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On social sensitivity to either zealot or independent minorities

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ABSTRACT

Individuals act in their own self-interest, but in so doing contribute to the observed wellbeing of society, as determined using the self-organized temporal criticality (SOTC) model. This model identifies the timing of crucial events as a new mechanism with which to generate criticality, thereby establishing a way for the internal dynamics of the decision making process to suppress the sensitivity of social opinion to either zealot or independent minorities. We find that the sensitivity to the influence of zealots is much smaller than in the case of criticality with a fine tuning control parameter and the action of independent minorities may affect temporal complexity so as to realize the condition of ideal $1/f$ noise.

1. Introduction

The role played by committed minorities, *zealots* or *fanatics*, in the behavior adopted by large groups, whether it is in the apparently frivolous taking on of a fad or fashion, or the more serious adoption of new social conventions, has attracted the attention of a significant number of sociologists [1–3], physicists [4,5], network scientists [6–10], and engineers [11], in addition to scientists working in many other disciplines. These investigators explore, using a variety of models from multiple vantage points, how in times of crisis, committed activists may produce political, or other, changes of significant importance to society, in spite of their relatively small number. A common feature of these models is criticality, at which point the aggregate of individuals becomes a collective with a single purpose, and under the right conditions the zealots can leverage the organized behavior to redirect the collective. We observe that in a system of finite size the global consensus state is not permanent and times of crisis occur when there is an ambiguity concerning a given social issue. The correlation function within the cooperative system becomes similarly extended as it is observed at criticality. This combination of independence (free will) and long-range correlation makes it possible for very small, but committed minorities to produce substantial changes in social consensus, see *e.g.* [10].

On the other hand, fluctuations are assumed to be generated by the same form of self-organization that brought the system to criticality in the first place. This assumption is frequently made by researchers studying the dynamics of the human brain [12–15] leaving open, however, the origin of criticality in this context. Allegrini et al. [16] emphasized that the intermittent nature of these fluctuations, according to the prediction that the inverse power-law (IPL) spectrum:

$$S(f) \propto 1/f^{\beta}, \tag{1}$$

with the IPL index,

$$\beta = 3 - \mu, \tag{2}$$

should lead to the ideal $1/f$ - noise condition $\beta = 1$ for $\mu = 2$. The IPL index μ labels the time intervals between crucial events [10] at the tipping point (critical point of a phase transition); the three dimensional Ising model [17] generates $\mu = 1.55$, whereas the decision making model (DMM) [4] yields $\mu = 1.5$ at criticality.

Xie et al. [18] studied the influence of inflexible individuals on social behavior, using the Naming Game to model the social interaction, and found that when the committed minority reaches a threshold of 10% of the population the opinion of the entire social network can be reversed to conform to that of the minority. The theoretical results were shown to be supported by laboratory experiment [1]. The theoretical influence of the minority was also shown to be largely independent of the structure of the interactions within the social model, but can be determined by as much as 10% to as little as 4% for a sparse network [19]. The percentage at which the tipping point occurs is clearly model dependent and can vary from 4% to 15% [20,21].

In this paper we consider also another kind of minority, the minority of *independents*. An independent is an individual who makes

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her choices with no influence from her nearest neighbor. In the long-time scale the behavior of the independent looks erratic and she exerts an influence on society, because their nearest neighbors make their choice taking into account also the erratic choices of the independent.

The analysis herein is based on the form of self-organization, called Self-Organized Temporal Criticality (SOTC) recently proposed in [22]. The individuals of this society have to make a choice between cooperation and defection. This paper shows that the bottom-up form of spontaneous organization described by SOTC strongly reduces the efficiency of the committed minority in redirecting the behavior of society. We show that the SOTC model also disrupts the action of independents, paying however the price of changing the IPL index μ that provides a measure of the system's complexity. This is an important finite size effect and its discussion makes this paper meet the request of the call for papers [23].

In Section 2 we adapt the linked concepts of intuition and deliberation by constructing a dynamic two-level network model, where single individuals are located at the two-dimensional lattice nodes of a composite network. The composite network consists of two interacting subnetworks. One subnetwork is based on the decision making model (DMM) [10] and leads to strategy choices made by the individuals under the influence of the choices of their nearest neighbors. The other subnetwork measures the Prisoner's Dilemma Game (PDG) payoffs of these choices [24]. The interaction between the two subnetworks is carried out by increasing or decreasing the individual imitation strength K_r according to the history of payoffs to that individual. This is a generalization of the self-organized criticality (SOC) model [25], called the self-organized temporal criticality (SOTC) model [22,26].

In the SOTC model the decisions made by individuals are assumed to be consistent with the criterion of bounded rationality [27], which were expanded by Kahneman [28], and more recently discussed from the perspective of evolutionary game theory [29,30]. Rand and Nowak [29] acknowledge the tension between what is good for the individual, what is good for society and they discuss the tension between them in the language of evolutionary game theory. Without reviewing the long history of studies into the nature of cooperation, defection, and the theoretical strategies that people may adopt to overcome their selfish urges, we note the meta-analysis of 67 empirical studies of cognitive-manipulation of economic cooperation games by Rand [30]. He concluded from his meta-analysis that all the experimental data could be explained using a dual-purpose heuristic model of cooperation, a model consisting of a dynamic interaction between deliberation and intuition. Deliberation is considered to be a rational process that always favors non-cooperation, whereas intuition is treated as an irrational process that can favor cooperation or non-cooperation, depending on the individual.

In Section 3 we present numerical results built on those presented earlier [22] to determine the social sensitivity to the uncompromising behavior of a small number of individuals holding either inflexible opinions or changing their opinion with no influence from their nearest neighbors. The committed minority individuals are assigned the state D and do not change their opinion. The independent change their choices in random way. In both these cases the minorities are totally independent of their nearest neighbors but their nearest neighbors are influenced by them according to the DMM rules. The remarkable result is that the SOTC approach to criticality turns out to be much less sensitive to the influence of these minorities that in the case of criticality is obtained by a fine tuning of the control parameter K. It is also remarkable that the independent minority does succeed in affecting the temporal complexity making it possible to realize $\mu = 2$, the condition that generates $1/f$ noise, produced by the brain in the wakefulness state.

2. Two-level network model

The dynamics of the model of interest consists of the interaction between two distinct subnetworks. The behavior of one subnetwork consists of decisions made by individuals influenced by their nearest neighbors and realized by the DMM [10]. The second subnetwork assesses the choice made by the individual and assigns a payoff based on the PDG model. The interaction between the two subnetworks is established by making the individual's imitation strength K_r increase or decrease, according to whether the average difference of the last two payoffs increase or decrease, in accordance with the corresponding changes in K_r. Although each of these imitation strengths is selected selfishly, which is to say the individual choices of imitation strengths are made in the best interest of the individual making the decision at that time, the social system is driven by the resulting internal dynamics towards the state of cooperation, which has the greatest social benefit, which is a unique property of the SOTC. The individuals of the two-level network are located at the nodes of a regular two-dimensional network, denoted by the symbol r, which is equivalent to the double index (i, j).

2.1. The DMM subnetwork

The intuition mechanism proposed by Rand [30] is realized through the dynamics of one subnetwork through the DMM. The DMM on a two-dimensional lattice is based on individuals imperfectly imitating the majority opinion of their four nearest neighbors, thereby biasing the probability of deciding to transition from being a cooperator (C) to being a defector (D):

$$g_{CD}^{(r)} = g_0 \exp\left\{-K_r \frac{N_C^{(r)} - N_D^{(r)}}{N}\right\}, \tag{3}$$

where $N_C^{(r)}$ is the number of nearest neighbors to individual r that are cooperators, $N_D^{(r)}$ the number of defectors, and each individual on the simple lattice has $N = 4$ nearest neighbors. In the same way the transition rate from defectors to cooperators $g_{DC}^{(r)}$ is obtained from Eq. (3) by interchanging indices. The unbiased transition rate is $g_0 = 0.01$ throughout the calculations, and $1/g_0$ defines the time scale for the process.

To realize SOTC, as we shall explain in Section (2.3), the imitation strength of the single individual changes in time, according to the interaction with the PDG subnetwork. The goal of this paper, as mentioned in Section 1, is to discuss the influence on the SOTC organization of a fraction ρ of individuals that do not fit the bottom-up approach to cooperation. These individuals are *zealots* (fanatics) or *independent* individuals. The zealots are individual who do not change their choice. In this paper they always select defection. The independent individuals exert a random perturbation on the SOTC organization. These individuals have an imitation strength $K_r = 0$, which does not change in time. Furthermore to enhance their random nature we assign to them $g_0 = 0.5$.

The DMM in isolation, with neither zealots nor independent individuals either, assigns to all the individual imitation strengths K_r the same value K, a control parameter that has been shown to make this theory undergo critical phase transitions and to be a member of the Ising universality class in which all the members of the network can act cooperatively, depending on the magnitude of K [10]. In the present two-level model the K_r can all be different. This decision making process is fast, emotional and in its original form does not involve any reasoning about payoff.

To denote the effect of imitation we assign to the units selecting the cooperation state the value $\xi_r = 1$ and to the units in the defection state the value $\xi_r = -1$. To establish whether cooperation or defection is selected by the social system we use the mean field

$x(t)$ defined by

$$x(t) = \frac{1}{M}\sum_{r=1}^{M}\xi_r(t). \tag{4}$$

For the isolated DMM if imitation strength K is less than the critical value $K < K_C$ the mean field vanishes, but at criticality, when $K = K_C$, the social system can select either the cooperation, or the defection, branch yielding for $K \gg K_C$, either the value $x = 1$ or $x = -1$. The same situation arises when the DMM is allowed to interact with the PDG, but the critical value of the imitation strength shifts to a new value. The critical value of the imitation parameter K is $K_C = 1$ in the all-to-all coupling configuration and $K_C = 1.5$ (for $M = 30 \times 30$) in the configuration of a regular two-dimensional lattice, with nearest neighbor coupling.

2.2. The PDG subnetwork

The connection with self-interest, according to the slow thinking, cognitive, mechanism of Kahneman [28] is established by a second subnetwork that determines the payoff for the choices made. To define the payoff we adopt rules based on the PDG [24], so that the second subnetwork becomes a realization of Rand's deliberative mechanism within the two-level network model.

Two players interact and receive a payoff from their interaction adopting either the defection or the cooperation strategy. If both players select the cooperation strategies, each of them receives the payoff R and their society receives the payoff $2R$. The player choosing the defection strategy receives the payoff T. The temptation to cheat is established by setting the condition $T > R$. However, this larger payoff is assigned to the defector only if the other player selects cooperation. The player selecting cooperation receives the payoff S, which is smaller than R. If the other player also selects defection, the payoff for both players is P, which is smaller than R. The game is based on the crucial payoffs $T > R > P > S$. Note that their choices are made continuously as the network dynamics unfold.

We adopt the choice of parameter values made by Gintis [24] and set $R = 1, P = 0, T - R = 0.9$ and $S = 0$. We evaluate the social benefit for the single individual, as well as, for the community as a whole as follows. We define first the payoff P_r for individual r as the average over the payoffs from the interactions with its four nearest neighbors. If both players of a pair are cooperators, the contribution to the payoff of the individual r, is $B_r = 2$. If one of the two playing individuals is a cooperator and the other is a defector, the contribution to the payoff of r is $B_r = T$. If both players are defectors the contribution to the payoff of r is $B_r = 0$. The payoff P_r to individual r is the sum over the four B_r's.

We work with a society of M individuals, so that on the global scale, the mean benefit to society of all the individuals is given by the average over all the payoffs P_r's:

$$\Pi(t) = \frac{1}{M}\sum_{r=1}^{M}P_r(t), \tag{5}$$

whereas the mean imitation strength is given by the average over all the imitation strengths $K_r(t)$:

$$K(t) = \frac{1}{M}\sum_{r=1}^{M}K_r(t). \tag{6}$$

2.3. The interaction

It is important to notice that K_r , the value of imitation strength adopted by the typical unit r to pay attention to the choices made by its four nearest neighbors, about selecting either the cooperation or the defection strategy, is not necessarily adopted by its four nearest neighbors. In other words, the imitation strength $K_r(t)$ is unidirectional and it determines how r reacts to all its nearest neighbors. The imitation strength $K_r(t)$ changes from individual to individual, as well as in time, and it is consequently very different from the control parameter K of the conventional DMM phase transition processes, where K has a single value throughout the entire network.

Each member of the present network is assigned a vanishing initial imitation strength, corresponding to complete independence of the choices made by its nearest neighbors. At each time step the units play the PDG and independently change their imitation strengths making the implicit assumption that the increase (decrease) of their individual payoff in the last two time steps makes it convenient for them to increase (decrease) their imitation strength. More precisely, they adopt the following rule. As stated earlier, time is discrete and the interval between two consecutive time events is $\Delta t = 1$. The imitation strength of the individual r changes in time according the individual choice rule as follows:

$$K_r(t) = K_r(t - \Delta t) + \chi\,\frac{[P_r(t - \Delta t) - P_r(t - 2\Delta t)]}{[P_r(t - \Delta t) + P_r(t - 2\Delta t)]} \tag{7}$$

where the parameter χ determines the intensity of interest of the individuals to the fractional change in their payoffs in time and is set to one in the calculations presented herein. The second term on the right-hand size of Eq. (7) is the ratio between two quantities that for special cases vanish. In this case we set the condition

$$K_r(t) = RK_r(t - \Delta t), \tag{8}$$

with $R < 1$. We select $R = 0.5$ but for other values $R < 1$ we get the same result.

The internal dynamics generated by the interaction of Eq. (7), that is between the two subnetworks, drives the average imitation strength and social benefit to the fluctuating plateau values shown in Section 3.

3. Results

In the case when a phase transition is generated by fine tuning of the control parameter, criticality generates non-Poisson renewal events characterized by an IPL probability density function (PDF) [10]. Critical behavior is manifest through events generating phase transitions, modeled by members of the Ising Universality class, as is the DMM. The occurrence of phase transition in a DMM network, with a finite number of interacting individuals, occurs at a critical value of the imitation parameter $K = K_C$. At criticality the mean field $x(t)$ fluctuates around zero and the time interval between two consecutive zero-crossings is described by a markedly non-exponential waiting-time PDF $\psi(t)$, with the IPL structure

$$\psi(\tau) \propto \frac{1}{\tau^{\mu}}, \tag{9}$$

where $\mu = 1.5$.

Here we activate SOTC for a two-dimensional regular lattice (with periodic boundary condition) having $M = 30 * 30$ units and we set $g_0 = 0.01$ and $T = 1.9$, with the mean social benefit, mean imitation strength and mean field starting from zero. The mean field of the two-level network is driven by internal dynamics toward criticality, where the time averaged value of $x(t)$, $\overline{x(t)}$, does not vanish, due to the fact that criticality in this case generates a majority of altruists. Before the interaction with either the zealot or independent minorities is turned on, the mean field reaches a critical state with $\overline{x(t)} = 0.7$, which is to say the social network has a steady state consisting of 85% altruists or cooperators, as depicted in Fig. 1.

At time $t = 5 \times 10^4$ after the calculation has been started, a number of individuals are selected at random positions on the

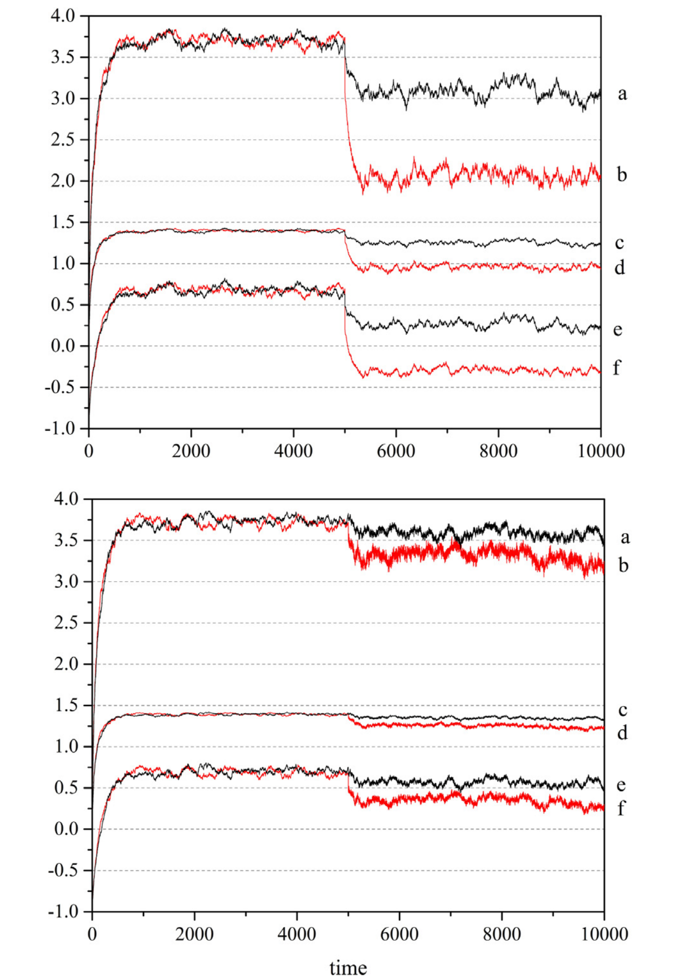

Fig. 1. Effect of fanatics (top) and Independents (bottom) on the 1D SOTC DMM system. At time 5*10[4] fanatics/independents started to act. Black and red correspond $\rho = 0.1$ and $\rho = 0.3$ respectively. (a) and (b) refer to Π; (c) and (d) refer to K; (e) and (f) refer to x. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

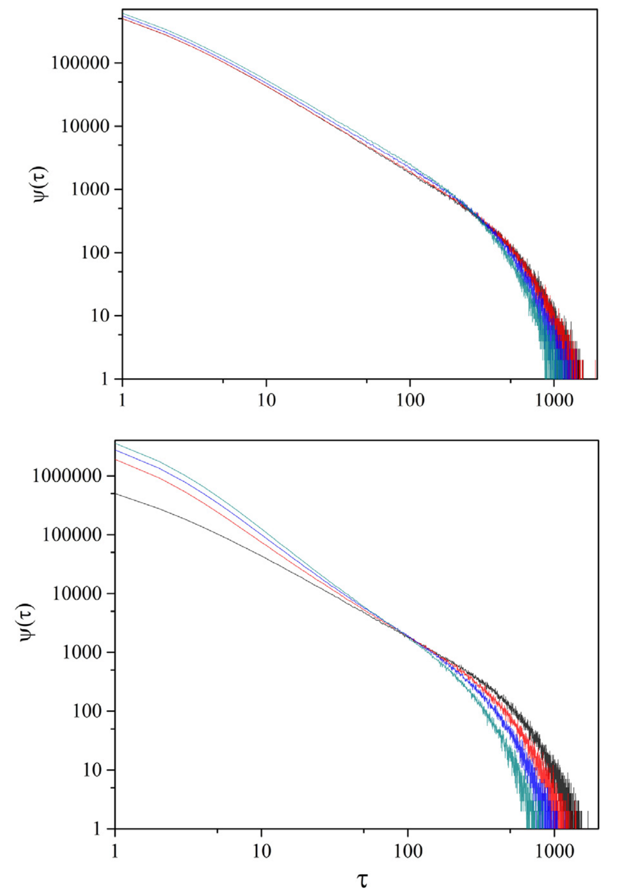

Fig. 2. Effect of fanatics (top) and independents (bottom) on complexity of the SOTC model system. Black, light blue, red, dark blue and purple correspond to ρ = 0, 0.1, 0.2, 0.3 and 0.4 fanatics/independents respectively. In top figure the slopes are approximately 1.35 and in the bottom figure slopes (from top to bottom) are approximately 1.35, 1.66, 1.81, 1.91 and 2.17. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

lattice and their behavior is modified. In the top panel of Fig. 1, these randomly chosen individuals are zealots and they are assigned the opinion state D and not allowed to change, although in every other way they interact with their nearest neighbors as usual. In the bottom panel the randomly selected individuals are independent. Their random behavior is totally independent of the choices of their nearest neighbors, but the choice of their nearest neighbors are influenced by them according to the rules defining the interaction between the two networks.

The two panels show the mean field $x(t)$, the mean global benefit $\Pi(t)$ and the mean imitation strength $K(t)$.

The black curve has $\rho = 0.1$ of the society selected at random to be fanatics, whereas for the red curve $\rho = 0.3$. There is a precipitous drop in the mean field once the modified behavior is introduced, falling from 0.7 to 0.35 and to -0.15 respectively. The dependence on the fraction of fanatics is remarkable.

In the lower panel of this figure we examine the influence on the mean field, not by individuals who do not change their opinion, but by independent individuals who capriciously change their opinions at random. The influence of this cohort group lacks the coherence of the fanatics and may be barely perceptible even at $\rho = 0.3$.

The mean field of the two-level network is driven toward criticality by its internal dynamics, where the time averaged value of the mean field $\overline{x(t)}$ does not vanish, due to the fact that criticality in this case generates a majority of altruists. To stress the occurrence of crucial events in a social system we adopt a method of event detection based on recording the times at which the mean variable crosses its time averaged value. Thus, there are fluctuations around $\overline{x(t)}$ and the IPL structure of Eq. (9) is obtained by evaluating the time distance between two consecutive re-crossings of $\overline{x(t)}$. As shown by Fig. 2, the time intervals between two consecutive crucial events is given by an IPL with index $\mu \approx 1.35$, a property shared by other systems at criticality, see, for instance [31].

It is important to stress that in addition to $x(t)$ also the variables $K(t)$ and $K_r(t)$ are characterized by the same property, namely, also the waiting time PDF of the time interval between two consecutive crossings by $K(t)$ of $\overline{K(t)}$ and by $K_r(t)$ of $\overline{K_r(t)}$, graphed versus time on log-log graph paper, yield an IPL index close to that of $x(t)$ [22]. This is a consequence of the fact that the behavior of the single individual is characterized by frequent collapses to vanishing and even negative values of $K_r(t)$. On the basis of the form of the transition rate given by Eq. (3) we interpret K_r negative as a single individual turning into a contrarian. The calculations done here and elsewhere [26] yields: $\overline{x(t)} \approx 0.7$, $\overline{K(t)} \approx 1.4$ and $\overline{K_r(t)} \approx 1.4$. This is the new phenomenon of self-organized temporal criticality.

Here again we are interested in the changes in the IPL PDF induced by the committed minorities in the social network. In the upper panel of Fig. 2 we see essentially no change in the slope of

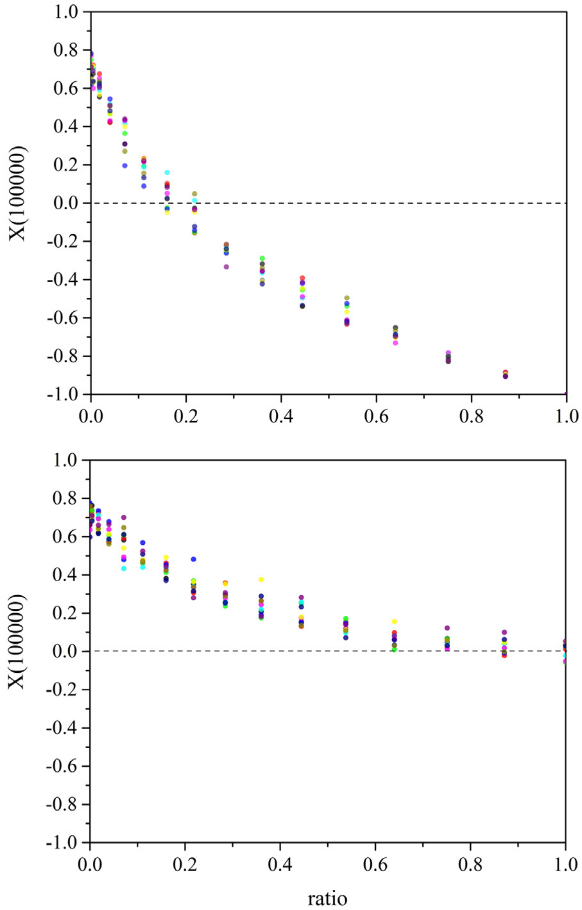

Fig. 3. The mean field of the SOTC model at time 10^5 versus the ratio of fanatics (top) and independent minorities (bottom) which was turned on at time 5×10^4.

the IPL PDF of approximately 1.35, even though the calculation has been done with the behavior of $\rho = 0$, 0.1, 0.2 and 0.3 assigned the permanent opinion *D*. The four calculation deviate slightly as the asymptotic exponential region is approached, since the finite number of individuals contributing to the exponential tempering of the IPL decreases as the number of fanatics increases. Note that this measure of sensitivity does not register the strength of the response to the change in the number of fanatics that the amplitude of the mean field records in Fig. 1.

The curves in the lower panel tell a different story. The slope for the IPL of unmodified network is 1.35, whereas when $\rho = 0.1$, 0.2, 0.3 and 0.4 ratio of the randomly selected individual change their opinion choices to noise the slopes denoting the IPL indices become 1.66, 1.81, 1.91 and 2.17 respectively. So, there exist a ρ between 0.3 and 0.4 with IPL index $\mu = 2$, thereby realizing according to Eq. (2) the ideal $1/f$ noise that is expected to correspond to the dynamics of the brain in the awake state [16]. Thus, the significance of the behavior modification depends on the measure employed. The mean field is relatively insensitive to a noisy minority, as depicted in the lower panel of Fig. 1, whereas the statistics of the mean field is quite sensitive to the noisy behavior as depicted in Fig. 2.

It is interesting to determine how the social response changes with the fraction of aberrant individuals is increased. In Fig. 3 the top panel records the asymptotic percentage of cooperators as a function of the fraction of randomly located fanatics within the social network. There is a monotonic decrease from a mean field of 0.7 with no fanatics to 1.0 with a 100% fanatics, with the mean field crossing the zero axis at approximately 20% fanatics. Note that the sensitivity of the social response is greatly suppressed compared to that of previously considered models in which a 10% contamination brought about a complete reversal of behavior.

The lower panel of Fig. 3 shows the effect of increasing the percentage of individuals who make random choice. It is necessary to force all the individuals to make random choice to totally disrupt the social organization generated by the SOTC bottom up approach. The maximum disruption of the social order is a reduction to subcritical behavior, where the individuals act independently of one another, which is to say they cease to act as a social group.

4. Discussion

In the SOTC model criticality is not forced upon the network by setting the individual imitation strength to a critical value. The critical value of the imitation strength is spontaneously reached without artificially enhancing the level of altruism within the network, but is dynamically attained by assuming that each individual selects the value of the imitation strength that assigns maximum benefit to themselves at a given time. The SOTC model does not require us to adopt the network reciprocity argument of Nowak and May [32] to prevent the infiltration of defectors in cooperation clusters, but instead establishes the emergence of cooperation by the mere use of the PDG payoff, thereby connecting the evolution of cooperation with the search for agreement between individuals and their nearest neighbors.

We think that the theoretical perspective advocated in this paper may afford a scientific perspective to address a debate on the failure of liberalism [33]. According to Brook [34]:

> The difficulties stem not from anything inherent in liberalism but from the fact that we have neglected the moral order and the vision of human dignity embedded within liberalism itself. As anybody who has read John Stuart Mill, Walt Whitman, Abraham Lincoln, Vaclav Havel, Michael Novak and Meir Soloveichik knows, liberal democracy contains a rich and soul-filling version of human flourishing and solidarity, which Deneen airbrushes from history.

Herein we have presented the SOTC model, which shows that the bottom-up approach to cooperation (solidarity) is the fundamental ingredient for the resilience of an organized society.

Acknowledgments

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