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A NEW NONLINEAR "RECONSTRUCTIVE" CONTROL APPROACH APPLIED TO THE AXIAL XENON OSCILLATION PROBLEM IN PWRs*

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ABSTRACT

The concept of dynamic equilibrium of control in conjuction with a proportional corrector constitutes the structure of the new control technique. The controller utilizes direct measurements and model-based state estimations in the feedback loop. The method includes recovering unanticipated parametric variations or partially unknown dynamics. An application of this approach to the axial xenon oscillation problem of PWRs was considered. A two-point xenon oscillation model was used in designing the controller and testing it through simulations.

1. INTRODUCTION

This paper presents a nonlinear control technique for trajectory following processes including unknown parametric deviations or partially unknown dynamics. Recent studies entitled "inverse problems in dynamics" [1] address solution techniques in which the controls are calculated for assigned trajectories. The embedded predictive models [2] use inverse models and calculate controls numerically.

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In this study, we introduce the use of dynamic equilibrium of control which exerts a dissipative control force against the open-loop dynamics. These forces appear when a system leaves its (minimum-energy) stable-equilibrium point due to some excess energy. The control is completed by reconstructing the dissipated dynamics for assigned trajectories. Compared with related methods in the literature, the dynamic equilibrium of control corresponds to a partial inverse dynamics of the complete trajectory following dynamics and recognizes the minimum effort to achieve continuous equilibria.

It is a well-known phenomenon that pressurized water reactors (PWR) exhibit xenon induced power oscillations. Analytical studies [3] showed that the nature of nonlinearities are quite complicated and may go beyond the control capability of reactor operators. During plant operations, the axial power oscillations are avoided by lowering the power to some economically undesired levels. In this paper, it is not intended to propose a new complete control strategy for the xenon oscillation problem of PWRs. This application introduces the concept of dynamic equilibrium for the compensation of complicated nonlinear behavior like the xenon oscillation using a simple model. However, the superior results indicate that the methodology may find a real application in this field.

2. CONTROL DESIGN

The method includes two design phases: control and adaptation, both utilizing a set of process measurements and model-based knowledge. The final form of controller is a combination of the two, determined by the availability of sensory information and the importance of the unknown part of process dynamics. Partially unknown dynamics is defined here as mismatches between actual process behavior and predictions obtained by analytical methods. Such mismatches may be due to time-dependent unknown parametric deviations, which are assumed to be constants in dynamic models, or by the lack of mathematical representation of the actual physical behavior. The control system is shown in Figure 1, including the flow of important system variables. The following describes the basics of these two design steps.

2.1 Control Phase

Consider a dynamic system bounded by a set of predefined trajectories. In state space representation, a nonlinear system is stated as

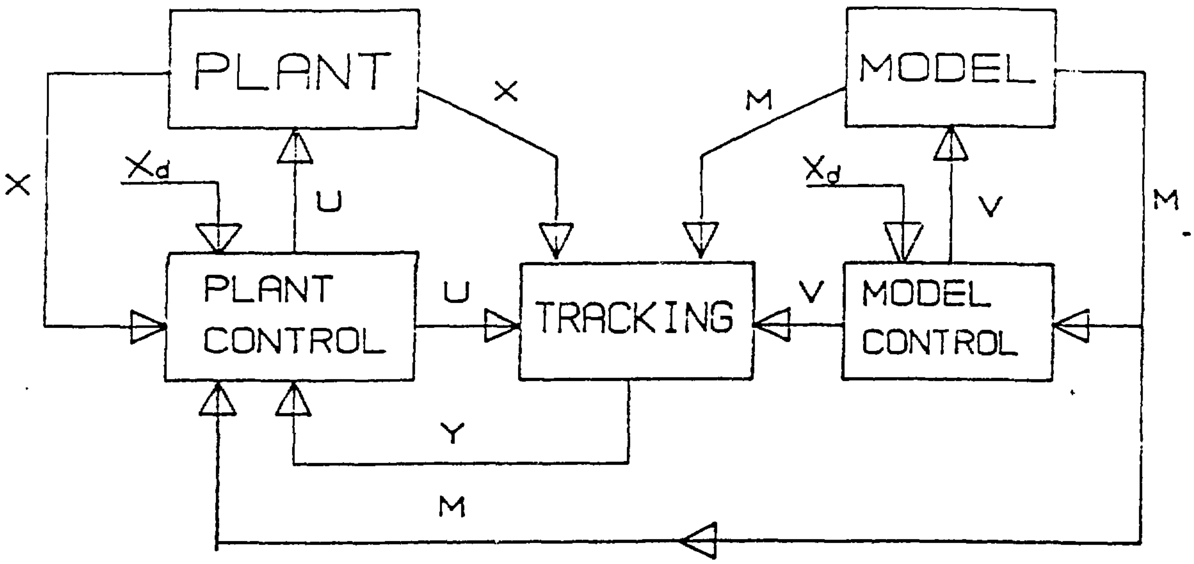


Figure 1. Block-structure of "Reconstructive" nonlinear control system.

$$\dot{\bar{X}} = f(\bar{X},\bar{U}) \quad (1)$$

where $\bar{X}$ and $\bar{U}$ are state and control vectors, respectively. We then concentrate on the i-th state equation assuming that a fixed trajectory was assigned to this state variable.

$$\dot{X}_i = f(X_i,X_j,U_i) \quad i\neq j \;,\; j=1,2,..,N \quad (2)$$

where

X_i = ith state variable,
X_j = coupling state variables,
U_i = ith control variable,
N = number of state equations.

Next we define the dynamic equilibria of U_i and of a selected coupling state X_j^* in case U_i does not exist provided U_j (any other control) exists which is an implicit function of the selected coupling state X_j^* (controllable path). Establishing dynamic equilibrium of a selected coupling state can also be used to overcome multiple-equilibrium problem (same control appearing in more than one dynamic equation). The dynamic equilibria are,

$$U_{iE} = f'(X_i,X_j,U_i)\Big|_{\dot{X}_i=0} \quad (3)$$

$$(X_j^*)_E = f''(X_i,X_j)\Big|_{\dot{X}_i=0} \quad (4)$$

where

X_j^*= selected coupling state variable,
E = subscript indicating equilibrium.

Evaluating the functions in Eqs (3) and (4) for $\dot{X}_i=0$ and solving for the control and for the selected state variable will yield time-variant equilibria that are not functions of U_i and X_j^*, respectively. We then introduce the desired trajectories to be followed by the process and associate it with control through the following definition of a total weighted errors,

$$E = [Q_i(U_i-U_{iE}) + R_i(X_i-X_{Ri})] \quad (5)$$

and

$$E^* = [Q_i(X_j^*-X_j{}^*{}_E) + R_i(X_i-X_{Ri})] \quad (6)$$

where

X_{Ri} = desired trajectory for i-th state variable,
R_i, Q_i = weights.

Note that the state variables and controls are normalized, nondimensional variables. Equation (5) does not imply a condition requiring the nonexistence of a target X_{Rj}^*. Such demand can be incorporated with another coupling state variable or by simply using U_j when U_j^* does not exist. The controls are solved from Eqs. (5) and (6) by forcing E or E^* to go to zero.

$$U_i = f'(X_i, X_j, U_i)\Big|_{\dot{X}_i=0} - K_i(X_i - X_{Ri}) \qquad (7)$$

or

$$U_j = g(X_i, X_j, X_j^*) \qquad (8)$$

$$X_j^* = f''(X_i, X_j)\Big|_{\dot{X}_i=0} - K_i(X_i - X_{Ri}) \qquad (9)$$

where

U_j = any other control ($j=1,2,\ldots,N$; $i \neq j$),
K_i = (R_i/Q_i) adjustable gain.

The final form of the control includes a dynamic equilibrium term which works towards dissipating the excess energy of a dynamic system between equilibrium and nonequilibrium points and a proportional term which forces the system to "rebuild" its dynamics in a desired form. It can be shown analytically that control (7) guarantees stability for linear systems.

The control design stated by Eqs.(7), (8) and (9) requires that the state variables that appear in these equations are directly measured or accurately estimated. If all the states are accessible through a measurement set, then the unanticipated transients (like state disturbance) will be recovered automatically by the dynamic equilibrium of control. If some of the state variables are estimated using mathematical models then the parametric deviations may affect the controller performance severely. This problem holds for every model-based control scheme and can be overcome using several adaptive techniques available in the literature [4]. We proceed with the second design phase, including adaptive features to predict the unknowns of the process.

2.2 Adaptive Phase

We introduce an adaptive philosophy for which the tracked parameters are used to update the control but not the model. In other words, the model provides a reference state similar to the model-reference adaptive control technique.

Consider process dynamics and its corresponding mathematical model in which the model assumes constant parameters that are time-variant in the actual process,

$$\dot{X}_i = f(X_i, X_j, U_i, Y_i) \tag{10}$$

and

$$\dot{M}_i = f(M_i, M_j, V_i, Y_m) \tag{11}$$

where

Y_i = time-variant parameter,
M_i = ith state variable of the model,
M_j = coupling state variables of the model,
V_i = control in the model,
Y_m = constant parameter used in the model.

Note that the control V_i of the model would exactly match U_i if Y_i did not change in time. Considering the first case where U_i exists, we present the following error criteria

$$E = [Q_i(Y_i - Y_m) + R_i(U_i - V_i)], \tag{12}$$

and a solution for Y_i^*

$$\begin{aligned} Y_i^* &= Y_m - K_y(U_i - V_i) \\ &= Y_m - K_y[(U_{iE} - V_{iE}) + K_{im}(M_i - X_{Ri}) - K_{ix}(X_i - X_{Ri})], \end{aligned} \tag{13}$$

where K_y, K_{im} and K_{ix} are adjustable constants. The solution (13) includes dynamic equilibria which may include unknown Y_i, and will be updated by Y_i^*. The control U_i is a function of Y_i^*.

The second case where U_i does not exist, the time-variant Y_i can be considered as partially unknown dynamics provided the corresponding state measurement is available. A solution can be obtained by treating Y_i as a control variable associated with a state trajectory, which is continuously provided by the model. An error expression is written in the following fashion.

$$E = \{Q_i(Y_i^* - Y_{iE}^*) + R_i(X_i - M_i)\} \tag{14}$$

A solution for Y_i^* is obtained,

$$Y_i^* = Y_{iE}^* - K_y(X_i - M_i) \tag{15}$$

where

$$Y_{iE}^* = f(X_i, X_j, Y_i^*)\Big|_{\dot{X}_i=0} \tag{16}$$

The Eqs. (15) and (16) are then used in U_j if it is a function of Y_i^*.

The design procedure stated above may require assigning additional trajectories not included in the frame of control tasks. This will be necessary when the estimated states include time-variant parameters and there is no trajectory following assignment to those states. In this case, the additional trajectory assignment can be selected intuitively provided it is close to its steady-state value. Note that the number of trajectory assignments can not exceed the number of degrees of freedom of the system.

3. AN APPLICATION TO THE XENON OSCILLATION PROBLEM

A two-point xenon oscillation model of a PWR [5] is used to demonstrate the method of "reconstructive" control. The model employs the nonlinear xenon and iodine balance equations and one group, one-dimensional, neutron diffusion equation having nonlinear power reactivity feedback. A two-term spatial, harmonic-series solution was assumed for the flux, xenon and iodine distributions. The system was made as close to critical as possible with assumed distributions using a variational principle. The xenon and iodine concentrations were then obtained from their governing differential equations. The input/output nature of the model makes it ideal for simulation of xenon-induced reactor transients. In the model, the spatial average of the normalized flux for the lower half and for the upper half are expressed by the following equations.

$$F_1(t) = 0.6366\ [1 - A(t)] \tag{17}$$

$$F_2(t) = 0.6366\ [1 + A(t)] \tag{18}$$

where

F_1, F_2 = upper and lower half average fluxes,
$A(t)$ = amplitude of the first harmonic.

Similarly, the average xenon and iodine oscillations in the upper and lower part of the core are expressed as

$$X_1(t) = 0.6366\ [1 - B(t)] \tag{19}$$

$$X_2(t) = 0.6366\ [1 + B(t)] \tag{20}$$

$$Y_1(t) = 0.6366\ [1 - C(t)] \tag{21}$$

$$Y_2(t) = 0.6366\ [1 + C(t)] \tag{22}$$

where

X_1 = average xenon concentration in lower-half,
X_2 = average xenon concentration in upper-half,
Y_1 = average iodine concentration in lower-half,
Y_2 = average iodine concentration in upper-half,
$B(t)$ = xenon amplitude function,
$C(t)$ = iodine amplitude function.

The amplitude functions described in the literature [5] satisfy the following equations.

$$-Z_2(t)\ A^2(t) + 2(Z_1 - Z_3)\ A(t) + Z_2(t) = 0 \tag{23}$$

$$\frac{dC(t)}{dt} = a_1\ A(t) - a_2\ C(t) \tag{24}$$

$$\frac{dB(t)}{dt} = [a_3 - a_4\ B(t)]\ A(t) - a_5\ B(t) + a_6\ C(t) \tag{25}$$

where

$$Z_2(t) = a_7[R_2(t) - R_1(t)] + a_8\ B(t) \tag{26}$$

and

Z_1, Z_3, a_i = neutronic and geometric constants,
$R_2(t)$ = reactivity input with upper partial-length control rod,
$R_1(t)$ = reactivity input with lower partial-length control rod.

The control task is determined such that the amplitude functions B(t) and C(t) remain constant (close to a fixed trajectory) which guarantees nonoscillatory flux response. The solution for $[R_2-R_1]$ (relative reactivity input with respect to upper and lower halves of the core) is obtained using the method described by Eqs. (7) and (9). In this example, two "coupling" states are chosen due to the fact that there are two differential state equations both without any direct entry of controls R_1 and R_2. First we select $C^*(t)$ in Eqn. (25) and establish dynamic equilibrium for B(t).

$$C^*_{EQ}(t) = a_5\, B(t)/a_6 - [a_3 - a_4\, B(t)]\, A(t)/a_6 \tag{27}$$

$$C^*(t) = C^*_{EQ}(t) - K_1\, [B(t)-B_d(t)] \tag{28}$$

where $B_d(t)$ is a desired trajectory, close to its steady-state value. Then we establish a dynamic equilibrium in Eqn.(24) for C(t) by selecting $A^*(t)$.

$$A^*_{EQ}(t) = a_2\, C^*(t)/a_3 \tag{29}$$

$$A^*(t) = A^*_{EQ} - K_2[C(t)-C_d(t)] \tag{30}$$

The final step is to use $A^*(t)$ in Eqs. (23) and (26) to solve for $[R_2-R_1]$.

$$[R_2(t) - R_1(t)] = \frac{2(Z_1 - Z_3)\, A^*}{a_7[1 - (A^*)^2]} - \frac{a_8\, B(t)}{a_7}\,. \tag{31}$$

Note that in Eqn. (31), A^* should not be equal to 1 at any time. It can easily be seen from Eqn.(23) that $A^*=1$ does not satisfy Eqn. (23) due to $Z_1 \neq Z_3$. In applications where such condition can not be established easily, the control signal can be protected by numerical methods or another strategy of achieving equilibrium can be used. The gains K_1 and K_2 were arbitrarily chosen both equal to 100 in this application.

4. SIMULATION RESULTS

Two-point axial xenon oscillation model is first simulated without control. Figure 2 shows the normalized flux amplitude in the lower half of the core for the Oconee plant.

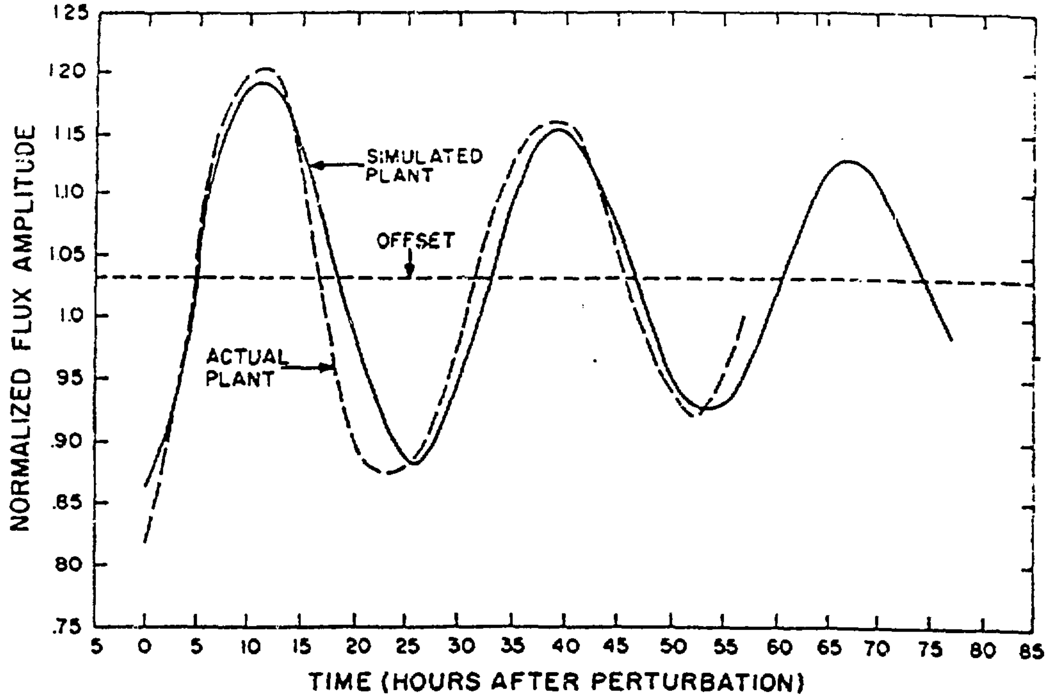


Figure 2. Time plot of model response compared with observed axial oscillation at Oconee Nuclear Station. [5].

Results of Ref[5] and our simulation are compared. The dotted curve shows the time response of the Oconee reactor during xenon oscillations. The reactor had been operating at 75% of full power with steady-state xenon prior to the perturbation (which lasted 2.5 hr.) by means of control rods. Figure 3 shows the xenon, iodine and flux oscillations in the upper half of the core for those conditions in which the dynamics exhibits a limit cycle behavior.

The controller is turned on 30 minutes after the 2.5-hr. perturbation. As shown in Fig.4, flux, xenon and iodine oscillations are compensated rapidly. Figure 5 shows the reactivity insertion using an upper partial-length control rod. The 2.5-hr. perturbation (about 1% worth of reactivity) is also performed using partial-length rods and appears as the initial pulse in the same plot. Comparing this "perturbation" pulse with the rest of the curve indicates that the amount of rod motion is reasonably small.

In order to demonstrate the effect of "dynamic equilibrium of control", the controller is turned off for 17 hours following the perturbation and then turned on. Figure 6 indicates that the controller suddenly perceives the dynamic equilibrium point and acts accordingly. As shown in Fig. 7, the required rod motion is much larger compared to the previous case which agrees with intuitive reasoning. Figure 8 shows xenon-iodine phase-plane graphs of uncontrolled and controlled (after 17 hrs.) reactor. The limit-cycle behavior of the uncontrolled plant is exhibited.

5. CONCLUSIONS

The application of the "reconstructive" control technique to an axial xenon oscillation problem clearly emphasizes the effectiveness of using dynamic equilibrium of control. The rod position (or reactivity input) changes in small amounts over a large interval of time, mapping the equilibrium trajectory of the overall dynamics. Obviously, such a trajectory is not easily perceivable by the reactor operator. Due to the unique structure of the xenon oscillation problem, adaptive features are not addressed. The controller uses calculated values for xenon and iodine concentrations or their corresponding amplitudes. There are fast algorithms for such calculations available in the literature [6]. In addition, the transient is quite slow enabling the system engineers to update their calculations. Unanticipated transients are automatically reflected in the dynamic equilibrium part of the controller, thus there is no need for an adaptive design for this purpose. However, the design can easily be extended using more

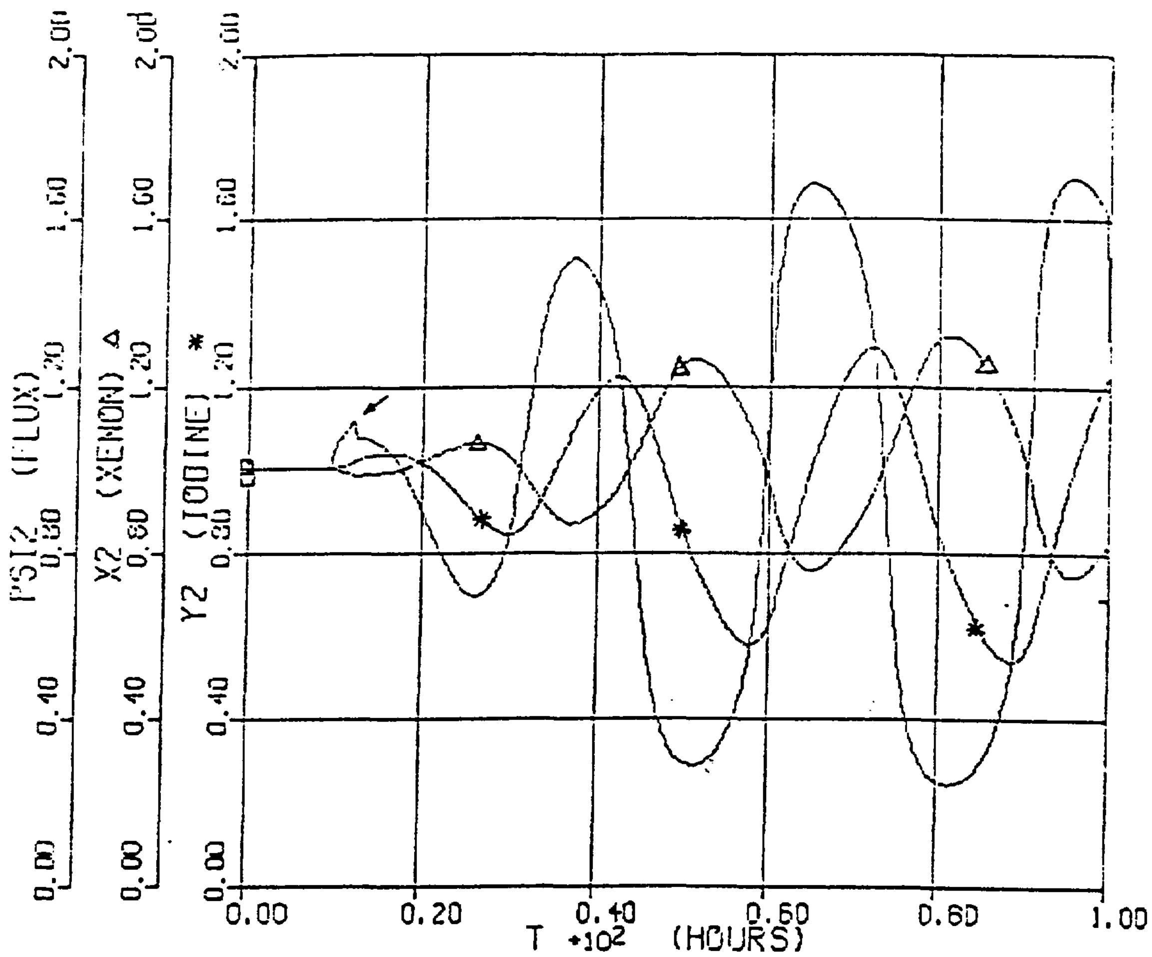


Figure 3. Limit-cycle behavior in the upper half of the core. Arrow indicates the perturbation.

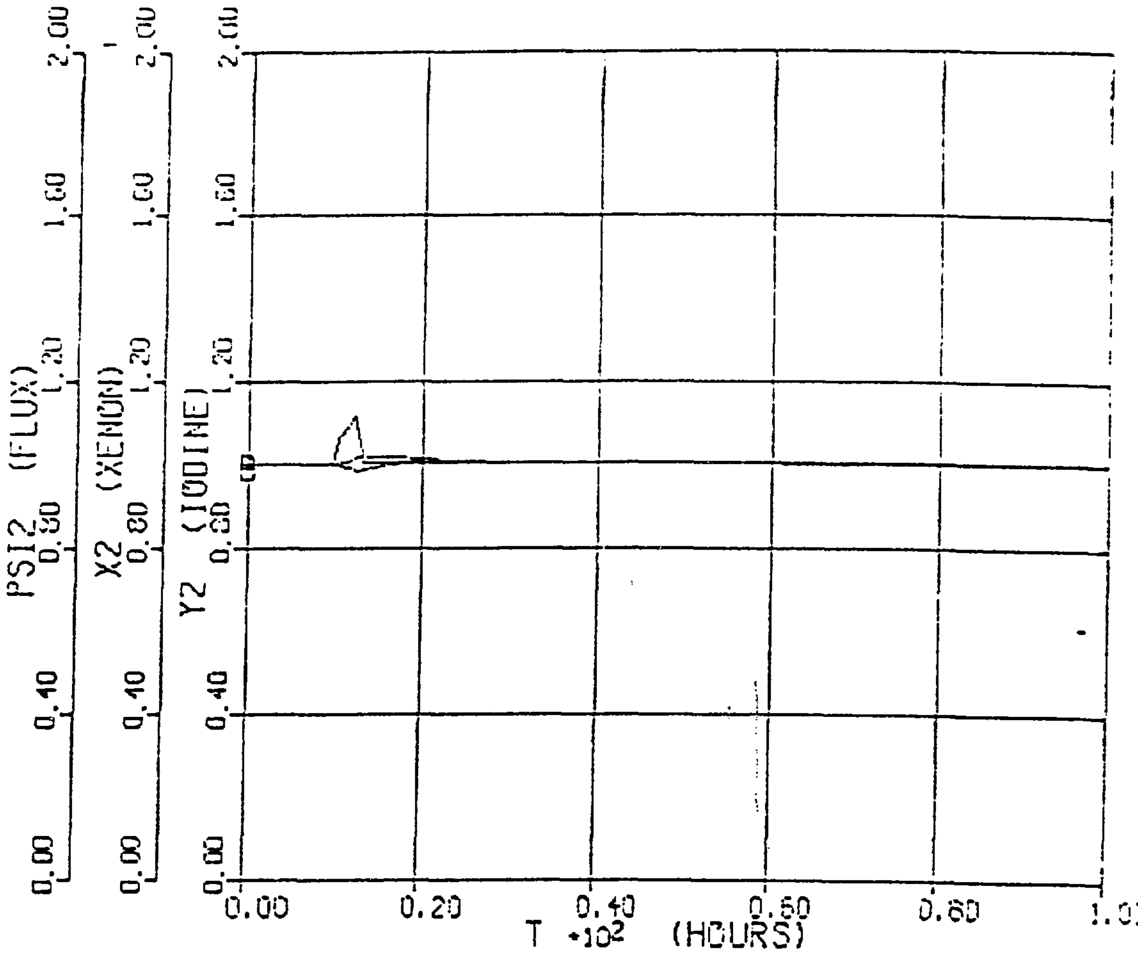


Figure 4. Flux, xenon and iodine behavior in the controlled plant (upper core region) Controller is turned on 30 minutes after the perturbation.

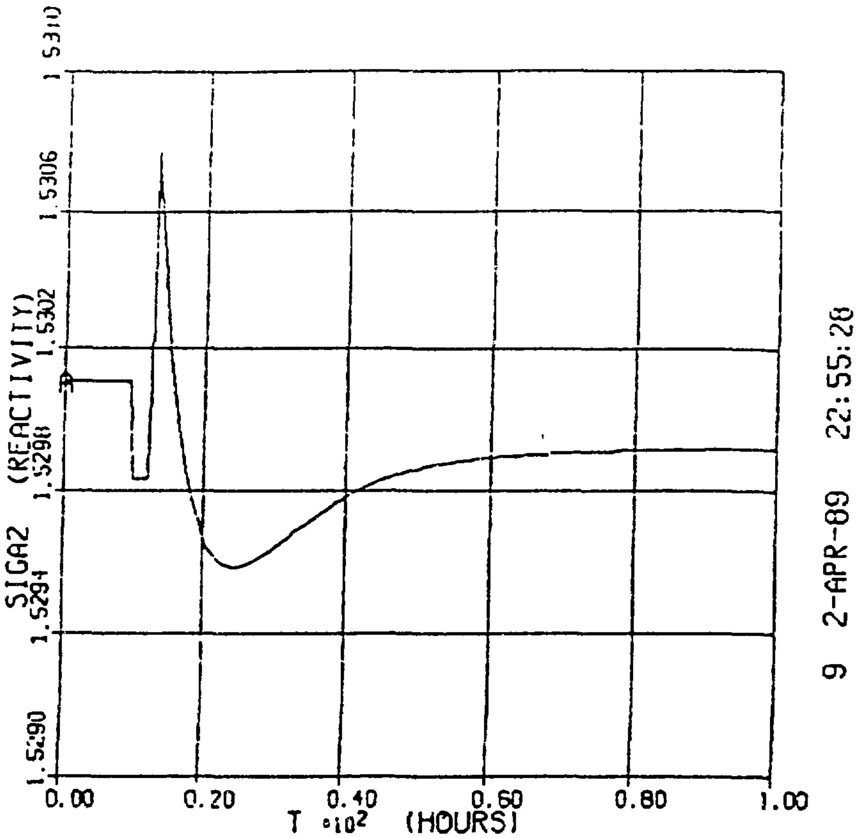


Figure 5. Reactivity input in the upper core region. Controller is turned on 30 minutes after the perturbation.

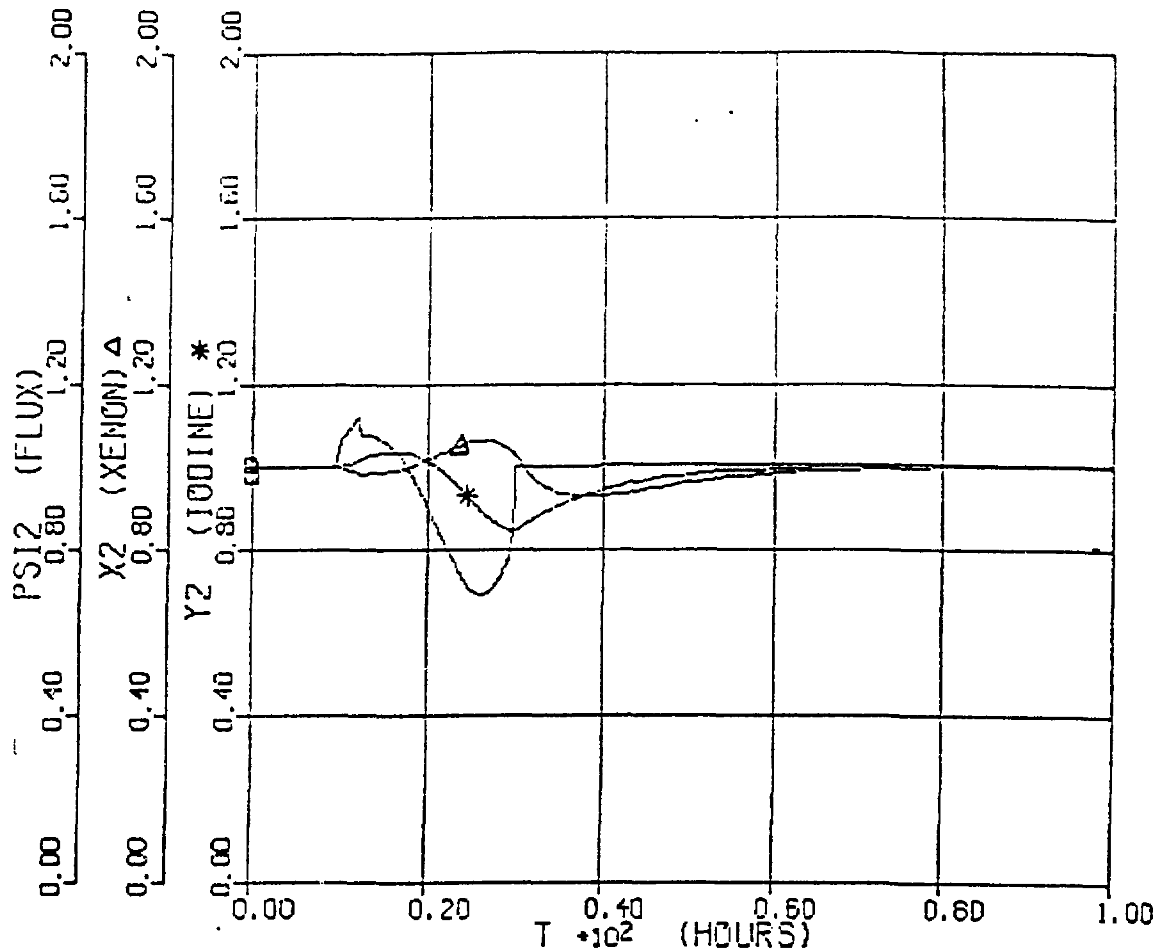


Figure 6. Flux, xenon and iodine behavior in the controlled plant (upper core region). Controller is turned on 17 hours after the perturbation.

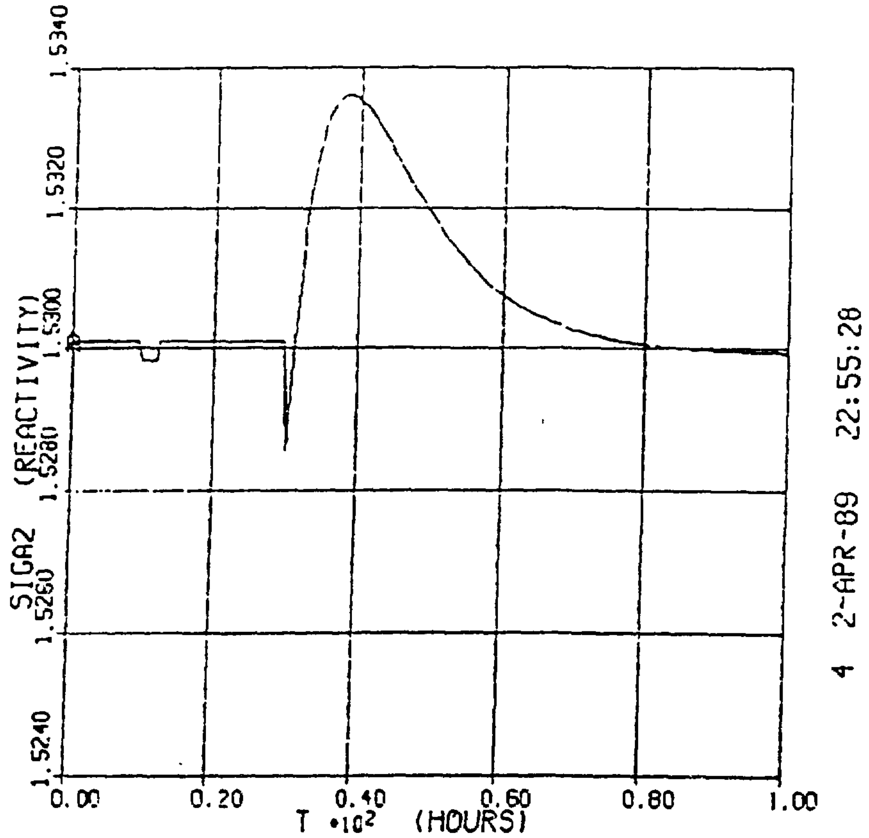
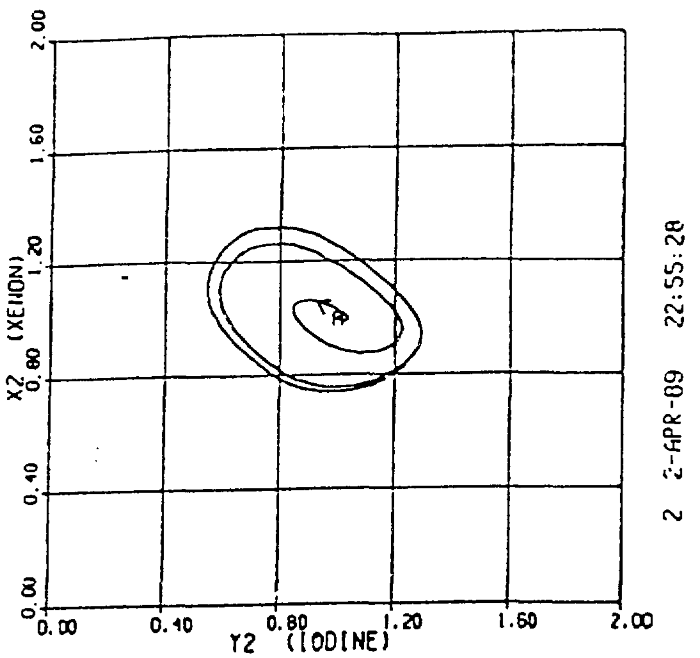


Figure 7. Reactivity input in the upper core region. Controller is turned on 17 hours after the perturbation.

(A)



(B)

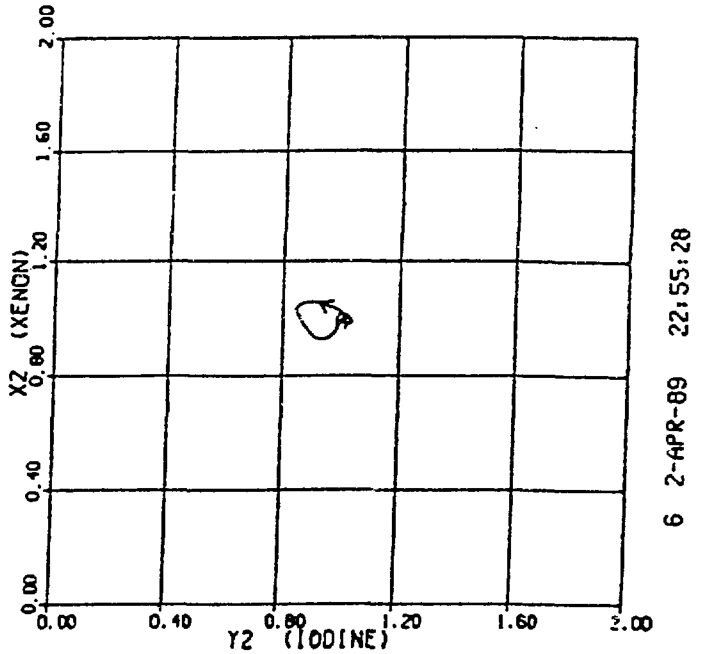


Figure 8. Xenon-iodine phase-plane graphs for (a) uncontrolled plant, (b) controller activated 17 hours after the perturbation.

sophisticated models and including the boron effect on the dynamic equilibrium.

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