Complexity measures of music

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ABSTRACT

We present a technique to search for the presence of crucial events in music, based on the analysis of the music volume. Earlier work on this issue was based on the assumption that crucial events correspond to the change of music notes, with the interesting result that the complexity index of the crucial events is $\mu \approx 2$, which is the same inverse power-law index of the dynamics of the brain. The search technique analyzes music volume and confirms the results of the earlier work, thereby contributing to the explanation as to why the brain is sensitive to music, through the phenomenon of complexity matching. Complexity matching has recently been interpreted as the transfer of multifractality from one complex network to another. For this reason we also examine the mulifractality of music, with the observation that the multifractal spectrum of a computer performance is significantly narrower than the multifractal spectrum of a human performance of the same musical score. We conjecture that although crucial events are demonstrably important for information transmission, they alone are not sufficient to define musicality, which is more adequately measured by the multifractality spectrum.

1. Introduction

One of the outstanding mathematicians of the twentieth century, George Birkhoff, argued that the aesthetics of art have mathematical, which is to say a quantitative, measure. The structure in various art forms, music in particular, that he discussed in his book [1] was largely overlooked by other scientists until the last quarter of the twentieth century, when Mandelbrot introduced the scientific community to fractals [2] and his protégé Voss applied these ideas to the mathematical analysis of music. Voss and Clark [3,4] used stochastic, or *1/f*, music, in which notes are selected at random and the frequency with which a particular note is used is determined by a prescribed distribution function, to gain insight into the structure of more conventional music. They determined that a variety of musical forms, jazz, blues, classical, have a blend of regularity and spontaneous change characteristic of *1/f*-music. Aesthetically pleasing music was found to have a *$1/f^\alpha$*-spectrum, with an inverse power-law index in the interval $0.5 < \alpha < 1.5$, thereby connecting the structure of music to the physical phenomena of *1/f*-noise [5].

In 1987 the newly developed concept of self-organized criticality (SOC) was used by Bak et al. [6] to explain the source of *1/f*-noise. Subsequently, *1/f*-noise has been found to be an ubiquitous property of complex networks near criticality, such as the brain. This suggests an exciting connection with the problem of cognition [7] because 1/*f* noise may represent the brain self-organizing through a vertical collation of the body's spontaneous physiological events. Soma et al. [8] have shown that the brain is more sensitive to *1/f*-fluctuations than to other forms of noise, resulting in higher information transfer rates in the visual cortex [9], pain-relief efficiency by electrical stimulation [10] and enhanced efficiency by biological ventilators [11]. West et al. [12] speculate that there is a complexity matching between Mozart's music (*1/f*-composition), the brain's organization (*1/f* -complex network) and the heartbeat (another *1/f*-process), to explain the result of Tsuruoka et al. [13] that listening to Mozart has the effect of inducing *1/f*-noise on heart beating. This also supports the conjecture that music mirrors the mind [14] in that its complexity is a reflection of the 1/*f*-complexity of the brain. The issue of complexity matching is closely related to the subject of the recent workshop [15] and we invite the readers to consult this reference for details on these interesting connections.

The observation that listening to Mozart's music enhances the reasoning skills of students [16] contributed to the ever-expanding circle of research interest centered on the possible complexity matching between Mozart's music and brain function. This is a thorny problem having aspects of a number of fundamental human issues, including but not limited to creativity, free will, determin-

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ism and randomness [17]. Our purpose here is to present a mathematical theory that explains these interesting aspects of music, which picks up where the above mentioned popular works leave off.

The approach presented herein uses the concept of a crucial event as a fundamental building block for the underlying time series, resulting in *1/f*-variability being the signature of complexity. The theory is an application of the recent work of Mahmoodi et al. [18], which contains an intuitive description of crucial events and develops a generalized form of SOC, *self-organized temporal criticality* (SOTC), based on the dynamics of complex networks. Experimental observation shows that moving from physics to biology is signaled by the emergence of the breakdown of ergodic behavior with increasing complexity. Ergodic behavior is one of the foundational assumptions of statistical physics, that being that time averages of system variables produce results equivalent to those obtained from ensemble averages of those variables [19]. Its almost ubiquitous breakdown in complex systems came as a surprise.

Ergodicity breakdown is caused by complex fluctuations being driven by crucial events. The time intervals between consecutive crucial events are statistically independent and described by a markedly non-exponential waiting-time probability density function (PDF). To realize the temporal complexity of crucial events requires the concept of an intermediate asymptotic region, characterized by an inverse power law (IPL) with index $\mu < 3$. These events are renewal [20] in the sense that their occurrence invokes a total rejuvenation of the system implying that sequential renewal events occur at times having no correlation with the times of occurrence of preceding events. SOTC shows that beyond the intermediate asymptotic region an exponential time region appears which entails that the system recovers the ergodic property in the long-time limit. This exponential truncation, generated by the same cooperative interaction responsible for the IPL nature of the intermediate asymptotic region, is often confused with the effects produced by the finite size of the observed time series.

We conjecture that music, being the mirror of mind, naturally reflects the brain's dynamics (which is a generator of 1/*f* -fluctuations [20]) thereby confirming the early observations of Voss and Clarke [3–5]. However, the arguments adopted by these pioneers are based on the assumption that 1/*f*-noise is generated by fluctuations with very slow, but *stationary* correlation functions. Whereas, the crucial events emerging from the statistical analysis of the time series generated by the brain [20], on the contrary, have non-stationary correlation functions. The significance of SOTC modeling is that the crucial events generated are the same as the 1/*f*-noise produced by the brain, that is, the fluctuations have non-stationary correlation functions.

The importance of crucial events for music composition was recognized in two earlier publications of our group [21,22]; the first paper illustrates an algorithm for composing music based on crucial events. The present paper is closer to the main goal of the second publication, which was the detection of crucial events in an existing musical composition. Vanni and Grigolini [22] assumed that the time at which a note change occurs is a crucial event and found that the IPL index was $\mu \approx 2$. Kello et al. [23] define events using the amplitude envelope of the data and study their properties. Out of the many possible ways to define events, we adopt a simple criterion for detecting events based on the volume of the music and we show that these events are crucial events.

Herein we confirm that music is driven by crucial events. Our analysis establishes a significant difference between computer and human performance of the same musical score, which is not surprising. The computer plays the notes as written by the composer, without interpretation. Humans, on the other hand, bring all their kno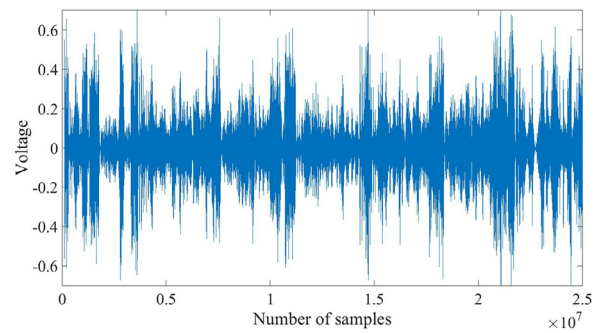wledge, experience and feeling for the music to their performance. The computer can provide the heart of the music, but only a human can make the heart beat.

Fig. 1. An example music signal.

We also make a preliminary attempt at establishing a connection between crucial events and multifractality. A time series without a characteristic time scale can be characterized by a scaling exponent, the fractal dimension. An even more complex time series can have a time-dependent fractal dimension, resulting in a spectrum of fractal dimensions. This spectrum defines a multifractal time series and the width of the multifractal spectrum is a measure of the variability of the time series' scaling behavior.

2. In search of crucial events

According to the theoretical perspective established in earlier work [24] we define *crucial events*, as events for which the time interval between two consecutive events is described by a waiting-time PDF $\psi(\tau)$ with the asymptotic IPL structure:

$$\psi(\tau) \propto \frac{1}{\tau^\mu}, \tag{1}$$

with an IPL index $\mu < 3$. In addition, every pair of time intervals between events must be uncorrelated

$$\langle \tau_i \tau_j \rangle \propto \delta_{ij}, \tag{2}$$

where the bracket indicates an average over the waiting-time PDF. The occurrence of crucial events establishes a new kind of fluctuation-dissipation process [25] and the transport of information from one complex system P to another complex system S is determined by the influence that the crucial events of P exert on the time of occurrence of the crucial events of S [26] (complexity management).

To search the crucial events in music we defined a threshold of 0.1 V for the signal of the musical piece under study and whenever signal reaches the threshold we call it an event. We measure the distances between these event and apply this analysis technique to the time series resulting from both computer and human performances of the music composition. The computer performance consisted of a MIDI (Musical Instrument Digital Interface) file and a FLAC recording (Free Lossless Audio Codec) provided a human performance. In Fig. 1 we depict the music selection for a orchestral performance of Mozart's *Concerto for Flute, Harp, and Orchestra, Allegro*. The music signal was sampled at 44,100 samples per second.

The IPLs in Fig. 2 have slopes of $\mu = 2.07$ and 2.2 for the human and computer performances, respectively (shown in black). To show that these events are renewal events we use the aging experiment [24]. Being renewal means that there is no statistical dependence between the lengths of any two intervals between events. To do aging we select a window size t_a, put the beginning of the window at an event and calculate the distance between the end of the window and the nearest subsequent event, this time is called
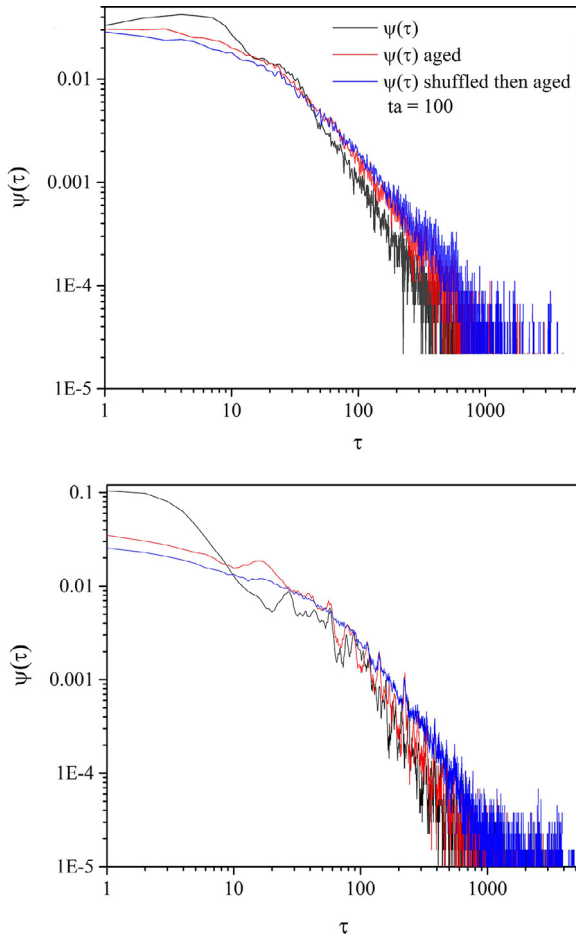
Fig. 2. The top panel is the waiting time PDF of the Human performance, the bottom panel is the PDF for the Computer performance. In each, the black line represents the waiting time PDF of the time intervals between two consecutive crossings of the threshold (0.1 V). The red and blue lines both represent the waiting time PDF of the corresponding aged time series, and the shuffled and aged time series, respectively. The size of the window used for the aging experiment is $t_a=100$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the aged τ. In the figure, the red line is the waiting time PDF of the aged τ's of the time series, using windows size of $t_a=100$. The blue line represents the PDF of the aged τ's of the time series that is shuffled prior to being aged. As can be seen in the figure, the red and blue lines essentially overlap one another. We interpret this overlap to mean the events defined by the crossings are renewal and are therefore crucial events.

3. Power spectrum

Another way to establish that the events detected are renewal is to evaluate the spectrum $S(\omega)$ to detemine if it is IPL. In this section, we will describe some of the theory behind this assertion and present the results of its application to the two time series in which we are interested. We find that the power spectra obtained lend further to support the assertion that both time series are renewal.

Lukovič and Grigolini [27] demonstrated that the power spectrum of a time series is a useful indicator of the presence of renewal events. The time series of a process with crucial events yields a process with a non-stationary correlation function. This non-stationarity is a consequence of ergodicity breakdown, which becomes perennial for $\mu < 3$. According to [27], the spectrum of fluctuations in this case cannot be derived from the Wiener–
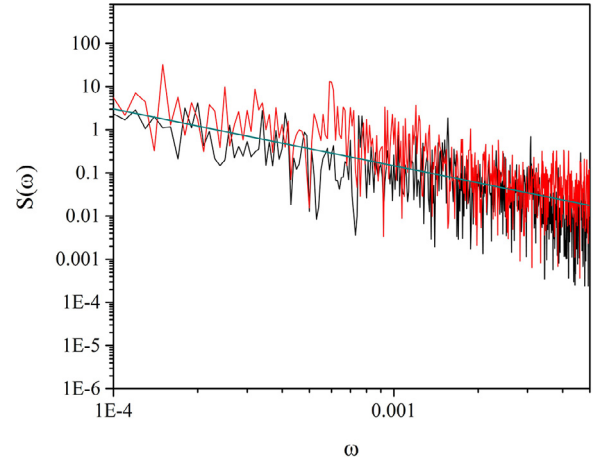

Fig. 3. Power spectra for human (black) and computer (red) data. The IPL indices are approximately one. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Khintchine theorem, relying as it does, on the stationarity assumption. It is necessary to take into account that in both cases of interest in this article ($\mu = 2.07$ and $\mu = 2.2$), the average time interval between two consecutive events diverges, thereby making the process driven by the crucial events non-stationary. This anomalous condition leads to a spectrum that is dependent on the length of the time series L [27]:

$$S(\omega) \propto \frac{1}{L^{2-\mu}} \frac{1}{\omega^{\beta}}, \tag{3}$$

with the IPL index

$$\beta = 3 - \mu. \tag{4a}$$

This result was obtained by going beyond the Wiener–Khintchine theorem adopted by Voss and Clarke in their analysis, but which cannot be applied to our condition if we make the reasonable assumption, based on the results depicted in Fig. 2, that the events we detected, are renewal. If they are renewal and they drive the signal $\xi(t)$, namely the music intensity, then the spectrum $S(\omega)$ is expected to follow the prescription of Eq. (3). In the case where the process yields a slow, but stationary correlation function, we would have $\beta < 1$ [27]. Evaluating the power spectrum in this case becomes computationally challenging because, as shown by Eq. (3), the noise intensity decreases with increasing L, the length of the time series. Nevertheless, the results depicted in Fig. 3 yield an IPL index $\beta \approx 1$, and Eq. (4a) yields $\beta = 3 - 2 = 1$, the agreement between these results is very encouraging.

Both the aging experiment and the spectrum satisfactorily support the claim that the events revealed using the threshold method are crucial events. To clarify this point Fig. 4 illustrates the waiting-time PDF of the intervals between two consecutive crossings of the threshold line, when the threshold is set equal to 0.002. This threshold is not large enough to filter out the events that are not crucial. We see that these non-crucial events produce a well pronounced exponential shoulder in the waiting-time PDF. The results of Fig. 2 have been obtained by filtering out these non-crucial events. We therefore conclude that the crucial events, which are the mechanism for information transport [26], also have the significant effect of determining the behavior of the spectrum for $\omega \to 0$.

4. Multifractality

The discovery of $1/f$ noise in music by Voss and Clarke [3] was interpreted assuming the music time series is stationary, which is consistent with Fractional Brownian Motion (FBM), and yields a
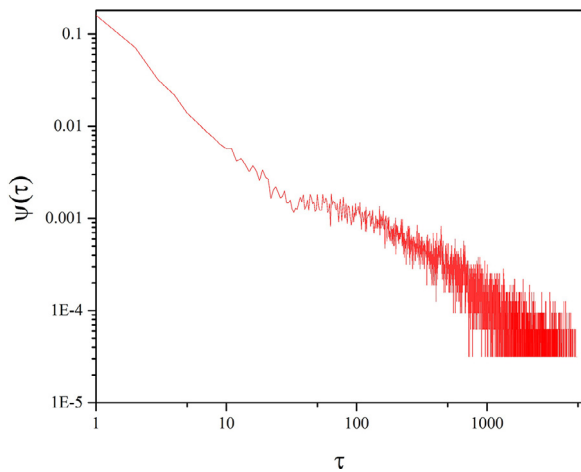
Fig. 4. The waiting-time PDF of the human performance time series, obtained using a threshold of 0.002.
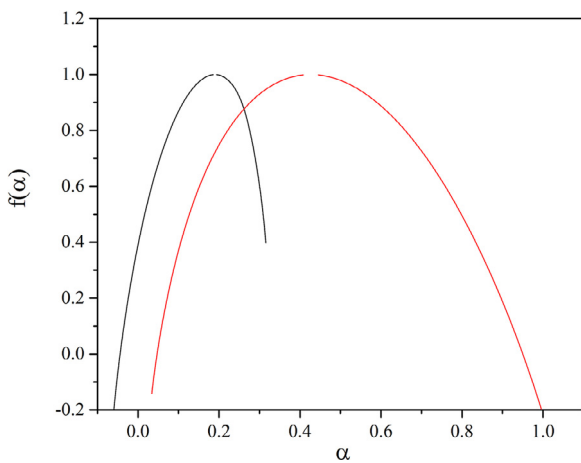

Fig. 5. The black line represents the multifractal spectrum of the computer performance, the red represents the human performance's multifractal spectrum. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

mono-fractal [2]. However, the present work goes beyond [3] and the ergodic assumption, by taking a non-stationary approach consistent with multifractality.

Using the method of Multifractal Detrended Fluctuation Analysis (MF-DFA) [28] to analyze each time series of the two performances gives the results shown in Fig. 5. The computer performance yields the narrow multifractal distribution, whereas the multifractal distribution of the human performance is significantly broader. This notable difference between the multifractal spectra indicates different levels of complexity in the two performances. The narrowness of the computer performance suggests a strict adherence to a single fractal dimension and consequently less complexity than observed in the human performance. In fact, the difference between the two performances may be better described by what the computer performance lacks compared to the human performance. This extra information, or musicality, contained in the human performance includes specific techniques that add to the complexity of the music through subtle variation in timing, intonation, articulation, dynamics, etc., which are likely a better match to the brain's complexity. The human performance is largely more aestheticly pleasing to the listener than is the computer performance.

Suppose the brain of Mozart contains a certain complexity, which is well described by SOTC as a generator of $1/f$-noise. Then Mozart transcribes his complexity, albeit incompletely, into the musical score of the chosen selection. To recover this lost musicality, the human performer injects their own interpretation of the lost complexity using their specific performance techniques. On the other hand, the computer performance is unable to interpret this lost component and delivers exactly what was transcribed, resulting in less variability in complexity. The computer performance is a partial record of the brain of Mozart even if Mozart himself would have produced a broader multifractal distribution when the piece was performed.

5. Concluding remarks

Crucial events exist in the changing of notes in music, as found by Vanni et al. [22]. Similarly, our analysis of the music signal via the change in volume, leads to the same conclusion. This difference in analysis is very important because the statistical analysis of the dynamics of the brain [20] shows that the brain is a generator of crucial events with the same IPL index. Additionally, we found that there is a noticeable difference in the fractal measures between human and computer performances.

Music is aesthetically pleasing to the brain [21,22] because of the crucial events described by $\mu = 2$. Multifractality may provide a clearer picture of which performance of Mozart would be more pleasing. The difference in musicality is obvious to the listener's ear and this difference can be quantified through the broader multifractal spectra. The increasing aesthetics of music favors a broader multifractal spectrum. Indeed, multifractality describes an additional measure of complexity. Crucial events measure the complexity of the intermediate asymptotics, whereas multifractality contains additional information, beyond the intermediate asymptotics, regarding the transient region and exponential truncation. All this subtlety in composition is experienced by the brain through the transfer of the music time series' multifracality.

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