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OPTIMIZATION OF TRANSPORT IN STELLARATORS*

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Here we relate two stellarator transport optimization schemes to single particle orbits. We also show that reducing transport in the $1/\nu$ regime reduces transport over a much broader range of collisionality.

In unoptimized stellarators, the orbits of deeply trapped particles usually have the greatest deviation from a flux surface and hence contribute most to diffusive transport losses. Thus it is of interest to focus on these particles. Using the adiabatic invariant J^* [1,2], one can show that the most deeply trapped particles closely follow surfaces given by

$$\mu B_{\min} + e\phi_E = \text{const} \tag{1}$$

Here $B_{\min}(\Psi, \theta)$ is the minimum of B with respect to the Boozer toroidal angle, ϕ, at fixed Boozer coordinates Ψ and θ. The electrostatic potential, the magnetic moment, and the charge are ϕ_E, μ, and e, respectively.

If we make the usual assumption that ϕ_E is independent of poloidal angle, θ, then all the poloidal dependence of $\mu B_{\min} + e\phi_E$ is through $B_{\min}$. Indeed, if $B_{\min}$ were independent of θ, then the contours of constant $\mu B_{\min} + e\phi_E$ would be contours of constant Ψ, and the bounce-averaged motion of deeply trapped particles would not deviate from the flux surfaces.

We can quantify these ideas by considering a simple model magnetic field of the form

$$B = B_0 \left\{1 - \epsilon_t \cos(\theta) - [\epsilon_h - \lambda\epsilon_t \cos(\theta)] \cos(M\phi - \ell\theta)\right\} \tag{2}$$

The minimum value of B with respect to ϕ is

$$B_{\min} = B_0 \left[1 - \epsilon_h - \epsilon_t(1 - \lambda) \cos(\theta)\right] \tag{3}$$

Here we note that the parameter λ is related to σ of Mynick et al. [3] by

$$\lambda = \sigma\epsilon_h/\epsilon_t \tag{4}$$

From Eq. (3) we see that there are two ways to make $B_{\min}$ independent of θ. One way is to set $\lambda = 1$, which is the optimization scheme introduced by Mynick et al. [3]. The second way to make $B_{\min}$ independent of θ is to arrange for ϵ_t to be equal to zero, i.e. the quasi-helical or "straight" stellarator enunciated by Nührenberg [4]. From J^* one can

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show that setting $\epsilon_t = 0$ causes the bounce- and transit-averaged motion of all particles (not just the deeply trapped particles) to follow the flux surfaces. Thus, while one would expect that the ϵ_t prescription would be more effective in reducing transport, it is still of interest to examine the λ prescription, since other design considerations may suggest the simultaneous use of both prescriptions.

To examine the dependence of transport on λ, we begin by considering the $1/\nu$ regime. This regime is formally independent of the magnitude of the electric field and is sufficiently simple that a large amount of magnetic geometric detail can be incorporated analytically. A sequence of analytic approximations has been developed for the $1/\nu$ regime [5]. For our purposes here, it is sufficient to use an approximation that is usually accurate to a few tens of percent. All the transport coefficients in this approximation have a common geometric factor of the form

$$G = \int d\theta \left(g_1 - 2g_2\lambda + g_3\lambda^2\right) \sin^2(\theta) \tag{5}$$

where the g_n are functions of the single parameter

$$\eta = 2\left[\epsilon_h - \lambda\epsilon_t \cos(\theta)\right] / \left[1 - \epsilon_h - \epsilon_t(1-\lambda)\cos(\theta)\right] \tag{6}$$

For $\epsilon_h \gg \epsilon_t$, $\eta \cong 2\epsilon_h/(1-\epsilon_h)$, and the integral in (6) is trivial. Minimizing the transport with respect to λ is then reduced to finding the minimum of the quadratic form

$$R_{1/\nu} = 1 - 2\lambda g_2/g_1 + \lambda^2 g_3/g_1 \tag{7}$$

The normalization of $R_{1/\nu}$ has been chosen so that it is unity at $\lambda = 0$, allowing us to treat it as a "reduction factor" for non-zero λ. Because the factor G includes the effect of all trapped particles, not just the deeply trapped ones, the minimizing value of λ, g_2/g_3, is not precisely equal to unity and varies with ϵ_h. For example, when $\epsilon_h = 0.3$, the minimizing value of λ is 1.3.

By using the DKES code [6] to calculate the transport flux, Γ_{11}, we have compared its $1/\nu$ regime dependence on λ with the dependence in other collisionality regimes. At collisionalities just below the $1/\nu$ regime, a peak in transport occurs as a function of collisionality. The sensitivity of this peak value of transport to λ is of considerable interest. To compare "peak" transport with $1/\nu$ transport, we define a reduction factor by

$$R_{\text{peak}} = \Gamma_{11}(\lambda)/\Gamma_{11}(\lambda = 0) \tag{8}$$

where both values of Γ_{11} are to be evaluated at the peak in transport as a function of collisionality (which may occur at different collisionalities for different values of λ).

Figure 1 shows the two reduction factors, $R_{1/\nu}$ and R_{peak}, for three different values of the electric field parameter "EFIELD" $\equiv E/v$. (The smallest value corresponds to $e\phi_E/T_e \simeq 1$, while the largest value corresponds to $e\phi_E/T_i \simeq 1$.) For the cases shown, $\epsilon_h = 0.3$ and $\epsilon_t = 0.1$. Also shown is an estimate of the λ dependence of the loss of high-energy particles due to unconfined drift orbits (proportional to $\epsilon_t|\lambda - 1|$). Notice that the diffusive transport results for "peak" and $1/\nu$ are quite similar and that the estimate of direct losses is qualitatively similar.

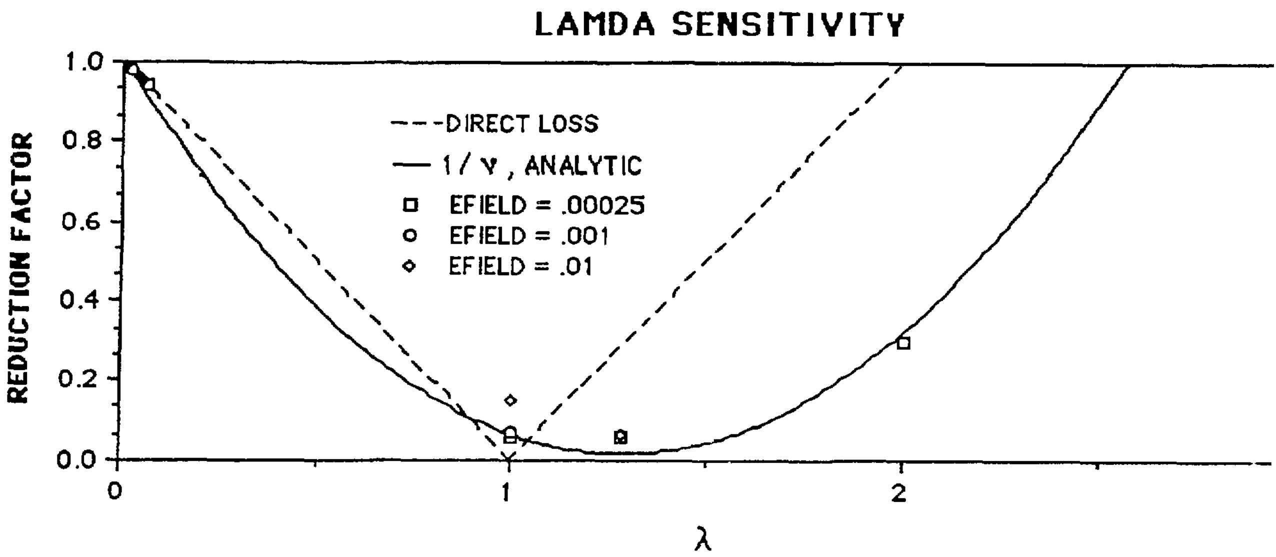
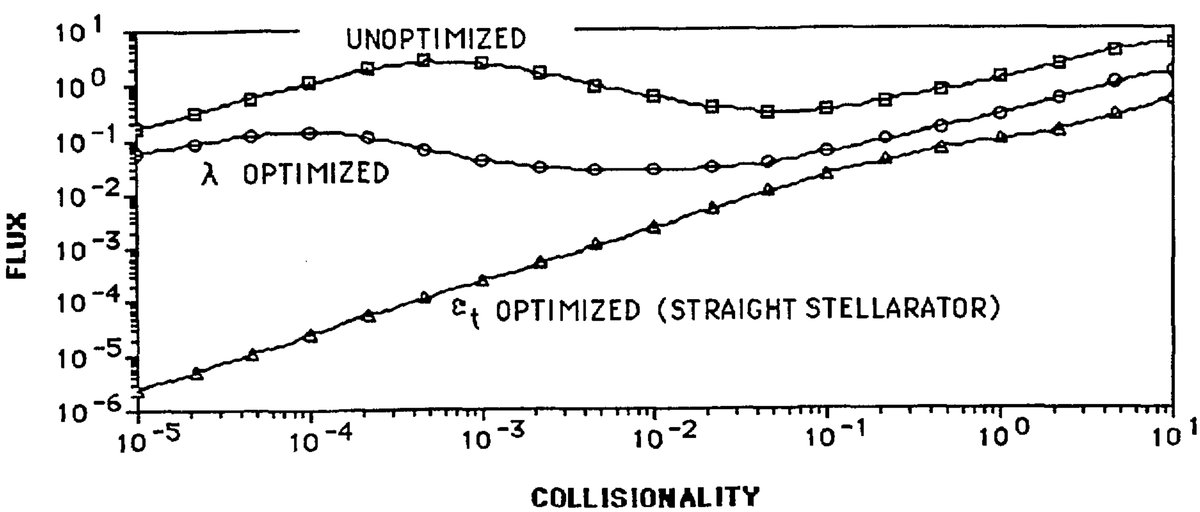


Figure 2 allows comparison of an unoptimized case ($\lambda = 0$) with a λ optimized case ($\lambda = 1.3$) and an ϵ_t-optimized case. All three cases have $\epsilon_h = 0.3$. The unoptimized and λ-optimized cases have $\epsilon_t = 0.1$, while the ϵ_t-optimized case has $\epsilon_t = 0$. The electric field parameter is $E/v = 0.001$. Other values of the electric field parameter give similar behavior. Notice that a broad range of collisionality (ν/v) is shown, and that both λ and ϵ_t optimizations significantly reduce transport over the entire range.



We conclude that both optimization techniques are effective at reducing transport over a broad range of collisionality. This suggests that the two techniques can be used simultaneously to reduce transport, while meeting other design constraints that are outside the purview of transport.

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