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Higgs Boson Spectrum From Infrared Fixed Points[1]

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Abstract

The fixed point structure of the renormalization group equations for the scalar quartic couplings in the one and two-doublet models is studied. Masses of the physical Higgs bosons can be determined by the infrared fixed points of the quartic coupling constants. The existence of these fixed points in the two-doublet model requires the presence of a heavy fourth generation in which quarks are coupled to both doublets. Otherwise, the potential can become quartically unstable at low energies for arbitrary initial stable values of the coupling constants.

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ABSTRACT

The fixed point structure of the renormalization group equations for the scalar quartic couplings in the one and two-doublet models is studied. Masses of the physical Higgs bosons can be determined by the infrared fixed points of the quartic coupling constants. The existence of these fixed points in the two-doublet model requires the presence of a heavy fourth generation in which quarks are coupled to both doublets. Otherwise, the potential can become quartically unstable at low energies for arbitrary initial stable values of the coupling constants.

Despite the successes of the standard model, the Higgs sector remains a mystery. The main reason is that the mass of the physical Higgs boson m_H depends on the quartic coupling constant λ in the scalar potential, which is a free parameter of the model. Many attempts [1-5] have been made to constrain or to predict λ (and hence m_H) all of which involve extra assumption(s). Pendleton and Ross[3] first suggested the interesting possibility that low energy physics may be dictated by the infrared (IR) fixed point structure of the renormalization group equations (RGE). If the RGE for the various couplings in a theory possess stable IR fixed points, the couplings will be swept towards these fixed points when evolving from high energy (*e.g.*, a unification scale Λ_U) to low energy (*e.g.*, the weak interaction scale $\Lambda_w \sim M_w$), irrespective of their initial values. Consequently predictions for low energy parameters can be obtained without knowledge of the symmetry conditions at Λ_U. Implicit in this approach is the assumption that a desert exists and perturbation theory is valid throughout the desert.

The work[6] I am going to describe was done in collaboration with Chris Hill and Sumathi Rao. We study numerically the IR fixed point structure of RGE for the scalar quartic couplings in the standard model and its extension to two Higgs two neutral scalars. Their (mass)^2 are, respectively,

$$\begin{aligned} m_{\pm}^2 &= -\frac{1}{2}(\lambda_4 + \lambda_5)\nu^2 \\ m_p^2 &= |\lambda_5|\nu^2 \\ m_1^2 &= \frac{1}{2}\eta_+\nu^2 \\ m_2^2 &= \frac{1}{2}\eta_-\nu^2 \ , \end{aligned} \tag{3}$$

where

$$\begin{aligned} \eta_{\pm} = {} & (\lambda_1 cos^2\beta + \lambda_2 \sin^2\beta) \\ & \pm [(\lambda_1 \cos^2\beta - \lambda_2 \sin^2\beta)^2 + (\lambda_3 + \lambda_4 + \lambda_5)^2 \sin^2 2\beta]^{1/2} \end{aligned} \tag{4}$$

and

$$tan\beta = \frac{\nu_2}{\nu_1} \ . \tag{5}$$

Here $\nu_1/\sqrt{2}(\nu_2/\sqrt{2})$ is the vacuum expectation value of $\phi_1(\phi_2)$ and $\nu^2 = \nu_1^2 + \nu_2^2$.

In order to have a residual $U(1)_{EM}$ symmetry so that the photon remains massless, λ_4 must be negative. Then the rquirement that the potential energy of the vacuum be bounded below necessarily implies the following conditions in tree-approximation:

$$\begin{aligned} &\lambda_1 > 0, \\ &\lambda_2 > 0, \\ \text{and} \quad &\sqrt{\lambda_1\lambda_2} > -\lambda_3 + |\lambda_4| + |\lambda_5| \ . \end{aligned} \tag{6}$$

The RGE for the gauge, Yukawa and quartic couplings are given in Ref. 6. The RGE are numerically integrated from $\Lambda_U = 10^{15}$ GeV to $\Lambda_w = 100$ GeV assuming large initial values for the Yukawa and quartic couplings (by large couplings we do not mean the saturation of unitarity bounds, $g^2 \sim 4\pi$, but rather g^2 "of order unity"). For simplicity the Yukawa couplings are assumed to be diagonal[7]. The Yukawa couplings of the light fermions (lighter than the t-quark) are set to be zero since they have negligible effects on the evolution of the coupling constants except for the counting in the gauge coupling beta-functions. Fixed points of the scalar quartic couplings are universal values attained from a sample of random (but satisfying Eq. (6)) initial values.

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We now discuss the results. Consider first the one-doublet model with three fermion generations. The evolution of the quartic coupling λ and the Yukawa coupling of the t-quark g_t is shown in Fig. 1a. For sufficiently large initial values of g_t λ reaches a fixed point. For smaller initial g_t it terminates on the dashed line of Fig. 1a. This can be turned into a relationship between m_H and m_t, the mass of the t-quark, which is shown in Fig. 1b. Curiously, m_H lies around 170 GeV for a large range of m_t. If more heavy fermions are present, m_H tends to increase.

In the two-doublet model fixed points exist only if both doublets are coupled to heavy quarks. This can be understood from the structure of the RGE. Suppose all Yukawa couplings are negligible. Then the RGE for λ_1 (and similarly λ_2) can be written as (g_2 and g_1 are the gauge couplings for SU(2) and U(1), respectively)

$$16\pi^2\mu\frac{\partial\lambda_1}{\partial\mu} = 12[\lambda_1 - \frac{1}{8}(3g_2^2 + g_1^2)]^2 + 2\lambda_3^2 + 2(\lambda_3+\lambda_4)^2 + 2\lambda_5^2 + \frac{9}{16}(g_2^4 + g_1^4) + \frac{3}{8}g_2^2g_1^2 \,. \quad (7)$$

Notice that the right hand side is always positive and thus no fixed point exists. Consequently $\lambda_1(\lambda_2)$ always decreases from its initial value when evolved to lower energy. In fact it decreases so fast that it can exit the stability region becoming negative and eventually negative infinite, thereby causing other couplings

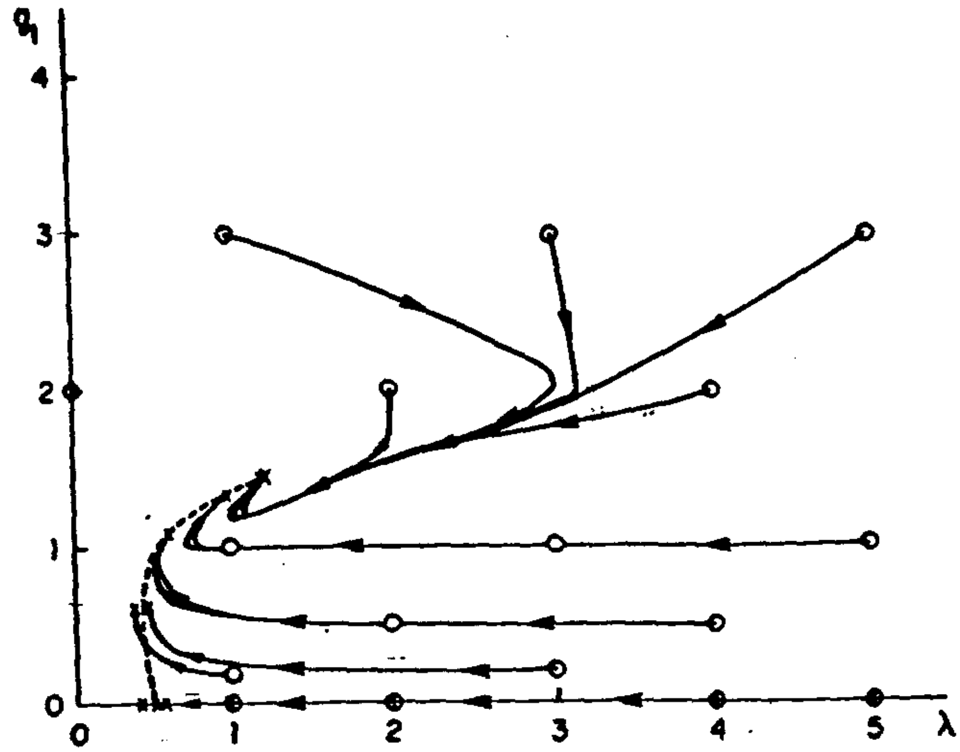

Fig. 1a. Flow of λ and g_t towards fixed points in the standard one-doublet model.

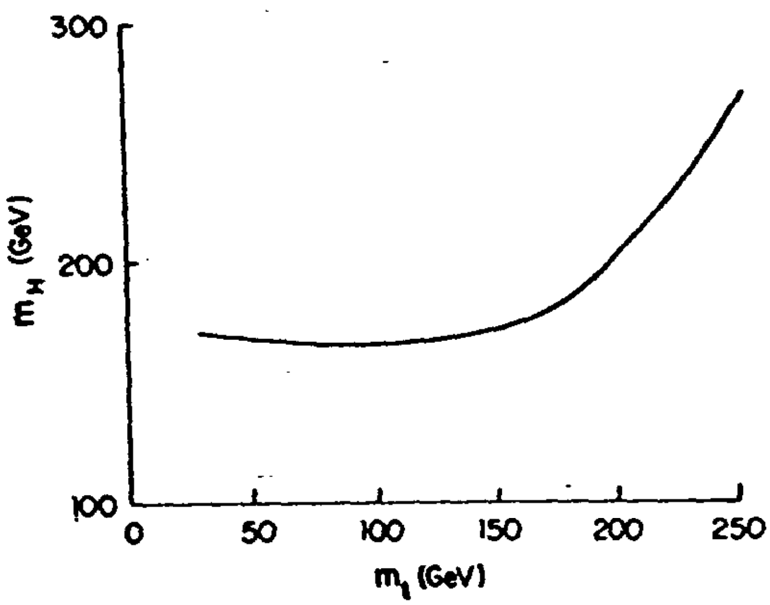

Fig. 1b. Relation between the Higgs mass and the t-quark mass in the one-doublet model.

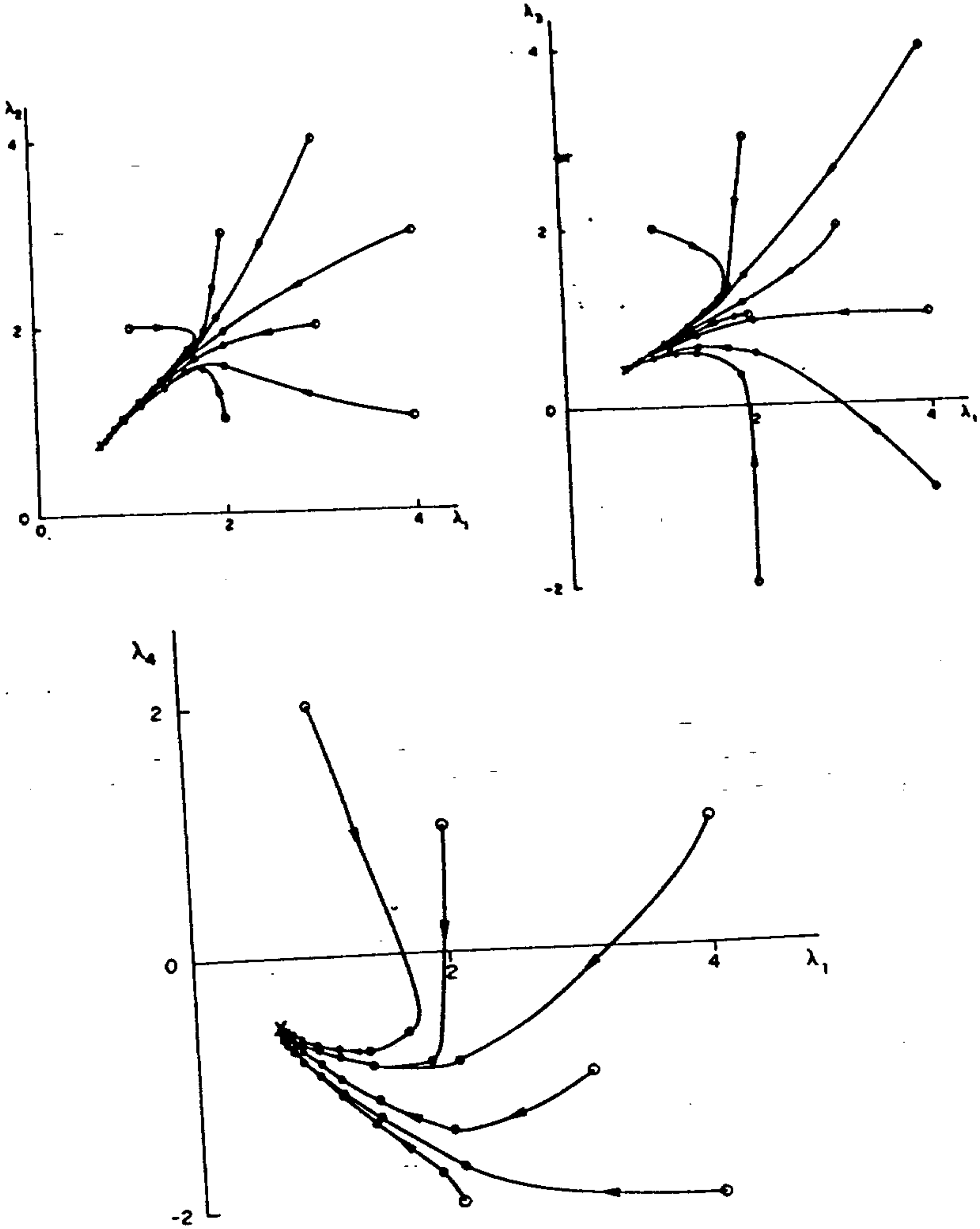

Fig. 2 Flow of $\lambda_1, \lambda_2, \lambda_3$, and λ_4 towards fixed points.

to diverge. This suggests the interesting possibility that a theory which is perturbatively weak at high energies can become non-perturbatively strong at lower energies. This is reminiscent of technicolor and might be applied to generate the breaking of the electroweak symmetries. Coupling ϕ_1 (or ϕ_2) to a heavy quark with Yukawa coupling f introduces to the RGE of λ_1 (orλ_2) the terms $12\lambda_1 f^2 - 12f^4$. The f^4-term is negative and hence a fixed point can exist if f is sufficiently large. Coupling to heavy lepton with equal Yukawa coupling introduces the similar terms but 3 time smaller, hence less effective in driving $\lambda_1(\lambda_2)$ to its fixed point.

Let us consider then the following Yukawa coupling scheme consistent with the natural suppression of off diagonal neutral couplings[8]

$$L_{yuk.} = \bar{Q}_L U u_R \phi_1^c + \bar{Q}_L D d_R \phi_2 + \bar{L}_L \ \mathcal{L} \ e_R \phi_2 + h.c. \,, \tag{8}$$

in which at least one of each charged fermion species has a large Yukawa coupling (corresponding to coupling scheme I in Ref. 6). This necessarily implies the existence of a fourth generation. Here $Q_L(L_L)$ is the left-handed quark (lepton) doublet, u_R, d_R, and e_R are, respectively, the right-handed up-quarks, down-quarks, and charged leptons, and U, D and $\mathcal{L}$ are Yukawa coupling matrices. Generation indices have been suppressed. Fixed points for the quartic couplings exist in this case and are shown in Fig. 2. Notice that the fixed point for λ_4 is negative (corresponding to a massless photon) for arbitrary initial values. It is somewhat remarkable that the renormalization group fixed point will select the physically interesting vacuum!

The masses of the physical Higgs particles can be determined from Eq. 3 with the fixed point values of the quartic couplings. These are shown in Table 1. The masses of the neutral scalars $m_{1,2}$ depend on the unknown vacuum expectation value ratio ν_2/ν_1. However, they lie within a finite region (as shown in Table 1) for ν_2/ν_1 ranging from 0 to ∞. (A recent renormalization group analysis[9] constrains $(\nu_2/\nu_1)^2$ to be less than 60. This does not affect the range shown in Table 1). The mass of the pseudoscalar m_p depends on λ_5 whose fixed point is zero (since the right hand side of the RGE is proportional to λ_5) but is never reached in the finite running time, although it tends to be small. The values of m_p shown in Table 1 are for typical values of λ_5 at Λ_w.

Also shown in Table I is the dependence of the Higgs masses on m_t. There is essentially no difference between $m_t = 0$ and $m_t = 50$ GeV, whereas a heavier

Table I. Fixed point masses (in GeV) of the physical Higgs particles (N = number of generations).

	m_t	$m_\pm$	m_1	m_2	m_p
$N = 4$	0	188	190 − 231	0 − 105	"61"
	50	186	188 − 225	0 − 105	"59"
	172	136	164 − 216	0 − 131	"25"
$N = 5$	0	142	144 − 191	0 − 106	"21"
	50	140	143 − 186	0 − 105	"21"
	135	118	133 − 174	0 − 110	"13"

t-quark tends to lower the masses. Similar effects occur with the addition of extra heavy generations as seen in the case with N=5. All the masses lie in an interesting range which is accessible to experiment in the near future.

Other distinct Yukawa coupling schemes are also considered in Ref. 6. We do not have the space-time to discuss them here.

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