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Abstract

Relativistic kinetic theory may be used to understand hot dense hadronic matter. We address the questions of collective flow and pion production in a 3 D relativistic fluid dynamic model and in the VUU microscopic theory. The GSI/LBL collective flow and pion data point to a stiff equation of state. The effect of the nuclear equation of state on the thermodynamic parameters is discussed. The properties of dense hot hadronic matter are studied in Au + Au collisions from 0.1 to 10 GeV/nucleon.

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The prospect of relativistic heavy ion experiments from 1 to 100 GeV/nucleon is exciting for heavy ion physics. Currently, among the sophisticated theoretical models for this energy regime are: the Time Dependent Dirac Equation[1], relativistic Nuclear Fluid Dynamics,[2] and Vlasov-Uehling-Uhlenbeck approaches.[3-8] Here we shall be concerned with the latter two models and discuss some results for the GeV/nucleon energy range. In both models we use the soft and stiff equations of state.[5-7]

Kinetic theory has become fundamental to nuclear physics in the Bevalac energy regime. Indeed, the Vlasov-Uehling-Uhlenbeck equation[3-8] has been successful in explaining single particle nucleon spectra,[4] pion and kaon production,[5] collective flow,[6] transverse momentum transfer,[7] and fragmentation.[8] Non-relativistically this equation may be written as

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} - \vec{\nabla} U \cdot \frac{\partial f}{\partial \vec{p}} =$$

$$1/(2m) \int d^3p_2 d^3p_1' d^3p_2' \; w \; [f_1' f_2' (1-f)(1-f_2) - f f_2 (1-f_1')(1-f_2')] \; \delta^3(\vec{p} + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \; \delta(E + E_2 - E_1' - E_2') \qquad (1)$$

where $w = w(\vec{p}, \vec{p}_2, \vec{p}_1', \vec{p}_2')$ is the appropriate transition matrix.[9,10] At higher energies, one must write the VUU equation relativistically[9] as

$$(1/m \; p^{\mu} \partial_{\mu} + F^{\mu} \partial_{p}^{\mu}) \; f(\underline{x}, \underline{p}) =$$

$$1/(2m) \int d^3p_2 d^3p_1' d^3p_2' \; W \; [f_1' f_2' (1-f)(1-f_2) - f f_2 (1-f_1')(1-f_2')] \; \delta^4(\underline{p} + \underline{p}_2 - \underline{p}_1' - \underline{p}_2') \, / E_2 E_1' E_2' \qquad (2)$$

where $F^\mu = dp^\mu/d\tau$ is the nuclear force field and $W = W(\underline{p},\underline{p}_2,\underline{p}_1',\underline{p}_2')$ is the Lorentz scalar transition matrix such that $w = W/EE_2E_1'E_2'$.[9] Furthermore, for energies beyond 10 GeV/nucleon, the parton degrees of freedom need to be included in the kinetic equation:[11]

$$(p^\mu\partial_\mu + Q_a F^a_{\mu\nu}\, p^\nu\partial^\mu_p + f_{abc}p^\nu A^b_\nu\, Q^c\partial^a_Q\,)\; f(\underline{x},\underline{p},\underline{Q}) = C(f) \qquad (3)$$

where A^b_ν is the local color field , Q_a ($a = 1,\ldots,8$) are the color vectors, and C is the appropriate collision integral.

One advantage of a kinetic theory is that a Monte Carlo approach may be used to solve the kinetic equation.[4-8,11] Also by integrating over the momentum space with weights of 1, p, and T, one obtains the fluid dynamic equations.[12] The collision term vanishes in the integration.

The relativistic fluid dynamic equations[13] reflect the conservation of baryon number, momentum, and energy:

$$\frac{\partial}{\partial t}(\rho\gamma) + \frac{1}{\sqrt{D}}\frac{\partial}{\partial x_i}(\sqrt{D}\rho\gamma v^i) = 0 \qquad (4)$$

$$\frac{\partial}{\partial t}S_j + \frac{1}{\sqrt{D}}\frac{\partial}{\partial x_i}(\sqrt{D}S_j v^i) + \frac{\partial}{\partial x_j}P = 0 \qquad (5)$$

$$\frac{\partial}{\partial t}(e\gamma) + \frac{1}{\sqrt{D}}\frac{\partial}{\partial x_i}(\sqrt{D}e\gamma v^i) + P\frac{\partial}{\partial t}(\gamma) + \frac{P}{\sqrt{D}}\frac{\partial}{\partial x_i}(\sqrt{D}\gamma v^i) = 0 \qquad (6)$$

here ρ is the proper baryon number density, $e = \rho(E_C(\rho) + E_T(\rho,T))$ is the proper internal energy density, P is the pressure, $S^\nu = (\rho + e + P)\gamma u^\nu$ is the momentum density, $u^\nu = \gamma v^\nu$ is the transport velocity, and D equals the determinant of the three metric. For the flat metric used here, D = 1. Note that $\gamma = 1/\sqrt{1-\beta^2}$ is the usual Lorentz factor. Both the nuclear and Coulomb potentials and the dissipative terms are difficult to handle in relativistic NFD where the covariant formulation adds time derivatives and implies retardation for the potential. These terms are neglected here except for the mean field and Fermi energies which are included in the compressional and thermal energy. A Coulomb or long range nuclear field could be included by overlaying a 3 D Poisson solver.

The VUU theory includes the mean field $U(\rho(r))$, special relativity, nucleon-nucleon collisions, and the Pauli principle.[4-8] The local gradient of the field is computed via a finite difference method analogous to Lagrange's method in fluid dynamics. The single particle distribution function $f(\vec{r},\vec{p},t)$ is obtained by ensemble averaging over the phase space distribution of test particles.[12]

In the LLNL relativistic NFD, Eulerian methods are employed to solve equations (3)-(5) simultaneously. The computational grid is fixed in space and the fluid flows through it. Two interwoven 3 D meshes of points, the cell centers and edges, are used for the grid. An explicit method of solution is used where values at one time step are calculated from those at the previous time step, the time step being limited by the CFL condition: no signal may propagate across more than one cell width in a time step.

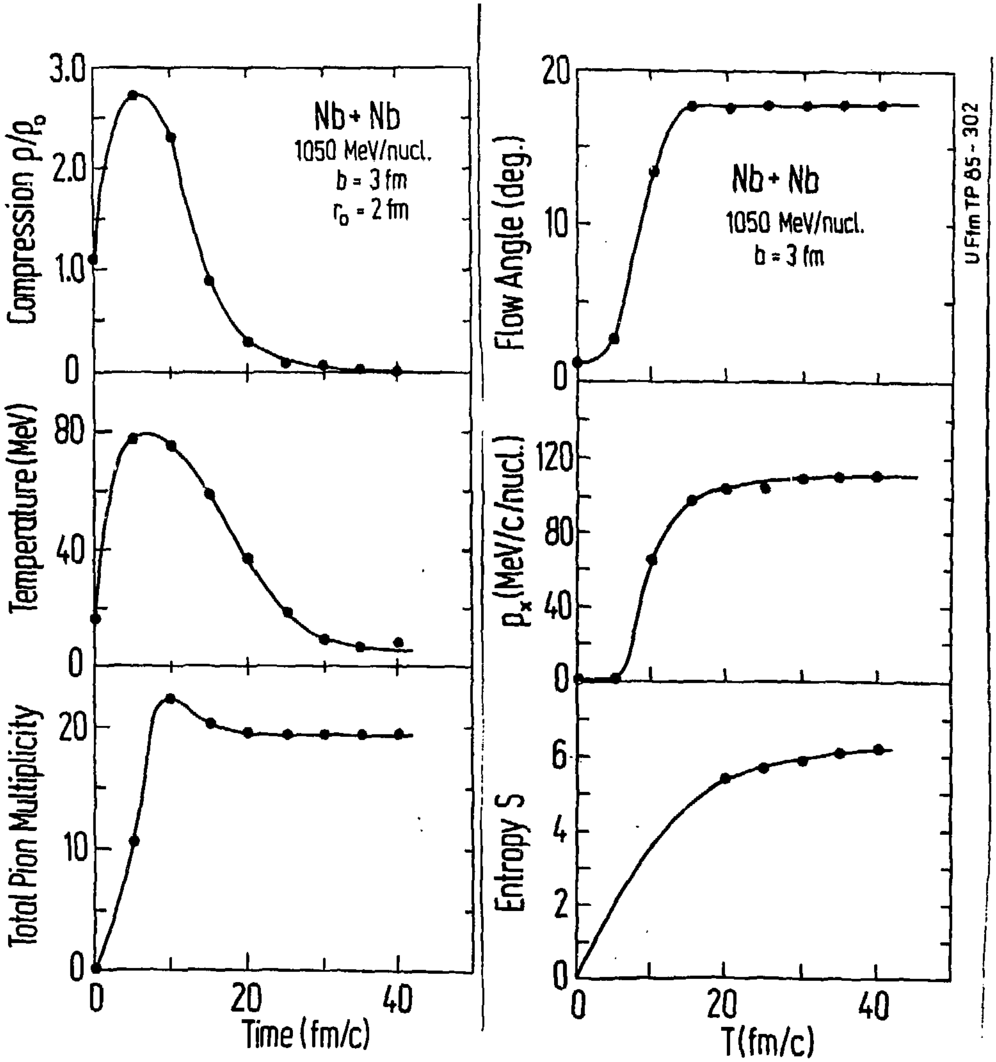


Figure 1: The time dependence of the central density and temperature, the pion multiplicity, the peak flow angle, the peak transverse momentum transfer, and the entropy is shown for Nb (1.05 GeV/nucleon) + Nb in the VUU model.

The algorithm has second order accuracy in space and time, which makes it more capable of handling shocks.

In Figure 1, we consider the collision Nb(1.05 GeV/nucleon) + Nb at impact parameter b = 3 fm in the VUU model with a stiff EOS.[5-7] A sphere of radius r_o = 2 fm is used to calculate central quantities. The central density rises to a maximum of 2.7 ρ_0 by t = 6 fm/c. Within one fm/c, the central temperature peaks at 80 MeV. About 10 fm/c after the time of maximum compression and temperature, the number of pions freezes out at n_π = 20. Of these 20 pions, there are 8.0 π^-, 5.8 π^o, and 5.7 π^+ on the average. There is a 15 % pion absorption effect. Note that the classical central temperature of 80 MeV is drastically different from the VUU nucleon slope parameter T_o = 130 MeV.

The invariant cross section slope parameter achieves its final state value shortly after the moment of maximum compression and temperature. This reflects an early freeze out of the nucleonic momentum distribution. This momentum distribution is however not completely isotropic; the isotropy[4] is R = 0.58. The well known directed sidewards flow[14] is predicted. The peak flow angle, peak transverse momentum transfer, and entropy in the VUU theory likewise saturate at $\theta_F = 18^o$, p_x = 110 MeV/c/nucleon, and S $\simeq$ 6 units per baryon. Note that the six dimensional integration used here to calculate the entropy overestimates by one unit;[15] the number 6 is hence an upper limit.

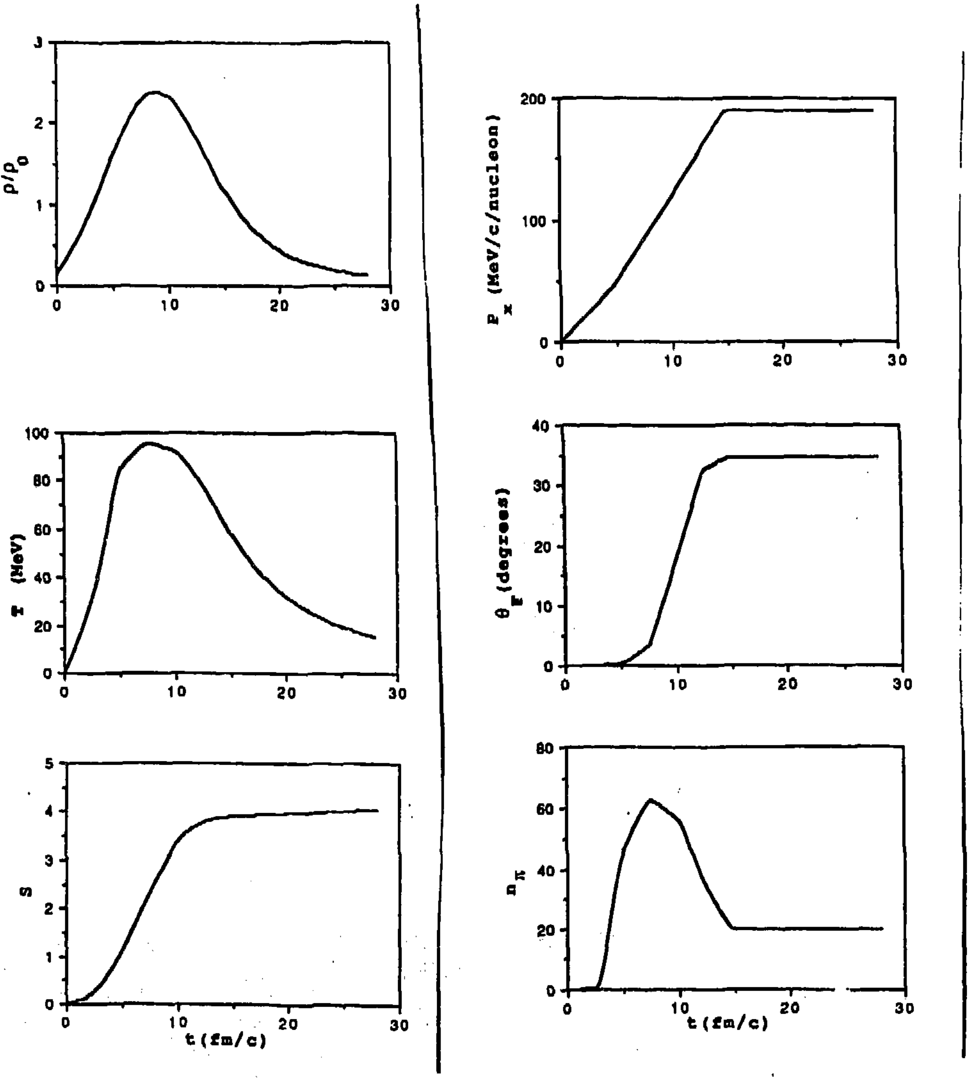
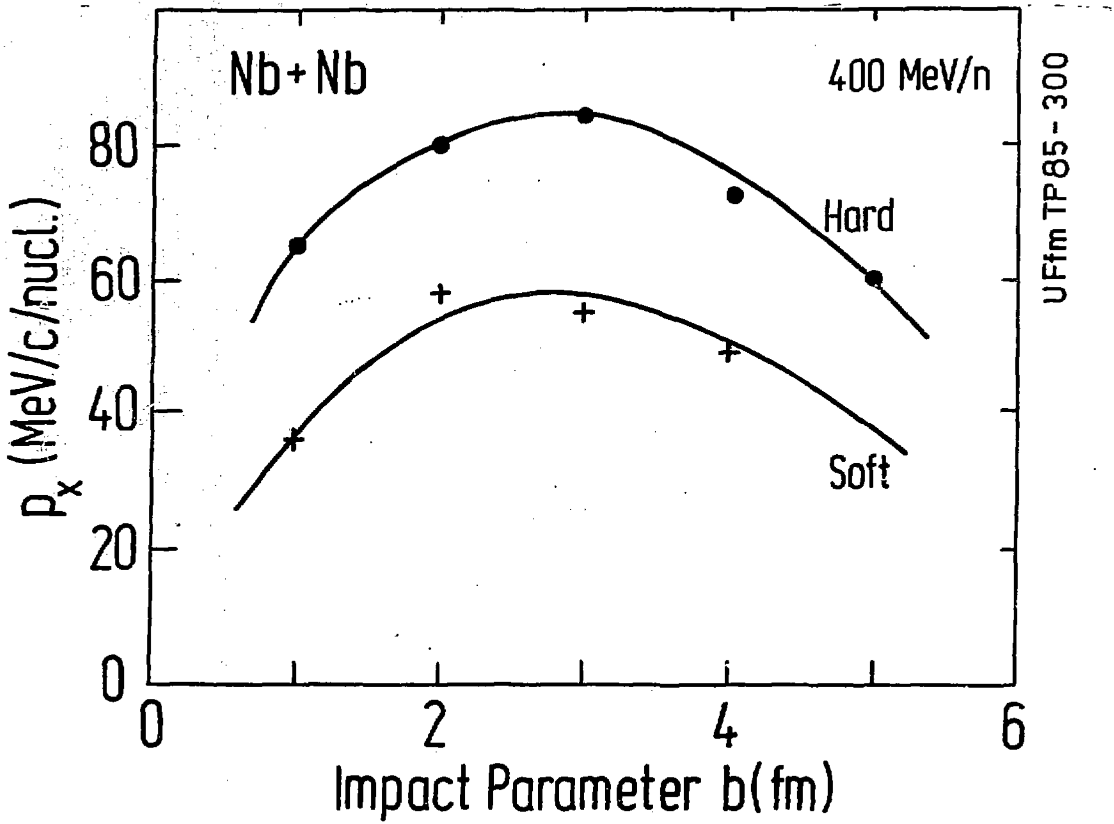


Figure 2: The time dependence of the same quantities with the same stiff EOS in the relativistic NFD model.

Next we turn to the same system with the same stiff EOS in the NFD model. See Figure 2. The central density and temperature rise to maxima of ρ/ρ_o = 2.4 and T = 96 MeV by t = 8 fm/c. The baryonic entropy, flow angle, and peak transverse momentum transfer saturate at S = 4.0, p_x = 185 MeV/c/nucleon, and $\theta_F = 35^o$ by t = 15 fm/c. The pionic contribution to the entropy, calculated from a simple equilibrium thermal model, would increase S by 0.4 units. The pion multiplicity shown in Figure 2 is calculated from a simple equilibrium thermal model[16] using the NFD central density and temperature. For reference, we have appended the VUU values for late times. Note that if the whole nucleus was shocked to the same maximum density and temperature, the thermal model overpredicts the number of pions by a factor of three. Currently two more realistic approaches to pion production are being worked on. The first is a thermal calculation using the density and temperature in every cell at a <u>unique</u> freezeout time. The second involves including pions as Monte Carlo particles which can exchange momentum and energy dynamically with the nuclear fluid.[17]

The NFD temperature shown in Figure 2 is lowered to 64 MeV by including the Δ and other known resonances into the thermal energy.[16] Thus the central temperature and density predicted by nuclear fluid dynamics are lower than the VUU values. In contradistinction, the flow angle and transverse

Figure 3: The impact parameter and EOS dependence of the peak transverse momentum is shown for Nb (400 MeV/nucleon) + Nb in the VUU model.

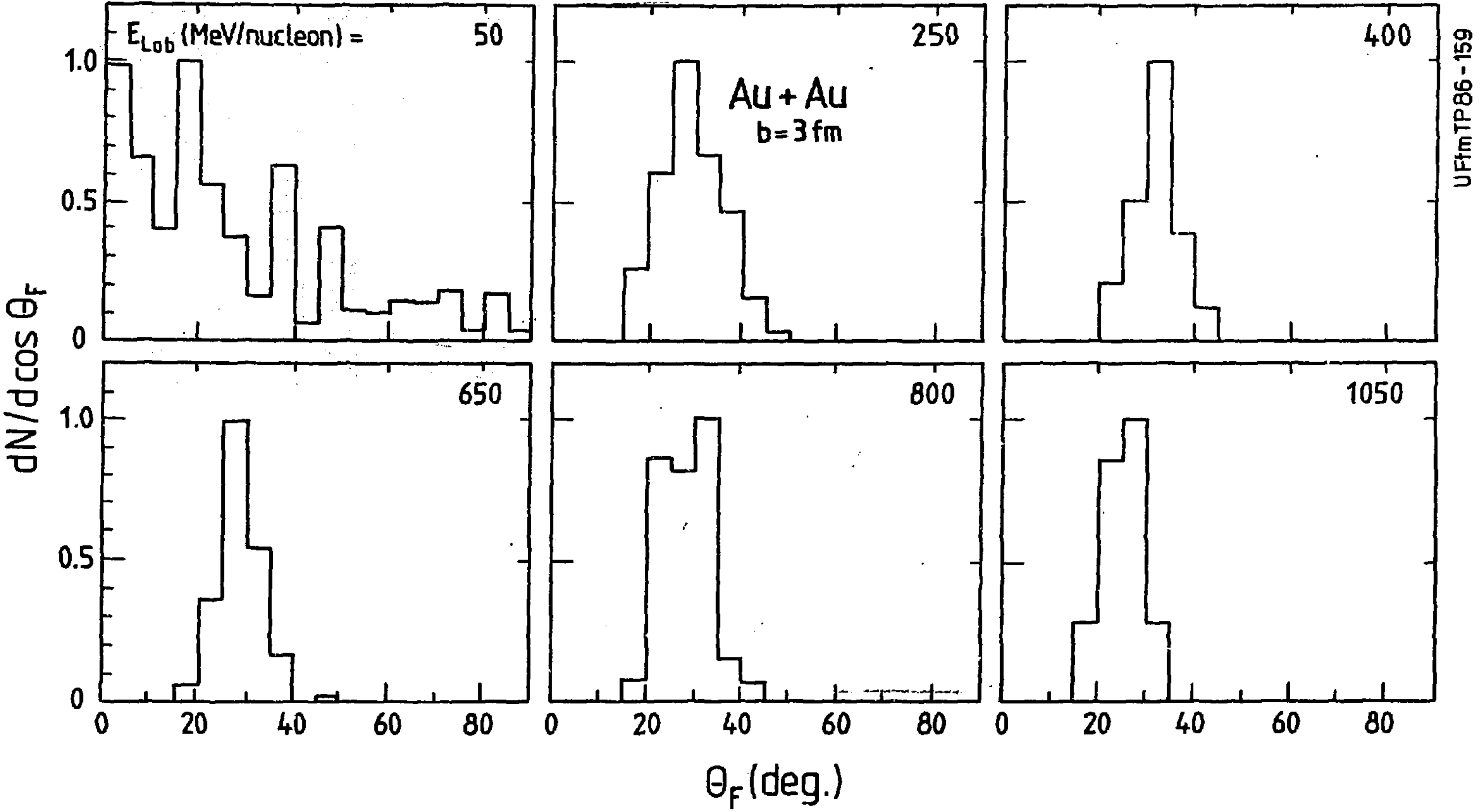


momentum transfer values are higher. Both results may be understood by the fact that the NFD lacks the Pauli principle except through the equation of state. Turning off the Pauli principle in the VUU model results in larger flow angles and higher p_x values;[18] the EOS is effectively harder without a Pauli principle and hence also lower central densities and lower temperatures result.

Nuclear fluid dynamics only mocks up quantum effects in that the compressional Fermi energy $E_{CF}(\rho)$ is assumed to be phenomenologically a part of the EOS of the fluid and the thermal Fermi energy $E_{TF}(\rho,T)$ and thermal Fermi pressure $P_{TF}(\rho,T)$ from the Δ and other resonances are used as the fluid's thermal energy and pressure. In the VUU model, the quantum effects enter through the collision integral: the Pauli principle modifies the transport properties of the nuclear matter and the equation of state.

We now turn to the impact parameter and energy dependence of the thermodynamic observables. First we show in Figure 3 the EOS sensitivity of the peak transverse momentum transfer in the VUU model for Nb (400 MeV/nucleon) + Nb. Note that the transverse momentum peaks at intermediate impact parameter just as it does experimentally.[19] The p_x values with the stiff EOS are 50 % - 200 % higher than those obtained with the soft EOS, depending upon the impact parameter. A comparison of the derivative dp_x/dy ($y = 0$) of the transverse momentum spectrum to the experimental values

Figure 4: Flow angle distributions are shown for Au + Au in the VUU model.



yields excellent agreement with the VUU stiff EOS for $T_{lab} <$ 500 MeV/nucleon.[19] For the higher energies, the VUU model predicts larger values than are measured; however the limitations of the experimental detector at higher energies preclude any definite conclusion. Indeed the VUU predictions with a stiff EOS agree with the lighter Ar (1.8 GeV/nucleon) + KCl data.[7]

Shown in Figure 4 are the predictions of the VUU model for Au + Au over the Bevalac energy range. Note that the flow angle distribution is broader at low energy $T_{lab} < 0.1$ GeV/nucleon, but that the peak flow angle decreases by just a few degrees as the bombarding energy is raised. From 0.25 to 1.05 GeV/nucleon at b = 3 fm, the VUU model predicts $\theta_F \simeq 30^o$. In the VUU model, the flow angle increases with higher atomic mass[12,20] and also rises from 0^o for peripheral collisions to higher angles for near central collisions.[20] This agrees with the experimental intermediate to high multiplicity data.[14]

Now we compare to NFD predictions for the same system with the same stiff EOS. The central density (see Figure 5) rises from ρ_o at low bombarding energies to 4.6 ρ_o at T_{lab} = 9 GeV/nucleon. The temperature rises from near 0 at low energies to 178 MeV at the highest energy. The entropy rises from S = 0 at T = 0 (by Nernst's theorem) to 6.3 at 9 GeV/nucleon. Note the general trend that $\rho_{INC} > \rho_{VUU} > \rho_{NFD}$ and $T_{VUU} > T_{NFD}$. The intranuclear cascade model lacks the

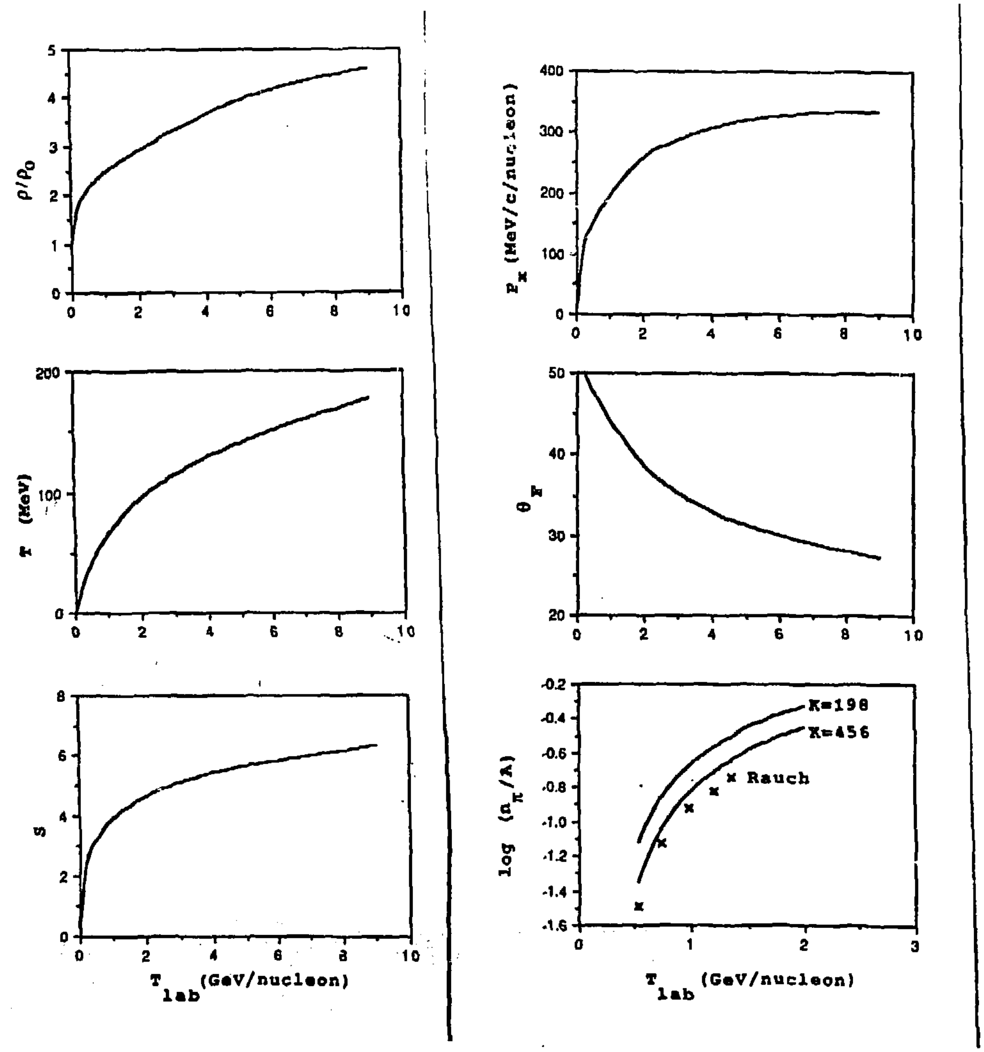


Figure 5: The energy dependence of the thermodynamic observables is shown for the relativistic fluid dynamic model for Au (0.1 - 10 GeV/nucleon).

compressional energy[18,21] whereas the fluid dynamic model lacks the Pauli principle. It is only for bombarding energies greater than a few GeV/nucleon that the effects of the Pauli principle are small enough for it to be neglected.[18]

The transverse momentum transfer rises from small values at low energies to a limiting value $p_x \simeq 335$ MeV/c/nucleon at $T_{lab} = 7$ GeV/nucleon. The flow angle falls from 50^o at 0.25 GeV/nucleon to 27^o at 9 GeV/nucleon. The collective flow observables in the 0.1 to 1.0 GeV/nucleon range are much higher in the NFD model than in the VUU model or in experiment.

Consider also the pion multiplicities per baryon (shown last in Figure 5) from a one dimensional relativistic fluid model.[16] We see that only an ultrahard EOS brings the theoretical pion predictions close to the experimental data.[22] In fact, the 3 D VUU model with a stiff EOS explains the La + La exclusive data very well[18] just as it explained the Ar + KCl data.[5] The n_π/A is essentially the same for A = 40 and A = 139. This VUU prediction and experimental result is precisely the A dependence expected according to the idea that pion production is a bulk nuclear matter probe rather than a surface probe.

In conclusion, much progress has been made in understanding nuclear matter under extreme conditions. The predictions of the NFD model beyond 1 GeV/nucleon are exciting and will be tested in forthcoming exclusive experiments. In order to understand and detect the quark gluon plasma, we must know what the conventional models like relativistic NFD and VUU predict.

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