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Research in Particle Theory

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ABSTRACT

In the test particle approximation, the scattering amplitude for two-particle scattering in (2+1)-dimensional Chern-Simons-Witten gravity and supergravity was computed and compared to the corresponding metric solutions. The formalism was then extended to the exact gauge theoretic treatment of the two-particle scattering problem and compared to 't Hooft's results from the metric approach.

We have studied dynamical symmetry breaking in 2+1 dimensional field theories. We have analyzed strong Extended Technicolor (ETC) models where the ETC coupling is close to a critical value. There are effective scalar fields in each of the theories. We have worked out how such scalar particles can be produced and how they decay.

The ϕ^4 field theory was investigated in the Schrodinger representation. The critical behavior was extracted in an arbitrary number of dimensions in second order of a systematic truncation approximation. The correlation exponent agrees with known values within a few percent.

1. Scattering in (2+1)-Dimensional Chern-Simons Gravity and Supergravity

A number of works related to the solution of the two-particle scattering problems in (2+1)-dimensional gravity and supergravity theories were completed. They are reported below in the order of their completion:

1.1 Test particle approximation in Chern-Simons-Witten Gravity[1]

In this work, the quantum scattering of a test particle by a fixed point-like source was studied from the point of view of Witten's Chern-Simons gauge theory of 2+1 dimensional gravity.[2] [3] Particles in Witten's approach are described by vectors of the Poincare' group, and all the observables of the theory are obtained from Wilson loops enclosing one or more of these particles. This problem had previously been studied by 't Hooft[4] and by Deser and Jackiw[5] in the framework of Einstein's metric theory of gravity. To make contact with these works, it was necessary to establish a connection between the classical phase space of the particles in the gauge theory and their space-time coordinates. It was shown that this can be achieved by noting that in the gauge theory approach the position vector, q^a, of the classical test particle transform like an $ISO(2,1)$ vector. Then by an appropriate gauge transformation, the space Mq of the position vectors was mapped into a conical space-time, thus making contact with the works of 't Hooft and of Deser and Jackiw.[4][5] An unusual feature of this transformation is that unlike most gauge transformations, it is not 2π-periodic. It is precisely due to these features that the transformed coordinates, q^a, satisfy the matching conditions of the coordinates on a cone and can be identified with them. The scattering amplitude was also explicitly computed.

1.2 SCATTERING IN TEST PARTICLE APPROXIMATION IN CHERN-SIMONS SUPERGRAVITY[6]

In this work the results of reference [1] were extended to the scattering of a test superparticle in the field of a massive superparticle. The motivation for undertaking this work was to find out the extent to which supersymmetry could account for the effects of spin. The natural framework for doing this was $N = 2$ Poincare' supergravity coupled to sources. In $N = 2$ supergravity, the gauge theory approach also plays a central role in restricting the nature of the charges which can be carried by the sources. By insisting that the coupling to sources be derivable from an action which is endowed with the same gauge invariances as in the sourceless Chern-Simons theory, one loses the freedom of assigning charges to the sources arbitrarily. For example, in the test particle approximation, we will show that it is possible to gauge away the Grassmann degrees of freedom of one, but not both, of the particles. So spin effects do appear in the scattering amplitude[6] . The scattering amplitude is recovered by summing the contributions from topologically distinct Wilson loops in the infinite dimensional representation of the Poincare' group carried by the test particle.

1.3 BEYOND THE TEST PARTICLE APPROXIMATION[7]

The works described in references [1] and [6] clearly demonstrated that the gauge theory approach provides an alternative picture for the description of particle scattering in the test particle approximation. The solution to a number of problems required further study in this approach. One of these was the solution to the two scattering problem beyond the test particle approximation. The main purpose of this work was to provide a different view of dealing with this problem. The scattering amplitude that we obtained agreed with that of 't Hooft in reference [4] in the limit in which both particles are slowly moving and very massive. There was also complete agreement when one of the invariants satisfied a quantization condition.[7],[8] For a 2+1 dimensional manifold, M, with topology $R\times\Sigma$, where R represents the time, the Chern-Simons action for Poincare' gravity can be written as[2][3]

$$I_{\rm CS} = \int dt \int_{\Sigma} \epsilon^{ij} e^{a}{}_{i}\partial_t\omega_{aj} - \eta_{ab}e^{a}{}_{0}F^{b}[\omega] - \eta_{ab}\omega^{a}{}_{0}F^{b}[e] \qquad (1.1)$$

$$ij = 1,2 \ ; \quad a,b = 0,1,2.$$

In this expression e^a_μ and ω^a_μ are components of the connection

$$A_\mu = e^{a}{}_{\mu}P_a + \omega^{a}{}_{\mu}J_a, \qquad (1.2)$$

where P^a and J^a are, respectively, the momentum and the angular momentum generators of the Poincare' group. Also,

$$F^a[A] = \epsilon^{ij}F^{a}{}_{ij}[A], \qquad (1.3)$$

where $F^{a}{}_{ij}$ are the spatial components of the field strength tensor for the connection A, and ϵ^{ij} is the antisymmetric tensor in two space dimensions. To couple a point-like source characterized by phase space coordinates (p^a, q^a) to the Poincare' gravity, one can supplement the action (1.1) by the source action[2][3]

$$I = \int_{C} dt\eta_{ab}p^a[\partial tq^b + t^\mu e^{b}{}_{\mu} + \epsilon^{b}{}_{cd}q^c\omega^{d}{}_{\mu})] + \lambda(p^2 + m^2) \qquad (1.4)$$

where the path C is the particle trajectory, λ is a Lagrange multiplier, and $t^\mu - dx^\mu/dt$ is the tangent to the path C.

To couple two or more sources to the Chern-Simons theory, it had been argued that one must add a term of type (1.4) for each of the point sources involved. This appeared reasonable since it regarded the interaction of a collection of point sources to be due to the separate coupling of the "charges" of each source to the gauge fields in a gauge invariant manner. Whether this way of coupling is suitable for describing a particular physical problem depends on the problem at hand. We pointed out that, for more than one source, the above method of coupling point sources to Poincare' gravity was not unique to the extent that the choice of canonical variables in a phase space corresponding to more than one particle was not unique. We took advantage of this non-uniqueness to write down an action which leads to 't Hooft's scattering amplitude and provides an exact solution to the 2-particle scattering problem in (2+1) dimensional gravity.

1.4 2+1 DIMENSIONAL CHERN-SIMONS GAUGE THEORIES COUPLED TO MANY SOURCES

First we shall give the precise connection between the internal coordinates, $q^a_{(n)}$ (conjugate to $p^a_{(n)}$), and spacetime for the special case of a single superparticle in spacetime. More precisely in the presence of a single source of mass m, spacetime is a case described by the metric solutions

$$ds^2 = d\tau^2 - dr^2 - \alpha^2 r^2 d\phi^2, \tag{1.5}$$

where $\alpha = -1 + m/2\pi$ is related to the deficit angle of the cone. On the other hand, the internal space, M_q, is globally Minkowskian and the connection between the Chern-Simons phase space and its spacetime counterpart is obscure. We established this connection by showing that the gauge transformation

$$q \rightarrow q' = e^{-\alpha\phi J_0} q, \tag{1.6}$$

where J_0 is the angular momentum generation of the Poincare' group, gives the set q' in $M_{q'}$ which are manifestly conical coordinates satisfying the same boundary conditions as the spacetime coordinates. The gauge transformation also gets rid of the spin connection, so the net effort as to trade the globally trivial matching conditions on M_q with a non-trivial connection for the space $M_{q'}$ which has non-trivial matching conditions q but a trivial connection.

Because the gauge transformation above lives in the covering group of the Poincare' group, it becomes necessary to view Witten's gravity as a gauge theory. not of the Poincare' group itself, but of its covering group. During the past year we have generalized (1.6) to the case of N sources in arbitrary relative motion.[9] The relationship between M_q with N holes and the multiconical spacetime is best stated as follows. Consider a test particle located at the spacetime point P away from sources. The appropriate gauge transformation, which is the generalization of (1.6) is the Wilson line linking the origin to the point P. We assert that the coordinates

$$q'(P) = \mathrm{Tr}P \exp\left(\int_{\alpha}^{P} A\right) q(P) \tag{1.7}$$

obey precisely the matching conditions of the spacetime coordinates given by the geometric constitution of Deser, Jackiw, and 't Hooft[10] . Indeed, by gauge transformations analogous to (1.7), one can go from any projection of the covering group to any other. (1.6) is the special transformation taking us from the Poincare' projection (the globally Minkowskian space M_q with one hole) to a manifest cone with deficit angle M. Equation (1.7) establishes the precise connection between the gauge theoretic and the spacetime approaches to gravity and supergravity in the pressence of isolated sources.

2. Dynamical Symmetry Breaking

Dynamical symmetry breaking plays a significant role in particle physics research. Starting with the work of Nambu and Jona-Lasinio.[11] it has been investigated as a mechanism for generating fermion masses. The breaking of the chiral symmetry of QCD is now widely accepted as a realization of this phenomenon. It is also proposed as a mechanism for electro-weak symmetry breaking in the standard model[12] Among its appealing features are the fact that it requires fewer arbitrary parameters than the alternative spontaneous symmetry breaking with elementary Higgs fields.

The examples discussed in the previous paragraph deal with dynamical breaking of continuous symmetries. It is also important to investigate the possibility of discrete symmetry breaking through dynamical means. Vafa and Witten[13] have shown that dynamical parity (P) or time reversal(T) breaking cannot happen in vector like gauge theories in 3+1 dimensions. Their arguments also applied to 2+1 dimensional QED if the number of fermion flavors is even and greater than two. Further analysis of dynamical parity breaking in QED3 in the large N flavor approximation was also carried out.[14] It was found that Parity breaking does not happen to the leading order in $1/N$. This analysis has been elaborated by Nash.[15] Analysis of dynamical symmetry breaking in realistic field theories requires nonperturbative methods. Commonly used are numerical lattice simulations, analytic approximation schemes such as the study of Dyson-Schwinger equations and variational methods. Another useful laboratory where much has been learned is the solution of models in lower dimensions, particularly in 1+1-dimensions where there are numerous solvable interacting quantum field theories with discrete symmetries broken by composite order parameters. We have studied the dynamical breaking of parity and time reversal and a certain discrete flavor symmetry in a strongly-coupled 2+1-dimensional field theory using a conventional effective potential approach and the large N approximation. Our method was especially suited to field theories with four-fermion interactions where the order parameter is the vacuum expectation value of a local composite operator.

We considered a continuum field theory with two kinds of four-fermion couplings defined by the action

$$S = \int d^3x \left\{ \sum_{a=1}^{N} A\bar{\psi}_a i\gamma_\mu \partial_\mu \psi_a + \frac{\lambda A^2}{2N\Lambda} \left(\sum_{a=1}^{N} \bar{\psi}_a \psi_a \right)^2 + \frac{\kappa A^2}{2N\Lambda} \left(\sum_{a=1}^{N} \bar{\psi}_a \tau^3 \psi_a \right)^2 \right\} \tag{2.1}$$

with N flavors of complex four-component fermions. Λ is the ultraviolet cutoff and A, λ, κ and c are dimensionless, cutoff dependent constants. $A^{\frac{1}{2}}$ is the fermion wavefunction renormalization parameter. λ and κ are the dimensionless four-fermi vertices.

We write down two kinds of fermion mass operators in this model, a scalar fermion mass $\bar{\psi}\tau^3\psi$, a pseudoscalar fermion mass $\bar{\psi}\psi$. If this model is gauged with a U(1) gauge field $\mathcal{A}_\mu$ it is also possible to have a Chern-Simons topological mass term for the photon field $\epsilon^{\mu\nu\lambda}\mathcal{A}_\mu\partial_\nu\mathcal{A}_\lambda$. The latter changes by an exact derivative under a gauge transform and therefore has a gauge invariant spacetime integral.[16] These mass terms are distinguished by the way in which they transform under discrete symmetries.

The action (2.1) has $U(N)\times U(N)$ symmetry. It also has a discrete Z_2 symmetry,

Under this transformation the scalar fermion mass operator $\bar{\psi}\tau^3\psi$ changes sign

$$Z_2 : \bar{\psi}\tau^3\psi \rightarrow -\bar{\psi}\tau^3\psi$$

and the mass operators $\bar{\psi}\psi$ and $\epsilon^{\mu\nu\lambda}\mathcal{A}_\mu\partial_\nu\mathcal{A}_\lambda$ are invariant

Since $\bar{\psi}\tau^3\psi$ is not invariant under Z_2 a non-vanishing expectation value, $<\bar{\psi}\tau^3\psi>\neq 0$, would indicate that the vacuum of the theory is not Z_2 symmetric. This operator is therefore an order parameter for Z_2 symmetry breaking.

The action in (2.1) is also invariant under the Euclidean three dimensional parity

Under parity transformation the mass operator $\bar{\psi}\tau^3\psi$ transforms like a scalar and $\bar{\psi}\psi$ like a pseudoscalar We also note that the topological mass term is parity odd.[16]

Either of the vacuum expectation values $<\bar{\psi}\psi>$ or $<\int\epsilon^{\mu\nu\lambda}\mathcal{A}_\mu\partial_\nu\mathcal{A}_\lambda>$ are order parameters for parity breaking. If we have both fermions and gauge fields in a theory if one of these parity odd operators has an expectation value, radiative corrections can induce an expectation value for the other one. In general this occurs when the physical fermion has a pseudoscalar mass larger in magnitude than the scalar mass.[17] Then the physical fermion and the physical photon would have parity violating masses and the Coulomb interaction would be short-ranged. We have only analyzed pure 4-fermi theories. We have indicated how these results could be generalized when gauge interactions are present.

The Z_2 and parity symmetries forbid the appearance of a bare mass for the fermions in action (2.1). If the physical fermion spectrum is to have a mass gap at least one of these symmetries must be broken dynamically. In the following we shall seek solutions of the model (2.1) which break either Z_2 or parity or both through the generation of fermion masses. We also found that, with the particular choice of four-fermion couplings taken in (2.1) either parity or chiral Z_2 symmetry can be broken but there is no solution where both are simultaneously broken.

To set up the $1/N$ expansion we introduced the Lagrange multiplier fields ϕ and χ and rewrote the continuum action in (2.1) as

$$S = \int d^3x \left\{ A\bar{\psi}i\gamma_\mu\partial_\mu\psi + iB\phi\bar{\psi}\psi + iC\chi\bar{\psi}\tau^3\psi + \frac{NB^2\Lambda}{2A^2\lambda}\phi^2 + \frac{NC^2\Lambda}{2A^2\kappa}\chi^2 \right\} \quad (2.2)$$

If we solve the equations of motion for the scalar fields ϕ and χ and substitute the solutions into (2.2) yields the four-fermion interaction terms in the action (2.1).

Using the usual manipulations we computed the effective action in terms of the expectation values of ϕ and χ

$$<\phi> = \frac{\delta}{\delta J_1} W[J_1, J_2] \equiv m_1 \quad (2.3)$$

and

$$<\chi> = \frac{\delta}{\delta J_2} W[J_1, J_2] \equiv m_2 \quad (2.4)$$

The computation of the free energy was carried out using the ususal functional methods but taking into account the fact that we are employing here a large N resummation of perturbation theory. For the details of this computation we refer the reader to the the original papers.[18] The leading large N effective action thus obtained is

$$\begin{aligned}\frac{\Gamma[m_1,m_2]}{V} &= \frac{N\Lambda b^2}{2\lambda a^2}m_1^2 + \frac{N\Lambda c^2}{2\kappa a^2}m_2^2 - \frac{N\Lambda}{\pi^2}\frac{b^2}{a^2}(m_1^2+m_2^2) \\ &+ \frac{N}{6\pi}\frac{b^3}{a^3}(|m_1+m_2|^3 + |m_1-m_2|^3) + \dots\end{aligned} \tag{2.5}$$

where, to leading order in $\frac{1}{N}$ we have taken $A = a$ and $B = b = C$ with a and b finite dimensionless constants. For now, it is sufficient to absorb the factors b/a into m_1 and m_2. Then, if we define the critical coupling constants

$$\lambda_c = \frac{\pi^2}{2} = \kappa_c \tag{2.6}$$

The leading order in large N part of Γ can be written as

$$\begin{aligned}\frac{1}{V}\Gamma[m_1,m_2] =& \frac{N\Lambda}{2}\left(\frac{1}{\lambda}-\frac{1}{\lambda_c}\right)m_1^2 + \frac{N\Lambda}{2}\left(\frac{1}{\kappa}-\frac{1}{\kappa_c}\right)m_2^2 \\ &+\frac{N}{6\pi}\left(|m_1+m_2|^3 + |m_1-m_2|^3\right) + \dots\end{aligned} \tag{2.7}$$

It is evident that if the either of the four-fermion couplings are sufficiently strong, either $\lambda > \lambda_c$ or $\kappa > \kappa_c$ the free energy is minimized by $m_1 \neq 0$ or $m_2 \neq 0$ and either the parity or Z_2 symmetry are broken dynamically. The completely symmetric phase with $m_1 = m_2 = 0$ is stable only in the region $\lambda < \lambda_c$ and $\kappa < \kappa_c$. When $\lambda > \lambda_c$ and also $\lambda > \kappa$ the stable minimum of (22) has $m_1 \neq 0$ and $m_2 = 0$. When $\kappa > \kappa_c$ and $\kappa > \lambda$ the stable minimum is where $m_2 \neq 0$ and where $m_1 = 0$. There are no stable minima where both m_1 and m_2 are simultaneously nonzero. On the symmetry line $\lambda = \kappa > \lambda_c = \kappa_c$ the symmetry breaking pattern is discontinuous and jumps from $m_1 \neq 0$, $m_2 = 0$ for $\lambda > \kappa$ to

$m_1 = 0$, $m_2 \neq 0$ for $\kappa > \lambda$. Further details of these calculations can be found in the paper of Semenoff and Wijewardhana.[18] A lattice version of this analysis has also been performed.[19]

3. Mass Enhancement in Extended Technicolor Theories

The recent revival of interest in technicolor theories has been stimulated by the observation that these theories should not necessarily be viewed as scaled up versions of QCD. Momentum components well above the confinement scale Λ_{tc} can play a more important role than they do in QCD, with important consequences such as the generation of large fermion masses. Walking technicolor is one example of this phenomena.[20] Another possibility is that the higher energy Extended Technicolor (ETC) interactions, which must be present to generate the masses of ordinary fermions, can play an important and direct role, along with the technicolor interactions, in electro-weak breaking, leading to even larger fermion masses.[21] This can take place only if the combination of the ETC coupling and the technicolor coupling at the ETC scale is sufficiently close to a certain critical value.[22]

It has recently been suggested that this ETC driven enhancement is associated with the appearence of composite scalars that are light compared to the ETC scale.[23]

The enhanced fermion mass arises from an effective Yukawa coupling of the fermion to the scalar, which develops a vacuum expectation value from the technicolor interactions. We have studied the properties of these composite scalar fields. We have concluded that unless the technicolor coupling at the ETC scale is unrealistically weak and the ETC coupling is very close to the critical curve, these light scalars have large widths.[24]

An important question is if the light scalar resonences are able to mediate Flavor Changing neutral current interactions. If we restrict our attention to

CP conserving interactions, then the possible off diagonal couplings of ordinary fermions to these resonences will not produce unacceptable Flavor Changing Neutral Currents if the zero momentum scalar masses M(0) are above 1.5 TeV. The contribution of these scalars to Flavor changing neutral processes involving the t quark may be much larger. This could be of immediate interest if the t quark is discovered in the next few years.

4. Field Theories in Schrodinger Representation

Since the Schrodinger representation of quantum field theories was proved to be renormalizable by Symanzik,[25] several attempts have been made to apply methods developed for the solution of the Schrodinger equation to quantum field theories. Various authors tried to use Lanczos' systematic variational method. The space of wave functionals in the Nth approximation of the Lanczos method is spanned by polynomials of the Hamiltonian applied to a Gaussian wave functional. Numerical approximations to various lattice field theories have been explored.[26] [27] of finite volume and critical slowing down in the critical regions. Briefly speaking, the matrix elements of powers of the Hamiltonian between Gaussian states can be calculated as superpositions of nonconnected Feynman diagrams for operator insertions. The infinite volume limit cannot be taken, because each connected subdiagram is proportional to the volume. Using simple considerations it is easy to show that the finiteness of the volume puts severe limitations on the value of the smallest mass one is able to reach in a given order of the approximation scheme.

One way to avoid the constraint of finite volume is to substitute the Gaussian functional with a nongaussian one of the form $\Psi_0[\phi] = \exp\{-S[\phi]\}$, where S is not quadratic. The exact ground state wave functional can be shown to have this form with a **connected** $S[\phi]$. The disadvantage of the above approach is that the expectation value of the Hamiltonian cannot be exactly calculated. Thus one has to rely on approximations, which are not variational in their character.

Among others, the approximate energy eigenvalues no longer approximate the true eigenvalue from above.

The substitution of $\Psi = \exp\{-S\}$ into the Schrodinger equation results in an infinite set of coupled equations for the Taylor expansion coefficients of S in field ϕ. One way to arrive at a managable approximation is to truncate the infinite Taylor series. It was shown that such a truncation leads to excellent numerical results for a ϕ^4 theory in zero spacial dimensions (the anharmonic oscillator).[28]

We have shown that the wave functional for the first excited (single particle) state can be written as $\Psi_1[\phi, p] = \chi[\phi, p]\Psi_0[\phi]$, where p is the momentum carried by the excited state.[29] The functional χ, just like S, is connected. The Schrodinger equation for the excited state wave functional leads to an infinite set of coupled linear integral equations for the Taylor expansion coefficients of χ. After truncating functional χ as well we obtained a set of integral equations for the remaining coefficients.

The excitation energy, $E(p)$ appears as a parameter in the integral equations for the Taylor expansion coefficients. It is easy to see that even if all coefficcents of S and χ could be eliminated from the system of equations, one would be left with a single integral equation for the excitation energy $E(p)$. The solution of that equation does not in general have the correct relativistic form $E(p) = \sqrt{p^2 + m_R^2}$, where m_R is the renormalized mass. On the other hand, if the truncations constitute a convergent series of approximations, then in increasing order the correct relativistic form should emerge.

It turns out that truncated set of equations are still too complicated to solve.[29] Thus a further approximation, the iteration of the integral equations, is performed. This leads to an integral equation for $E(p)$ alone, which would agree with the perturbational approximation for $E(p)$, provided $E(p)$ itself was expanded in a power series of the coupling constant. As it stands, it is a highly non-linear integral equation for $E(p)$. The equation is reminiscent to the Schwinger-

Dyson equation in the Lagrangian theory on one hand, and to Wigner-Brillouin perturbation theory on the other hand, but it is distinct from both.

Besides a trivial perturbative solution, there is a nontrivial solution of the intgeral equation for $E(p)$. It is linear for large p, approaches m_R for small p, but it has a power behavior for $\mu >> p >> m_R$, where μ is the scale. The solution leads to a nontrivial relation between the unrenormalized and renormalized masses. This relation determines the critical correlation exponent, ν. We have found exponent ν in the second (two loop) approximation. As expected, we obtained $\nu = 1/2$, the correct mean field value in three spacial dimensions ($D = 3$), while for $D = 1$ and 2 we obtained critical exponents, which agree with the known exact and numerically generated values 1 and 0.631, respectively, within a few percent.

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