln \tau$ and $x^* = \ln \tau^*$, and completion of squares in the exponents finally yields after some manipulation

$$\langle V\rangle = \left(\frac{\gamma+\gamma^*}{\pi}\right)^{\frac{1}{2}} \left\{ \int_{-\infty}^{\infty} C \exp\left[-(\gamma+\gamma^*)\left(x - \frac{2\gamma^* x^* + \beta + 3/2}{2(\gamma+\gamma^*)}\right)^2\right] dx + \int_{-\infty}^{\infty} D \exp\left[-(\gamma+\gamma^*)\left(x - \frac{2\gamma^* x^* + \beta + 5/2}{2(\gamma+\gamma^*)}\right)^2\right] \exp\left[\frac{\gamma^* x^* + \frac{1}{2}\beta + 1}{(\gamma+\gamma^*)}\right] dx \right\} \quad . \qquad [60]$$

Integration then results in the interesting and remarkably simple expression

$$\langle V\rangle = C + D\tau'' \qquad [61]$$

where

$$\tau'' = \exp\left[\frac{\gamma^* \ln \tau^* + \frac{1}{2}\beta + 1}{(\gamma+\gamma^*)}\right] \quad . \qquad [62]$$

Next, the maximum value of <V> must be found by means of Eq. 11. First, however, define

$$f_i \quad = \quad a_{i1} + a_{i2}\,\tau" \quad , \tag{63}$$

so that

$$\langle V\rangle \quad = \quad \sum_{i=1}^{4} f_i m^{i-1} \quad . \tag{64}$$

Then

$$\frac{\partial\langle V\rangle}{\partial m} \quad = \quad 0 \quad = \quad f_2 + 2f_3 m + 3f_4 m^2 \quad , \tag{65}$$

or solving

$$m" \quad = \quad \frac{-\,f_3 - \sqrt{f_3^2 - 3f_2 f_4}}{3f_4} \quad , \tag{66}$$

where m" is the flow rate which maximizes <V> and m" is a function of $\tau"$ through the coefficients f_i.

Note here the analogy between Eq. 37 and Eq. 66. The expressions are identical if $\tau"$ in the latter case is considered to be analogous to τ in the former. This analogy can then be exploited by recognizing that the linearization of Eq. 37 will also hold for Eq. 66 if $\tau"$ is substituted for τ:

$$m" \quad = \quad d_{11} + d_{12}\,\tau" \quad . \tag{67}$$

Following the analogy through Eqs. 39 and 40 demonstrates that

$$V'' = \sum_{i=1}^{5} c_i (\tau'')^{i-1} \quad , \qquad [68]$$

where the coefficients c_i have the same numerical values as before.

The last step is to evaluate Eq. 12 for the prior expected value if an imperfect experiment is performed by use of Eqs. 54, 55, 62, and 68. Substitution of these latter gives

$$\langle V'' \rangle = \int_o^\infty \left\{ \sum_{i=1}^{5} c_i \left[\exp \left(\frac{\gamma^* \ell n \ \tau^* + \tfrac{1}{2}\beta + 1}{(\gamma + \gamma^*)} \right) \right]^{i-1} \right. \qquad [69]$$

$$\left. \alpha_{eff} \tau^{*\beta_{eff}} \exp \left[- \gamma_{eff} \ (\ell n \ \tau^*)^2 \right] \right\} d\tau^* \quad .$$

After expansion term-by-term, a change of variables such that $x = \ell n \ \tau$ and $x^* = \ell n \ \tau^*$, completion of the squares in the exponents, another change of variables as in Eq. 26, integration, and further algebra involving Eq. 55, this equation becomes

$$\langle V'' \rangle = c_1 + c_2 \exp[\frac{2\beta+3}{4\gamma}] + c_3 \exp \left[\frac{\beta+2}{\gamma} - \right.$$

$$\left. \frac{1}{2(\gamma+\gamma^*)} \right] + c_4 \exp \left[\frac{6\beta+9}{4\gamma} + \frac{3\gamma^*}{2\gamma(\gamma+\gamma^*)} \right]$$

$$+ c_5 \exp\left[\frac{2\beta+4}{\gamma+\gamma^*} + \frac{2\beta\gamma^*+6\gamma^*-6}{\gamma(\gamma+\gamma^*)}\right] \quad . \qquad [70]$$

Finally, by utilizing the definitions of Eqs. 17 and 19, this equation can be shown to be identical to

$$\begin{aligned} \langle V'' \rangle \;=\; & c_1 + c_2\langle\tau\rangle + c_3\langle\tau\rangle^2(\nu^2+1)^{[\gamma^*/(\gamma+\gamma^*)]} \\ & + c_4\langle\tau\rangle^3(\nu^2+1)^{[3\gamma^*/(\gamma+\gamma^*)]} \qquad [71] \\ & + c_5\langle\tau\rangle^4(\nu^2+1)^{[6\gamma^*/(\gamma+\gamma^*)]} \quad . \end{aligned}$$

Note from the form of the definition of $\nu(\tau^*|\tau)$ that as the precision of the experiment improves, $\nu(\tau^*|\tau)$ decreases and γ^* increases. Thus, in the limit as the imperfect experiment approaches the precision of the perfect experiment, Eq. 71 approaches the form of the analogous expression Eq. 43.

If it is assumed as before that

$$\begin{aligned} \langle\tau\rangle &= 75 \\ \nu^2(\tau) &= 0.01 \end{aligned} \qquad [72]$$

and additionally that

$$\nu^2(\tau^*|\tau) = 0.005 \quad , \qquad [73]$$

then

$$\gamma^* = 100.25 \qquad [74]$$

and

$$\langle V'' \rangle = 3.547 \text{ (dollars} \times 10^{-6}) \quad . \qquad [75]$$

Finally, from Eq. 13 the expected value of imperfect information is

$$\begin{aligned} \text{EVIPI} &= \$3.547 \times 10^{6} - \$3.521 \times 10^{6} \\ &= \$26,600 \quad . \end{aligned} \qquad [76]$$

REFERENCES

1. For additional reading concerning the value of information, see Raiffa, Howard, Decision Analysis - Introductory Lectures on Choices Under Uncertainty, Reading, MA: Addison Wesley, 1968.

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