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Nuclide Migration from Areal Sources Into a Fracture

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Abstract

We show analytic solutions to the problem of hydrogeologic transport of radionuclides released from finite areal sources into a planar fracture. We illustrate the solutions through numerical and graphical displays of the spatial and temporal distribution of the radionuclides as a result of advection in the fracture, transverse dispersion and surface sorption, as well as diffusion into and sorption in the rock matrix. The numerical illustrations indicate that sufficient distances away from the sources equivalent single sources give acceptable approximations.

Introduction

In constructing nuclear waste repositories in rock, it may be necessary to place a waste package across a rock fracture, or a rock fracture may develop some time after a waste package has been emplaced. It is necessary to predict the spatial and temporal distribution of radionuclides from a line of waste packages facing a rock fracture, since such fractures may now be considered a main path for released radionuclides to re-enter the biosphere. The situation studied is shown in Figure 1, where a line of waste packages, here assumed to be corroded and releasing radionuclides, is exposed to a planar rock fracture in a fluid-saturated porous rock.

Analytic Solutions - A Single Areal Source

Consider an areal source in Figure 1. After a radionuclide is released, several transport processes are important in tracing its eventual fate. First, there is advection in the rock fracture. Second, there is transverse dispersion in the planar fracture. We established in an earlier study that longitudinal dispersion is of rather minor significance (Ahn, P. L. Chambré and T. H. Pigford, 1985). The radionuclide can sorb to the rock surface in the fracture, and it can migrate through diffusion into pores in the rock. The latter process is known as matrix penetration in nuclear waste disposal. After a nuclide has migrated into a pore, it is exposed to large amount of surfaces and sorption can take place. These transport processes can be described in the following coupled differential equations. We write a governing equation for the mass balance of nuclides in water in the rock fracture

$$R_f \frac{\partial N}{\partial t} + v \cdot \frac{\partial N}{\partial z} - D_T \frac{\partial^2 N}{\partial x^2} + R_f \lambda N + q/b = 0 \qquad t > 0, z > 0, -\infty < x < \infty \qquad (1)$$

where $N(x, z, t)$ is the nuclide concentration in the water in the rock fracture at time t and at location (x, z) $[m\ nuclide/\ell^3\ fracture\ water]$

K_f is the nuclide distribution coefficient in dimensions of $[(m\ nuclide/\ell^2\ fracture\ wall)/(m\ nuclide/\ell^3\ fracture\ water)]$

$2b$ is the aperture of the fracture $[\ell]$

v is the pore velocity in the fracture $[\ell/t]$

D_T is the transverse dispersion coefficient $[\ell^2/t]$

λ is the nuclide's decay constant $[t^{-1}]$

R_f is the dimensionless retardation coefficient in the fracture, defined as $R_f = 1 + K_f/b$, and

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We write a similar mass balance for the nuclide inside the porous rock matrix

$$R_p \frac{\partial M}{\partial t} - D_p \frac{\partial^2 M}{\partial y^2} + R_p \lambda M = 0 \qquad t > 0, y > b, z > 0, -\infty < x < \infty \tag{2}$$

where $M(x, y, z, t)$ is the concentration of the nuclide in the matrix-pore water
$[m\ nuclide/\ell^3\ pore\ water]$
D_p is the nuclide diffusion coefficient in the rock matrix $[\ell^2/t]$
R_p is a dimensionless retardation coefficient defined as $R_p = 1 + a_p K_p/\epsilon$
K_p is a distribution coefficient in dimensions of
$[(m\ nuclide/\ell^2\ pore\ surface)/(m\ nuclide/\ell^3\ pore\ water)]$
a_p is the pore surface area per unit volume of rock $[\ell^2 pore\ surface/\ell^3\ rock\ matrix]$
ϵ is the matrix porosity.

The two governing equations are coupled by two relationships. The first is flux continuity at the surface of the rock fracture

$$q(x, z, t) = -\epsilon D_p \frac{\partial M}{\partial y}\bigg]_{y=b}, \quad t > 0, z > 0, -\infty < x < \infty \tag{3}$$

We solve the governing equations (1) and (2) and subject to (3) and the following initial conditions

$$N(x, z, 0) = 0, \qquad z > 0, -\infty < x < \infty, \tag{4}$$
$$M(x, y, z, 0) = 0, \qquad y > b, z > 0, -\infty < x < \infty, \tag{5}$$

and boundary conditions

$$N(x, 0, t) = \varphi(t)[h(x + a) - h(x - a)], \qquad t > 0, -\infty < x < \infty, \tag{6}$$
$$N(x, \infty, t) = 0, \qquad t > 0, -\infty < x < \infty, \tag{7}$$
$$N(\pm\infty, z, t) = 0, \qquad z > 0, t > 0, \tag{8}$$
$$M(x, b, z, t) = N(x, z, t), \qquad t > 0, z > 0, -\infty < x < \infty, \tag{9}$$
$$M(x, +\infty, z, t) = 0, \qquad t > 0, z > 0, -\infty < x < \infty \tag{10}$$

where $\varphi(t)$ represents the time-dependent concentration at the surface of an areal source and can be in the form $N^\circ e^{-\lambda t}\{h(t) - h(t - T)\}$, with $h(\cdot)$ the Heaviside step function and N° the initial concentration. Equation (6) states that the center of an areal source of dimensions $2a \times 2b$ is located at the origin of the system. Equation (9) states concentration continuity at the rock/fracture interface. The solution to the above system of governing equations is given in Ahn *et al.*, (1987) and summarized below.

$$N(x, z, t) = N^\circ F(b, z, t) \cdot G(x; \theta, a, 0), \quad t \geq 0, -\infty < x < \infty, \tag{11}$$
$$M(x, y, z, t) = N^\circ F(y, z, t) \cdot G(x; \theta, a, 0), \quad t \geq 0, y \geq b, z \geq 0, -\infty < x < \infty, \tag{12}$$

where

$$F(y, z, t) = f(y, z, t) - e^{-\lambda T} f(y, z, t - T), \tag{13}$$

$$f(y, z, t) = e^{-\lambda t} h(t - ZA) \cdot erfc\left[\frac{Z + B(y - b)}{2\sqrt{t - ZA}}\right], \tag{14}$$

$$G(x; \theta, a, x_1) = \frac{1}{2} \cdot \left[erf\left(\frac{x - x_1 + a}{2\theta}\right) - erf\left(\frac{x - x_1 - a}{2\theta}\right)\right] \tag{15}$$

$$Z = \frac{R_f z}{vA}, \qquad A = \frac{bR_f}{\epsilon\sqrt{D_p R_p}}, \qquad B = \sqrt{R_p/D_p}, \qquad \theta = \sqrt{\frac{zD_T}{v}}. \tag{16}$$

These equations give the concentration field of the radionuclide in both the fracture and the rock matrix, as a function of space and time.

Analytic Solution - Multiple Areal Sources

If there are multiple areal sources along the fracture, as shown in Figure 1, the solution is obtained by superposing the solutions for single areal sources. For areal sources each of width $2a$ separated by a *pitch* d, and the same release characteristics, the solution is

$$M(x, y, z, t, m) = N^\circ F(y, z, t) \sum_{k=1}^{m} G(x; \theta, a, x_k), \tag{17}$$

where m is the number of areal sources, and
x_k is the location of the center of the k^{th} areal source on the transverse axis x and is
$x_k = d[k - (m+1)/2], \quad k = 1, \ldots, m$ $[L]$

Numerical Illustrations

We now provide numerical illustrations of equation (17). In Figure 2 we show the normalized concentration profile in the transverse direction. The ordinate shows the concentration predicted by equation (17) normalized by the concentration predicted by an infintely long source, defined as

$$M^\infty(y, z, t) = N^\circ(2a/d) \cdot F(y, z, t) \tag{18}$$

Combining equations (17) and (18), the ordinate in Figure 2 shows

$$\frac{M(x, y, z, t, m)}{M^\infty(y, z, t)} = (d/2a) \sum_{k=1}^{m} G(x; \theta, a, x_k), \tag{19}$$

The normalized concentrations in Figure 2 are for four values of the distance parameter $\theta = \sqrt{(zD_T)/v}$ and for ten areal sources separated on ten-meter centers, or a total "repository" of 100 meters. The following parameter values were used in the calculations,
a, half-width of the areal sources, 0.14 m,
b, half-aperture of the fracture, 0.005 m,
d, pitch or separation between areal sources, 10 m,
D_p, diffusion coefficient, 10^{-2} m^2/yr,
D_T, transverse dispersion coefficient, 0.05 m^2/yr,
m, number of areal sources, 10,
N°, source strength, 1,
R_f, fracture retardation coefficient, 1
R_p, matrix retardation coefficient, 100
v, pore velocity, 10 m/yr,
ϵ, porosity, 0.01.

The differences between the concentrations fields predicted by the two models are illustrated more clearly in Figure 3. Here the concentration ratio of Equation (19) is plotted against the distance parameter, with a constant spacing of 10 meters apart, for the number of areal sources up to 80.

Near the areal sources, that is for small values of θ, the plumes are evident and the concentration field for discrete areal sources differs considerably from that for the infinitely long source. The "peaks" are the local plumes, opposite the location of the areal sources and the "valleys" are where the local plumes are absent, opposite the region in between areal sources. However,

farther away from the areal sources, $\theta \approx 3$ meters, the plumes from the areal sources have merged completely due to transverse dispersion. At greater distances and until the concentrations ratio becomes less than unity in Figure 3, the simpler, infinite-source model predicts the concentration field just as well as the detailed areal-source model. For larger values of θ, when the concentration ratio becomes less than unity, the infinite-source model predicts a concentration that is higher than is accurate but on the conservative side. In this region the concentration field can be predicted accurately by replacing the discrete areal sources with a single finite-areal source of equivalent strength and with the same overall dimensions as the array of discrete sources. These observations are independent of nuclide, rock type and time.

Conclusions

We present analytic solutions for the dispersion of radionuclides released from multiple areal sources into a rock fracture. We illustrate the analytic solutions with graphical display of plumes. We ask the question of when should such detailed solutions be used, and when can simpler mathematical approximations give reasonably accurate predictions. From the numerical implementation of the analytic solutions we find that three regions exist, in terms of a distance parameter $\theta = \sqrt{(zD_T)/v}$. For the parameter values used in this study, at $\theta \leq 3$ meters the detailed multiple-areal-source solution should be used to give accurate predictions of the concentration field. For θ between 3 meters to 100 meters, the concentration field predicted by a simpler infinite-line source model is identical. For θ greater than 100 meters, the simpler infinite-line source model overestimates conservatively and an equivalent single finite-areal source will yield the correct concentration field.

References

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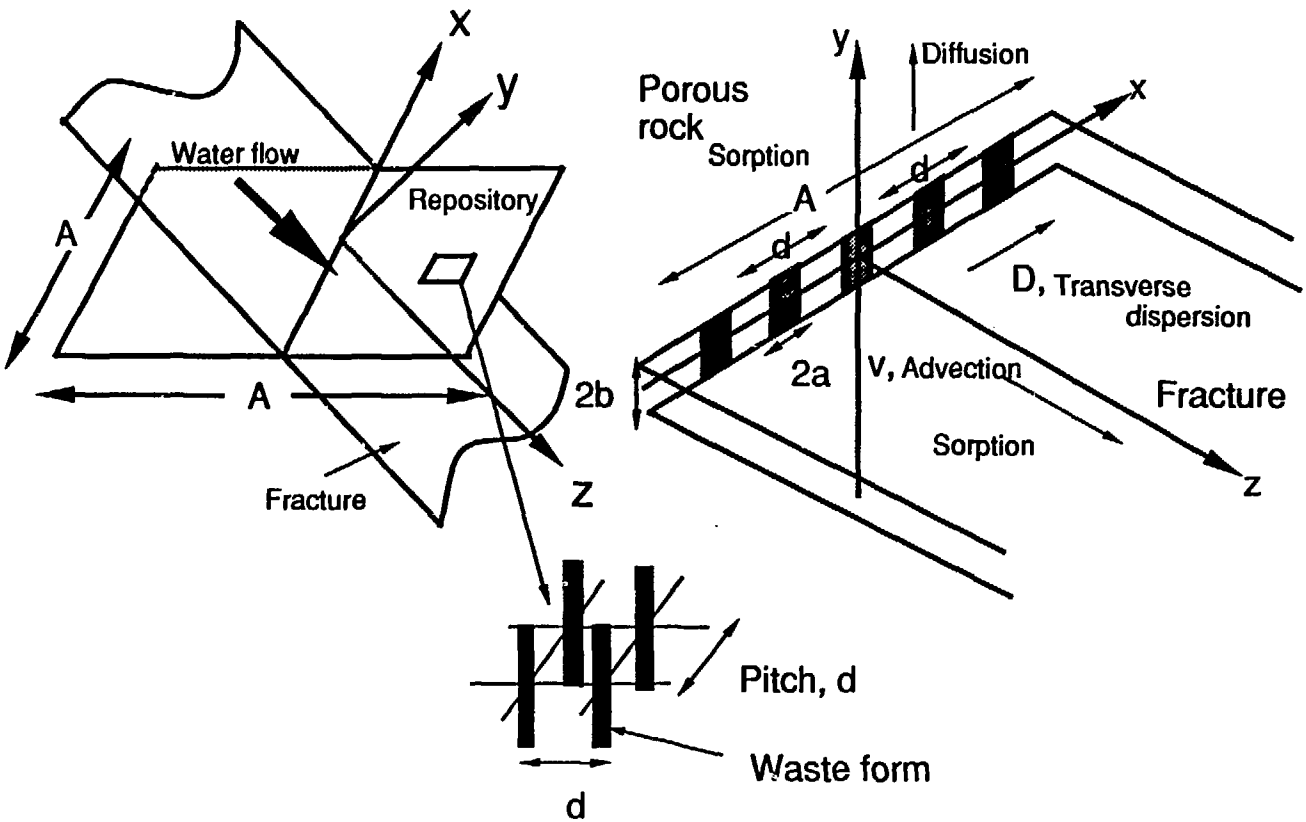

Figure 1. Relation between a planar fracture and a geologic repository, showing the processes considered in the model.

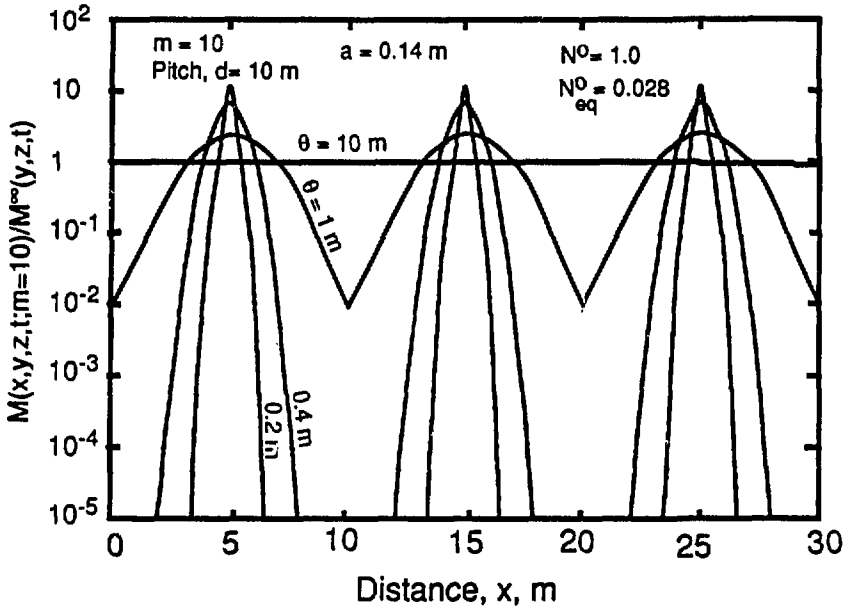

Figure 2 Distribution of the relative nuclide concentration (multiple-patch sources/ an infinite source) for four values of the distance parameter along the transverse coordinate x.

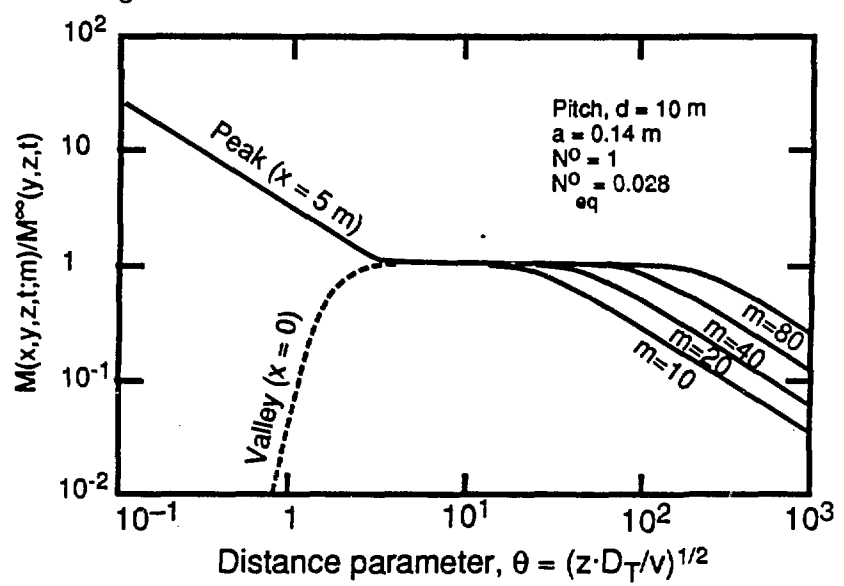

Figure 3 Effect of the size of multiple-patch sources on relative nuclide concentration profiles as a function of the distance parameter. For physical meanings of the peak and valley, refer to Figure 2.