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Simulation of Coupled Bunch Mode Growth Driven by a High-Q Resonator: A Transient Response Approach*

S. Stahl and S. A. Bogacz
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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Simulation of Coupled Bunch Mode Growth Driven by a High-Q Resonator: A Transient Response Approach

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Introduction

In this article the use of a longitudinal phase-space tracking code, ESME[1], to simulate the growth of a coupled-bunch instability in the Fermilab Booster is examined. A description of the calculation of the resonant response is given, and results are presented for the growth of the coupled bunch instability in a ring in which all of the rf buckets are equally populated an in one in which several consecutive buckets are empty.

Parameters

The Fermilab Booster accelerates protons in 84 bunches from injection at a momentum of .650 GeV/c to extraction at 8.889 GeV/c. The guide field is approximately harmonic at 15 Hz, thus the time from injection to extraction is approximately 33 msec. The total voltage maintained by 18 rf cavities varies primarily from 300 to 800 MVolts during the cycle. The rf frequency varies from 30.1 MHz to 52.8 MHz. The operating intensity is approximately 2.2×10^{12} protons. In this simulation, the operating intensity is taken to be somewhat higher, approximately 4×10^{12}, in order to gauge the possible performance of the Booster at the higher intensity at which it is intended to be operated following the Fermilab Linac Upgrade.

It has been noted that the Booster is subject to a coupled bunch instability following transition, which occurs 17.3 msec after injection ($\gamma_t = 5.446$). Certainly, some element in the rf environment of the Booster is responsible. The primary suspects are the rf cavities themselves, since they can sustain modes with high enough Q's to allow the bunch-to-bunch communication required for the coupled bunch instability. J. Crisp has measured impedance values for a number of parasitic resonances in a Booster Cavity. The resonance treated in this article, modeled on those measurements, has a Q of 3311, a resonant frequency of 87.7 MHz, and a shunt impedance of 914 kΩ.

For M equally spaced bunches there are M possible coupled dipole modes. Let the mode number be m ($1 \leq m \leq M$). According to Sacherer[2], the resonant condition for the mth coupled bunch mode, with ω_0 the resonator frequency and ω_s the synchrotron frequency, is

$$\omega_c = (nM + m)\omega_0 \pm \omega_s \tag{1}$$

in which n is an integer, and ω_c is the frequency of the coupled bunch mode. It is certainly possible for a number of modes (characterized by n and m) to satisfy this condition. In the case of the Booster above transition and the resonanance parameters used in this article, however, the only values of m and n satisfying this relation are 56 and 1, respectively. The relevant coupled bunch mode is thus mode 56. That harmonic of the revolution frequency is equal to the resonant value at 22.42 msec, and takes 0.59 msec to sweep through the full width at half maximum of the resonance.

Simulation

In previous work[3] the resonance was treated in the frequency domain, and the voltage was calculated according to

$$V_i(0) = e\omega_0 \sum_k \rho_k Z(k\omega_0) e^{ik\theta} \tag{2}$$

in which ρ_k was computed via a discrete Fourier transform. The impedance presented to the beam by the resonator was modeled as

$$Z(\omega) = \frac{R}{1 + iQ(\omega/\omega_c - \omega_c/\omega)} \tag{3}$$

i.e., the steady state response. The use of the Fourier Transform and its implicit periodic boundary conditions resulted in the explicit absence of the synchrotron sidebands in the simulated beam spectrum, while the steady state approximation neglected the transient response of the resonator.

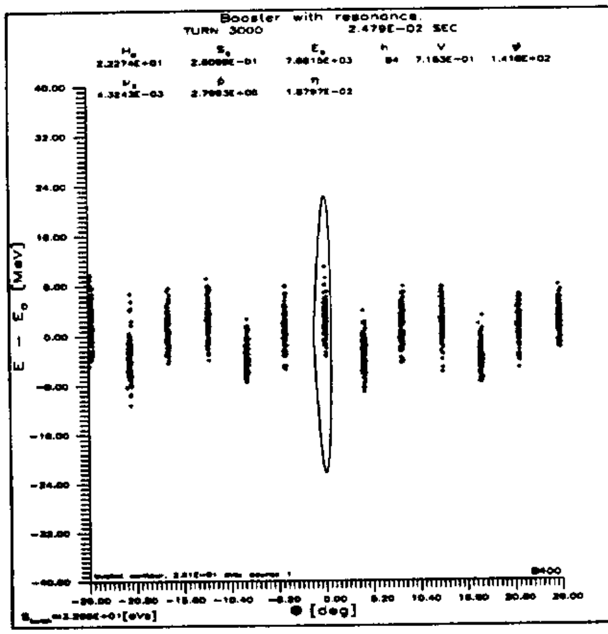

Figure 1: Phase-space plot of 12 bunches at 24.8 msec., depicting mode 56 coupled bunch instability.

The results presented here represent an attempt to treat the resonant response in a less approximate manner. Instead of handling the problem in the frequency domain, the differential equation representing the response of a parallel RLC circuit to the beam current is calculated via Laplace Transform[4], and the resulting voltage determined on a turn-by-turn basis in ESME. The response due to the current is expressed as the integral

$$V_i = \frac{1}{C}\int_0^t I(t-\tau)e^{-\alpha t}(\cos\beta t - \frac{\alpha}{\beta}\sin\beta t)d\tau \tag{4}$$

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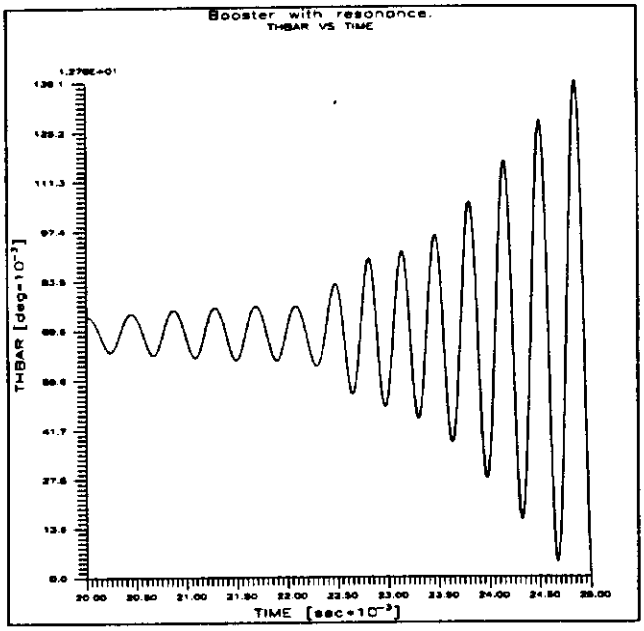

Figure 2: Amplitude of the oscillation, depicted as the mean azimuthal position of a bunch vs. time, for the symmetric distribution.

while the response due to the initial conditions (i.e., those prevailing at the end of the preceding turn) is

$$V_j = V_0 e^{-\alpha t}(\cos\beta t + \frac{\alpha}{\beta}\sin\beta t + \frac{1}{\beta}\sin\beta t(\dot{V}_0 - I_0/C)) \qquad (5)$$

where C is the capacitance and V_0, $\dot{V}_0$, and I_0 are the initial conditions. The parameter $\alpha = 1/2RC$, while $\beta = \sqrt{\omega_0^2 - \alpha^2}$.

Two simulations are presented here. In the first, all 84 rf buckets are populated with a Bi-Gaussian distribution of emittance 0.02 eV-sec. Each bunch is represented by 100 macro-particles. The entire distribution (the ring) is divided into 3000 bins for the purposes of computing and applying the voltage due to the resonance. The charge corresponding to each bunch is set to 6.0×10^{10} protons. The existence of a coupled bunch mode is demonstrated by the phase space plot in Fig. 1, in which twelve bunches are shown. The oscillations of a single bunch are depicted in Fig. 2.

In the second simulation, the parameters are the same as the first, except for the fact that five buckets are not initially populated. The absence of these bunches enhances the growth of the mode considerably. The induced voltage in the second case rises to an amplitude an order of magnitude larger than in the first, 200 KVolts vs. 7 kVolts (see Figs. 3 and 4). A vivid representation of the growth of the instability is given in the mountain ranges in Figs. 5 and 6. The larger growth of the mode in the gapped-beam case may be attributed to the enhancement of the harmonic component of the current at mode 56 initially, whereas in the symmetrical case the instability grows out of noise.

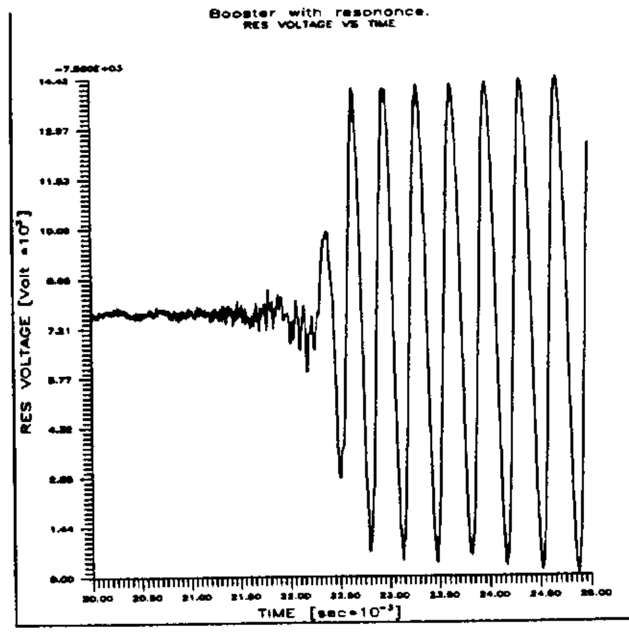

Figure 3: Voltage vs. time induced at one position in the distribution for all buckets filled.

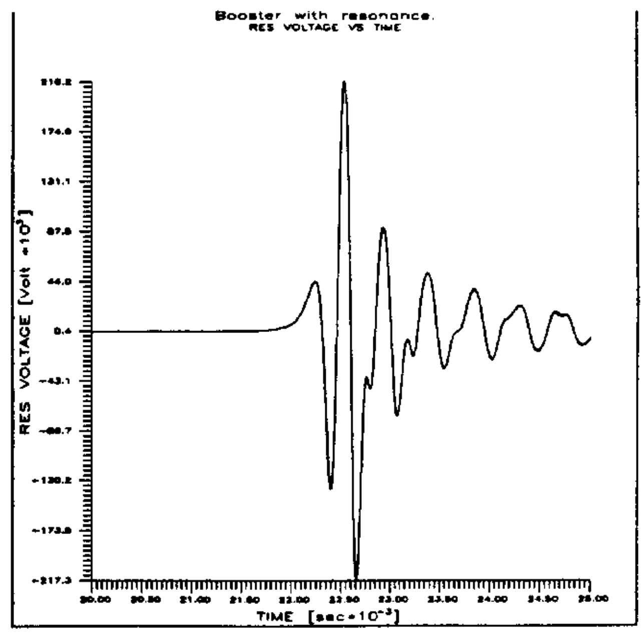

Figure 4: Voltage vs. time induced at one position in the distribution for a "gapped" distribution.

References

[1] J.A. MacLachlan, Fermilab TM-1274 (May 1984), unpublished (largely obsolete, to be superseded by documentation by S. Stahl and J.A. MacLachlan).

[2] F. Sacherer, IEEE Trans. Nucl. Sci **24**, 1393 (1977).

[3] S. Stahl and S.A. Bogacz, Phys. Rev. D **37**, 1300 (1988).

[4] J.A. MacLachlan, Fermilab FN-481 (April 1988), unpublished.

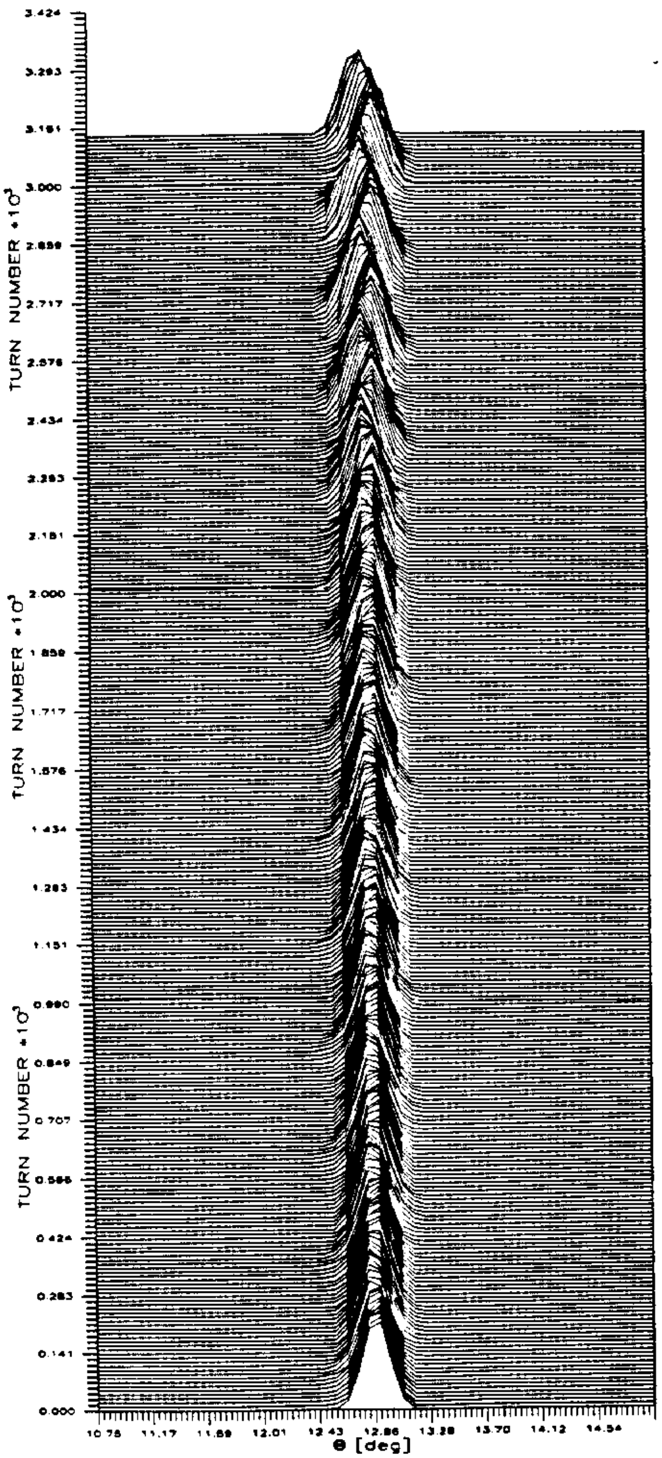

Figure 5: Mountain range illustrating the growth of the coupled bunch mode 56 from 20 to 25 msec. The traces are separated by 10 turns. All buckets are equally populated.

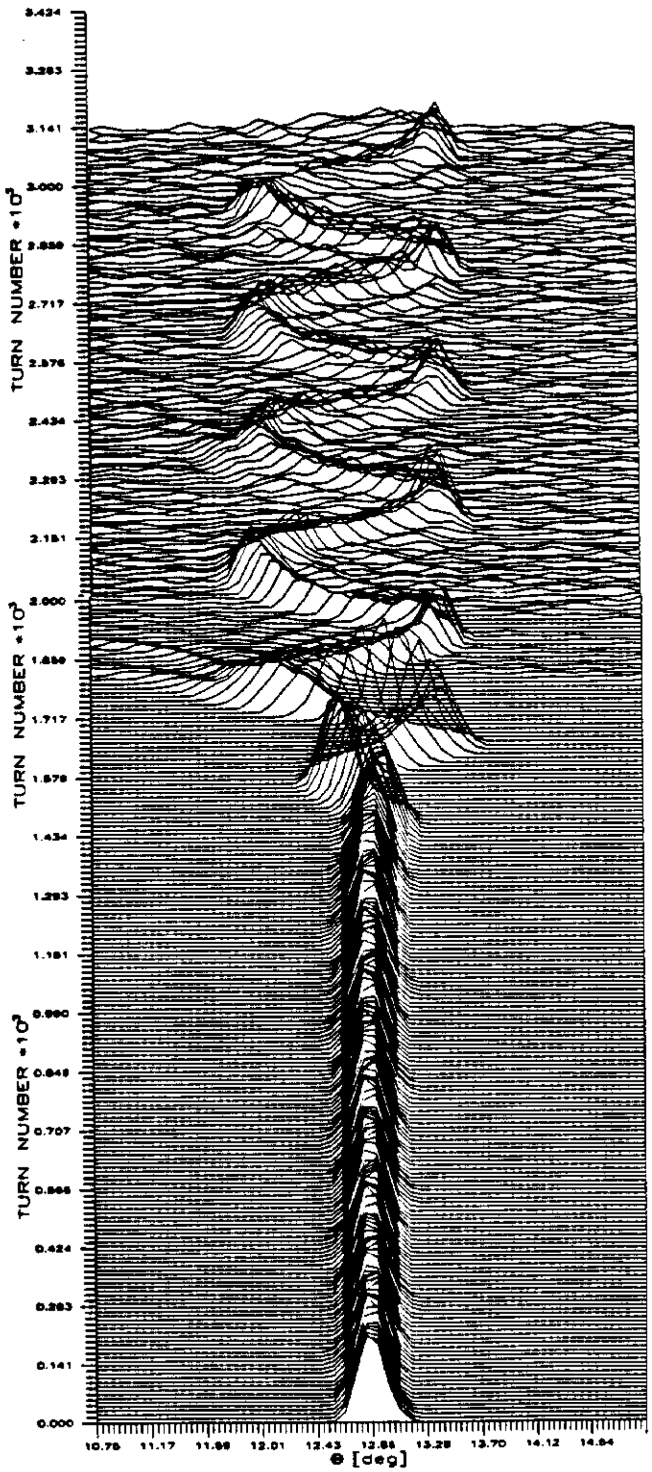

Figure 6: Mountain range illustrating the growth of the coupled bunch mode 56 from 20 to 25 msec. The traces are separated by 10 turns. In this simulation, five consecutive buckets were left empty. The bunch depicted is immediately adjacent to the gap.