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3.6 SIMPLIFIED METHODS FOR DESIGN

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ABSTRACT

Simplified design analysis methods for elevated temperature construction are classified and reviewed. The motivation for the development of the simplified methods is the cost, both in terms of computational time and associated engineering labor, when a rigorous accounting of time-dependent material behavior is combined with detailed two- and three-dimensional geometric models of equipment operating at elevated temperature. These costs remain high in spite of significant advances in finite element software during the past decade. An additional motivating factor is the skill demanded of the analyst in order to produce reliable results within a nonlinear framework.

Because the major impetus for developing elevated temperature design methodology during the past ten years has been the Liquid Metal Fast Breeder Reactor (LMFBR) program, considerable emphasis will be placed upon results from this source. The operating characteristics of the LMFBR are such that cycles of severe transient thermal stresses can be interspersed with normal elevated temperature operational periods of significant duration, leading to a combination of plastic and creep deformation.

Here the various simplified methods are organized into two general categories, depending upon whether it is the material, or constitutive, model that is reduced, or the geometric modeling that is simplified. Because the elastic representation of material behavior is so prevalent, an entire section is devoted to elastic analysis methods. Finally, the validation of the simplified procedures is discussed.

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INTRODUCTION

The current design by analysis approach embraced by the elevated temperature code case [1] of the ASME Boiler and Pressure Vessel Code has evolved over a period of fifteen to twenty years [2]. Additional deformation and failure modes were considered. For construction materials, geometries and loading conditions* of interest prior to this, the failure mode of predominant concern was ductile rupture caused by excessive internal pressure. However, as operating temperatures of the equipment ~~because~~ became sufficiently high to cause significant time-dependent inelastic deformation and as operating environments degraded material performance, design procedures to protect against other failure modes were included. Probably the two most important additional failure modes, from the point of view of inelastic material behavior, were: (1) creep-fatigue and ductile crack extension, driven by plastic deformation; and (2) creep instability and rupture, driven by time-dependent inelastic deformation.

The essence of design is the treatment of uncertainty — either in material properties, geometry, or loading — in such a way that conservatism prevails and public safety is assured. Design rules or elastic analysis methods can often be developed that account for the various levels of uncertainty through safety factors, so that conservatism is unquestioned. However, when competing physical processes are present, as in the case of low-cycle fatigue and creep, a rule or method that is con-

*The term loading or loading conditions is generalized here to include environmental effects, such as corrosion, high temperature, and irradiation that promote material damage either in themselves or through accompanying stress and deformation states.

servative for one process may be unconservative for another. In addition, experimental or operational proof is difficult to generate because test times must be comparable to the lifetime of the component. These considerations have led to a requirement for detailed inelastic analysis.

Performance of detailed inelastic analysis has become more routine in recent years, but it is still usually reserved for the relatively few components that cannot be shown to meet design or safety requirements on other bases. Inelastic analysis is considerably more expensive than elastic analysis, but cost is not the only disadvantage. Because of the complexity of the constitutive models, a significantly greater amount of material data is required, such as temperature-dependent yield data, hardening moduli, and creep properties. Because the time history of events affects the results of inelastic analysis to a much greater degree than it affects the results of elastic analysis, simplification of the loading history included in the design specifications is more difficult.

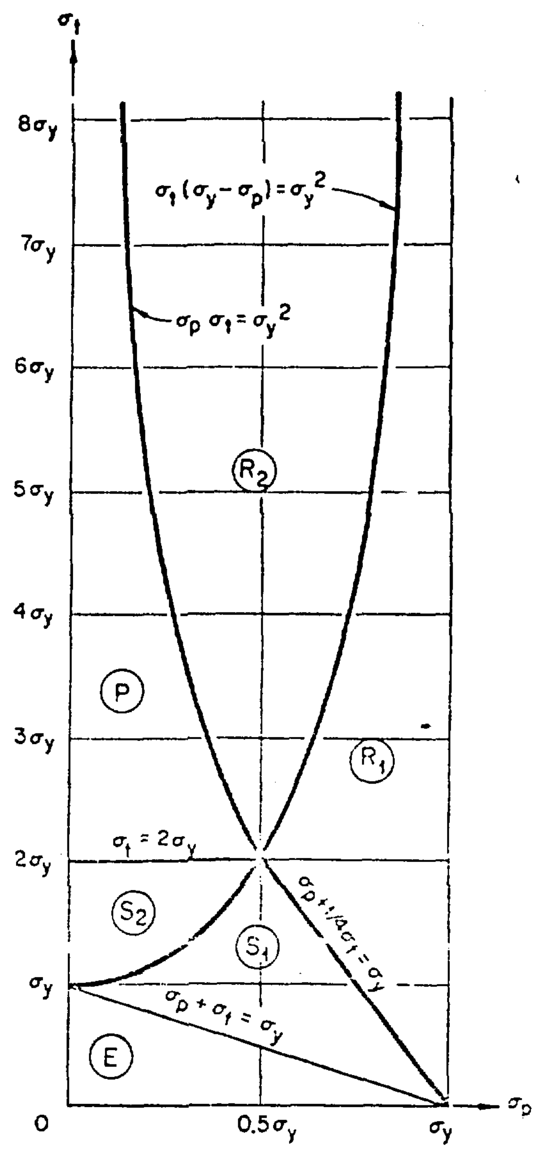
The advantages of simplified analysis methods are well known. In addition to their low cost of application, which includes both computational and engineering costs, the analyst can more readily interpret the results, in terms of design criteria, without resorting to extensive post-processing. However, many simplified methods that are generally applicable are extremely conservative, and conservatism for more realistic simplified methods can be difficult or impossible to prove. It is this one issue, more than any other, that has dampened the enthusiasm for simplified analysis.

Even methods which occasionally produce unconservative results can be useful for preliminary design of components, ...lly when the major concerns are the estimation of design ...n, screening and sensitivity studies, and guidance for more detailed analysis. However, when the aim is to provide information for final stress reports, the conservatism of the method must be adequately demonstrated.

DESIGN METHODS BASED ON ELASTIC ANALYSIS

"Elastic analysis methods" refers here to simplified methods which require only an elastic analysis of a component even though inelastic behavior is known to occur. Let us first look at the elastic analysis methods in Code Case N-47 [1].

Miller [3] considered the problem of thermal ratchetting in the absence of creep. He identified the demarcation line (in a pressure-induced primary versus thermally-induced secondary stress space) between ratchetting and nonratchetting for a thin-walled cylinder subjected to a cyclic uniform heat flux through the wall and a sustained internal pressure for an elastic-perfect-plastic material. Bree [4,5] extended Miller's work to include the effect of creep. He also showed that in the absence of creep there were six distinct regions of behavior as shown in Fig. 1. The boundaries of these regions were determined by analysis of the stress and strain distribution through the wall



Stress regime	Behavior
R_1 and R_2	Ratchetting
S_1 and S_2	Shakedown after first half-cycle
P	Plastic cycling
E	Elastic

FIG. 1 BREE DIAGRAM

as the thermal stress was cycled. Note that Miller's demarcation line between ratchetting and nonratchetting regions corresponds to the line in Fig. 1 that separates regions P and R_2 and regions S_1 and R_1.

The elastic analysis methods for ratchetting in Code Case N-47 [1] have evolved over the last decade from the work by Bree. Bree actually used a simplified, uniaxially-stressed restrained strip model of the thin tube in his analysis. Subsequent work by O'Donnell and Porowski [6], which is the basis for Test No. 3 of Code Case N-47, also makes use of Bree's one-dimensional model of a thin-walled tube. For this reason the use of the elastic ratchetting rules, except for those that assure that the structure never yields, are restricted to axisymmetric structures subject to axisymmetric loading away from local structural discontinuities.

In studying the Bree stress and strain distribution through the wall after the first cycle, O'Donnell and Porowski observed that, in the S_1, S_2, and P regions, some portion of the wall always remains elastic. This is defined to be the elastic core. The maximum stress in that region at the hot end of the cycle was called the "core stress". O'Donnell and Porowski reasoned that the average strain accumulation would be restricted by the elastic core. In particular, the creep strain that would accumulate due to the "core stress" is used to bound the average creep ratchetting strain in the S_1, S_2, and P regimes.

In a second paper [5], Bree considered the effects of creep. He generated bounds on the average strain accumulation for all regions of the *Bree diagram using the assumption that complete* relaxation to the primary stress level would occur during each cycle. Those bounds are the basis of the Bree complete relaxation method, which provides bounds for both the average and linear bending strains in all six regimes of the Bree diagram.

More recently O'Donnell and Porowski [7] have improved their creep ratchetting bounds to include the effects of strain hardening and temperature-dependent yield strength. Furthermore, they have developed bounds for the average ratchetting strain under creep conditions in the plastic ratchetting regimes R_1 *and* R_2 by introducting the concept of a relaxed core stress. Their method includes an explicit consideration of the enhanced creep strain* that occurs because of the nonlinear dependence of creep strain on stress. The Bree complete relaxation method does not explicitly account for enhanced creep. Instead, it relies on the over-conservatism of the assumption of complete relaxation to be sufficient to accommodate any enhanced creep. These recent improvements by O'Donnell and Porowski are now being considered for inclusion in Code Case N-47.

There have not been as many major modifications of the elastic creep-fatigue rules during the past decade. The elastic creep-fatigue rules in Code Case N-47 continue to use the linear damage rule

$$\sum_{j=1}^{p}\left(\frac{n}{N_d}\right)_j + \sum_{k=1}^{q}\left(\frac{\Delta t}{T_d}\right)_k \leq D \qquad (1)$$

where

D = total creep-fatigue damage
n = number of applied cycles of loading condition j
N_d = number of design allowable cycles of loading condition j
Δt = time duration of loading condition k
T_d = *allowable time at loading condition* k

Some important improvements in the procedures for predicting creep-fatigue damage from elastic analysis results have been made. In the area of fatigue damage, Severud [8] developed an improved method for estimating the strain-range at a stress concentration from elastic analysis, based on Neuber's work. Equation (7) of Code Case 1592-7 [9] which was intended to account for increased strain due to inelastic behavior, is:

$$\epsilon_T = K_\epsilon \epsilon_e + K_\epsilon^2 \epsilon_p + K_T \epsilon_F \qquad (2)$$

where

ϵ_T = the derived maximum strain for the loading condition
ϵ_e = the elastic strain in the regions under consideration - exclusive of strain concentrations,
K_ϵ = the theoretical elastic strain concentration factor
ϵ_p = the inelastic strain in the region under consideration - exclusive of strain concentrations and peak thermal strains
ϵ_F = peak thermal strain associated with the peak thermal stress intensity
K_T = strain concentration factor applied to peak thermal strain component ϵ_F. The procedure for applying Eq. (2) (described in detail by Campbell ([10]) is quite laborious, since ϵ_T must be found for each of six strain components (ϵ_x, ϵ_y, ϵ_z, γ_{xy}, γ_{xz}, γ_{yz}) before determining the component strain ranges and the equivalent strain range. In addition, Eq. (2), which was developed to approximate the Neuber equation without requiring an iterative solution technique [11], did not give any strain concentrating effect until the net section yielded. This was contrary to the Neuber [12] method and to known structural response. Strain concentrations relative to the elastically-calculated *stress concentration* initiate as soon as local yielding in the notch occurs. However, after net section yielding, Eq. (2) gives a rapid increase in strain concentration that can lead to large over predictions of maximum strain.

The new Eq. (7) of Code Case N-47 [1] developed by Severud [8] is:

*The term enhanced creep refers to the difference between the creep strain in a member with a nonuniform stress distribution and the creep strain in a member with a uniform stress distribution, both distributions having the same average stress.

$$\epsilon_T = \frac{(S^*)}{\bar{S}} K^2 \epsilon_n + K\epsilon_c + K_T\epsilon_F \qquad (3)$$

where

ϵ_T = the derived maximum total equivalent strain for the loading cycle

K = the theoretical elastic stress concentration factor for the local geometric discontinuity or the fatigue strength reduction factor applicable for the local geometric discontinuity considering both the temperature and the loading history

S^* = the nominal stress indicator exclusive of the stress concentration factor and is determined by entering the stress-strain curve at ϵ_n

$\bar{S}$ = the stress indicator determined by entering the stress-strain curve at strain $K\,\epsilon_n$

ϵ_n = the nominal total equivalent elastic plus plastic strain for the loading condition exclusive of the geometry-induced local stress concentration, exclusive of peak thermal strain, and exclusive of creep strain

ϵ_c = the creep strain from load controlled stresses and is determined from the isochronous stress-strain curve using the load controlled stress intensity. Creep strain is the difference in strain obtained from the time-dependent isochronous stress-strain curve for the time of the load under consideration minus the strain obtained from the zero-time isochronous stress-strain curve

ϵ_F = peak thermal strain associated with peak thermal stress intensity

K_T = elastic stress concentration factor applied to peak thermal strain component, ϵ_F, to account for interaction with local geometric discontinuities

The stress-strain curve used for obtaining values of S^* and $\bar{S}$ is developed by taking the appropriate hot tensile (zero time) stress-strain curve from those given in the Code Case N-47 and transforming the stress-strain coordinate system.

Other improvements in the elastic fatigue procedures suggested by Severud [6] in addition to the revised Eq. (7), include: (a) a variable Poisson's ratio in calculating stresses and strains using elastic equations to account for local thermal strain concentrations; (b) a modified approach for determining the equivalent strain range; and (c) an adjusted design fatigue curve. The use of a variable Poisson's ratio (between 0.3 for purely elastic behavior and 0.5 for purely plastic behavior) helps overcome the underprediction of equivalent strain range obtained using elastic methods. The modified approach for determining the equivalent strain range gives more conservative values, but is easier to apply because less computation and stress-strain data are required. The design fatigue curves were lowered by 15 percent in the high cycle region because the equation for computing the equivalent strain range assumes a Poisson's ratio of 0.5, which results in a 15 percent underprediction of the maximum principal strain range during elastic cycling.

Perhaps the most novel approach recently put forth as a creep-fatigue damage assessment is that of Houtman [13-14]. The approach stems from the observation of the similarities between one-dimensional inelastic analysis of infinite plates and thin-walled cylinders subjected to cyclic thermal loading. The crucial display of this similarity is a plot of an inelastic strain correction factor versus an elastic stress ratio for a number of companion elastic and inelastic analyses of plates and cylinders with varying wall thicknesses and temperature histories. From each elastic analysis, the peak effective stress, the metal temperature (at the time and the location of peak effective stress), the elastically-calculated effective strain, and time are recorded. From the corresponding inelastic analysis, the effective inelastic strain at that time and location is recorded. The inelastic strain correction factor, K_ϵ, is the ratio of the effective inelastic strain to the elastically-calculated effective strain. The elastic stress ratio is the peak elastically-calculated effective stress, S_n, normalized by the 0.2 percent offset yield stress, S_Y, at the temperature of the peak stress point. A typical plot used by Houtman [14] is shown in Fig. 2. He states that K_ϵ is virtually repeatable for additional cycles, since no mean stress due to mechanical loading is available to

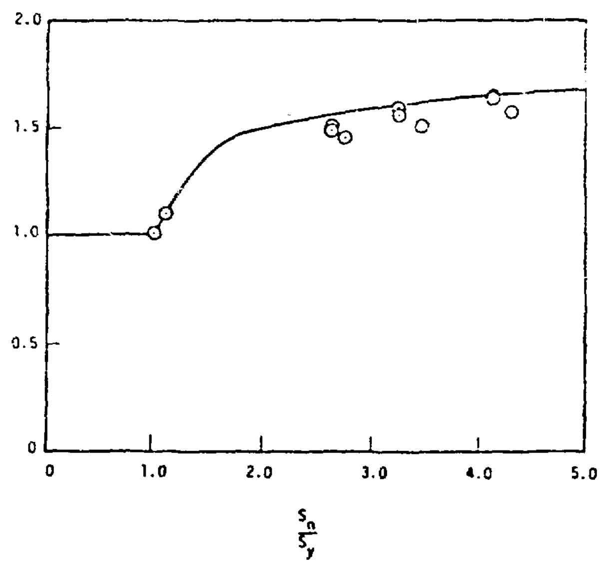


FIG. 2 INELASTIC STRAIN CORRECTION FACTOR, K_ϵ, VS ELASTIC STRESS RATIO, S_n/S_y, TAKEN FROM REF. [14]. S_n IS THE ELASTICALLY-CALCULATED EFFECTIVE STRESS AND S_y IS THE 0.2% OFFSET YIELD STRESS

drive the inelastic strain. He also points out that substantial variations in plastic properties (±20 percent in the yield stress and order-of-magnitude changes in the ratio of tangent modulus to Young's modulus) do not alter the K_ϵ significantly.

In order to use this information in a creep-fatigue evaluation, one begins with the elastic stresses and strain for a cyclic temperature history. At the time of peak stress (strain), the elastic stress ratio is computed and the total inelastic strain estimated from K_ϵ. The maximum stress at the beginning of the creep hold period is found by using this inelastic strain and its corresponding stress at temperature, kinematic hardening unloading to reverse yielding at the steady-state operating temperature, and complete reversal of strain through plastic flow at the operating temperature. The relaxation of this stress is then calculated during the hold period. Strain range and stress versus time during the hold period are then available for calculating fatigue and creep damage increments.

GEOMETRIC SIMPLIFICATION

Geometric simplifications takes many forms. The most common is a reduction in the number of spatial dimensions being considered, such as one- or two-dimensional analysis, rather than full three-dimensional calculations. A slight variation would involve the use of generalized deformation parameters in one or more dimensions.

A restriction on the magnitude of deformation, such as infinitesimal strains or moderate rotations, is also considered to [illegible] geometric simplification. In view of the general design requirement to limit strains in order to avoid fatigue and ductility exhaustion, designers are seldom faced with finite strain conditions. However, finite rotations are occasionally needed to carry out buckling calculations for imperfection-sensitive structures. Therefore, a method for taking imperfections into account, without actually conducting a geometrically nonlinear analysis, would represent a geometric simplification.

ONE-DIMENSIONAL ANALYSIS

A considerable amount of preliminary design work is based upon one-dimensional analysis, especially in regions of vessels and components that are devoid of abrupt cross-sectional changes. A common approach is to use two- or three-dimensional elastic analysis in order to derive the equilibrium values of the resultants to be used in one-dimensional inelastic analysis. Usually the resultant of interest is the axial membrane force which is required to sa[illegible]y pressure equilibrium. Generalized plane strain conditions are applied to the one-dimensional analysis model, so that the axial resultant is effectively applied to all axial degrees of freedom at the same time.

efforts

One of the first effects to develop a one-dimensional (axisymmetric generalized plane strain) computer program of some generality is described by Barsoum [15] who built upon the work of Miller [3], Bree [4,5], and Burgreen [16]. This simplified analysis tool, called CREPLACYL, has been employed in design evaluation [17]. Two-dimensional and three-dimensional models are elastically analyzed in order to determine the appropriate far-field loading conditions for the one-dimensional cylindrical geometry. The inelastic strain range and the nominal stress intensity are determined so that creep-fatigue and ratchetting can be evaluated.

Such extended one-dimensional analysis tools are now widely available. In addition, any general purpose inelastic program can be executed, relatively inexpensively, in the axisymmetric generalized plane strain mode, as indicated by a recent Japanese study [18]. Unfortunately, while the simplified methods involving these tools are sometimes conservative, no general rationale for numerically establishing the stress resultants (including shear resultants, as well as internal pressure and axial force) is available. An inability to capture the character ~~to~~ of creep strain concentrations caused by elastic follow-up would seem to be inevitable.

An exhaustive study of the ability of one-dimensional inelastic methods to simulate the behavior of a two- or three-dimensional structure was conducted by Sartory and Yahr [19]. Several alternative types of generalized force deformation simulations were attempted, but they were either significantly nonconservative (in many instances) for both ratchetting strain and creep-fatigue damage or were so overly conservative as to be impractical. In every case, the elastically-calculated local primary stress intensity was reproduced in the one-dimensional inelastic analyses by adjusting the pressure loading, keeping the axial loading unchanged. An elastically-calculated edge moment or rotation was applied to simulate the secondary stress due to through-the-wall thermal gradients. The applied rotation tended to be unconservative, due to stress relaxation, while methods for prescribing values of moment were vastly overconservative, due to the prevention of stress relaxation. Thus, although one-dimensional analyses are widely used, rules that assure their conservation have yet to be developed.

COARSE GRID ANALYSIS

The cost of a finite element analysis can be reduced by decreasing the number of elements that are used to model the structure. Thus two-dimensional geometries can be inelastically analyzed at a reasonable cost and in a reasonable time if an extremely coarse grid is used. Although peak values may not be predicted well, this approach has the definite advantage of accounting for the low-order interaction of the various portions of the structure. By low-order, we mean that average values of the two-dimensional stress field are probably reasonable, but that bending stresses require scrutiny. Accurate values of bending stresses depend upon the grid's ability to capture curvature, normally a weak point in a coarse grid analysis.

Steep thermal gradients through the wall of a component are even more troublesome for a coarse-grid analysis. The strain strain distribution that can be modelled by the few elements across the thickness does not approximate the thermal expansion strains produced by the steep thermal gradients. The error in matching these thermal expansion strains is projected, in some cases (depending upon the finite element formulation used), onto the higher dilatational deformation modes of the elements, resulting in fictitiously large hydrostatic stresses. For low-order elements, the highest deformation modes are coupled dilatational-shear (keystone) modes. Studies by Barsoum and Loomis [20] indicate that deviatoric stresses and strains are unaffected by the high hydrostatic stresses, but this finding can only be true under a restricted set of conditions, such as the absence of shear, and should not be generalized.

It should also be pointed out that a converged finite element solution contains virtually no strain energy in the highest deformation modes. Thus, the presence of the fictitiously large hydrostatic stresses is a signal that the results obtained are questionable. The deviatoric stresses may be fortuitously correct for two reasons: (a) the axial and circumferential bending are captured entirely by element extensional deformation (true only for low-order elements); or (b) the shear stress through the cross-section is negligible because the structure is thin. If this is the case, since the effective stress and the inelastic strains are functions only of the deviatoric stresses, the strain limit and creep-fatigue damage evaluations may be reasonable.

SPECIAL PIPE AND BEND ELEMENTS

The application of simplified analysis to piping components remains in a very preliminary state. An elastic analysis of a full piping run is often used to locate highly-stressed regions and local inelastic analysis of these regions (e.g., an elbow) are then performed to provide detailed information on stress levels and strain concentrations. This clean separation is less than in actual practice because of uncertainty in the feedback mechanisms between local and global regions.

A more direct approach has been proposed by Workman and Rodabaugh [21]. The piping computer program PIRAX2 represents an attempt to extend simplified elastic-plastic piping analysis, where ovalization of elbows due to elastic-plastic deformation is accounted for by flexibility factors [22] and where curved beam theory describes the longitudinal response to creep/relaxation behavior. The important additional assumptions required are that: (a) the structural response of a curved pipe can be approximated by curved beam theory with flexibility factors to account for ovalization due to elastic, plastic, and creep deformation; (b) only steady-state creep is modeled; and (c) inelastic strains due to axial, bending, and torsional loading can be uncoupled. Boyle and Spence [23] have questioned some of these assumptions and have argued that the problem cannot be legitimately *simplified in this way*. Nevertheless, they have produced a code which employs similar assumptions [24]. Other alternatives have become available, largely due to various extensions of the finite element method. They are discussed below.

The extension of axisymmetric finite element formulations to pipe bends was first documented by Marcal [25]. Conventional axisymmetric shell elements model the deformation of the pipe in a cross-sectional plane. In this respect, the formulation resembles that for a complete toroidal shell. However, additional deformation modes, in the form of bending and stretching longitudinally along the pipe, are coupled to the axisymmetric shell modes through the addition of a nodal point with three degrees of freedom [26]. These three degrees of freedom represent relative normal motion, relative in-plane rotation, and relative out-of-plane rotation of one end plane of the pipe bend with respect to the other. The additional nodal point is assumed to be connected to each pair of nodal points defining an axisymmetric element in the cross-sectional plane. Because of this connectivity, phenomena such as elbow ovalization due to in-plane bending are captured, with the accuracy of the model dependent upon both the axisymmetric shell modeling of the ovalization and the longitudinal modeling along the bend.

For the particular simplified elbow finite element in the MARC general purpose program, the choice of one additional node per elbow section with degrees of freedom implies that stretch and curvature are constant along each section. This, in turn, implies that compatibility of deformation is not maintained from section to section, which implies that monotonic convergence of the approximate solution cannot be guaranteed as a function of decreasing element size.

An alternative method has been suggested by Ohtsubo and Watanabe [27, 28] that is referred to as the finite ring element method. Rather than assuming constant stretch and curvature, the ring element deformations are approximated by second-order Hermitian polynomials along the longitude of the pipe bend between two transverse cross-sectional planes. The deformation in the circumferential direction around the cross section is approximated by trigonometric functions. The strain-displacement relations of thin shell theory are adopted; however, while shearing strains through the thickness are assumed negligible, shearing strain in the shell surface is retained. In-plane and out-of-plane deformations are assumed to be uncoupled, with complementary sets of trigonometric functions chosen accordingly. Each ring element has 12 m degrees of freedom, where m is the number of trigonometric Fourier coefficients. Integration of the stiffness matrix requires, for reasonable accuracy, some thirty-two Gauss points in the circumferential direction and four Gauss points in the longitudinal direction. Interelement compatibility is presumably satisfied.

This element may or may not fall into the simplified methods category, depending upon comparative computational effort with full shell models. For instance, the number of Gaussian integration points and the number of degrees of freedom are roughly commensurate with a relatively crude, doubly-curved shell grid. Another point that requires investigation is the effect that the lack of adequate rigid-body motion will have on convergence.

Other variations of the Ohtsubo-Watanabe formulation are under development. One, by Bathe and Almeida [29], employs isoparametric coordinates along the longitude of the elbow in order to describe both the geometry, the curved-beam deformations, and the longitudinal variation in ovalization. The circumferential variation in ovalization is treated by trigonometric functions, similar to Ohtsubo and Watanabe. At present, four nodes and four ovalization displacement patterns are used in the longitudinal direction for each elbow element. This corresponds to a cubic interpolation pattern. Again, the number of integration points in both the circumferential (trigonometric) and longitudinal (isoparametric) directions in order to achieve accuracy may be such as to prescribe the use of this element as a simplified analysis method. Bathe and Almeida currently recommend Newton-Cotes integration formulas, instead of Gaussian integration, with three points through the wall thickness, five points longitudinally, and either twelve or twenty-four points circumferentially, depending upon the [illegible] in-plane or out-of-plane). Other alternatives might [illegible] lower-order approximation to the longitudinal variation in ovalization.

CONSTITUTIVE SIMPLIFICATION

There are many possible constitutive simplifications. In this section we discuss only those simplified models that relate to the two fundamental stress analysis situations of elevated temperature design: (1) the redistribution of stress due to time-dependent material behavior, and (2) the redistribution of stress due to combined time-dependent and time-independent material behavior.

When the loading on the structure is monotonic, solutions for either situation are fairly straightforward, provided that any residual stresses existing prior to loading are known, or are insignificant. If the equilibrating stresses are below yield, their distribution is due solely to time-dependent relaxation of the highest values, with a corresponding transfer of load to the adjacent lower stressed regions. Calculations for this situation are relatively simple with modern computational tools, since the initial strain method [30] does not require modification of the stiffness matrix. When an implicit treatment of creep behavior is attempted [31], the stiffness matrix is modified and larger time steps can be taken in the analysis; however, implicit formulations are the exception rather than the rule.

When time-independent inelasticity accompanies the time-dependent inelasticity, the problem is a little more complex. At stresses near yield, the creep rates are sufficiently high to sharply affect convergence of the explicit initial strain method. Also, under some conditions, additional yielding can occur during time-dependent stress redistribution. Both these effects cause computational problems. One approach that is used is to lump the primary creep, where the creep strain rates and stress relaxation rates are highest, with the plasticity.

An example of a simplified method of inelastic analysis for monotonic loading is the isochronous stress-strain method, used by Sobel for creep analysis of piping elbows and discussed in Ref. [32]. In attempting to evaluate the time-dependent deformation characteristics of elbows tested at elevated temperature, it was observed that creep analysis was considerably more expensive than elastic-plastic (time-independent) deformation analysis. To reduce the analysis costs, the monotonic creep response was predicted using a related elastic-plastic analysis. This isochronous stress-strain curve for the desired test time was used as the stress-strain curve in an elastic-plastic analysis. The calculated deformation was interpreted as the deformation that would occur at the test time that corresponded to the isochronous curve. Fig. 3 shows the comparison between results from an actual creep analysis and this simplified method in terms of rotation versus normalized load.

When the loading is cyclic, the issue of simplified inelastic analysis is much more complex. After loading into the plastic range, with perhaps some time-dependent stress redistribution and further inelastic deformation as the result of a hold period at elevated temperature, removal or reversal of the load leads to residual stress states that strongly influence the subsequent response of the structure. These residual stresses can be help-

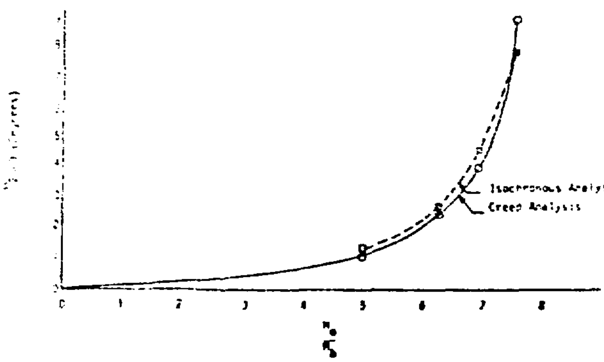


FIG. 3 ROTATION AT 2000 h VS LOAD LEVEL IN AN ELBOW SUBJECTED TO AN IN-PLANE ELBOW-CLOSING MOMENT [32]

ful in shaking the structure down to elastic behavior. Time-dependent material response has the effect of relieving these residual stresses however.

A number of simplified approaches to this interaction between residual stresses and time-dependent deformation are now covered by the rules of Code Case N-47. An extremely conservative approach is not to take any credit for favorable residual stresses after unloading. This method, termed the Bree complete relaxation approach was mentioned previously and is discussed in some detailed in Ref. [32].

Bounding techniques, which have been adequately reviewed elsewhere [32], make use of idealizations of the material behavior in order to develop bounding solutions. For example, elastic-perfect-plastic behavior and a Norton creep law (strain rate, an exponential function of stress) are often assumed.

VALIDATION OF SIMPLIFIED METHODS

The Oak Ridge National Laboratory (ORNL) and selected subcontractors have, for the past several years, been assessing the conservatism of existing simplified elastic analysis methods. Their approach has been to compare results from the simplified methods with results from detailed inelastic analyses. The simplified methods are said to be conservative and valid if the accumulated strains predicted by the simplified methods are greater than those predicted by detailed inelastic analysis, and if the simplified (elastic) creep-fatigue rules yield a smaller number of allowable cycles than do the inelastic creep-fatigue rules. This approach assumes that the inelastic analysis results are valid. Validation of the inelastic analysis results has been addressed elsewhere [33]. Some of the early work [34] produced baseline inelastic results for a series of infinite pipes and of an intersection of a pipe and spherical shell. Assessment of thermal ratchetting rules were emphasized in the early work, but the program has been expanded to examine the ability of simplified methods to predict creep-fatigue damage as well as ratchetting. That work has been carried out in two phases.

Yahr and Sartory [19] have summarized the first phase of the ORNL program. Nine axisymmetric geometries were examined, including notched and stepped-wall cylindrical shells, subject to a reference duty cycle involving a thermal downshock, depressurization, repressurization, reheat to temperature, and a creep hold period. Fig. 4 illustrates the thermal and mechanical load histograms for this cycle. The transient temperature and the inelastic response were determined for ten duty cycles. Then, each geometry and loading was elastically analyzed for one duty cycle. The elastic results were used in the evaluation of ratchetting strains, according to the O'Donnell-Porowski method [1] and the Bree complete relaxation method [5], and in elastic creep-fatigue evaluations according to the ASME CC N-47 rules [1]. Also, one-dimensional axisymmetric (generalized plane strain) inelastic solutions were obtained.

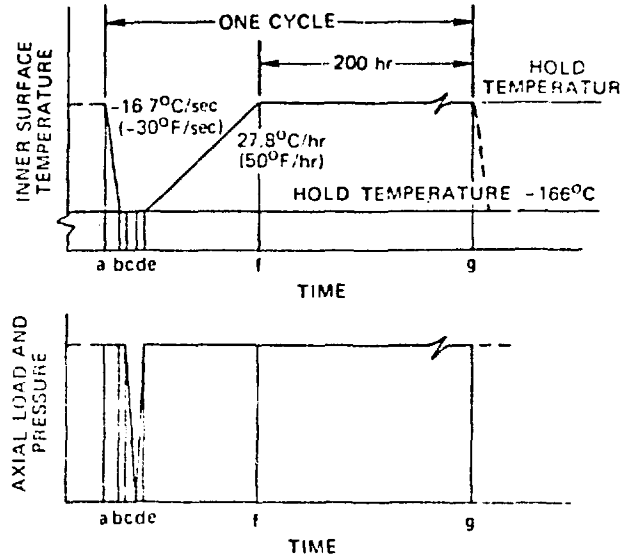


FIG. 4 THERMAL AND MECHANICAL LOAD HISTOGRAMS FOR SEVERAL OF THE ORNL AXISYMMETRIC GEOMETRIES. TAKEN FROM REF. [19]

The major findings of this study were:

1. The generation of repetitive cycles after the third or fourth cycle was apparent; this was true in spite of the observation that increasing patterns of strain were present in many cases.

2. The O'Donnell-Porowski and Bree methods were generally conservative, sometimes extremely conservative, in comparison to detailed inelastic results; two instances of nonconservatism (a notched section and several thick-walled sections) were easily resolved through slight modifications of procedure.

3. One-dimensional inelastic solutions did not produce uniformly conservative predictions.

A set of three-dimensional inelastic analyses was also included in the ORNL program to test the validity of simplified methods for such structures. Combustion Engineering, with a group under R. S. Barsoum [20,35], computed the response of a nozzle-to-cylinder geometry for a little more than three cycles. At Atomics International Division of Rockwell International, Y. S. Pan [36] computed one cycle of the response of a nozzle-to-spherical shell geometry. The latter problem was nonaxisymmetric because a bending moment was applied to the nozzle in addition to internal pressure and thermal downshocks on the internal surface. The geometries analyzed in the two studies are shown in Figs. 5 and 6. The elastic ratchetting and creep-fatigue rules were always conservative in the intersection regions of both the nozzle-to-shell problems.

Winkel [37] has described experience at Hanford Engineering Development Laboratory (HEDL) with simplified inelastic

NOZZLE-TO-CYLINDRICAL SHELL
(IHX INLET NOZZLE)

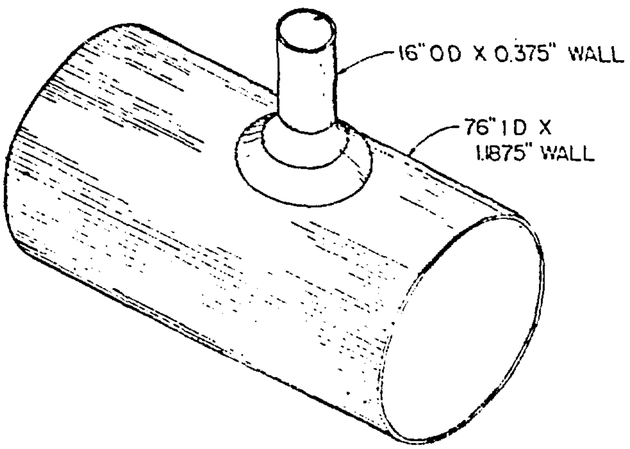


FIG. 5 NOZZLE-TO-CYLINDRICAL SHELL GEOMETRY ANALYZED BY COMBUSTION ENGINEERING (UNDER ORNL SUBCONTRACT), SEE REFS. [20,35]. THE GEOMETRY IS THE SAME AS THE FFTF IHX INLET NOZZLE, AND THE LOADING INVOLVED AN OUT-OF-PLANE MOMENT LOADING ON THE NOZZLE, AN INTERNAL PRESSURE, AND A REPEATED INTERNAL THERMAL DOWNSHOCK

NOZZLE-TO-SPHERICAL SHELL

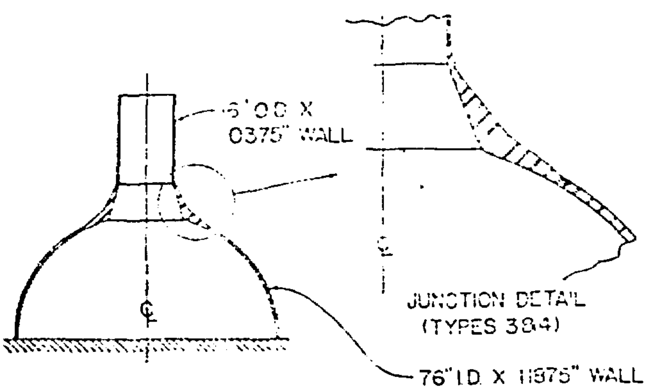


FIG. 6 NOZZLE-TO-SPHERICAL SHELL GEOMETRY ANALYZED BY ATOMICS INTERNATIONAL (UNDER ORNL SUBCONTRACT), SEE REF. [36]. THE LOADING INCLUDED A NOZZLE BENDING MOMENT, AN INTERNAL PRESSURE, AND A REPEATED INTERNAL THERMAL DOWNSHOCK. THE PROBLEM WAS THUS NONAXISYMMETRIC.

analysis methods that have been developed during the design of the Fast Flux Test Facility (FFTF). One of the validation exercises compared the ratchetting predicted using the Bree [5], O'Donnell-Porowski [1], and one-dimensional cylinder analysis methods with the ratchetting predicted by rigorous inelastic analyses for various plant components. Only the Bree method was conservative for all of the 20 cases examined. Both the O'Donnell-Porowski and one-dimensional cylinder analyses were generally conservative; both were unconservative in five of the cases, however.

Another HEDL study described by Winkel [37] involved a straight pipe subjected to a constant bending moment and rapid cooldown transients on the inside surface. The only departure from a one-dimensional cylinder problem was the nonaxisymmetric bending moment. The objective of this study was to investigate the conservatism of replacing the beam bending stress with an axisymmetric stress equal to the peak bending value. One-dimensional cylinder approximations were performed for both the tensile and compressive sides of the beam, with the peak bending stress "smeared" axisymmetrically. In all damage categories the rigorous solution was bounded by at least one of the one-dimensional cylinder analyses. Based upon this case study and related experiences at HEDL, it was concluded that axisymmetric smearing of a beam bending stress is a reasonable simplification. However, consideration must be given to both the tensile and compressive sides of the beam to obtain conservative results.

Gangadharan *et al.* [36] used increasingly complex analyses in their structural evaluation of the primary sodium inlet nozzle of the Fast Flux Test Facility's intermediate heat exchanger (FFTF-IHX) design. An axisymmetric model of the nozzle and outer shell was first analyzed elastically. (Their justification for replacing the cylindrical outer shell by an axisymmetric spherical shell with twice the radius is based upon the argument that the system mechanical loads are minor contributors to the stresses and deformations; instead, thermal gradients across the wall and the resulting stresses, which they believed are influenced only slightly be geometry, are of major concern.) Critical locations identified from the elastic analysis were then examined by the Bree method [5] using the average circumferential stress across the section as the primary stress component and the maximum principal stress difference (for thermal stress only) as the secondary component. These stresses were normalized by the strength at the highest temperature in the histogram. The estimated ratchetting strains, which later proved to be drastically conservative, exceeded the design limit of 1 percent, so additional analysis was required. One-dimensional inelastic analyses were then carried out in a second effort to demonstrate that the IXH design was adequate. The thick-walled cylinder was given an inner radius corresponding to the distance along a normal drawn from the nozzle throat inner surface (at the critical location) to the nozzle axis. An internal pressure sufficient to equilibrate an average circum-

...tial stress equivalent to that calculated elastically from axisymmetric analysis was applied. An axial stress resultant equivalent to the average, elastically-calculated meridional stress at the section was also applied. These mechanical loadings were varied during a cycle as follows: (a) for a reactor scram transient the mechanical loads vanish during the thermal downshock, but are held at full value during heat-up and elevated temperature hold conditions; (b) for a control-rod drop, the mechanical loads are maintained at full value. The linear thermal gradient is calculated from the maximum linearized circumferential thermal stress during the cycle. All material properties are the values at maximum cycle temperature. One would expect such conservative assumptions to yield conservative results; in this case, the "conservative" results indicated that no ratchetting would occur and that cumulative creep-fatigue damage was negligible.

Finally, a detailed axisymmetric model of the IHX was inelastically analyzed. As was true for the one-dimensional inelastic analyses, these results showed that the damage due to creep-fatigue and the ratchetting strains were within the allowables. Unfortunately, no detailed numerical comparisons between the detailed inelastic analysis and the one-dimensional analysis were provided, so the study cannot be used to justify the use of one-dimensional analysis.

Kamal, Chern, and Pai [39], under the sponsorship of ORNL, have used detailed inelastic analysis results [40] obtained in support of the design of the Clinch River Breeder Reactor Project intermediate heat exchanger (CRBRP-IHX) as the basis for evaluating the use of simplified structural models. The simplified models considered include the generalized plane strain, axisymmetric thick-walled cylinder [41], which is applicable when the thermal loading in the actual structure is dominated by the through-the-wall temperature gradient, and the finite cylindrical shell [42], which is appropriate when the meridional temperature gradient becomes dominant. From the thermal elastic analysis results of the detailed finite element model, critical areas were identified, dimensions of the simplified model were selected, and associated loading histograms were derived. Inelastic analyses using efficient computer codes were then performed to simulate the stress-strain conditions of these critical areas. The method was applied to the primary inlet nozzle and "Y" junction of the CRBRP-IHX for which detailed inelastic analysis results for six unit load cycles were available for comparison.

It was concluded [39] that the simplified approach using cylindrical structural models has the capability to simulate the trend of the stress variations for critical areas of complex structural shapes subjected to time varying loads, but techniques assuring conservatism of the method were not found. Less satisfactory results, but still useful for screening analysis, were obtained for the "Y" junction. The one-dimensional model used to assess creep damage shows promise but needs further development. The results obtained using the one-dimensional (cylindrical) computer codes demonstrated the usefulness of this type of simplified analysis approach for screening analyses.

An international piping benchmark effort [43] was recently completed which included both simplified and detailed solutions to three piping problems for which experimental data were available. In general, detailed inelastic analysis methods provided greater predictability of deformation than the simplified methods. The simplified methods, as a rule, predicted greater amounts of deformation than measured, although the results varied widely. Some detailed methods encountered difficulty where nonconstant loading, large deformations, or nonlinear structural behavior were present. The simplified methods were less capable of predicting structural response to load changes (subsequent to initial loading) involving plasticity or creep. This lack of capability sometimes led to large disagreement between analysis and experimental results. It was concluded, however, since relative expense of performing a simplified analysis may be as much as an order of magnitude less than that for a detailed analysis, that there is an important role for such simplified techniques where applicable. A thorough understanding of the simplifications and the ramifications on results obtained is essential in utilization of such techniques.

The Pressure Vessel Research Committee's (PVRC) Subcommittee on Elevated Temperature Design is presently conducting an international benchmark project in which simplified methods are applied by various investigators to three benchmark problems: (1) thermal ratchetting of a cylinder that is being tested by the French; (2) cyclic creep ratchetting of a cylinder that has been tested by ORNL [44]; and (3) cyclic thermal loading of a nozzle-to-sphere configuration that has been analyzed extensively by ORNL. The objective of the PVRC project is to provide a comparison of a variety of simplified methods of inelastic analysis with experimental results and detailed inelastic analysis. Extensive computational results have been obtained from workers in the U.S., United Kingdom, France, and Japan. An interpretive report has been prepared on the first two benchmark problems [45] to make the results available and provide assessment of simplified methods of analysis based on the detailed comparisons.

Examination of each of these problems reveals that the simplified methods, since they are "local" in nature, cannot readily capture the load transfer from adjacent regions of differing flexibility. It seems clear that good results (both conservative and accurate) can be obtained when flexibilities are relatively uniform. An improved method is needed for those areas where this requirement is not met.

CONCLUSIONS

This review of simplified methods of design analysis at elevated temperature has focused on historical developments of the past decade, with occasional references to prior pioneering developments. This section of the review will attempt to summarize the current state of the art, followed by a projection of the role that advanced developments may play in the future.

We begin by observing that the three major issues are assured conservatism, cost effectiveness, and interpretation of results. The situation with respect to conservatism of simplified methods is improving, primarily as the result of comprehensive studies such as those conducted at ORNL and HEDL. Currently recommended methods have been evaluated and, in some cases, extended. Such studies have provided the impetus for a critical evaluation of bounding techniques, which can be rigorously defended on conservative grounds. Many of these bounding techniques have been discussed by Nickell [32] with the general conclusion that the Bree and O'Donnell-Porowski results can be better understood within a bounding method framework.

Cost effectiveness is an item that is also of high priority in design. These costs can be attributed to computation itself and to pre- and postcomputation engineering costs. Simplified methods have had a significant effect in reducing both cost [illegible] components by enabling the design analyst to produce approximate results quickly and efficiently prior to, and sometimes precluding the need for, detailed nonlinear analysis. First, the simplified methods generally require less preanalysis preparation of data. Second, the simplified techniques tend to be less subject to error and inexpensive to execute. As a consequence, when detailed nonlinear analysis is required, the designer is guided by previous simplified calculations which, in turn, provide a check on the results from more complex models. Another useful application of simplified analysis methods is the extent to which the effects of variations in loading, geometry, or material behavior can be investigated in parametric form. Thus, trends can sometimes be established, with perhaps only a final verification provided by detailed nonlinear analysis.

Such parametric studies lead naturally to the third concern — the interpretation of results. The principal consideration of the analyst is the satisfaction of design allowables, such as those for inelastic strain ratchetting and creep-fatigue damage. For detailed nonlinear analysis, the volume of output generated is massive and postprocessing is essential for screening and transforming the data into a visually-tractable format. At the same time, the postprocessor can evaluate the results according to the design rules and criteria. Simplified methods play a role in this case by providing the designer with independent results that help to identify any discrepancies in the results.

Simplified analysis methods are apt to become more complex, blurring the distinctions between simplified and detailed analysis. For example, as the ratio of computing cost to engineering cost decreases, two-dimensional inelastic analysis will become more routine, gradually replacing some of the one-dimensional (generalized plane strain) calculations that are now performed. Bounding methods will likely be routinely applied in two dimensions with an occasional application to three-dimensional problems. Three-dimensional elastic calculations will probably remain the norm.

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