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ALTERATION OF THE SUPERCONDUCTING PROPERTIES OF A15 COMPOUNDS AND ELEMENTARY COMPOSITE SUPERCONDUCTORS BY NON-HYDROSTATIC ELASTIC STRAIN

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ABSTRACT

Elastic strains alter (usually, but not always, adversely) the critical temperatures, magnetic fields, and current densities of superconducting A15 compounds; non-hydrostatic strain states are particularly effective in this regard. This paper is a review of the experimental evidence, obtained by a variety of techniques, concerning the strain dependence of the critical properties of a number of A15 compounds and a discussion of theoretical models for describing such effects.

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INTRODUCTION

It is now well known that stress alters the properties of superconductors; the frequency of appearance of sessions entitled "Stress Effects on Superconductors" in the ICMC and Applied Superconductivity conferences bears witness to the practical importance of this topic. In addition, studies of changes in the critical properties of superconducting compounds as the size and shape of their crystallographic architecture is varied by external stress provide helpful insights into the relations between superconductivity and electronic and vibrational structure, interatomic bonding, etc. It is the purpose of this paper to review briefly and discuss the salient features of what is known about the dependence of the superconducting critical temperature, magnetic field, and current density of compounds with the A15 crystal structure (i.e., Nb_3Sn, V_3Si etc.) upon elastic strain, with emphasis on "non-hydrostatic" strain states in which the shape, as well as the size, of the unit cell is varied. Furthermore, the implications of the strain-dependent properties of the compounds for the behavior of elementary composite conductors will be examined; in addition to its relevance to technological applications, this is important because many of the experiments on strain dependence have been made with composite rather than free-standing specimens. Finally, this review is not meant to be exhaustive in its coverage (see the recent and excellent review by Koch and Easton (1) for a wide-ranging and thorough review of stress effects),

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and I will draw heavily and unabashedly upon the recent work of my colleagues at Brookhaven to illustrate a number of points.

THE ELASTIC STRAIN DEPENDENCE OF THE CRITICAL TEMPERATURE

Neither the magnitude nor the functional form of the strain dependence of the critical temperature, T_c, is unequivocally well established for any A15 compound. This is due in part to two factors: to the existence of the cubic $\leftrightarrows$ tetragonal martensitic phase transition which occurs in many A15 compounds (2) and to the related occurrence of nonlinear stress-strain relations under some circumstances (3). Before discussing the uncertainties however, let us first establish a phenomenological framework to characterize the strain dependence of T_c in both single and polycrystals.

Following Testardi (4), one might reasonably expect the strain dependence of T_c for a single crystal to be described by:

$$T_c(\underline{\varepsilon}) = T_c(0) + \underline{\Gamma} \cdot \underline{\varepsilon} + \frac{1}{2}\,\underline{\varepsilon} \cdot \underline{\underline{\Delta}} \cdot \underline{\varepsilon} \qquad (1)$$

where $\underline{\varepsilon}$ is the strain (written as a 6-component vector) and $\underline{\Gamma}$ and $\underline{\underline{\Delta}}$ are tensors for which the components must be found experimentally. The number of distinct, nonzero elements are governed by crystal symmetry (e.g. $\underline{\underline{\Delta}}$ has the same symmetry as the elastic moduli); for crystals with cubic symmetry, Eq. (1) becomes:

$$\begin{aligned} T_c(\underline{\varepsilon}) = T_c(0) &+ \Gamma_1(\varepsilon_1+\varepsilon_2+\varepsilon_3) + \tfrac{1}{2}\Delta_{11}(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2) \\ &+\Delta_{12}(\varepsilon_1\varepsilon_2+\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_3) + \tfrac{1}{2}\Delta_{44}(\varepsilon_4^2+\varepsilon_5^2+\varepsilon_6^2) \quad . \end{aligned} \qquad (2)$$

Thus one requires, in this framework, four coefficients to describe the strain dependence of T_c for a cubic crystal.

If, as in several A15 compounds, the crystal, through the martensitic transition, becomes tetragonal above the superconducting transition, more

constants are required. For example, terms linear in $(\varepsilon_1-\varepsilon_2)$, $(\varepsilon_1+\varepsilon_2-2\varepsilon_3)$, etc. are now permitted by symmetry, and more than three distinct constants appear in $\underline{\underline{\Delta}}$. Furthermore, such an expansion is only appropriate for a single domain crystal. In general, crystals exhibiting martensitic transformations will be multi-domained and, in most practical applications, multi-domained polycrystals will be encountered. Finally, the martensitic transition in A15 crystals can be suppressed by stress as well as disorder (2), so that the degree of transformation in a polycrystal subject to a triaxial strain state with strains of the order of 1% is problematic. Because of these complications we shall confine our discussion to non-transforming crystals for simplicity, but we expect that the principal conclusions will be independent of this restriction.

An equation describing the T_c of a random polycrystalline aggregate, for which each of the grains obeys Eq. (1) is obtained by the following angular average. Suppose that the polycrystal is subjected to a triaxial strain state with principal strains ε_{1p}, ε_{2p}, and ε_{3p}. A particular grain, denoted by i, has strain components referred to the crystal axes of that grain given by $\underline{\varepsilon}(i)=\underline{\underline{L}}(i)\cdot\underline{\varepsilon}_p$, where the elements of the tranformation matrix $\underline{\underline{L}}$ are direction cosines between the crystal axes and the principal axes (5). Thus by Eq. (1) the critical temperature of that grain, $T_c(i,\underline{\varepsilon})$, is given by:

$$T_c(i,\underline{\varepsilon}) = T_c(0) + \underline{\Gamma}\cdot\underline{\underline{L}}(i)\cdot\underline{\varepsilon}_p + \frac{1}{2}\underline{\varepsilon}_p\cdot\underline{\underline{L}}^{+}(i)\cdot\underline{\underline{\Delta}}\cdot\underline{\underline{L}}(i)\cdot\underline{\varepsilon}_p \quad . \tag{3}$$

The critical temperature for the aggregate of such grains is obtained by averaging over all possible axis orientations; this yields an equation equivalent to Eq. (1) with $\underline{\Gamma}$ and $\underline{\underline{\Delta}}$ replaced by:

$$\underline{\Gamma}_{\text{polycrystal}} = \langle\underline{\Gamma}\cdot\underline{\underline{L}}\rangle \tag{4a}$$

$$\underline{\underline{\Delta}}_{\text{polycrystal}} = \langle\underline{\underline{L}}^{+}\cdot\underline{\underline{\Delta}}\cdot\underline{\underline{L}}\rangle \tag{4b}$$

where the brackets denote angular averages. (This approach is analogous to the Voigt constant strain approximation for the elastic moduli of polycrystals.)

For a random distribution of grains of cubic symmetry, one obtains by this procedure an analog of Eq. (2):

$$T_c(\underline{\varepsilon}^p) = T_c(0) + \Gamma_1(\varepsilon_{1p}+\varepsilon_{2p}+\varepsilon_{3p}) + \left(\frac{3\Delta_{11}+2\Delta_{12}+\Delta_{44}}{10}\right)\left[\varepsilon_{1p}^2+\varepsilon_{2p}^2+\varepsilon_{3p}^2\right]$$
$$+\left(\frac{2\Delta_{11}+8\Delta_{12}-\Delta_{44}}{10}\right)\left[\varepsilon_{1p}\varepsilon_{2p}+\varepsilon_{1p}\varepsilon_{3p}+\varepsilon_{2p}\varepsilon_{3p}\right] \quad . \tag{5}$$

This can be put in more compact form by writing the strains in terms of the dilatation, $e\equiv\varepsilon_{1p}+\varepsilon_{2p}+\varepsilon_{3p}$, and deviatoric components, (i.e. non-hydrostatic) $\hat{\varepsilon}_i\equiv\varepsilon_{ip}-\frac{1}{3}e$,

$$T_c(\underline{\varepsilon}) = T_c(0) + \Gamma_1 e + \frac{1}{2}\Delta_v e^2 + \frac{2}{5}\Delta_t[\hat{\varepsilon}_1^2+\hat{\varepsilon}_2^2+\hat{\varepsilon}_3^2] \tag{6}$$

where

$$\Delta_v \equiv \frac{\Delta_{11}+2\Delta_{12}}{3} \tag{7a}$$

$$\Delta_t \equiv \left(\frac{\Delta_{11}-\Delta_{12}}{2}\right)+\frac{3}{8}\Delta_{44} \quad . \tag{7b}$$

(The components of Δ have been grouped in this way to stress their analogy with the grouping of elastic constants in the bulk and shear moduli of cubic crystals: $(C_{11}+2C_{12})/3$, $(C_{11}-C_{12})/2$, and C_{44}.) Equations (6) and (2) now provide the desired phenomenological framework to characterize the strain dependence of T_c in both polycrystals and single crystals.

The relative efficacy of non-hydrostatic components of strain in altering T_c will be reflected in the size of Δ_t compared with Δ_v and Γ_1 in Eqs. (6) and (7). Let us examine the available experimental evidence on this point (which is summarized in Table I). By means of a thermodynamic argument, Testardi (4) related the coefficients of $\underline{\underline{\Delta}}$ to the behavior of elastic moduli in the vicinity of the superconducting transition and by this means determined values for V_3Si

and V_3Ge; Γ_1 and Δ_v and Δ_t computed from these results are listed in Table I. It may be seen that Testardi's results yield $\Delta_v \approx \Delta_t$. Studies of the hydrostatic pressure dependence of the T_c of V_3Si (6,7) contradict these findings: while Testardi's coefficients suggest that a substantial quadratic depression of T_c should be observable for compressions of $\sim 2\times10^{-3}$ ($p\sim400$ MPa), experiments by Chu and Diatschenko (7) show an essentially linear increase of T_c with pressure up to $\sim$2500 MPa (at which point there is a pressure-induced phase transformation to a low T_c phase). Since the bulk modulus does not exhibit any softening effects on passing through the martensitic phase transition (2), one would expect reasonably linear pressure-dilatation relation for A15 compounds, in marked contrast to the uniaxial stress-strain behavior (3). Therefore the results of Chu and Diatschenko suggest that there is no appreciable quadratic term in T_c vs ε for ε up to 10^{-2} and thus that $|\Delta_v| \ll 2\times10^4$ K, in comparison with Testardi's value of -11×10^4 K. Although the published data do not permit such a quantitative comparison of values of Δ_v for V_3Ge, Smith (6) suggests that Testardi's value of $+2\times10^4$ is probably too high. Finally, Chu and Vieland (12) observed a linear hydrostatic pressure dependence of T_c for Nb_3Sn for pressures up to 1500 MPa ($\varepsilon\sim10^{-2}$). Analysis of their results using a linear pressure-dilatation relation suggests that for this material $|\Delta_v| \ll 5\times10^3$ K; this should be compared with $\Delta_t \sim 10^5$ K as suggested by experiments on composite wires, to be discussed below.

There have been several measurements of the effects of uniaxial compressive stress on the T_c of single crystals of V_3Si, and Nb_3Sn (8,9,13). All of these experiments showed that T_c decreased linearly with increasing compression up to stresses of $\sim$50 MPa (9), and that the severity of the T_c depression was markedly anisotropic: greatest for stress along <100> and least along <111> with <110> intermediate. (These results are summarized in Table I.) The linear decrease

with stress led Weger et al. (8) to propose that the form of the stress dependence (for stress along <100>, <010>, etc.) was:

$$\Delta T_c = -k \left\{ |\sigma_1-\sigma_2| + |\sigma_2-\sigma_3| + |\sigma_3-\sigma_1| \right\} . \qquad (8)$$

The exact relation between this form and the strain dependence, e.g., Eq. (2), depends on the stress-strain relation, which is known to be highly nonlinear for stress along <100> in single crystals of V_3Si at temperatures near T_c (3). Testardi (10) proposed an approximate stress-strain relation based on these results and showed how the quadratic strain dependence was consistent with an essentially linear stress dependence over the experimentally studied range of stress (see also (9) on this point). If Testardi's stress-strain relation is used to obtain the triaxial strain state and thus to obtain a value of Δ_t from the $\partial T_c/\partial\sigma_{<100>}$ for V_3Si, one obtains $\sim -9\times10^4$ K, in quite good agreement with the value of $\sim -10\times10^4$ K obtained by Testardi (4) from elastic modulus behavior near T_c and the value of -8×10^4 K obtained from experiments on composite wires, to be discussed below. Finally, the nature of the anisotropy in stress sensitivity mentioned above (see Table I) is consistent with the relative values of the coefficients of strain dependence Δ_{44} and $(\Delta_{11}-\Delta_{12})/2$ obtained by Testardi (4). Thus all of the data for V_3Si are consistent with a non-hydrostatic strain dependence of T_c of the form of Eqs. (2) and (6) (provided Δ_{11} is $\sim -\Delta_{12}$, i.e. $\Delta_v \sim 0$) and with reasonable agreement on the magnitude of the coefficients $(\Delta_{11}-\Delta_{12})/2$, Δ_{44}, and thus of Δ_t. Because there are no data on stress-strain relations in Nb_3Sn, one cannot analyze the results of McEvoy (13) in the same detail as was done for V_3Si, but the fact that $\partial T_c/\partial\sigma_{<100>}$ is of the same order of magnitude for the two materials is consistent with the near equality of Δ_t for the two (see Table I).

We see then that the existing experimental data are consistent with Eqs. (2) and (6) as a description of the strain dependence of T_c for V_3Si, Nb_3Sn, and

V_3Ge. Furthermore, the preponderence of evidence suggests that $\Delta_v \ll \Delta_t$ for these materials (and perhaps for all A15 compounds) and thus, from the relative values of Γ_1 and Δ_t in Table I, that the effects of non-hydrostatic components of strain greatly outweigh the effects of hydrostatic components for strains $\gtrsim 10^{-3}$. Let us now examine the behavior of simple composite conductors in this context. To aid in this consideration, a brief discussion of the nature of the strain state in such conductors is given in the Appendix.

The strain dependence of the T_c of Nb_3Sn in monofilament composite wires produced by the bronze process has been studied by Luhman et al. (14,19). They find essentially an inverted parabolic variation of the T_c with externally applied strain, the apex of the inverted parabola occurring at values of strain interpreted by these authors (as Rupp (20) had done originally, on the basis of critical current studies) as the point at which the external strain just cancels the differential thermal contraction induced residual strain. A "universal plot" of the results based on this interpretation is shown in Fig. 1. The agreement between the external strain at which the T_c peaks with that of the maximum in the critical current is shown in Fig. 2.

Let us examine these results in the context of Eq. (6). As discussed in the Appendix, the nature of strain state in the cylindrically symmetric composite permits two distinct components of principle strain: axial, ε_{zz}, and radial (or tangential), $\varepsilon_{rr}=\varepsilon_{\theta\theta}\equiv\xi\varepsilon_{zz}$. The parabolic variation of T_c with strain indicated that the linear term $\Gamma_1 e$ in Eq. (6) is negligible and, although the evidence discussed above suggests that $\Delta_v \ll \Delta_t$, we include Δ_v for generality. Thus, for composite wires; Eq. (6)

$$T_c - T_c(0) = \varepsilon_{zz}^2 \left\{ \frac{1}{2}\Delta_v (1+2\xi)^2 + \frac{4}{15}\Delta_t (1-\xi)^2 \right\} \qquad (9)$$

where $\varepsilon_{zz}=\varepsilon_{zz}^i+\varepsilon$, ε being the applied axial strain, ε_{zz}^i the initial residual

axial strain, and ξ is a function of both ε^i_{zz} and ε.

First, let us examine the validity of Rupp's conjecture (20) that T_c is a maximum when $\varepsilon=-\varepsilon^i_{zz}$. To do this it is necessary to find the strain at the maximum, ε^*, by differentiating Eq. (9) with respect to ε, making use of Eqs. (A3-A5) in the Appendix, setting it to zero, and then finding the value of $T_c(\varepsilon^*)$ from Eq. (9). The results of this procedure are a function of a large number of parameters (i.e., R, E_f, E_m, ν_f, ν_m), and their validity depends on the approximations used in the derivations described in the Appendix, especially the use of linear elasticity theory. Nonetheless, they may provide some insight. Some representative results are listed in Table II for two "limiting" cases: a) the bronze matrix and the ($Nb+Nb_3Sn$) filament both behave elastically ($x\equiv(1+\nu_m)\frac{E_m}{E_f}\simeq 4/3$) and b) the matrix yields plastically giving an effective modulus $E_m/E_f \sim 1/3$ (x=4.5). In both cases the maximum occurs at a strain somewhat less than that required to cancel the initial residual strain; for the most likely value of Δ_v, $\sim$0, T_c takes on the strain-free value at this point, but it is seen that this may not be so for nonzero values of Δ_v. These results suggest that an evaluation of Δ_t based on Rupp's conjecture ($\varepsilon^*=-\varepsilon^i_{zz}$) and Eq. (9) (with $\Delta_v\sim$0)

$$\Delta_t \simeq \frac{15[T_c(\varepsilon^*)-T_c(i)]}{4(1-\xi)^2(\varepsilon^*)^2} \qquad (10)$$

may overestimate $|\Delta_t|$ since $|\varepsilon^i_{zz}|<\varepsilon^*$, but it is difficult to estimate the necessary corrections very accurately. Estimates of ξ_i, the ratio of the radial to axial strains in the initial state, yield $\sim$ -.1 for a range of plausible parameters for the elements of the composite. With this value and the data in Fig. 1, Eq. (10) yields $\Delta_t\simeq-7.4\times10^4$ K, bearing in mind from the discussion above that this may be too large.

Another method for estimating Δ_t from the properties of composite monofilament conductors is suggested by the form of the dependence of the residual strains upon the matrix-to-filament (or "bronze-to-niobium") ratio, R, given by Eqs. (A1-A3) of the Appendix. Combining these equations with Eq. (9), neglecting small terms proportional to $(\nu_m - \nu_f)$, and using the large R limit of the ratio of radial to axial stress, ξ_∞, yields an approximate relation, good for large R:

$$|T_c(R)-T_c(0)|^{-1/2} = \frac{1 + R^{-1}\frac{E_f}{E_m}}{\delta\sqrt{\frac{4}{15}|\Delta_t|(1-\xi_\infty)^2}} \xrightarrow[R\to\infty]{} \frac{|\Delta_t|^{-1/2}}{\sqrt{\frac{4}{15}\delta^2(1-\xi_\infty)^2}} \quad . \tag{11}$$

This suggests a means of plotting the variation of T_c with R to obtain a linear extrapolation to R=∞ where the unknown terms (δ,ξ_∞) in the multiplier of $|\Delta_t|^{-1/2}$ are readily estimated with fair accuracy. An experiment based on this idea has been carried out by Aihara et al. (11) for monofilament bronze-processed composite conductors of Nb_3Sn, V_3Ga, V_3Si, and V_3Ge. Composite wires of different initial ratio, R_o, were fabricated. The ratio of each of these wires was continuously decreased by etching away the bronze matrix, and T_c was measured as a function of R. Data obtained in this way for Nb_3Sn composite wires are plotted as suggested by Eq. (11) in Fig. 3. Straight lines are indeed produced by such a plot; however, the intercept of each plot varies with the initial ratio R_o, an effect caused by plastic deformation in the bronze during cooling from the reaction temperature. Such plastic flow effects should decrease as $R_o\to\infty$, but this improvement is counteracted by an increasing amount of plastic flow in the Nb core so that the Nb_3Sn is not in a strain-free state even when the bronze is etched away. (No such core-plasticity effects were observed for V cores.) However, corrections for these effects are possible, and the resulting value of Δ_t, $-(7\text{-}8)\times10^4$ K, compares well with the value deduced from the external stress experiment described above, -7.4×10^4 K.

Δ_t was determined by similar methods for V_3Si, V_3Ge, and V_3Ga. The results are listed in Table I, and the relative size of the strain dependence for the various compounds is graphically illustrated in Fig. 4. (The Nb core plasticity effect mentioned above causes the ΔT_c shown for Nb_3Sn to be too small; without this, the curves for Nb_3Sn and V_3Si would be very similar.) Of particular interest is the increase in the T_c of V_3Ge by strain, an effect predicted by Testardi (4) and first demonstrated experimentally by Bussiere et al. (21). As may be seen from Table I the values of Δ_t determined for V_3Si and V_3Ge by the etching experiment agree reasonably well with those calculated from the coefficients deduced by Testardi from the behavior of elastic moduli (4).

The variation of the strain dependence of the T_c of Nb_3Sn with radiation-produced disorder has been studied by C. L. Snead, Jr. of BNL, who measured the dependence on the fluence of reactor-spectrum neutrons of the change in T_c which occurred upon removal by etching of the matrix from bronze-processed composite wires. These results are summarized in Table III and show that the strain dependence becomes more pronounced with increasing disorder.

The change in T_c of a composite tape is considerably larger than that of a composite wire having the same axial strain in the superconductor. The reason of course is that the triaxial strain state is different even though the axial strain is the same. The analogous equation to Eq. (9) for a tape composite (whose strain state is discussed in the Appendix) is:

$$T_c - T_c(0) = \varepsilon_{zz}^2 \left\{ \frac{\Delta_v}{2}(2+\xi_{tape})^2 + \frac{4\Delta_t}{15}(1-\xi_{tape})^2 \right\} \qquad (12)$$

with $\xi_{tape} = -2\nu_f(1-\nu_f)^{-1}$, from Eq. (A7). Combining Eqs. (9) and (12) shows that for a superconductor with $\Delta_v \sim 0$, the relative change in T_c is:

$$\frac{\Delta T_c(\text{tape})}{\Delta T_c(\text{wire})} = \frac{(1-\xi_{tape})^2}{(1-\xi_{wire})^2} \simeq 2.3 - 3.3 \quad . \tag{13}$$

Jean Bussiere of BNL and I have compared the behavior of Nb_3Sn tapes and wires and find that $\Delta T_c(\text{tape})/\Delta T_c(\text{wire}) \sim 2$, in reasonable agreement with Eq. (13). Finally, recent data on T_c of A15 Nb_3Ge deposited on various substrates (15) can be analyzed with the aid of Eq. (12) to yield $\Delta_t \sim -8\times10^4$ K, a value comparable to that of Nb_3Sn.

The values of Δ_t for several A15 compounds, determined by the methods discussed in detail above, are collected in Table I. Do these values follow any systematic pattern? This is best discerned in the context of a "McMillan-type" equation which approximately relates the T_c to several physically interpretable and, in principle, calculable parameters. The archetype of such equations was first proposed by McMillan (22), and it has since been refined and extended by a number of authors (23). For our purposes we will use the variation due to Dynes (23):

$$T_c = \frac{\hbar\langle\omega\rangle}{1.20\ k}\, e^{-\left\{\frac{1.04(1+\lambda)}{\lambda(1-.62\ \mu^*)-\mu^*}\right\}} \tag{14}$$

where $\lambda \equiv N(0)\langle I^2\rangle/M\langle\omega^2\rangle$ is the electron-phonon coupling parameter, experimental values of which are collected in Table IV, $N(0)$ is the electronic density of states at the Fermi level, $\langle I^2\rangle$ is an average electron-phonon matrix element, M the average atomic mass of the compound, $\langle\omega\rangle$ and $\langle\omega^2\rangle$ are appropriately weighted averages of the phonon spectrum, and μ^* is the electron repulsion pseudopotential, which we will assume to be independent of strain and to have the "canonical value" of 0.13. The connection of these quantities to the Δ_t coefficients of Table I is made by differentiating Eq. (14) with respect to

the square of a strain ε which changes the unit cell tetragonality at constant volume (see Eq. (2)):

$$\frac{\Delta_t}{T_c} = \frac{d\ell nT_c}{d\varepsilon^2} = \frac{d\ell n\langle\omega\rangle}{d\varepsilon^2} + F(\lambda)\,\frac{d\ell n\lambda}{d\varepsilon^2} \qquad (15)$$

where $F(\lambda)\equiv\lambda(1.06+.04\lambda)(.92\lambda-.13)^{-2}$. Values of $\frac{d\ell nT_c}{d\varepsilon^2}$ derived from the values of Δ_t and T_c tabulated in Table I are collected in Table IV along with values of the strain dependence of the electron-phonon coupling constant $\frac{d\ell n\lambda}{d\varepsilon^2}$ derived from them with Eq. (15), assuming $\langle\omega\rangle$ is independent of strain. The values of $\frac{d\ell n\lambda}{d\varepsilon^2}$ so obtained correlate well with the theoretical values of the electron-phonon matrix element $\langle I^2\rangle$ calculated by Klein et al. (24). Such a correlation would be expected if the strain dependence of λ is dominated by the variation with strain of the electronic density of states N(0).

Although such an interpretation of the strain dependence is consistent with those models, such as that of Weger, Labbé, and Friedel (2), which emphasize the role of sharp peaks in the density of states in the superconductivity of A15 compounds, other investigators, most notably Testardi (25), emphasize the importance of phonon anharmonicity in causing the high T_c of many A15 compounds. Equation (15) can be rewritten so as to separate effects due to anharmonicity and those due to the strain dependence of the electronic density of states:

$$\frac{d\ell nT_c}{d\varepsilon^2} = \left\{\frac{d\ell n\langle\omega\rangle}{d\varepsilon^2} - 2F(\lambda)\,\frac{d\ell n\langle\omega^2\rangle^{1/2}}{d\varepsilon^2}\right\} + F(\lambda)\,\frac{d\ell nN(0)\langle I^2\rangle}{d\varepsilon^2} \quad . \qquad (16)$$

If Hopfield's arguments (26) that the product $N(0)\langle I^2\rangle$ is an "atomic" property, relatively insensitive to crystal structure, then the first term (in braces), due to phonon anharmonicity, would be the principle source of the

strain dependence of T_c. Values of this term divided by $(1-2F(\lambda))$, obtained assuming $N(0)\langle I^2\rangle$ to be constant, are collected in Table IV under the heading $d\ell n\bar{\omega}/d\varepsilon^2$; they also vary systematically with $\langle I^2\rangle$. At this moment the actual relative contributions of anharmonicity and the strain sensitivity of N(0) are not known; however we shall see in the next section that the strain dependence of the critical magnetic field H_{c2} cannot be explained by $dN(0)/d\varepsilon^2$ alone.

THE ELASTIC STRAIN DEPENDENCE OF THE CRITICAL MAGNETIC FIELD

Much of the discussion of the last section about the critical temperature applies also to the critical magnetic field; there are quantitative differences however, and these provide some insight into the relative contribution of various mechanisms on a microscopic scale to the strain dependence.

Rupp (27) and Ekin (28) have both shown that the upper critical field, H_{c2}, of commercially produced, bronze-processed, multifilamentary Nb_3Sn composite wires depends essentially quadratically on axial strain and that a "universal curve", analogous to that shown in Fig. 1 for T_c, can be constructed. Depending on the heat treatment of the conductor, H_{c2} is reduced by strains of $\pm 10^{-2}$ to 65-80% of the strain-free value. From Fig. 1, or the data in Table I, it is seen that the T_c for Nb_3Sn is reduced to about 86% of its former value, i.e., the reduction by strain of H_{c2} is more severe than that for T_c. Similar behavior has been observed by Suenaga et al. (19) for bronze-processed Nb_3Sn monofilament conductors and is shown graphically in Fig. 5.

Such an effect is understandable at least qualitatively on the basis of the Ginzburg-Landau-Abrikosov-Gor'kov (GLAG) theory of type-II superconductivity uncluding corrections for the electron-phonon interaction. (An excellent recent discussion of the critical fields of A15 compounds in terms of GLAG theory has been published by Orlando et al. (29); see also (30)). The desired relation between critical field and critical temperature is obtained by first relating

the thermodynamic critical field to the critical temperature by equating the condensation energy, which introduces H_c, to the superconducting gap energy, which introduces T_c. The thermodynamic critical field H_c is then multiplied by $\sqrt{2}\,\kappa$ to obtain the upper critical field H_{c2}, and finally we use Goodman's approximation for κ (31) to obtain:

$$\frac{H_{c2}}{T_c} \simeq \frac{c}{\langle \ell \rangle}\sqrt{N(0)(1+\lambda)}\left\{1 + 1.32\,\frac{\langle \ell \rangle}{\xi_o}\right\} \tag{16}$$

where in addition to the parameters defined previously, c is a numerical constant, which is not expected to depend very much on elastic strain, $\langle \ell \rangle$ is the conduction electron mean-free path and ξ_o is the coherence length; the latter is dependent on the critical temperature and electronic Fermi velocity (31) and hence is strain-dependent. In this equation and the following discussion, H_{c2} is meant to be the upper critical field at zero temperature. On the basis of the preceding discussion of T_c and the observations of (27) and (28), we expect H_{c2}/T_c to vary quadratically with tetragonal strain, thus:

$$\frac{d\ell n\,(\frac{H_{c2}}{T_c})}{d\varepsilon^2} = \frac{1}{2}\,\frac{d\ell N(0)}{d\varepsilon^2} + \frac{1}{2}\,\frac{\lambda}{(1+\lambda)}\,\frac{d\ell n\lambda}{d\varepsilon^2} - \frac{1.32\,\frac{\langle \ell \rangle}{\xi_o}}{1+1.32\,\frac{\langle \ell \rangle}{\xi_o}}\,\frac{d\ell n\xi_o}{d\varepsilon^2} \quad . \tag{17}$$

The various terms on the rhs of the equation can be estimated based on the discussion of the strain-dependence of T_c. Let us examine various limiting cases.

Suppose that the density of states N(0) is the principal source of strain dependence. In this case, and in the dirty limit $\langle \ell \rangle << \xi_o$, Eq. (17) becomes, together with Eq. (15):

$$\frac{d\ell n(H_{c2}/T_c)}{d\varepsilon^2} = \frac{1}{2}\frac{(1+2\lambda)}{(1+\lambda)}\frac{d\ell n\lambda}{d\varepsilon^2} \simeq \frac{1}{2}\frac{(1+2\lambda)}{F(\lambda)(1+\lambda)}\frac{d\ell nT_c}{d\varepsilon^2} \tag{18}$$

where $F(\lambda)$ is defined after Eq. (15). For Nb_3Sn, Eq. (18) says that since T_c is reduced 14% by a strain of 1%, (H_{c2}/T) should be reduced by 10%; this is at variance with the experimental value of the reduction of the latter, ~25%, to be seen in Fig. 5. Thus the results are inconsistent with the assumption that the strain dependence of T_c and H_{c2} is due only to a variation of the electronic density of states. The assumption that phonon anharmonicity is solely responsible for the strain dependence results in an even worse discrepancy. It is only possible to account for the strain dependence of H_{c2}/T_c if both effects contribute.

THE ELASTIC STRAIN DEPENDENCE OF THE CRITICAL CURRENT DENSITY

Understanding the elastic strain dependence of the critical current density, J_c, in high transverse magnetic fields for A15 compounds is made easier by the existence of a scaling law, proposed by Kramer (32), which seems to be obeyed by these materials:

$$J_c \propto \frac{[H_{c2}(T)]^{2.5}}{H} h^{1/2}(1-h)^2 \tag{19}$$

where H is the applied magnetic field.

$$H_{c2}(T) \simeq H_{c2}(0)\left[1-\left(\frac{T}{T_c}\right)^2\right] \tag{20}$$

and

$$h \equiv \frac{H}{H_{c2}(T)} \tag{21}$$

While there have been some criticisms (33) of the validity of the assumptions made by Kramer in its derivation, the scaling law is observed to hold quite

well for polycrystalline Nb_3Sn in composite conductors produced by the bronze process (19,28) as well as by so-called "in-situ" methods (34), for applied fields greater than about 4 T.

Simple manipulation of Eq. (19) shows that

$$(J_c H)^{1/2} H^{-1/4} \propto (H - H_{c2}) \quad . \tag{22}$$

Thus a plot of the lhs versus H for samples with varying degrees of strain should yield a series of parallel straight lines intercepting the H axis at the different strain-dependent values of H_{c2}. Such behavior is illustrated in Fig. 6 for monofilament bronze-processed Nb_3Sn wires with varying degrees of internal strain. Furthermore, both Rupp (27) and Ekin (28) have shown how measured strain-dependent values of H_{c2} for multifilamentary Nb_3Sn composite conductors can be used to predict the strain dependence of J_c using Kramer's scaling law. Thus it appears that, at least at high fields, the strain-dependence of the critical current is adequately accounted for by the strain dependence of H_{c2} and T_c.

ACKNOWLEDGEMENT

I am grateful to my colleagues J. F. Bussiere, T. Luhman, and M. Suenaga for stimulating discussions on this topic and to them as well as to K. Aihara and C. L. Snead, Jr. for permission to use their unpublished data.

APPENDIX. THE NATURE OF THE STATE OF STRAIN IN ELEMENTARY TWO-COMPONENT COMPOSITES

In a substantial number of the experiments on strain dependence, the A15 compounds are present as elements of composite superconductors. In order to interpret these experiments properly it is necessary to have some understanding of the triaxial nature of the internal strains present as a consequence of differential thermal expansion among the elements of the composite and of how this strain state is altered by externally applied stress. We shall illustrate the general features by a brief, approximate discussion of the strain state present in two idealized composite conductors: a cylindrical conductor containing a single superconducting filament within a normal matrix and a two component tape conductor.

A. Cylindrical Composite

Consider first an internally strained cylindrical conductor, for example a single filament Nb_3Sn conductor made by the "bronze process". Strictly speaking this is a three component composite (Nb, Nb_3Sn, and bronze), but for simplicity we shall treat the (Nb+Nb_3Sn) core as a single element, the "filament", whose properties will be denoted by a subscript f, since the thermal expansion coefficients of Nb and Nb_3Sn are identical. The properties of the bronze matrix will be denoted by a subscript m. It is straightforward to derive the strains in a three element composite wire by extending the methods described below, but the results are rather complicated; the additional accuracy does not justify the additional complexity for our present purposes. We shall treat each component as an isotropic, linear elastic continuum.

Because the thermal expansion coefficient of bronze substantially exceeds that of Nb and Nb_3Sn (or V and V_3Si, etc.), cooling from the reaction temperature for the formation of Nb_3Sn causes a residual strain state with the filament

in compression and the matrix in tension. We calculate the strain state by the following process. Imagine the matrix to be a hollow cylinder separated from the cylindrical filament. At the reaction temperature the filament exactly fills the hole in the hollow cylinder. Cool the separated matrix and filament to the temperature at which the superconducting properties are to be measured. The dimensions of the filament now differ fractionally from those of the hole in the hollow cylinder by an amount δ, given by the integral of the difference in the linear thermal expansion coefficients over the interval between the reaction and test temperatures. Now apply an internal pressure to the walls of the hole in the cylinder and an equal external pressure to the cylindrical surface of the filament. Apply a uniform tensile stress to the ends of the hollow cylinder and a different uniform compressive stress to the ends of the filament. Increase the pressure and two axial stresses until three conditions are met: i) the radius of the filament matches the radius of the hole in the matrix, ii) the length of the filament and matrix are equal, and iii) the integral of the axial stress over the area of the filament plus matrix is zero. When these conditions are met, the strains in the filament and matrix are the same as those present when the two elements are "glued" together to form the composite. It is straightforward to carry out this procedure explicitly by combining Hooke's law with the cylindrically symmetric, plane strain radial elastic solutions in (17) and assuming the axial strain to be uniform in each component. These approximations are sufficiently accurate for our purposes and are similar to those used in the theory of the mechanical properties of two component composites developed in (18). The resulting axial strain in the filament is:

where $\varepsilon(\sigma)$ is the axial strain due to the applied stress alone, i.e., the measured "external" strain arising from the applied stress.

B. Planar Composite (Tape)

The residual strains arising from differential thermal contraction in a thin tape "sandwich" conductor are obtained as follows. As above, consider the conductor to consist of layers of two materials: external layers of bronze, with properties denoted by a subscript m and a central layer of ($Nb+Nb_3Sn$ or $V+V_3Si$, etc.) treated as an effectively homogeneous material with properties denoted by subscript f. As in part A above, we calculate the strains by imagining the separated elements to be cooled from the reaction temperature, where they are the same length and breadth, to the test temperature where differential thermal contraction causes a mismatch in size. We shall assume that the thickness of the tape is very small compared to the length and breadth so that a state of plane stress is a good approximation to reality. Apply uniform tensions and compressions in the plane of the layers until: i) the layers match in length and breadth and ii) the integral of stress over a cross section of the entire tape is zero. The resulting strains are those that will occur when the elements are then "glued" together. The strains in the central layer of superconductor are:

$$\varepsilon_{f\parallel} = \frac{-\delta}{R^{-1}\,\dfrac{E_f(1-\nu_m)}{E_m(1-\nu_f)} + 1} \tag{A6}$$

$$\xi_f \equiv \frac{\varepsilon_{f\perp}}{\varepsilon_{f\parallel}} = \frac{-2\nu_f}{1-\nu_f} \tag{A7}$$

where the two principal strains in the plane of the tape are denoted by the symbol $\parallel$ and that perpendicular to it by the symbol $\perp$.

The strains in the external layers of matrix are:

$$\epsilon_{m\,II} = \frac{+\,\delta}{1 + \dfrac{RE_m(1-\nu_f)}{E_f(1-\nu_m)}} \tag{A8}$$

$$\xi_m \equiv \frac{\epsilon_{m\perp}}{\epsilon_{m\,II}} = \frac{-2\nu_m}{1-\nu_m} \quad . \tag{A9}$$

These results can be used (with the obvious alterations required) to describe a composite tape consisting of external layers of superconductor deposited on a central substrate layer.

$$\varepsilon_{zz}^{f}(0) = \frac{-\delta\left\{1+2R^{-1}\left(\frac{E_f}{E_m}\right)(1+\nu_m)(\nu_m-\nu_f)G^{-1}\right\}}{1+R^{-1}\left(\frac{E_f}{E_m}\right)\left[1+2(1+\nu_m)(\nu_m-\nu_f)G^{-1}\right]} \qquad \text{(A1)}$$

where the argument zero indicates no external strain has been applied, δ is the fractional mismatch and is positive for bronze-processed wires, E is Young's modulus, ν is Poisson's ratio, R is the volume ratio of bronze matrix to filament and

$$G \equiv (1+\nu_f)(1-2\nu_f) + R^{-1}\frac{E_f}{E_m}(1+\nu_m)(2+R-2\nu_m) \quad . \qquad \text{(A2)}$$

The radial and tangential components of strain in the filament are given by:

$$\xi(0) \equiv \frac{\varepsilon_{rr}(0)}{\varepsilon_{zz}(0)} = \frac{\varepsilon_{\theta\theta}(0)}{\varepsilon_{zz}(0)}$$

$$= -\nu_f + \frac{G^{-1}(1+\nu_f)(1-2\nu_f)\left\{R\left(\frac{E_m}{E_f}\right)(1+\nu_f)+(1+\nu_m)\right\}}{\left\{R\left(\frac{E_m}{E_f}\right)+2G^{-1}(1+\nu_m)(\nu_m-\nu_f)\right\}} \quad . \qquad \text{(A3)}$$

To find the strains due to the differential thermal contraction, three boundary conditions were invoked, as described above. If an axial tensile stress is applied to the composite, only condition iii) above is changed: now the integral of the axial stress over the cross-sectional area of the filament plus matrix must equal the external stress. The resulting strains are:

$$\varepsilon_{zz}(\sigma) = \varepsilon_{zz}(0) + \varepsilon(\sigma) \qquad \text{(A4)}$$

$$\xi(\sigma) \equiv \frac{\varepsilon_{rr}(\sigma)}{\varepsilon_{zz}(\sigma)} = \frac{\varepsilon_{\theta\theta}(\theta)}{\varepsilon_{zz}(0)} = \frac{\xi(0)-\left\{(\nu_m-\nu_f)(1+\nu_f)(1-2\nu_f)G^{-1}+\nu_f\right\}\frac{\varepsilon(\sigma)}{\varepsilon_{zz}(0)}}{1+\frac{\varepsilon(\sigma)}{\varepsilon_{zz}(0)}} \qquad \text{(A5)}$$

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Table I.

COEFFICIENTS FOR STRESS AND STRAIN DEPENDENCE OF THE CRITICAL TEMPERATURE FOR SEVERAL A15 COMPOUNDS

Compound, T_c°	Γ_1, K	Δ_v, K	Δ_t, K	$\frac{2\Delta_{44}}{(\Delta_{11} - \Delta_{12})}$	$\frac{\partial T_c}{\partial \sigma}$, K Pa^{-1}	Reference	Specimen	Comment
V_3Si, 16.8 K	$\lvert\Gamma_1\rvert < 50$	-11×10^4	-9.9×10^4	.1		4	s.c., non-trans.[a]	from elastic properties
	$-(45 - 64)$[b]	$\lvert\Delta_v\rvert < 2 \times 10^4$ [b]			$-(2.6 - 3.7) \times 10^{-10}$	6,7	s.c., non-trans.	hydrostatic, $\sigma = -p$
	-70[b]	$\lvert\Delta_v\rvert < 2 \times 10^4$ [b]			-4.0×10^{-10}	7	s.c., trans.	hydrostatic, $\sigma = -p$
			(-9×10^4)[c]		$-(5.1 - 9.7) \times 10^{-9}$	8,9	s.c.	uniaxial stress, <100>
					-4.5×10^{-9}	9	s.c.	uniaxial stress, <110>
					~ 0	8	s.c.	uniaxial stress <111>
			-8×10^4			11	poly.	triaxial; composite wire etching experiment
V_3Ge, 6.1 K	$\lvert\Gamma_1\rvert < 45$	$+1.9 \times 10^4$	$+5.8 \times 10^4$	.1		4	s.c.	from elastic properties
	-141[b]				-8.1×10^{-10}	6	poly.	hydrostatic, $\sigma = -p$
			$+4 \times 10^4$			11	poly.	triaxial; composite wire etching experiment
V_3Ga, 16.5 K			-9×10^4			11	poly	triaxial; composite wire etching experiment
					-1.0×10^{-10}	6	poly.	hydrostatic, $\sigma = -p$
Nb_3Sn, 18 K	$+23$[b]	$\lvert\Delta_v\rvert < 5 \times 10^3$ [b]			$+1.4 \times 10^{-10}$	6, 12	poly; s.c. trans.	hydrostatic, $\sigma = -p$
					-1.1×10^{-8}	13	s.c., trans.	uniaxial stress, <100>
			$-(7 - 8) \times 10^4$			11	poly.	triaxial; composite wire etching experiments.
			-7.4×10^4			14	poly.	triaxial; composite wire, external tension
Nb_3Ge, ~20 K			-8×10^4			15	poly.	triaxial; composite tape

[a] "Trans." or "non-trans." denotes whether or not specimen crystal exhibits the martensitic transition, "s.c." and "poly." denote single or polycrystalline.

[b] Calculated with linear elasticity theory and the bulk modulus of Ref. 4.

[c] Calculated with Testardi's (10) nonlinear triaxial stress-strain relation based on the data of Patel and Batterman (3).

Table II.

CALCULATED VALUES OF THE FRACTIONAL RECOVERY OF THE CRITICAL TEMPERATURE AT THE EXTREMUM ($\varepsilon=\varepsilon^*$) OF THE T_c VERSUS APPLIED STRAIN CURVE FOR MONOFILAMENT COMPOSITE WIRES

Δ_v/Δ_t	$-\varepsilon^*/\varepsilon^i_{zz}$		$\frac{[T_c(\varepsilon^*) - T_c(i)]^b}{[T_c(o) - T_c(i)]}$	
	x^a = 4/3	4.5	4.3	4.5
-.4	.64	.86	2.77	1.06
0	.75	.90	1.00	1.00
+.4	.85	.94	.77	.96

[a] $x \equiv (1+\nu_m)\frac{Ef}{Em}$; ν_f=1/3 for all cases

[b] $T_c(\varepsilon^*)$, $T_c(i)$, $T_c(o)$ are the critical temperatures for, respectively, the strain at the maximum or minimum in the T_c versus ε curve, in the initial as-fabricated state, and in the strain-free state.

Table III.

EFFECT OF NEUTRON-IRRADIATION
ON THE STRAIN DEPENDENCE OF T_c IN Nb_3Sn[a]

Fluence 10^{17} ncm^{-2}, E>1 MeV	T_c, K (Midpoint)	$\Delta T_c/T_c$[b]
0	16.55	.029
1.3	16.60	.028
6.6	16.52	.025
13.0	16.27	.042
23.4	15.92	.066
62.4	13.00	.076
130.0	10.05	.084

[a]Bronze-processed, 700°C/96 hrs; Bronze/niobium ratio $\cong$15. Unpublished data of C. L. Snead, Jr. see also Ref. 16.

[b]ΔT_c is the increase in critical temperature upon removing the bronze matrix by etching.

Table IV.

SYSTEMATIC BEHAVIOR OF THE UNIAXIAL STRAIN DEPENDENCE OF THE CRITICAL TEMPERATURE OF A15 COMPOUNDS IN THE CONTEXT OF A "McMILLAN-TYPE" EQUATION

Compound	T_c, K	$\frac{d\ell n\ T_c}{d\varepsilon^2}$ [a]	λ [b]	$\left(\frac{d\ell n\lambda}{d\varepsilon^2}\right)_{\langle\omega\rangle}$ [c]	$\langle I^2\rangle$, [d] Arbitrary Units	$\left(\frac{d\ell n\bar{\omega}}{d\varepsilon^2}\right)_{max}$ [e]
V_3Ga	16.5	-600	1.17	-420	260	+319
V_3Ge	6.1	+6100	.70	+2100	330	-1300
V_3Si	17.1	-4700	1.12	-3100	334	+2300
Nb_3Sn	18.0	-4100	1.44	-3600	402	+3300
Nb_3Ge	19	-3900	1.80	-4500	472	+5200

[a] Calculated from the Δ_t data of Table I.

[b] Experimental values tabulated in Ref. 24.

[c] Calculated with Eq. (15) assuming $\langle\omega\rangle$ is constant.

[d] First principles calculation in Ref. 24.

[e] Calculated with Eq. (16) assuming $N\langle I^2\rangle$ is constant.

FIGURE CAPTIONS

Figure 1. "Universal" strain dependence of T_c for bronze-processed monofilament Nb_3Sn conductors. The plot was obtained by matching curves of T_c versus applied axial strain at the maximum T_c and the strain at this maximum. Data are from References 14 and 19.

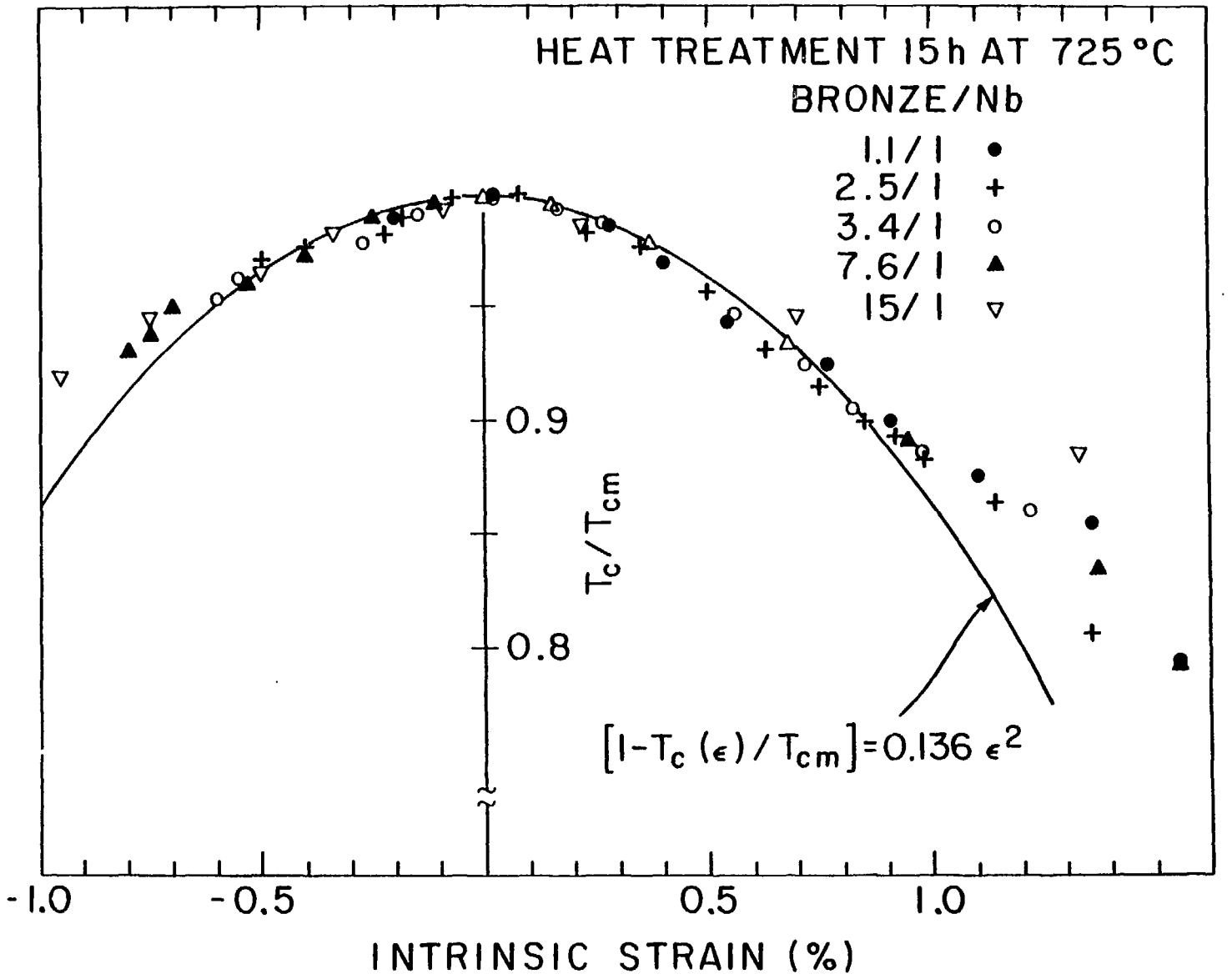
Figure 2. Applied strain at the maximum of curves of critical current density, J_c, and of critical temperature, T_c, versus applied axial strain for bronze-processed monofilament Nb_3Sn conductors. The bronze/niobium ratios are indicated. (Unpublished data of T. Luhman and M. Suenaga.)

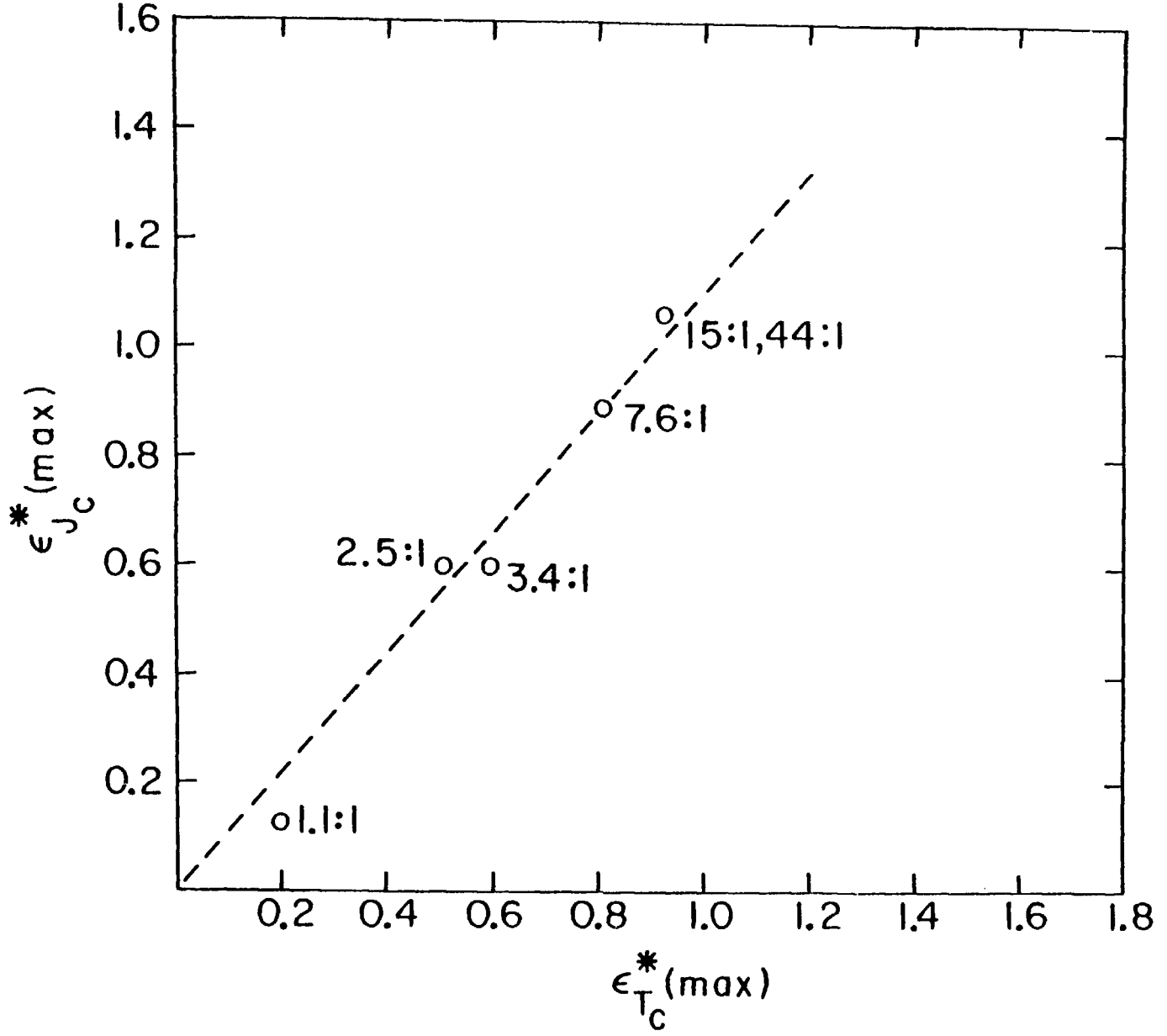
Figure 3. Variation of the differential thermal contraction induced depression of the critical temperature of bronze-processed monofilament Nb_3Sn conductors as the bronze matrix is etched away. The form of the plot is suggested by Equation (11). As-fabricated bronze/Nb ratios, R_o, are as indicated. Data are from Reference 11.

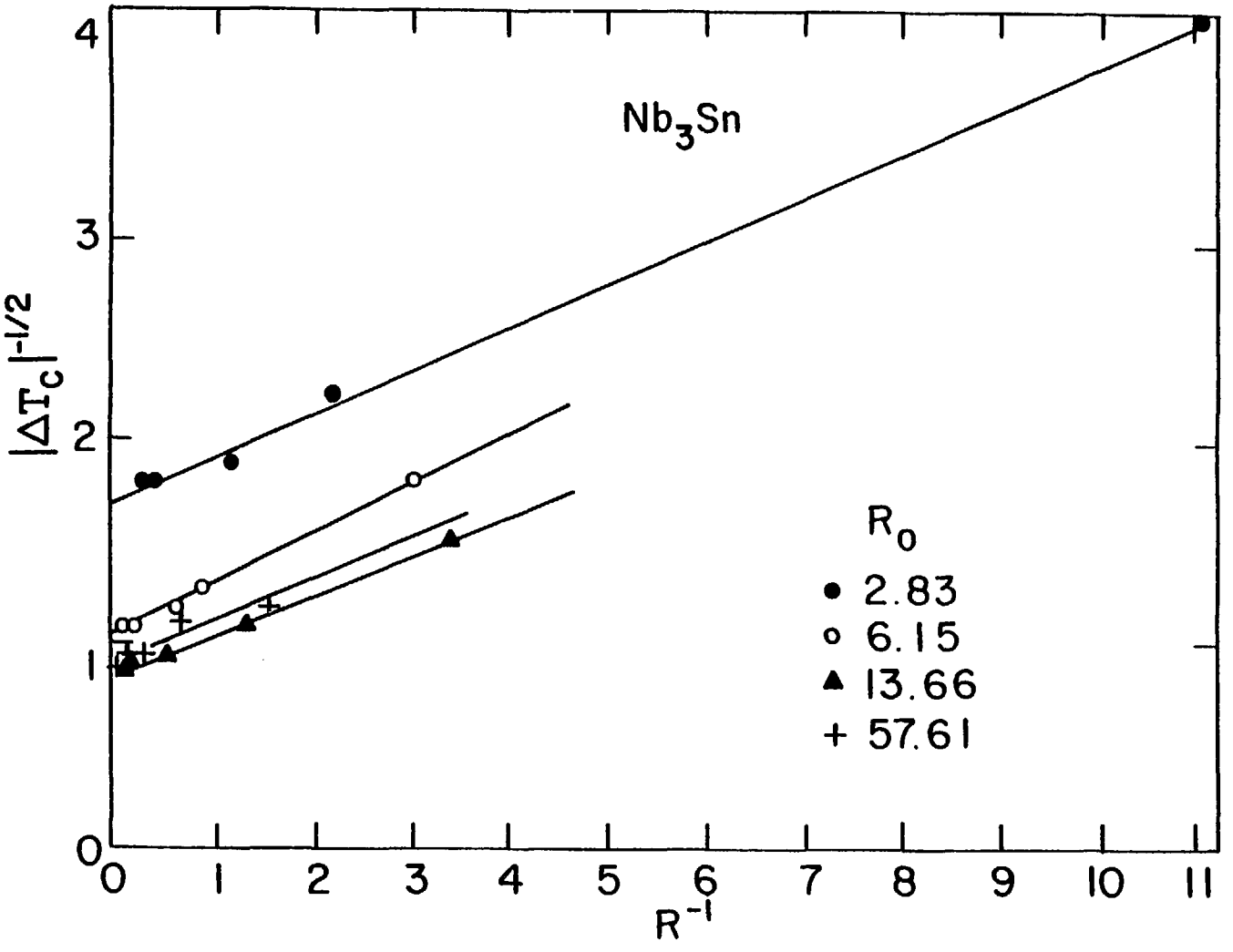
Figure 4. Variation of the differential thermal contraction induced change in the critical temperature of bronze-processed monofilament conductors of various A15 compounds. Data are from Reference 11.

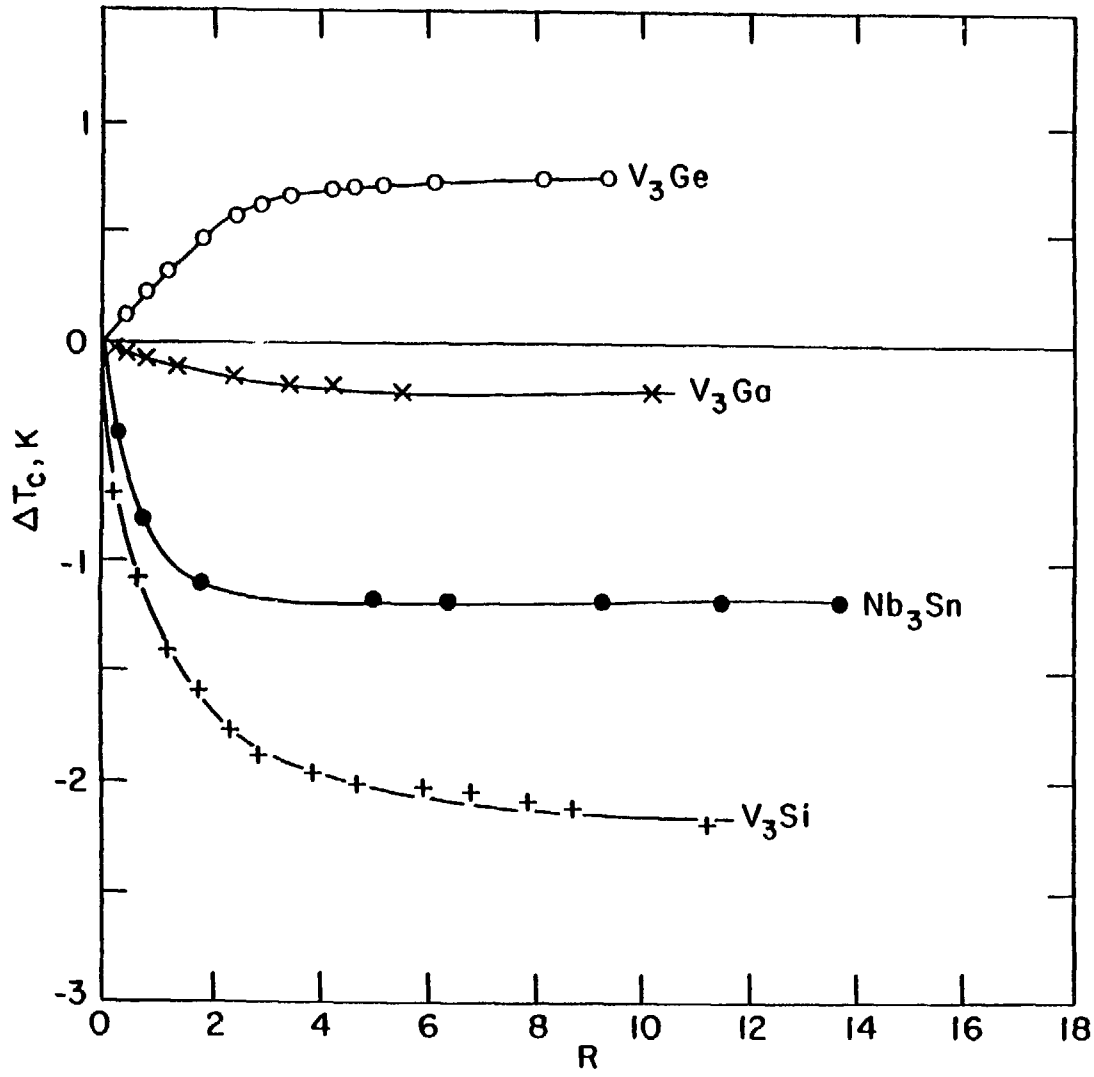
Figure 5. Variation of the ratio of the upper critical field at 4.2 K to the critical temperature for bronze-processed monofilament Nb_3Sn conductors with the bronze to niobium ratio and thus the degree of strain. Values of strain corresponding to these ratios may be read from Figure 2. Data are from Reference 19.

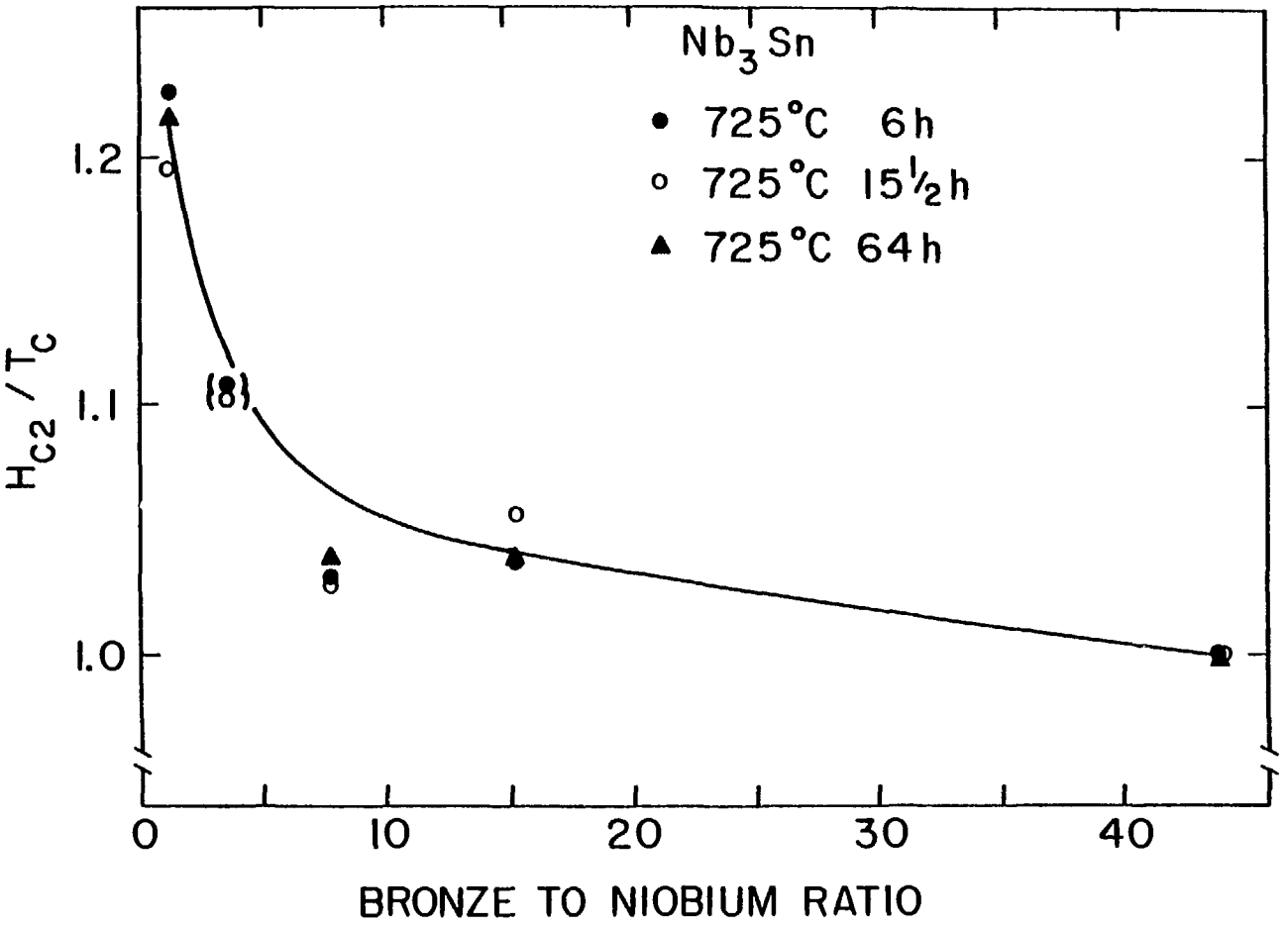
Figure 6. A "Kramer-scaling" plot of critical current density at 4.2 K for bronze-processed monofilament Nb_3Sn conductors with bronze to niobium ratios as indicated. Data are from Reference 19.

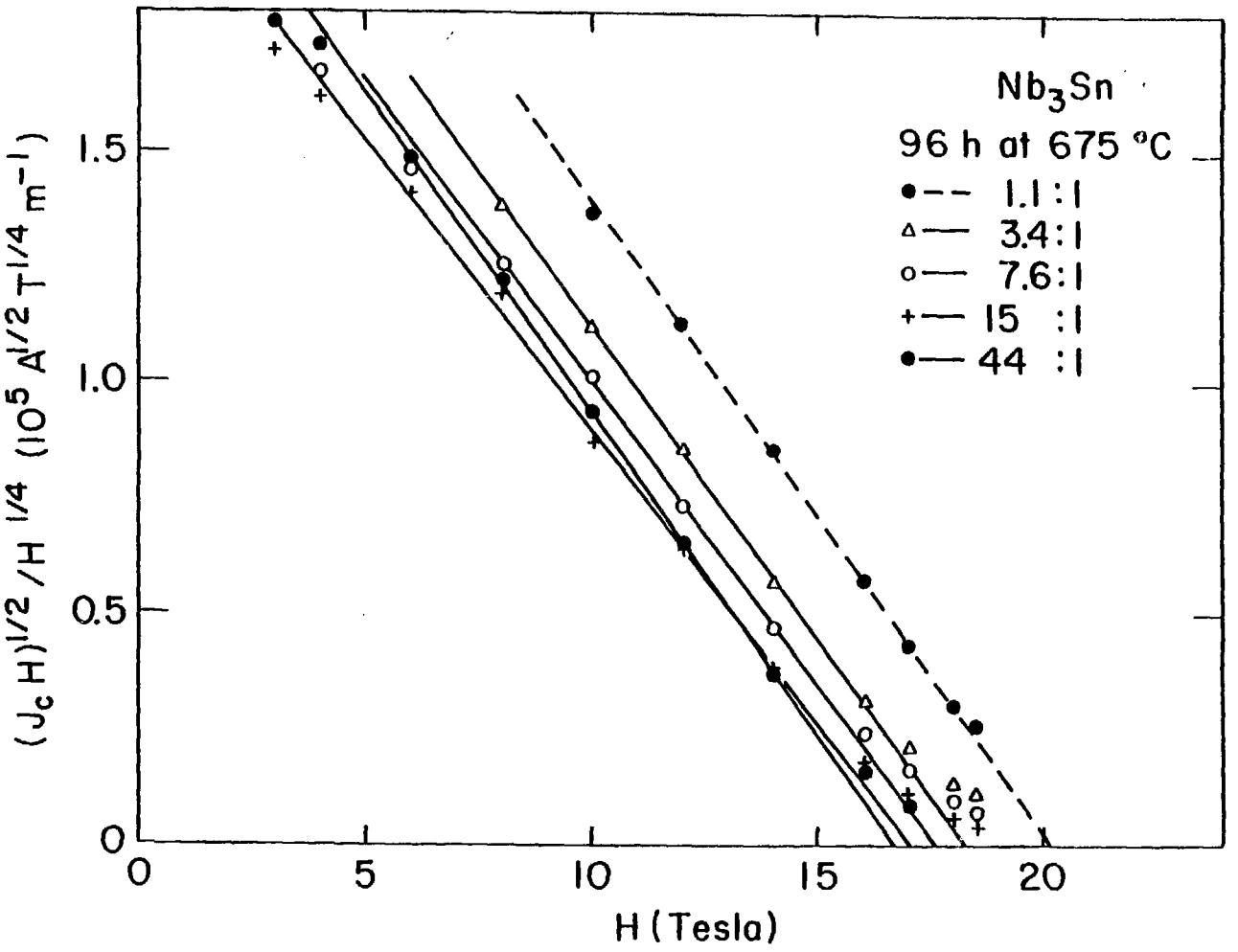

HEAT TREATMENT 15h AT 725°C
BRONZE/Nb
1.1/1
2.5/1
3.4/1
7.6/1
15/1
0.9
0.8
T_c/T_{cm}
$[1-T_c(\epsilon)/T_{cm}]=0.136\,\epsilon^2$
-1.0
-0.5
0.5
1.0
INTRINSIC STRAIN (%)


1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\epsilon^*_{J_C}$(max)
15:1,44:1
7.6:1
2.5:1
3.4:1
1.1:1
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
$\epsilon^*_{T_C}$(max)


Nb3Sn
|ΔTc|-1/2
R-1
R0
2.83
6.15
13.66
57.61
0
1
2
3
4
5
6
7
8
9
10
11


1
0
-1
-2
-3
ΔT_c, K
0
2
4
6
8
10
12
14
16
18
R
V_3Ge
V_3Ga
Nb_3Sn
V_3Si


Nb_3Sn
● 725°C 6h
o 725°C 15½h
▲ 725°C 64h
H_{c2}/T_c
1.2
1.1
1.0
0
10
20
30
40
BRONZE TO NIOBIUM RATIO


Nb_3Sn
96 h at 675 °C
1.1 : 1
3.4 : 1
7.6 : 1
15 : 1
44 : 1
$(J_c H)^{1/2} / H^{1/4}$ $(10^5 A^{1/2} T^{1/4} m^{-1})$
H (Tesla)
0
0.5
1.0
1.5
5
10
15
20