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CONE BEAM 3D RECONSTRUCTION WITH DOUBLE CIRCULAR TRAJECTORY

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ABSTRACT

In X-ray cone beam tomography the only planar source trajectory which do not produce incomplete data is the infinite line. This kind of source trajectory is not experimentally doable. To ensure a complete data acquisition with cone beam radiographs, a set of non planar trajectory has been studied. Among the trajectories proposed in the literature a simple one is the set of 2 circular trajectories with intersection of the two trajectory axis. The angle between the two axis is related to the maximum aperture of the cone beam. We propose here an exact method to perform this reconstruction using the 3D radon transform of the object. The modulation transfer function (MTF) of this algorithm remain identical to the MTF on the central slice of reconstruction with single circular trajectory. The density relative mean square error stays within 2% for an aperture of ±30°. With single circular trajectory The relative mean square error may reach 20% at the same aperture. With double circular trajectory, horizontal artifacts are almost suppressed.

INTRODUCTION

Lately 3D cone beam tomography became an interesting method for nondestructive evaluation of advanced materials. The main field of application is the control of structural ceramics [1]. The study of such materials imply high density resolution and high sensitivity to cracks [2,3]. As a matter of fact, with circular source trajectory when the cone-beam aperture increases, the density is underestimated and some horizontal crack like artifacts may appears in respect of interfaces in the sample [4]. These artifacts limit the thickness we can examine with planar source trajectory. To maintain the reconstruction accuracy, with circular source trajectory, angular aperture must stay under ±10° in the best cases [4]. To increase examined thickness and to maintain resolution, we have to widen the cone beam aperture. This aperture widening allows to reduce source-object distance, photon noise is then reduced. In fact if we want to reduce the volume of the elementary voxel by 8, to maintain the reconstruction signal to noise ratio constant we have to multiply the flux in the voxel by 16. Thus, the shorter the source object distance will be, the better the signal to noise ratio. An aperture close to ±30° allows to examine cubic volumes in good conditions.

Until now most of the experiments presented in the literature were performed with planar source trajectory [5,6]. Recently a new method was presented by B.Smith [7] and have been applied to non planar source trajectories by Kudo.[8] The inversion presented by Kudo uses Hilbert Transform. We present in this paper an exact reconstruction method using 3D Radon transform inversion with a double circular source trajectory. This work is an application of P.Grangeat's work [9,10] who established a general mathematical relation between X-ray transform of and 3D radon transform. Section 2 of this paper we recall the mathematical relation between X-ray transform and 3D radon transform. Section 3 we show how two circular source trajectories can give access to a complete information on the sample. We give the relationship between the size of the reconstructed object and the angle between the axis of the two trajectories. Section 4 we study on simulated data the Modulation Transfer Function (MTF) of the dual axis reconstruction method. We gives some comparatives results with single planar trajectory reconstructions.

THEORY

P.Grangeat [9] shown that we can link exactly the 3D Radon transform of an object and the X-ray transform of the same object. With this relation we can directly determine points of the first derivative of the Radon transform and this independently of the source trajectory. For each radiograph, the set of point filled in the Radon space is a hollow sphere.

Given an object function f(M) where M is a given point of the space, let us define the X-ray transform Xf(S,A), the radiographic reading at the point A corresponding to a source position S, as:

$$Xf(S,A) = \int_{a=0}^{a=+\infty} f(S + a.\frac{SA}{||SA||})da \quad , \qquad (1)$$

If we consider the point P of the Radon space of origin O, the Radon transform of f in P is given by:

$$Rf(P) = \int_{(OP.OM)=0} f(M)dM \quad , \qquad (3)$$

and call SYf(S,n) the integral over the line D(S,n) intersecting the detector and the plane passing through S and perpendicular to n (fig: 1)

$$SYf(S,\vec{n}) = \int_{A\in D(S,\vec{n})} Yf(S,A)dA \tag{4}$$

where,

$$Yf(S,A) = \frac{||SO||}{||SA||} Xf(S,A) \tag{5}$$

Then the Grangeat formula can be written [9]:

$$\frac{||OS||^2}{||OS\wedge\vec{n}||^2} \frac{\partial(SYf)}{\partial\rho'} = \frac{\partial(Rf)}{\partial\rho}(OS.\vec{n},\vec{n}) \tag{6}$$

with ρ = OC and ρ' = OP'as defined in Fig. 1.

Kirillov [11] and Tuy [12] showed that to perform an exact 3D cone beam tomography all the planes passing through each point of the object must cut at least once the source trajectory. Obviously in the case of the circular trajectory the planes which are parallels to the trajectory and cross the sample do not follow this condition

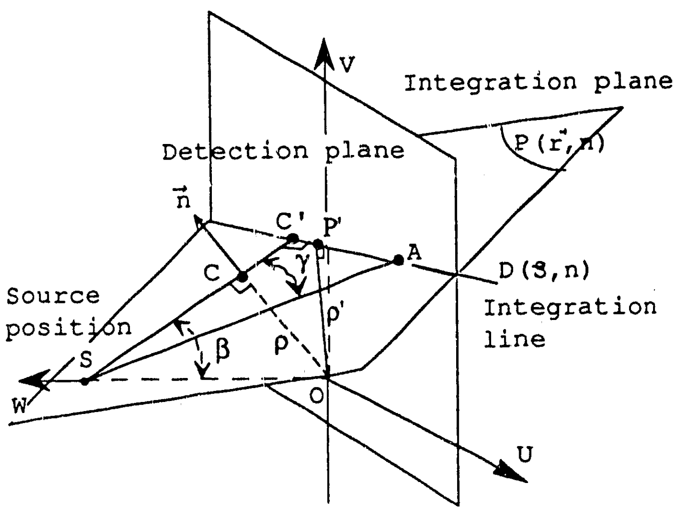

Fig. 1: Acquisition geometry for the relationship between X-ray transform and Radon transform

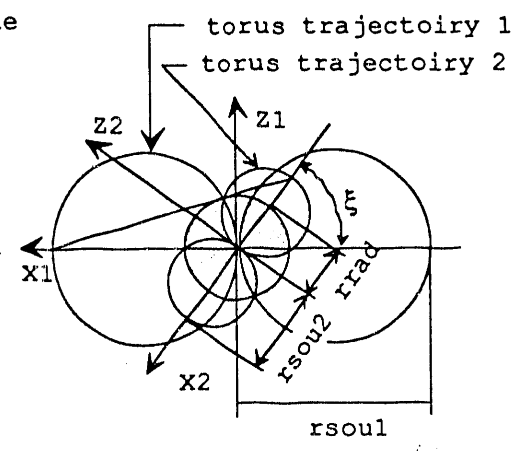

Fig. 2: Maximum object radius with double circular trajectory of radius rsou1 and rsou2

METHOD

The set of the points of the Radon space which are addressed by a circular trajectory defines a torus [9]. The Radon transform of a spherical object is different from zero in a sphere of same diameter that the object. Thus in order to reconstruct an object of same diameter than this sphere we have to measures these points in the Radon space. The principle of the double circular trajectories is to define a torus whose the axis intersect the axis of the first one and which is defined in the area where the first one is not know. The simplest trajectory on a mathematical point of view is to built the second torus at 90 degree of the first one [8]. This trajectory provides the largest possible radius of the object to reconstruct and allows a total angular aperture of ±45°, but is seldom compatible with hardware problems. Most of the time the possible angle between the two trajectories is lower than ±45°. As shown in Fig. 2, let assume that the angle between the two trajectories is ξ, then if rsou1 and rsou2 are the radius of the trajectories and rrad is the maximum radius of the object that can be examined without approximation,

$$\mathrm{rrad} = \frac{\mathrm{rsou1.rsou2.sin}\xi}{\sqrt{\mathrm{rsou2}^2 + 2.\mathrm{rsou2.rsou1.cos}\xi + \mathrm{rsou1}^2}} \qquad (7)$$

RESULTS

Using the same methods we have used in [4] we have measured the MFT of the dual axis reconstruction. This MFT (Fig. 3) is identical to the MTF of the Radon algorithm on the central slice and is independent of the aperture. In the case of single axis reconstruction the MTF is very perturbed at large aperture whatever the algorithm we use. The uniformity of the MTF with double circular trajectory shows that we have suppressed all the effect of the shadow area without reducing the geometrical resolution.

The density resolution was evaluated the same way that it has been done in [4]. A set of spheres is located on the axis of one of the two trajectory, and the average density on each sphere is compared to the sphere of the central slice. With a single circular source trajectory, the 3D backprojection algorithm and the Radon algorithm underestimate the density of the spheres located far from the trajectory plane. Fig. 4 we see that, for an aperture of ±30°, with a single circular source trajectory the relative mean square error (RME) may reach 20%. With dual axis resolution the RME stay under 2%.

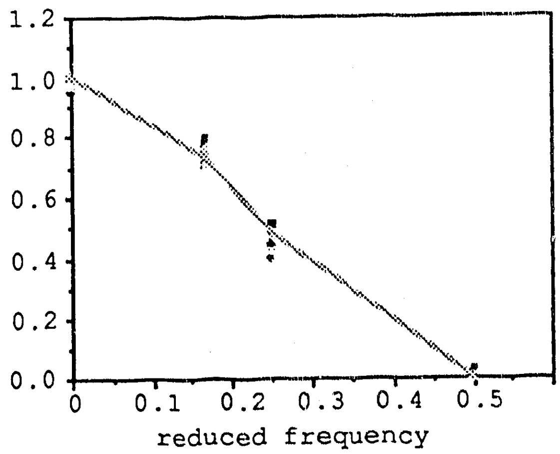

Fig. 3. Modulation function transfert of the dual axis reconstruction.
Radon 1 axis central slice
Radon 1 axis 10° position slice
Radon 2 axis 30° position
Apodisation function of Radon

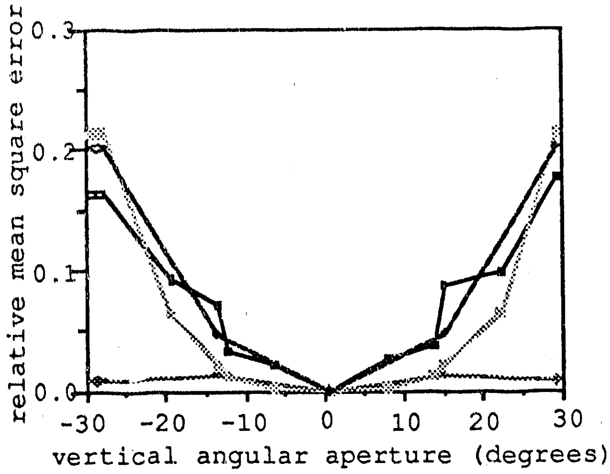

Fig. 4. Relative mean square error of the differents cone beam reconstruction codes.
3D backprojection
Radon 2 axis
Radon 1 axis
Radon 1 axis 0. in shadow area

Effects of dual axis reconstruction on horizontal artifacts has been studied on a set of parallelipedic simulated samples. Fig. 5 shows the improvement on the reconstruction with double circular trajectory. This improvement is sensitive on each interfaces and mainly at the top and bottom of the sample.

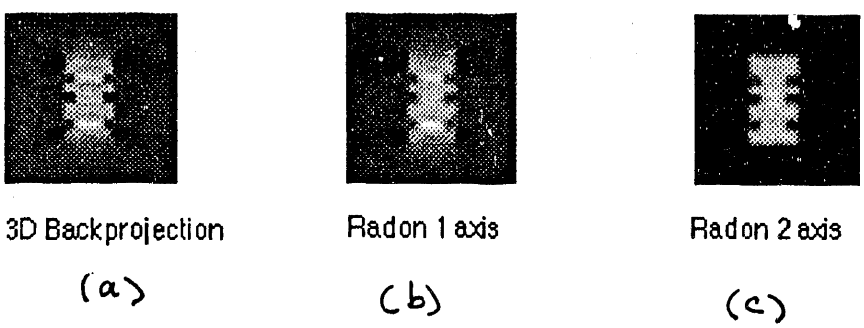

Fig. 5: Effect of the double trajectory on the horizontal artifacts. (a) 3D backprojection reconstruction, (b) Radon Reconstruction with single circular trajectory (Radon 1 axis)Radon reconstruction with double circular trajectory (Radon 2 axis), (c)

CONCLUSION

We have shown that 3D cone-beam tomography with double circular source trajectory suppress problems of missing information encountered with single circular trajectories. Mainly the density resolution is independent of the interest area position, and streak artifacts characteristics of single trajectories reconstructions are suppressed. This method can be implemented without approximation using the Grangeat formula. Such a method allows to design 3D cone beam tomograph for small parts with good efficiency of the photons. We are upgrading our experimental setup to acquire data on double circular trajectory.

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