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RELATIVISTIC 3-D NUCLEAR HYDRODYNAMICS WITH MONTE CARLO PIONS*

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Abstract

A model for relativistic three-dimensional hydrodynamical nuclear fluids has been coupled to a Monte Carlo pion model which treats the production, scattering, and absorption of pions in relativistic nuclear fluids. The model is dynamic and allows us to explicitly follow the temporal and spatial development of pion components through an entire collision process and into the final state. Such calculations will be necessary to extract meaningful information from measured RHIC pion distributions. We present preliminary results and discussion for ^{139}La + ^{139}La collisions at 1350 MeV/nuc (lab) and at various impact parameters.

Introduction

The success of collective theories describing heavy-ion collisions at low and intermediate energies[1] suggests that extensions of these theories are likely to be useful for explaining RHIC data. The prediction several years ago of bulk hydrodynamic flow in nuclei,[2] later confirmed at the Bevalac,[3] has led to the development of nuclear hydrodynamics as an effective phenomenological description of many aspects of heavy-ion collisions.[4,5,6] We are developing a relativistic three-dimensional hydrodynamic model applicable from energies below 1 GeV per nucleon up to those found at the AGS. We expect to extend this model to CERN and

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RHIC energies.[7] Since meson production is an important part of these interactions, we have coupled to the hydrodynamics a Monte Carlo meson model in which the particles are produced, scattered, transported and absorbed by the baryonic fluid.[8] In this paper, we will briefly describe these two components of our model, and present some results indicating its applicability and usefulness.

The Relativistic 3D Hydrodynamics Model

The solution of the full strongly-interacting, quantum-mechanical, many-body problem that is a heavy-ion collision system is far beyond our current abilities. Thus approximations must be made in order to investigate these systems theoretically. Since there are likely to be large numbers of interactions between individual nucleons in these collisions, kinetic models should be applicable.[9] At these energies, the correct description must be relativistic. Thus relativistic hydrodynamics, which is derivable from kinetic theories, has already had success in describing aspects of the collisions.[10] We have developed a new hydrodynamic model, with a different solution scheme than previously used, as well as the new pion model described below.[7,8]

The link between non-relativistic hydrodynamics and the Schroedinger equation is well known, and thus we can take the collective limit in the relativistic case as well to be hydrodynamic. In this case, the equations of ideal hydrodynamics result from the conservation of the stress-energy tensor,

$$T^{\mu\nu} = u^{\mu}u^{\nu}(\rho + \rho\varepsilon + p) + pg^{\mu\nu} ,$$

and of the baryon flux ρu^{μ}. Here, u^{μ} is the 4-velocity, ε the proper energy, p the pressure, ρ the proper density and $g^{\mu\nu}$ the metric. With the definitions of the coordinate energy density $E= \varepsilon\rho u^0$, baryon density $D= \rho u^0$

and momentum density, $S^{\mu} = \rho(1+\varepsilon+p/\rho)u^{0}u^{\mu}$, the equations for the model are conservation equations for these quantities and have the obvious non-relativistic reductions. We solve these equations on a grid fixed in space with a continuous distribution of matter flowing through the grid. In contrast to earlier work of others,[11] we do not use a particle-in-cell method. Further details of our solution scheme can be found in ref. (7). The methods we use are known to be second-order accurate in time and space, although the scheme is not explicitly energy conservative. We have exercised our code against a battery of tests, however, and find the energy accurate to better than a few percent with sufficiently fine gridding.

Thus far the model is independent of the nuclear physics of the system. We have incorporated the nuclear physics through the compressional part of the equation of state. The equation of state of hot nuclear matter is poorly understood, although essential to our problem. Our equation of state extrapolates from the known compressibility (200 MeV) and binding energy (-8 MeV/nuc) of finite nuclear matter to treat more highly compressed systems.[7] We have built considerable flexibility with respect to the equation of state into the model since so little is known about it. We extract the effect of the equation of state on the dynamics by studying the systematics of our results.

In such a complicated calculation as three-dimensional relativistic hydrodynamics, it is important to understand the limitations and accuracies of the numerical model on which the physics relies. Since we have solved our equations using an operator splitting algorithm, it is easy to test the various components of the model. Many tests have been made on our code.[7] We have verified the accuracy of our evaluation of the advection, compression, pressure heating and acceleration terms. In addition, we have used the code in a one-dimensional configuration to compare against analytic solutions of the relativistic hydrodynamic equations such as the relativistic wall shock and shock tube. It is from this extensive testing that we are confident of the applicability of the model in the energy regime described above.

The Pion Model

Pions provide a natural probe of relativistic heavy-ion collisions. Since baryon number is conserved, and there are no pions in the initial state, any pions emitted have drawn their energy content from the available kinetic energy in the center of mass of the colliding systems. The reduction in kinetic energy of outgoing nucleons and massive fragments can be appreciable. For example, in ^{139}La +^{139}La collisions at 1350 MeV/nuc (lab), pions carry roughly 30% of the initial kinetic energy from the system.[12]

Accurate models of high-energy heavy-ion collisions must incorporate pionic degrees of freedom. Relativistic hydrodynamics alone does not describe particle production. To study pion production and propagation, we have developed a Monte Carlo approximation which directly couples pions to hydrodynamics. Our model is dynamic and allows us to explicitly follow the temporal and spatial development of pion components through the entire collision process and into the final state. We may thus address whether or not the final pion states actually provide information about the environment of hot, dense nuclear matter formed during the central collision time. Such calculations will be necessary to extract meaningful information from measured RHIC pion distributions.

In our model pions have been treated as particles with relativistic kinematics that follow rectilinear trajectories between collisions with the nuclear fluid. We allow production to occur when enough thermal energy is available in a region of 5-10 fm^3 to provide mass and momentum for a pion. In actual collisions at BEVALAC energies only a few tens of pions are produced. A Monte Carlo treatment for this number of particles would be inappropriate. That is, we would need to run a collision many times to get results comparable to experiments. We thus calculate a mean collision in terms of pion possibilities. We therefore produce fractional pions to increase statistical accuracy. The probabilities of production and subsequent interactions have been constructed to be independent of the fraction we let each code-pion represent. The use of fractional pions also helps the numerical approximations of the pion coupling to the hydrodynamics.

Our pion model requires effective pion-nucleon cross sections as input for the probability functions. For production we choose a pion momentum randomly from a relativistic Bose-Einstein distribution, mapped to the most probable product of pion velocity times absorption cross section. We have used detailed balance to relate absorption to production, and we have chosen the absorption cross section as the more natural quantity to describe interactions deep inside a nucleus. Functionally, we have great flexibility in the forms we use for all cross sections. Our only restriction is that momentum and density enter in a factorized fashion, e.g., $F(\rho) * G(p)$. We dynamically couple the pions to the fluid by exchange of momentum and energy. We require local conservation of $T^{\mu\nu}$ (fluid + pion) over the same volume for production, scattering, and absorption.

Once a pion has been produced we follow its trajectory through the fluid. It may be reabsorbed or scattered, or simply transported by its velocity. We describe scattering of a pion with fractional mass from a nucleon of similarly reduced mass so that kinematically we mimic true pion-nucleon collisions. Scattering has been treated like the isotropic elastic scattering of a pion from a free nucleon. In the context of heavy-ion collisions this would more appropriately be referred to as inelastic scattering. In elastic pion-nucleus scattering, the entire nucleus recoils, while with inelastic scattering there is excitation of the nucleus or knockout. In the fluid rest frame, the nucleons have a thermal velocity distribution. To incorporate this relative pion-nucleon velocity in our scattering probability, we have convoluted σv into the finite-temperature relativistic Boltzmann distribution and we use the resulting expression in the probability formula. Thus there remains a non-zero probability of scattering for a pion of zero velocity in the fluid frame. Prior to scattering, the nucleon is assigned a random momentum chosen from the Boltzmann distribution.

Various tests[8] of our pion model have determined its statistical accuracy and sensitivity to various input quantities. A straightforward test of the Bose statistics has been constructed by taking a few zones of uniform mass and energy density, and a volume large enough to prevent escape of pions. This static, hot fluid was allowed to produce, absorb, and scatter pions. The system was large enough to prevent appreciable momentum transfer from the pions back to the fluid, so the fluid remained static, having cooled slightly due to production energy loss. The pions formed a Bose-Einstein distribution characteristic of the temperature of the fluid bath. Scattering alone would force pions into a distribution with the statistics of the underlying fluid. Scattering coupled to production and absorption keeps the pions in a Bose distribution. The equilibrium distribution was reached independent of the values of the scattering and absorption cross sections (and independent of the fluid density also). However, the equilibration time was directly dependent upon the cross sections. We found that our Bose test problem is useful in determining equilibration times for any postulated cross sections. This time must be compared to that available in heavy-ion collisions to determine if equilibration is possible.

Preliminary Results:

As an example, we present calculations of ^{139}La + ^{139}La collisions at 1350 MeV/nuc (lab). We used a Skyrme-type equation of state with an incompressibility K = 198 MeV at normal nuclear density. For the thermal pressure we assumed a nonrelativistic Fermi gas. Our temperature model reproduced both the degenerate and non-degenerate

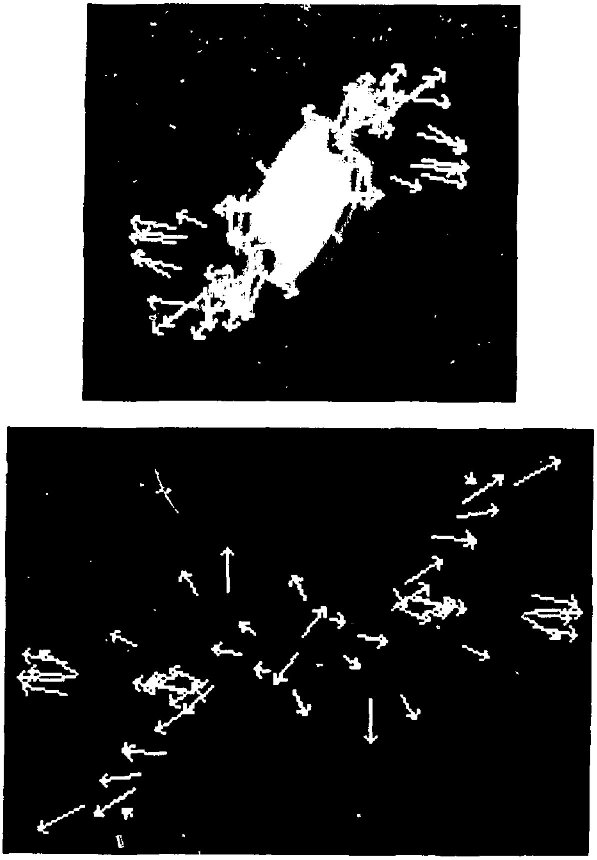

Figure 1. Density plots in the reaction plane for $^{139}La + {}^{139}La$ collisions at 1350 MeV/nuc with an impact parameter of 3 fm. The grey scales indicate densities from 0 to 4 times normal nuclear density. The arrows are marker particles projected into the reaction plane. The tail of the arrow is at the projected location, with its length proportional to the momentum. The top plot is at 10 fm/c, which is near the time of maximum compression, while the lower is at 15 fm/c when the fragments have started to separate, and many of the marker particles are free-streaming.

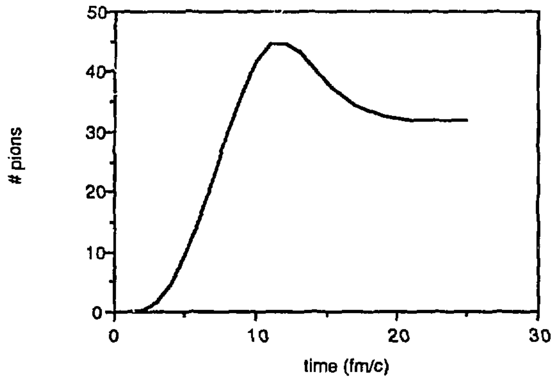

Figure 2. Pion multiplicity as a function of time for a zero impact parameter collision of ^{139} La + ^{139} La at 1350 MeV/nuc.

Fermi-gas limits.

Figures 1-a and 1-b are contour plots of ^{139} La + ^{139} La collisions (1350 MeV/nuc) at collision times of 10 fm/c and 15 fm/c and at an impact parameter of 3 fm. Notice that many marker pions have already escaped the bulk of the fluid at these times.

In Figure 2 we show the number of pions present in the entire system as a function of time. We have removed the scaling of fractional mass so that the values plotted correspond to true pions. We observe an increase in number as a function of time, with a peak just after 12 fm/c. The time of maximum density and temperature is about 10 fm/c. The pion number decreases with time due to a net reabsorption . (Recall that we do not use a "freeze out" model for pions.) Production continues until roughly 17 fm/c, with absorption discontinuing at about 19 fm/c.

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We track each pion until it leaves the numerical grid on which the fluid dynamics were calculated. The last interaction of each pion marker is recorded, including the fluid element density and speed, and the coordinates where interaction occurred. The total number of scattering events for each particle is also recorded, so that we may extract a mean number of scatterings per emitted pion. Determination of this quantity is essential for addressing whether or not the observed pions retain information about the central collision conditions, or whether they equilibrate with the expanding nuclear matter, losing all history of the hot, highly compressed state. For the ^{139}La + ^{139}La case at zero impact parameter, we found 3.8 scatterings per emitted pion.

This model currently underpredicts the pion multiplicity (32 pions) as compared with experimental results (51 pions)[13] for ^{139}La + ^{139}La. The fraction of initial kinetic energy transferred to outgoing pions was also underpredicted by roughly 30%. We have a program under way to understand the sensitivities of these quantities to the equation of state and the cross sections.

Summary and Plans

At LLNL we have developed a 3-dimensional relativistic hydrodynamics model with a dynamic Monte Carlo pion treatment which directly couples pions to the hydrodynamics. Our code tracks pion production and propagation through the dynamic hot nuclear fluid. Since mesons are one of the primary observable signals from relativistic collisions of heavy nuclei, such calculations will be necessary to extract meaningful nuclear fluid information from measured pion distributions from RHIC, the Bevalac, and other relativistic heavy ion colliders.

We are currently in the process of running simulations of most of the available heavy ion collision data, with emphasis on La on La, Nb on Nb, Au on Au, and Ca on Ca. We are also beginning to incorporate more detailed pion-nucleus cross sections.

We plan to proceed with a number of other related simulations which now seem feasible. Calculations of astrophysical significance will be carried out to explore astrophysical sensitivity to the nuclear equation of state. By setting up the appropriate initial conditions, we could dynamically simulate quark-gluon plasmas. With extensive upgrading, we could directly simulate RHIC collisions. Work will soon begin on dynamic modeling of K-meson production and propagation. Other possiblities will no doubt appear as we continue to appreciate the breadth of application of this model.

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