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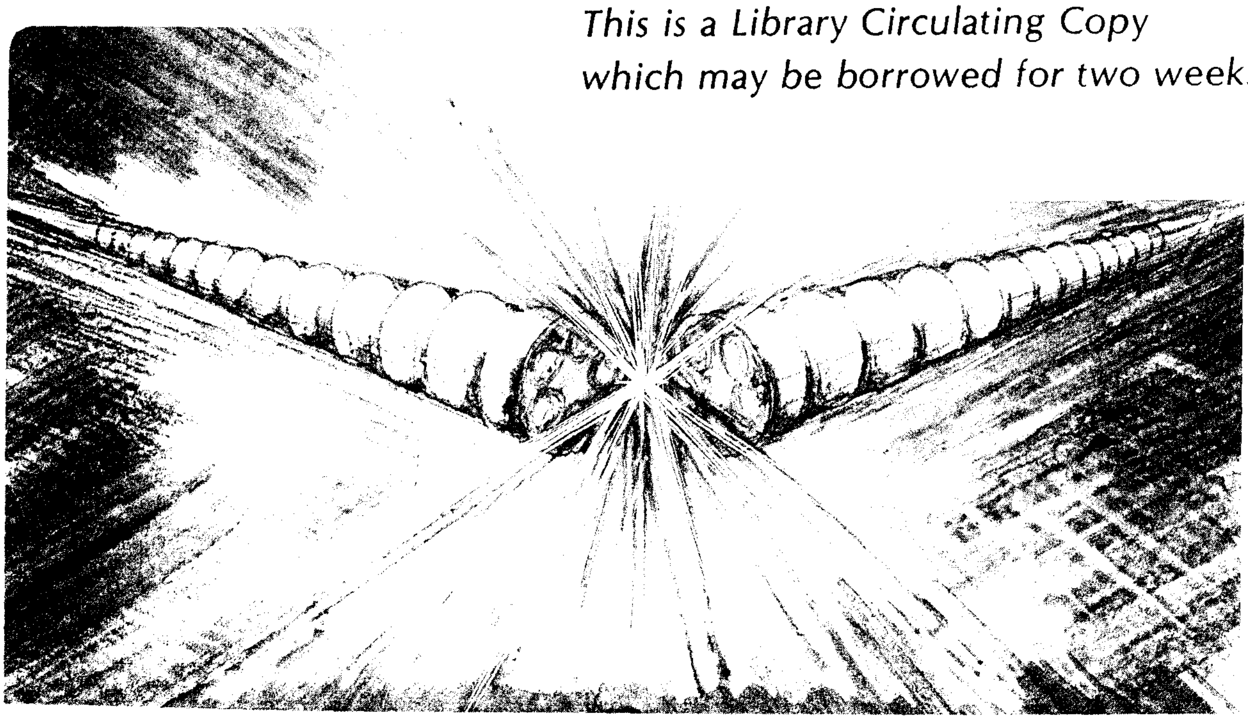

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ON THE RE-ACCELERATION OF BUNCHED BEAMS

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ABSTRACT

We examine the re-acceleration of a bunched beam through a linear induction accelerator (LIA) cavity, with attention to the energy lost through coupling to the TM modes of the structure. We find that the energy lost at 1 kA peak current is a small fraction of the boost which the LIA is designed to impart. We discuss implications for a Relativistic Klystron or Free Electron Laser (FEL) version of the Two-Beam Accelerator (TBA).

INTRODUCTION

The Relativistic Klystron[1] and Free Electron Laser[2] versions of the Two Beam Accelerator require the re-acceleration of a 1-3 kA, 40-50 ns electron beam (the "drive" beam), a few mm in radius, bunched on the scale of the klystron output cavity rf wavelength. This re-acceleration is to be accomplished by the use of linear induction accelerator (LIA) cavities (see Figure 1) which impart a boost of order ~ 100 keV to each electron passing through the cavity.[3] One problem with this scheme is that an rf beam passing through the LIA cavity will couple to the various TM modes of the structure and lose energy ("beam loading").

Our concern in this paper is to compute that energy loss or, equivalently, the longitudinal impedance of the LIA cavity.[4] We will not be addressing the beam break-up problem[5] and we will not be

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considering beams arriving off-axis, which would, of course, couple differently to cavity modes.

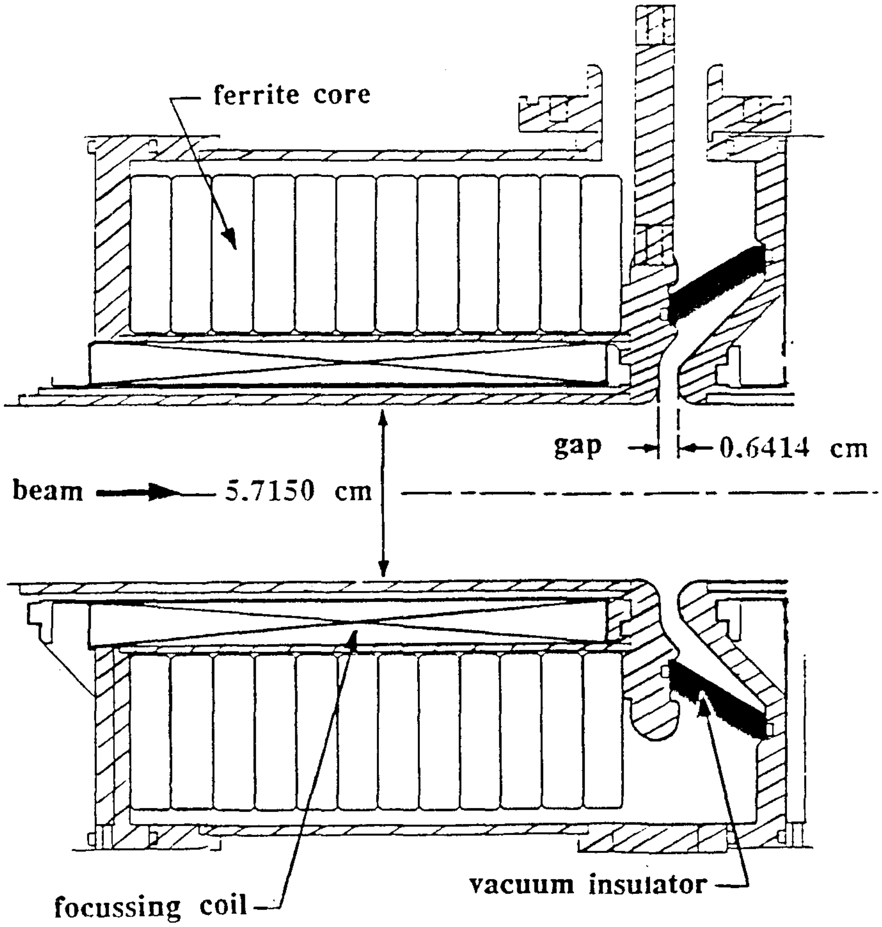

Figure 1. The LIA (linear induction accelerator) cavity geometry. Pictured is the SNOMAD II LIA cavity to be used in upcoming experiments at the Accelerator Research Center (ARC) at Lawrence Livermore National Laboratory (LLNL).

The organization of this paper is as follows. In the next section we compute a lower bound on the energy loss of an electron passing through the LIA cavity, by modelling the cavity as an open gap. In the third section we compute an upper bound to the energy loss modelling the LIA cavity as an rf cavity. In the fourth section we make use of the "idealized LIA model," consisting of a pillbox terminated in an impedance. In the fifth section, we compare analytic results with the longitudinal impedance determined numerically via AMOS[6]. For examples, we will refer to the SNOMADII and Advanced Test Accelerator (ATA) cells[7]. In the last section, we draw conclusions.

Table I. Notation

ω_λ=angular frequency of the TM_λ mode
Q_λ= Q of the mode TM_λ
L= gap length
b=beam pipe radius
f_c=cut-off frequency for TM_{01}=2.405 c/ 2 π b
v_b=beam velocity
c=speed of light
λ=(m,n,p)=mode index
m=azimuthal mode number=0 for this work
n=radial mode number
p=z mode number

τ=pulse length ~ 40 - 50 ns
f=beam bunching frequency ~ 11.42 - 17.14 GHz
ω=2 π f
I=peak current ~ 2 kA
θ=transit angle=$\omega L/v_b$
γ = beam energy / rest energy ~ 10 - 20

OPEN GAP MODEL

As a first estimate of the energy loss in the LIA cavity, we estimate the power radiated through an open gap by an rf beam. By superposition, the effect of the gap is to provide a virtual current source given by

$$\vec{J}(\vec{r},t) = \hat{z}\frac{\delta(r-b)}{2\pi b}\theta(z)\theta(L-z)I(t-z/v_b)$$

i.e., just the opposite of the return current that if present would result in no radiation at all. Solving Maxwell's equations using the Green's function corresponding to boundary conditions at ∞,[8] we have

$$\vec{A}(\vec{r},\omega) = \frac{1}{c}\int d^3\vec{r}\,' \frac{\exp\left(ik\left|\vec{r} - \vec{r}\,'\right|\right)}{\left|\vec{r} - \vec{r}\,'\right|}\vec{J}(\vec{r}\,',\omega)$$

where $k=\omega/c$, and we are interested in ω corresponding to the harmonics of the beam.

To estimate the total power radiated, we compute the Poynting vector,

$$\vec{S} = -\hat{r}_\perp \frac{c}{4\pi} E_z B_\varphi$$

and integrate this over a cylindrical surface at some large radius. We express the result in terms of an impedance for the gap[9]

$$P(\omega) = R(\omega)\left|I(\omega)\right|^2$$

and we find

$$R(\omega) = \frac{1}{2c}\left| k\int_0^L dz \exp\left(i\frac{kz}{\beta}\right) J_0\left(k\sqrt{z^2+b^2}\right)\right|^2$$

and $\beta=v_b/c$.

In the limit $b>>L$ and $kL^2/2b << 1$ we have

$$R(\omega) \approx \frac{2}{c}\beta^2 \sin^2\left(\frac{kL}{2\beta}\right) J_0^2(kb)$$

For the SNOMADII and the parameters of Table I, this result gives an impedance of order 3 Ω. The rf current is typically ~ 1 kA, so that the impedance in Ohms corresponds to the average energy loss in keV, and an energy loss ~ 3 keV would be significant, but acceptable.

Having obtained an estimate for a lower bound on the impedance, we proceed to obtain an upper bound.

CAVITY MODEL

Next, we make use of an rf cavity model to obtain an upper bound on the impedance of the LIA cavity. We write the electric field as a superposition of cavity modes

$$\vec{E}(\vec{r},t) = \sum_{\lambda} q_{\lambda}(t)\vec{E}_{\lambda}(\vec{r})$$

where λ is the mode index (0,n,p) and q_λ is the normalized field amplitude.[10]

Maxwell's equations reduce to[11]

$$\frac{d^2 q_\lambda}{dt^2} + \frac{\omega_\lambda}{Q_\lambda}\frac{dq_\lambda}{dt} + \omega_\lambda^2 q_\lambda = -4\pi\frac{\partial}{\partial t}\int_{\text{cavity}} d^3\vec{r}\,\vec{J}(\vec{r},t)\bullet\vec{E}_\lambda^*(\vec{r})$$

$$= -4\pi\frac{\partial}{\partial t}\int dz\, I\left(t-\frac{z}{v_b}\right)E^*_{\lambda z}(r_\perp = 0, z)$$

where we explicitly neglect the beam spot size and any off-axis displacement of the beam.

Defining -eV(t) to be the change in energy experienced by an electron entering the cavity at time t, we have

$$V(t) = \int dz\, E_z\left(r_\perp = 0, z, t+\frac{z}{v_b}\right)$$

$$= \sum_\lambda V_\lambda(t)$$

$$= \sum_\lambda \int dz\, E_{\lambda z}(r_\perp = 0, z)\, q_\lambda\left(t+\frac{z}{v_b}\right)$$

For the Fourier components of the voltages V_λ, we have then, from the equation for the mode amplitudes, q_λ,

$$\left(\omega_\lambda^2 - \omega^2 - i\frac{\omega\omega_\lambda}{Q_\lambda}\right)\tilde{V}_\lambda(\omega) = i\omega\omega_\lambda\frac{Z_\lambda}{Q_\lambda}\tilde{I}(\omega)$$

where Z_λ/Q_λ, the "surge impedance", is defined according to[12]

$$\frac{Z_\lambda}{Q_\lambda} = \frac{4\pi}{\omega}\left|\int_{\text{cavity}} dz\, E_{\lambda z}(r_\perp = 0, z)\exp\left(i\frac{\omega z}{v_b}\right)\right|^2$$

The beam then deposits energy into the TM_λ mode at a rate

$$\begin{aligned} P_\lambda(\omega) &= \frac{1}{2}\mathrm{Re}\left(V_\lambda(\omega)\, I^*(\omega)\right) \\ &= R_\lambda(\omega)\left|I(\omega)\right|^2 \end{aligned}$$

where we characterize the action of the mode λ in terms of an impedance $R_\lambda(\omega)$[13]:

$$R_\lambda = \frac{1}{2}Z_\lambda \frac{\left(\frac{\omega\omega_\lambda}{Q_\lambda}\right)^2}{\left(\omega_\lambda^2 - \omega^2\right)^2 + \left(\frac{\omega\omega_\lambda}{Q_\lambda}\right)^2}$$

For example, for the TM_{0n0} mode of a closed cylindrical cavity we have[14]

$$\frac{Z_{0n0}}{Q_{0n0}} = \frac{4\pi}{c}\frac{L}{R}\left(\frac{\sin\,(\theta/2)}{\theta/2}\right)^2$$

where $\theta = \omega L/v_b$ is the transit angle and R is the cavity radius. We then have

$$R_{0n0} = \frac{2\pi}{c}\frac{L}{R}\left(\frac{\sin(\theta/2)}{\theta/2}\right)^2 \frac{Q_{0n0}\left(\frac{\omega\omega_{0n0}}{Q_{0n0}}\right)^2}{\left(\omega_{0n0}^2 - \omega^2\right)^2 + \left(\frac{\omega\omega_{0n0}}{Q_{0n0}}\right)^2}$$

For SNOMADII parameters the surge impedance as given above is ~ 20 Ω at 11.42 GHz and ~ 15 Ω at 17.14 GHz. This result gives an impedance, R_{0n0}, of order 5 - 100 Ω, depending on how close the beam frequency is to a resonance and depending on the mode Q. Taking this together with the result of the last section, we have upper and lower bounds on the effective impedance of the actual LIA cavity.

IDEALIZED LIA MODEL

A somewhat more realistic model than the closed cavity or the open gap, is the pillbox terminated in an impedance, on a beam pipe. This model has been examined by Briggs, et al in conection with the "dc" beam at ATA.[15]

In Reference 7, it is shown that the impedance of this idealized LIA cavity is

$$Z(\omega) = i\frac{1}{2\pi} Z_0 \frac{L}{b}\left(\frac{\sin(\theta/2)}{\theta/2}\right)^2 \frac{1}{H}$$

where $Z_0 = 4\pi/c \sim 377\ \Omega$ and what we have denoted $R(\omega)$ above is 1/2 Re $Z(\omega)$. H is given by

$$H = \frac{J_0'(\frac{\omega b}{c})}{J_0(\frac{\omega b}{c})} - \frac{G'(\frac{\omega b}{c})}{G(\frac{\omega b}{c})} + \frac{2\,\omega b^2}{cL}\sum_{n=1}^{\infty}\frac{1-\exp(-\eta_n L/b)}{\eta_n^3}$$

where $\eta_n^2 = j_n^2 - (\omega b/c)^2$ and

$$G(x) = J_0(x) + CN_0(x)$$

with

$$C = \frac{i\frac{Z_s}{Z_0}J_0'(\frac{\omega R}{c}) - J_0(\frac{\omega R}{c})}{N_0(\frac{\omega R}{c}) - i\frac{Z_s}{Z_0}N_0'(\frac{\omega R}{c})}$$

We exhibit the real part of the impedance as a function of frequency for R=10 cm, b=2.8575 cm and Z_s/Z_0=2 in Figure 2 (SNOMADII parameters).

Examining Figure 2, we see that this model predicts Re (Z) ~ 5 - 15 Ω for SNOMADII, depending on how close the beam frequency is to a resonance. This corresponds to an R(ω) ~ 3 - 8 Ω. The lower value, corresponding to a beam bunching frequency between resonances, is consistent with the open gap result.

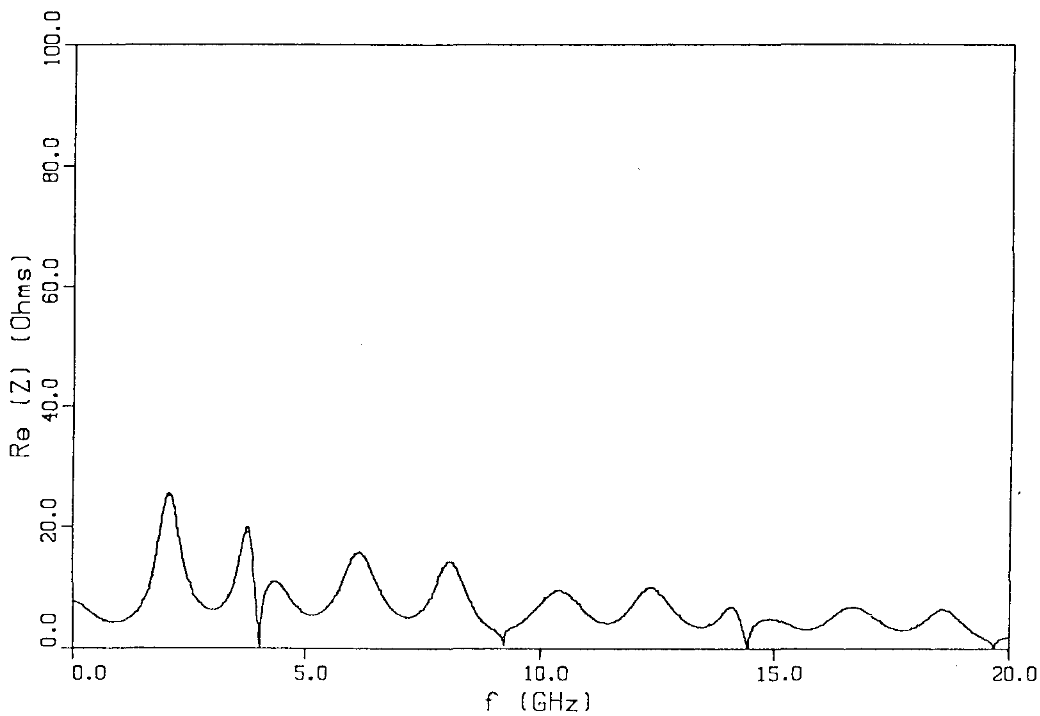

Figure 2. Impedance of the idealized SNOMADII cell for R=10 cm and Z_s/Z_0=2.

Note that the idealized LIA result assumes E_z is constant across the gap, i.e., it is analogous to considering only p=0 modes. This leads to an underestimate of the impedance at frequencies f > c/2L. For the SNOMADII cell, c/2L ~ 23 GHz, larger than frequencies of interest; thus we would expect the idealized LIA model to be rather good for such short gap cavities.

In Figure 3, we exhibit the real part of Z(ω) for R=27 cm, b=6.725 cm and Z_s/Z_0=2 (ATA parameters), as computed from the idealized LIA model. In the next section, we will compute the impedance for the same cavity numerically, as a further check on our estimates.

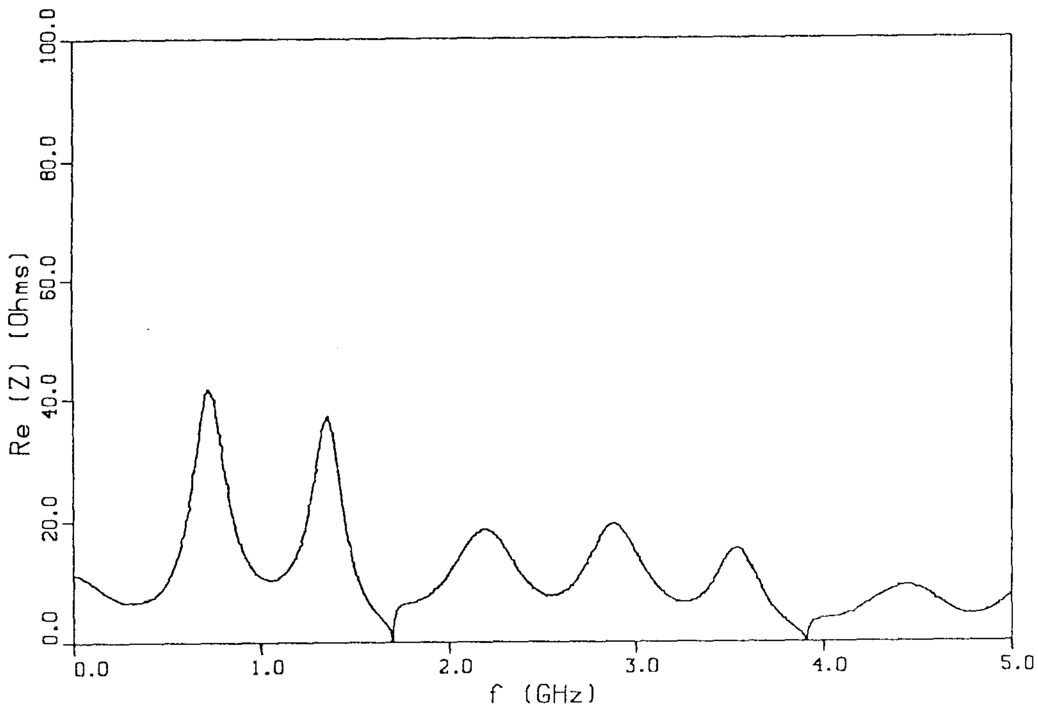

Figure 3. Impedance of the idealized ATA cell for R=27 cm and Z_s/Z_0=2.

NUMERICALLY MODELLED CAVITY[16]

It is useful to compare these analytic estimates to the results of numerical work. Two results for the impedance of the ATA cavity are depicted in Figures 4 and 5. The difference between the two results is due to the method of terminating the beam pipe. In Figure 4, the beam pipe is terminated in the free space impedance of 377 Ohms, while in Figure 5, the beam pipe is terminated in a conducting wall.

Evidently, the use of conducting boundary conditions at the some distance down the beam pipe introduces many additional spikes in longitudinal impedance. These correspond to TM_{0np} modes, with $p > 0$, of the entire structure viewed as a single cavity. In fact, any impedance mismatch at the pipe termination will result in spikes. Thus it is not surprising to see spikes near cut-off, where impedance mismatch is unavoidable.

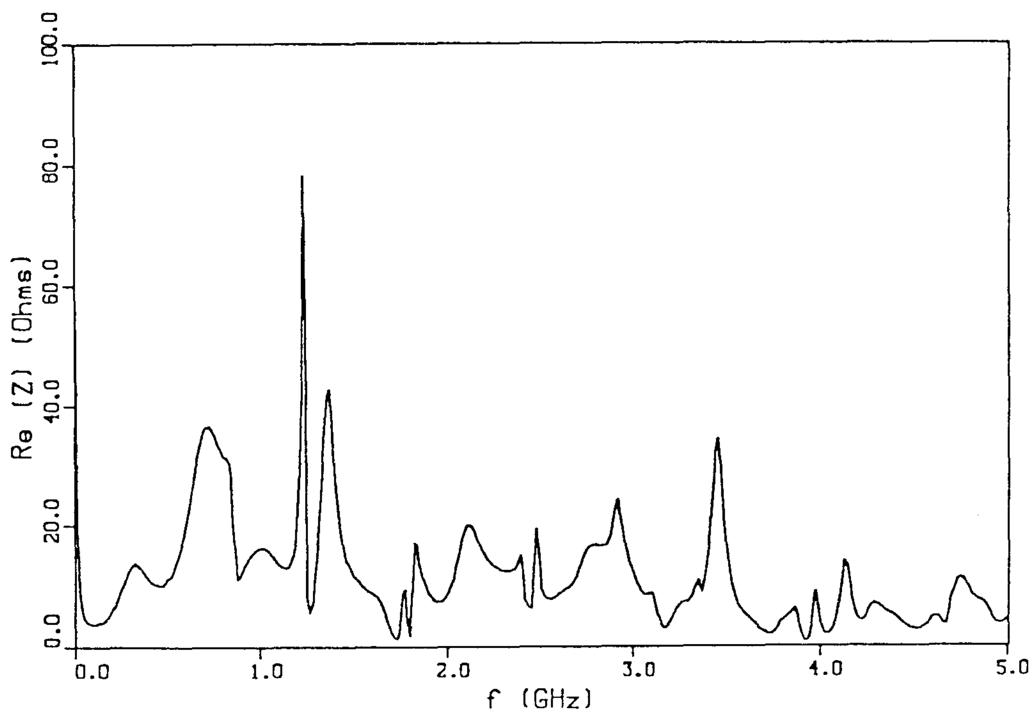

Figure 4. Impedance of the ATA cell as determined by the AMOS code, using a free space impedance termination of the beam pipe.

Thus to accurately predict the impedance of the cavity, the termination must be accurately modelled, and this is the subject of ongoing numerical work. For the purposes of this paper it is enough to see that the idealized cavity model gives a fair estimate of peak impedance for p=0 modes (below 6 GHz).

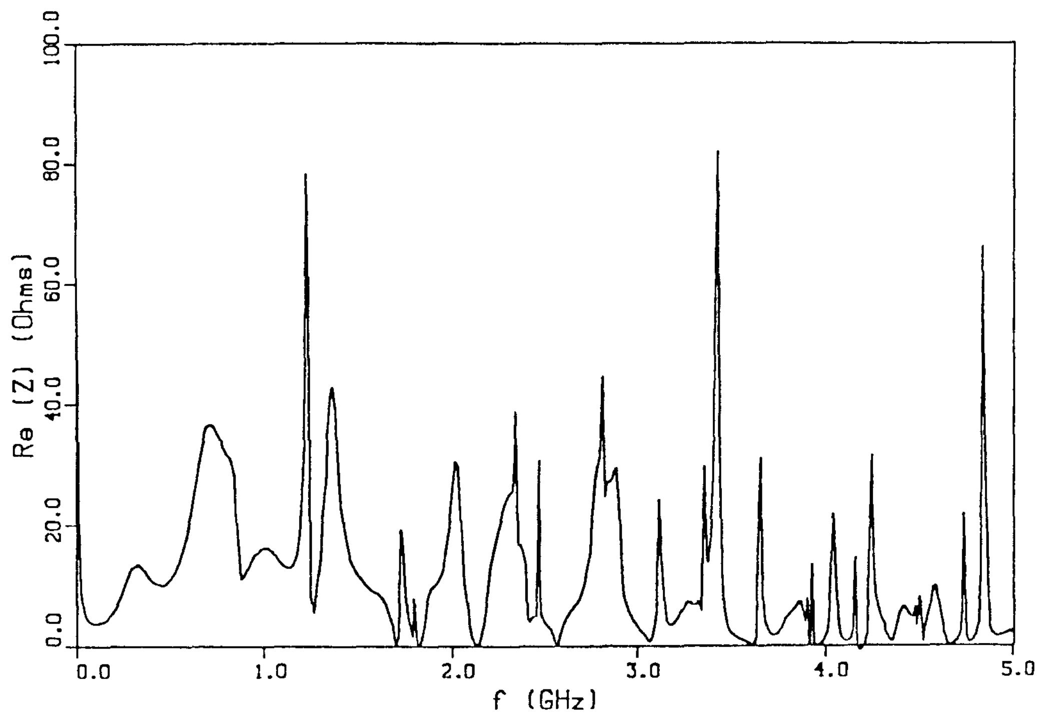

Figure 5. Impedance of the ATA cell as determined by the AMOS code, using a conducting boundary termination of the beam pipe.

CONCLUSIONS

We have found that energy loss in reacceleration of a bunched beam through an LIA cavity will be appreciable, but acceptable. Evidently, this beam loading can vary significantly, depending on whether the beam frequency is on a resonance, or between resonances. Spurious TM_{0n0} resonances should therefore be a consideration in any LIA design. We also found that , between resonances, the impedance is of order that of an open gap.

We note also that electrons at the beam head will suffer less boost degradation, so that a spread in energy along a length ~ Qc/ω at the beam head will result. However, this length is of order a few centimeters, and represents only a small fraction of the entire 12 - 15 meter beam.

More significantly, there will be an energy spread within each bunch of order the average energy loss, and this must be considered in any TBA design.

The authors acknowledge an instructive work by P. B. Wilson [17] and comments by R.L. Gluckstern.[18]

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[1] A. M. Sessler and S. S. Yu, "Relativistic Klystron Version of the Two-Beam Accelerator," Phys. Rev. Lett., 58, 2439 (1987).

[2]A.M.Sessler, E.Sternbach, and J.S. Wurtele, "A New Version of a Free Electron Laser Two-Beam Accelerator," LBL-25937.

[3]J.E. Leiss, "Induction Linear Accelerators and Their Applications," IEEE Trans. Nuc. Sci., NS-26, 3, 3870-3876, June (1979).

[4]See, for example, K. L. F. Bane, AIP Conf. Proc. No. 153, 1987.

[5]For a discussion of BBU in the drive beam of the TBA, see D.H. Whittum, G. A. Travish, A.M Sessler, G. D. Craig, and J. F. DeFord, "Beam Break-Up in the Two-Beam Accelerator," Proceedings of the 1989 IEEE Particle Accelerator Conference, Chicago, Illinois, March 20 - 23, 1989.

[6]J. F. DeFord and G. D. Craig, "The Azimuthal Mode Simulation Code (AMOS)", ibid.

[7]R.J. Briggs, D.L. Birx, G.J. Caporaso, V.K. Neil, and T.C. Genoni, Part. Acc., **18**, 41, 1985.

[8]A more exact treatment of the boundary conditions is given in section 4.

[9]$R(\omega)$ is one-half the real part of the longitudinal impedance of the structure evaluated at the beam bunching frequency.

[10]Our normalization is such that

$$\int d^3\vec{r}\, \vec{E}_\lambda(\vec{r}) \bullet \vec{E}^*_\lambda(\vec{r}) = 1$$

[11] We make the approximation, valid for Q >> 1, that the various modes are uncoupled. See, for example, John C. Slater, *Microwave Electronics*, D. Van Nostrand and Co., Inc., 1950.

[12] See Ref. 7. This surge impedance may be related to the longitudinal impedance corresponding to the mode λ, according to

$$\tilde{Z}_\lambda = Z_\lambda \frac{-\frac{i\omega\omega_\lambda}{Q_\lambda}}{\left(\omega_\lambda^2 - \omega^2 - i\frac{\omega\omega_\lambda}{Q_\lambda}\right)}$$

And this notation may be related to the longitudinal impedance as defined by Chao (Alexander W. Chao, "Coherent Instabilities of a Relativistic Bunched Beam," SLAC-PUB-2946, also, *Physics of High Energy Particle Accelerators*, Proceedings of the SLAC Summer School, Aug 2- Aug 13, 1982) according to

$$Z^{\parallel}_{m=0} = \sum_\lambda \tilde{Z}_\lambda$$

The surge impedance for a closed cavity is calculated in a straightforward manner from the standard result for the TM_λ modes of a closed pillbox. (See, for example, J.D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, New York, USA, 1975, 2nd ed.) The modes are given by

$$E_z = E_\lambda J_0\left(j_n \frac{r}{R}\right) \cos\left(\frac{p\pi z}{L}\right)$$

$$\vec{E}_\perp = E_\lambda \frac{p\pi}{L} \frac{R}{j_n} J_1\left(j_n \frac{r}{R}\right) \sin\left(\frac{p\pi z}{L}\right)\hat{r}$$

$$\vec{H}_\perp = -i E_\lambda \frac{\omega}{c} \frac{R}{j_n} J_1\left(j_n \frac{r}{R}\right) \cos\left(\frac{p\pi z}{L}\right)\hat{\phi}$$

where j_n is the n-th zero of J_0, and other notation is as defined in Table I. E_λ is given by

$$E_\lambda = \sqrt{\frac{2}{\pi LR^2}} \frac{1}{|J_1(j_n)|} \frac{1}{\sqrt{1+\left(\frac{p\pi R}{j_n L}\right)^2}}$$

and we invoke the normalization given in footnote 10.

[13]This R_λ is related to the m=0 longitudinal impedance according to

$$\sum_\lambda R_\lambda = \frac{1}{2} \mathrm{Re}\, Z^{\parallel}_{m=0}$$

[14]Where we have taken $j_n J_1(j_n)^2 \sim 2/\pi$, which is accurate to within 2% even for n=1.

[15] See Ref. 7. For this work, we consider only the Fourier component of the current at the beam primary frequency. Thus we replace the 40 - 50 ns macropulse (of perhaps 200 - 300 bunches) with an infinite train of bunches. The current may then be represented as a Fourier series. We neglect the higher harmonics, and consider only the first component, at the beam bunching frequency. Typically, this component is about one-half the peak current.

[16] See Ref. 6.

[17]P. B. Wilson, "High Energy Electron Linacs: Applications to Storage Ring RF Systems and Linear Colliders," SLAC-PUB-2884, also, AIP Conference Proceedings No. 87, 450, 1982

[18]See also, R.L. Gluckstern, "High Frequency Behavior of the Longitudinal Impedance for a Cavity of General Shape," and, by the same author, "Longitudinal Impedance of a Periodic Structure at High Frequency," (both to be published in Phys. Rev. D).

Gluckstern finds, for the high frequency limit of the longitudinal impedance of a pillbox on a beam pipe,

$$Z(\omega) \approx (1+i) Z_0 \frac{1}{2\pi^{3/2}} \frac{L^{1/2}}{k^{1/2} b}$$

in agreement with the Lawson diffraction model. This gives $R(\omega) = 1/2$ Re $(Z) \sim 6\ \Omega$ for SNOMADII parameters, in fair agreement with the other estimates above.

This high frequency result (the "optical model") gives a good estimate for the envelope of the actual impedance curve at frequencies of interest, where resonances tend to be less pronounced.