CHARACTERIZATION OF NONSTATIONARY RANDOM PROCESSES

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AUTOBIOGRAPHY

Thomas Paez is a native of Albuquerque, New Mexico. He received his B.S. and M.S. degrees from The University of New Mexico in 1971. He then enrolled at Purdue University to pursue his doctoral studies, and received his Ph.D. in 1973. He has been employed at Kaman Sciences Corporation (1973-75), Sandia National Laboratories (1975-77), and The University of New Mexico (1977-84). He is currently a Member of the Technical Staff at Sandia National Laboratories where he is engaged in analysis and testing activites in the areas of structural dynamics and probabilistic structural mechanics.

KEYWORDS

Dynamics, Shock Test, Random Excitation, Random Environment, Probabilistic Mechanics, Stochastic Models

ABSTRACT

Current methods for shock test specification and shock testing treat the shock environment as a deterministic source. The present study proposes to treat shock sources as nonstationary random processes. A model for a realistic nonstationary random process shock source is specified, and the effect of variation of parameters in the shock source is shown. A method for estimating the parameters of the random process is established, and some numerical examples show that the method yields reasonable results. The use of this model in shock testing is discussed.

INTRODUCTION

A fundamental objective of some mechanical tests is to simulate a field environment and subject a structure to the simulation. When a structure survives the simulated field environment in the laboratory, then it is assumed that an identical structure would survive the same environment in the field. When a specific field environment can be accurately predicted, and when this environment can be faithfully reproduced in the laboratory, any structure that survives the environment is certain to survive the field input.

However, it is rare that field environments can be exactly predicted because, usually, the factors that influence them are random. This fact is accounted for when random vibration environments are measured in the field and applied in the laboratory. Stationary random vibration environments occur frequently in the field, and their measured samples can be used to estimate their spectral densities. In the laboratory, stationary random vibration signals with approximately normal amplitude distributions can be generated and used to test structures. The specific excitation generated in the laboratory differs in precise form from the excitation measured in the field and those anticipated in future field realizations, but the probabilistic character of the excitation is matched accurately, and this is considered to constitute a reasonable simulation.

Shock environments are treated differently. Shock excitations realized in the field are also unpredictable, but shock test specification techniques do not usually take their randomness into consideration, directly. Rather, laboratory excitations are sought that are deterministically obtained and conservatively represent measured field shock environments. The technique used most commonly for shock test specification is the method of shock response spectra. This method seeks to establish, through a simple peak response criterion, a laboratory shock test that is more severe than an ensemble of field excitations. Other techniques for shock test specification are used less frequently, but none of the methods in common use classifies the field environment as a nonstationary random process--in most cases, what the environment is. A nonstationary random process is any random process that is not stationary. Random processes whose statistical characteristics vary rapidly with time are generally treated as nonstationary random processes.

A comprehensive, probability-based, test specification technique for shock and short nonstationary random vibration must have sevral components. First, the technique must establish a method for estimating the statistical parameters of a nonstationary random signal source. For the method to be general, the random process model must be general so that a reasonable variety of excitation sources can be modelled. For the technique to be useful, it must be possible to estimate the parameters of the nonstationary random process using one (or a few) measured realization(s). Second, the random process characteristics must be fully understood so that it is possible to modify the estimated parameters to obtain a test with a known level of conservatism. Third, it must be feasible to use the parameters of the test source to specify a test that can be executed on available test equipment.

The present investigation addresses these matters, with special emphasis on the first item, modeling a signal source and estimating its parameters. Several approaches are available for characterizing nonstationary random sources. Among these are techniques that require a measured ensemble of signals, time averaging techniques, and techniques that use the energy spectral density. (See References 1 and 2.) The present investigation starts with a parametric model of the nonstationary random source and uses the method of maximum likelihood (Reference 3) to estimate its parameters.

THE NONSTATIONARY RANDOM PROCESS MODEL

The model used in the present study permits the simulation of random sources that superimpose a rapidly varying oscillatory shock pulse on a background oscillatory random signal with slowly varying amplitude characteristic. The random process source is a sum of narrowband, nonstationary random process components. Let $\{X(t), t\geq 0\}$ denote the shock source random process. $\{X(t)\}$ is a zero mean, nonstationary, normal random process that can be expressed

$$X(t) = \sum_{j=1}^{N} X_j(t), \qquad t\geq 0. \qquad (1)$$

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The $\{X_j(t),\ t>0\}, j=1,\dots,n$, are the narrowband components of $\{X(t)\}$. Each $\{X_j(t)\}$ is itself a zero mean nonstationary, normal random process with parametric form

$$X_j(t) = Z_j(t)\{[1-a_j(t-t_c)] + b_j(t-t_s)H(t-t_s)\exp(-\alpha_j(t-t_s))\}H(t), \quad j=1,\dots,N, \quad -\infty<t<\infty. \qquad (2)$$

The $\{Z_j(t),\ -\infty<t<\infty\}$ is a normally distributed, band-limited white noise random process with spectral density

$$S_j(\omega) = \begin{cases} S_j\ , & \Omega_j - \frac{\Delta\Omega}{2} \le |\omega| \le \Omega_j + \frac{\Delta\Omega}{2}\ , \quad j=1,\dots,N. \\ 0\ , & \text{elsewhere} \end{cases} \qquad (2a)$$

The $\Delta\Omega$ is a positive frequency constant and

$$\Omega_j = (j-\tfrac{1}{2})\Delta\Omega \qquad j=1,\dots,N. \qquad (2b)$$

The component white noise random processes are independent of one another. The a_j and b_j are arbitrary constants, and α_j is a positive constant. The t_c and t_s are positive time constants. The t_c can be taken as the central value of a time period of interest where the random process realizations are observed. (The start time is taken as $t=0$.) The t_s is the start time of the superimposed pulse. $H(.)$ is the Heaviside unit step function.

Using the model given in (1) and (2) a wide variety of random process sources can be simulated. Note that the narrowband random process component, (2), in the frequency band $(\Omega_j-\Delta\Omega/2, \Omega_j+\Delta\Omega/2)$ is simply the product of a band-limited white noise random process and a modulating function. The parameters of the white noise random process and the modulating function can be adjusted to yield a variety of behaviors in each component. For example, Figure 1 shows a 4.096 sec segment of a realization of a band-limited white noise random process. This was generated using the technique of Reference 4, the superposition of randomly phased harmonic components. The spectral density of the random process is $S = 1.0$; the center frequency of the spectral density is $\Omega_j=600$ rad/sec; the bandwidth is $\Delta\Omega=300$ rad/sec. (The random process has signal content in the frequency band (450, 750) rad/sec.) When the spectral density is increased, the mean square value of the random process increases, and when the spectral density is decreased, the mean square value of the random process decreases.

In order to demonstrate the range of shapes the modulating function can be given, let $t_c=2.048$ sec and $t_s=1.0$ sec in the following. The spectral density of the underlying white noise random process and the constant a_j are adjusted to establish the character of the background random process. The constants b_j and α_j are adjusted to establish the character of the superimposed pulse. To start, let $b_j=0$. (Then the value of α_j is arbitrary. The units of the a_j's, b_j's and α_j's are units of $X(t)$/sec; this notation is omitted in the following.) Figure 2 shows the modulating function for values of $a_j=-0.1,-0.05,0.0,0.05,0.1$. The background signal can obviously be adjusted to yield a random signal with linearly decreasing or increasing amplitude characteristic. The amplitude, duration

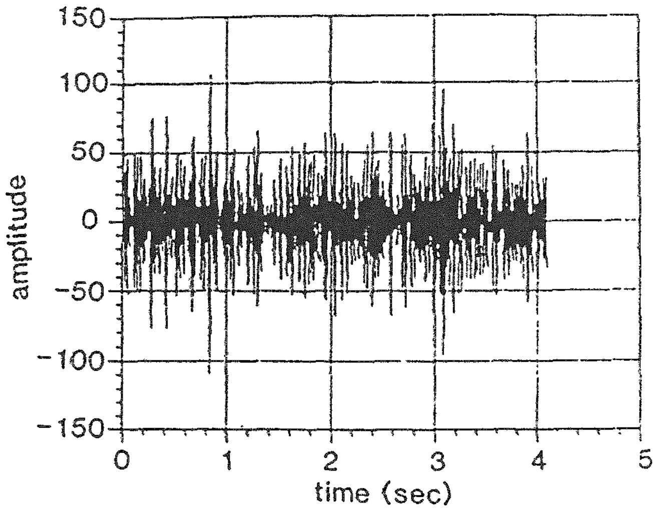

Figure 1 Band-limited White Noise Realization. $S=1$, $T=4.096$ sec, $\Omega=600$, $\Delta\Omega=300$ rad/sec.

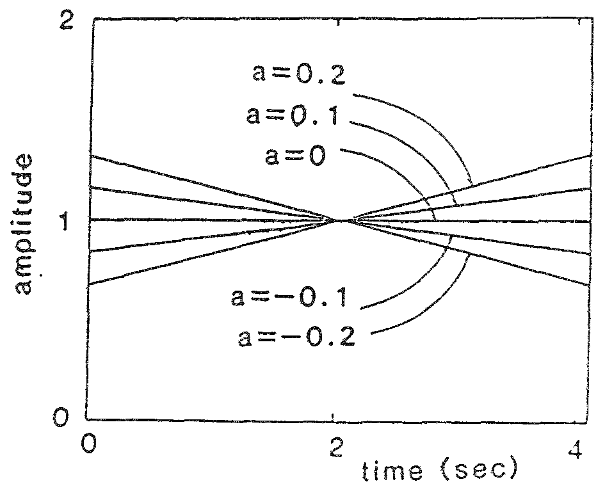

Figure 2 Modulating Function, b=0.

and decay rate of the pulse are controlled by b_j and α_j. Figure 3 shows the modulating function when these constants take specific values. In particular, a_j is set to 0, and the combinations $b_j=12$ and $\alpha_j=2$, $b_j=25$ and $\alpha_j=3$, $b_j=24$ and $\alpha_j=4$, are shown.

Some examples of the nonstationary random processes generated when the white noise is multiplied by a particular modulating function are shown in Figures 4 and 5. In Figure 4, the white noise random process realization of Figure 1 is multiplied by the modulating function of (2) where $a_j=-0.10$, $b_j=0$ and α_j is arbitrary. In Figure 5, the white noise random process realization of Figure 1 is multiplied by the modulating function where $a_j=0$, $b_j=25$, and $\alpha_j=3$.

Numerous components like those shown in Figures 4 and 5 may combine to create a random process whose realizations are similar to shock signals measured

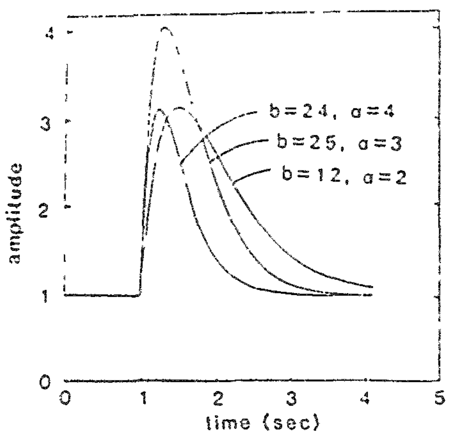

Figure 3 Modulating Function a=0.

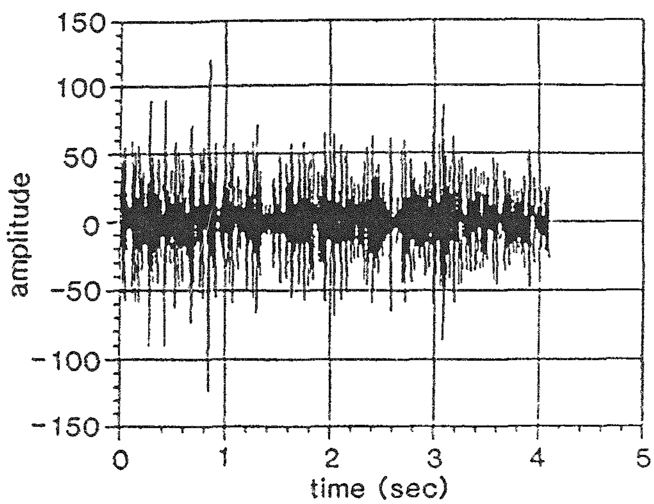

Figure 4 Nonstationary Random Process Realization. Ω=600 rad/sec, ΔΩ=300 rad/sec, a=-0.10, b=0, S=1

in the field. Figure 6 shows the kind of random process realization that can be created by superimposing the components described in (2). This realization has five nonstationary, narrowband components. For all components, ΔΩ=300 rad/sec. All the other parameters are given in Table 1.

TABLE 1 Parameters of the nonstationary random process of Figure 6. ΔΩ=300 rad/sec.

Component No.	Ω_j	S_j	a_j	b_j	α_j
1	150	1.0	0.0	10.0	3.0
2	450	2.0	0.0	20.0	3.0
3	750	0.5	-0.1	15.0	2.0
4	1050	0.2	0.05	10.0	4.0
5	1350	0.1	0.0	5.0	2.0

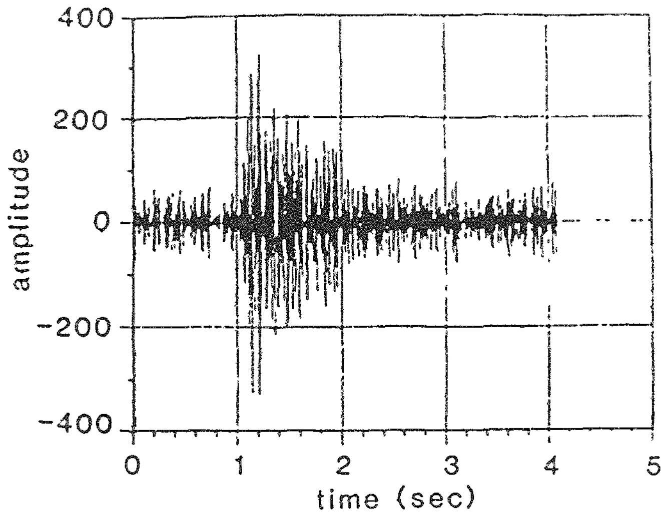

Figure 5 Nonstationary Random Process Realization. a=0, b=25, α=3, S=1, Ω=600 rad/sec, ΔΩ=300 rad/sec.

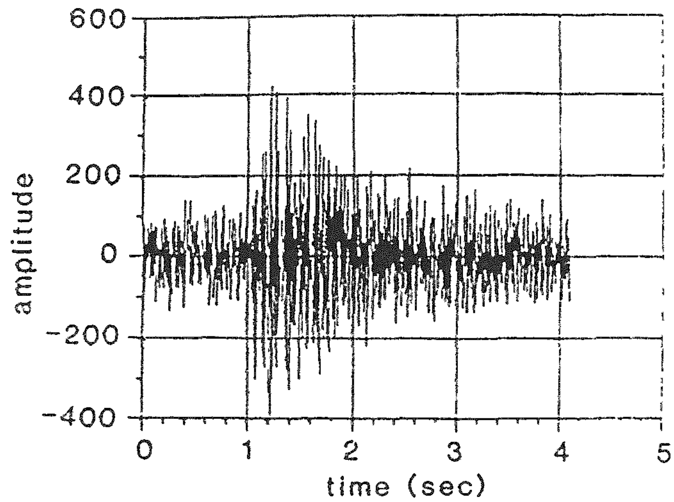

Figure 6 Random Process Realization. Characteristics Given in Table 1.

Because the random process defined in (1) and (2) and its components are not stationary, it does not have a spectral density. However, the random process and its components do have autocorrelation functions. The autocorrelation function of the jth component random process, $\{X_j(t)\}$, is

$$R_{x_jx_j}(t_1,t_2) = \frac{4S_j}{t_1-t_2} \cos \Omega_j(t_1-t_2) \times \sin\frac{\Delta\Omega}{2}(t_1-t_2)H(t_1)H(t_2)\{1+a_j(t_1-t_c) + b_j(t_1-t_s)H(t_1-t_s)\exp(-\alpha_j(t_1-t_s)\} \times \{1+a_j(t_2-t_c)+b_j(t_2-t_s)H(t_2-t_s)\exp(-\alpha_j(t_2-t_s))\},$$

$$-\infty < t_1, t_2 < \infty \qquad (3)$$

The mean square of the random process component is obtained by evaluating the autocorrelation function at $t_2 = t_1$. This is

$$\sigma^2_{x_j}(t) = 2S_j\Delta\Omega\{1+a_j(t-t_c)+b_j(t-t_s)H(t-t_s)$$
$$\exp(-\alpha_j(t-t_s))\}, \quad 0\leq t<\infty. \tag{3a}$$

The synthesized random process, $\{X(t)\}$, has an autocorrelation function that depends on the component autocorrelation functions. It is

$$R_{XX}(t_1,t_2) = \sum_{j=1}^{N} R_{X_jX_j}(t_1,t_2), \quad -\infty<t_1,t_2<\infty \tag{4}$$

The mean square of the random process $\{X(t)\}$ is simply

$$\sigma^2_X(t) = \sum_{j=1}^{N} \sigma^2_{X_j}(t), \quad t\geq 0. \tag{4a}$$

The autocorrelation function given in (4) and the fact that the random process has mean zero and normal distribution are sufficient to completely specify the random process.

PARAMETER ESTIMATION FOR THE NONSTATIONARY RANDOM PROCESS

The model defined in the previous section was shown to have a wide range of realizations. Some of these realizations might realistically represent a random process source encountered in a specific application. If an analyst is willing to assume that the random source observed in a particular application has the underlying model given in (1) and (2), then this model can be used to represent the random source in analyses and tests, and the analyst has only to estimate its parameters from measured data. Various approaches can be used in the estimation of parameters of probability models; the method of maximum likelihood is used frequently and will be applied in the present investigation.

In order to use the method of maximum likelihood, it is assumed that a sequence of measured random process realizations is available. Further, it is assumed that the probability density function (pdf) governing these data is known and that the pdf has certain parameters. When the measured data values are substituted into the pdf expression, then it is interpreted as a likelihood function. The method of maximum likelihood chooses the parameters of the pdf as those which jointly maximize the likelihood function.

In the present application, it is assumed that a single random process realization can be filtered to obtain N narrowband component realizations. It is assumed that each narrowband compoent is a mean zero, normally distributed random process with mean square given by (3a). The model in (1) and (2) simulates a transient pulse superimposed on a background signal; therefore, it is assumed that the measurement includes a start-up period when only background signal is present, and that the measurement is of great enough duration to include a period at the end when the effects of the pulse have become small. From these assumptions, a multi-stage process is developed to estimate the parameters, S_j, a_j, b_j, and α_j. It is assumed that the parameters $\Delta\Omega$, Ω_j, t_c and t_s can be specified by the analyst.

Let x_i, i=1,...,n, represent a narrowband component of the measured signal. These x_i's are filtered values of the measured random process realization. The time interval between measurements is Δt. Because the component is assumed normally distributed, the joint pdf of the random variables X_i, i=1,...,n, can be approximated by

$$p_{X_1\ldots X_n}(x_1,\ldots,x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma_X(S,a,b,\alpha,t_i)}$$
$$\exp\left[\frac{-x_i^2}{2\sigma^2_X(S,a,b,\alpha,t_i)}\right], \quad -\infty<x_i<\infty. \tag{5}$$

where $\sigma^2(S,a,b,\alpha,t)$ is given by (3a). The j subscripts on the parameters have been dropped because only one signal component is analyzed in the following, and there is no danger of confusion. This is an approximation because it assumes independence between random variables in the sequence, and they probably are not usually independent, in practice. However, it leads to accurate results in this application; therefore, it is used. When (5) is taken to be a function of S,a,b, and α, it is a likelihood function denoted $L(S,a,b,\alpha)$.

Recall that the beginning portion of the measured signal and the end of the measured signal are times when the superimposed pulse has little influence on the overall source; during these times it can be assumed that only the background random source operates. Based on this assumption, the parameters S and a of the background signal can be estimated in a preliminary analysis. Let t_a denote the time when the superimposed pulse begins to operate, and let t_b denote the time when the effects of the superimposed pulse become small. Then in the time intervals $(0,t_a)$ and (t_b,T) only the background signal is present. T is equal to $n\Delta t$. During these time intervals, the mean square value of the random process is given by

$$\sigma^2_X(S,a,t) = 2S\Delta\Omega\{1+a(t-t_c)\} \quad t\geq 0 \tag{6}$$

This mean square expression and the points in the measured signal from the time intervals $(0,t_a)$ and (t_b,T) can be used in the likelihood function to estimate S and a. The likelihood function is

$$L(S,a) = \prod_i \frac{1}{\sqrt{2\pi}\,\sigma_X(S,a,t_i)} \exp\left[\frac{-x_i^2}{2\sigma^2_X(S,a,t_i)}\right], \tag{7}$$

where the product includes only those values from the appropriate time intervals. The values of S and a that maximize L(S,a) are the maximum likelihood estimators of S and a. Because (7) is an exponential expression, its logarithm can be maximized to maximize the function itself. The log likelihood function is

$$\ln L(S,a) = \sum_i \{-\frac{1}{2}\ln(2\pi)-\ln\sigma_X(S,a,t_i)-\frac{x_i^2}{2\sigma^2_X(S,a,t_i)}\} \tag{8}$$

The maximum of this function may occur at the point where the partial derivatives of $\ln L$, with respect to S and a, equal zero. We assume that it does. The partial derivative of $\ln L$ with respect to S is

$$\frac{\partial}{\partial s}\ln L = \sum_i \left[\frac{1}{S} - \frac{x_i^2}{2S^2\Delta\Omega(1+a(t_i-t_c))^2}\right] \tag{9}$$

This is zero where

$$S = \frac{1}{2n\Delta\Omega}\sum_i \frac{x_i^2}{(1+a(t_i-t_c))^2}. \tag{10}$$

The partial derivative of $\ln L$ with respect to a is

$$\frac{\partial}{\partial a}\ln L = \frac{-n\sum_i[x_i^2(t_i-t_c)/(1+a(t_i-t_c))^3]}{\sum_i x_i^2/(1+a(t_i-t_c))^2} + \sum_i \frac{(t_i-t_c)}{1+a(t_i-t_c)}. \tag{11}$$

A numerical procedure must be used to determine where this is zero, and a simple computer program has been written to execute this computation. Examples of its use are given later.

Once the estimates of S and a are obtained, estimates of b and α can be sought. The same basic technique established above can be used here. In the present analysis, though, the data values taken in the time interval (t_a, t_b) are used, and the mean square function used is that given in (3a). The log likelihood function is

$$\ln L(b,\alpha) = \sum_i \left\{-\frac{1}{2}\ln(2\pi) - \ln\sigma_x(S,a,b,\alpha,t_i) - \frac{x_i^2}{2\sigma_x^2(S,a,b,\alpha,t_i)}\right\}. \tag{12}$$

It is taken to be a function of b and α because S and a have already been estimated. It is assumed that the maximum occurs where the partial derivatives of $\ln L$ with respect to b and α are zero. The partial derivative of $\ln L$ with respect to b is

$$\frac{\partial}{\partial b}\ln L = \sum_i \frac{2t_i e^{-\alpha(t_i-t_s)}}{[(1+a(t_i-t_c))+b(t_i-t_s)e^{-\alpha(t_i-t_s)}]}\left\{1 - \frac{x_i^2}{2S\Delta\Omega[(1+a(t_i-t_c))+b(t_i-t_s)e^{-\alpha(t_i-t_s)}]}\right\}. \tag{13}$$

and the partial derivative with respect to α is

$$\frac{\partial}{\partial \alpha}\ln L = -\sum_i \frac{2bt_i^2\, e^{-\alpha(t_i-t_s)}}{[(1+a(t_i-t_c))+b(t_i-t_s)e^{-\alpha(t_i-t_s)}]}\left\{1 - \frac{x_i^2}{2S\Delta\Omega[(1+a(t_i-t_c))+b(t_i-t_s)e^{-\alpha(t_i-t_s)}]}\right\}. \tag{14}$$

A numerical procedure must be used to find the minimum, and a computer program has been written to do this. The program starts at an arbitrary point on the surface of $\ln L$. The gradient at that point is computed. The surface of $\ln L$ along the gradient is assumed parabolic and the minimum is computed. At this point, a new gradient is computed and the computation cycle is repeated. The procedure is repeated until some convergence criterion is satisfied. The final location yields the values of b and α.

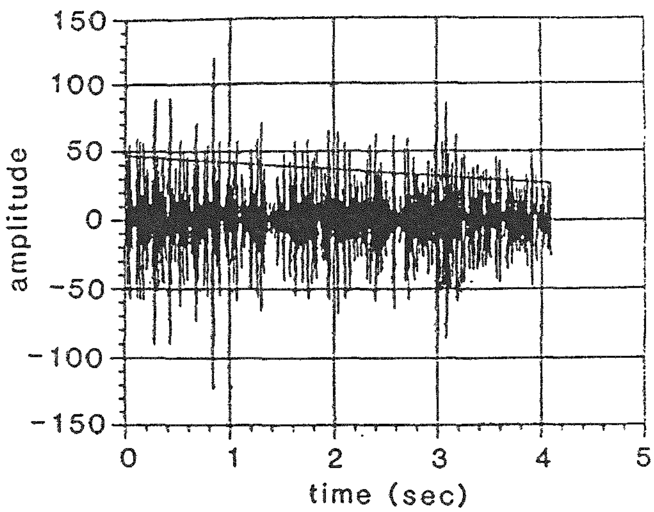

Figure 7 Nonstationary Random Signal and Scaled Modulating Function

The estimation procedure outlined above was used on some specific signals. In particular, some of the signals used in the previous section to demonstrate the random source model were analyzed. First, the signal shown in Figure 4 was analyzed using the above procedure. Table 2 summarizes the results showing actual underlying parameters and estimated parameters. Figure 7 shows the signal and the modulating function based on the estimated parameters. (The modulating function is scaled by the root-mean-square value of the random process.)

TABLE 2 Underlying and estimated parameters of a nonstationary random process.

	S	a	b
Actual parameters	1.0	-0.1	0
Estimated parameters	1.118	-0.136	-

In the second analysis, the signal shown in Figure 5 was analyzed. Table 3 summarizes the results showing the actual underlying parameters and the estimated parameters. Figure 8 shows the signal and the modulating function based on the estimated parameters. (The modulating function is scaled by the root-mean-square value of the random process.)

TABLE 3 Underlying and estimated parameters of a nonstationary random process.

	S	a	b	α
Actual parameters	1.0	0	25	3
Estimated parameters	1.092	0.019	36.4	3.76

These examples show that the maximum likelihood parameter estimation procedure can be used to accurately estimate the parameters of a nonstationary random process.

DISCUSSION AND CONCLUSIONS

Several elements are required to establish a shock test specification and shock testing procedure. Among these may be the establishment of a model for

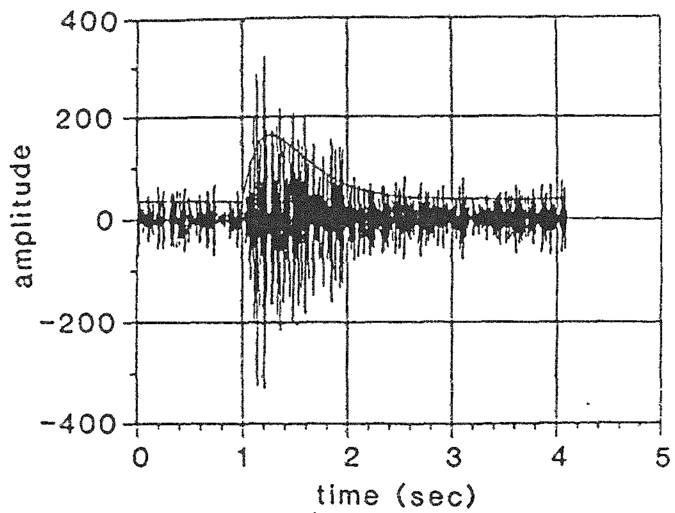

Figure 8 Nonstationary Random Signal and Scaled Modulating Function.

the shock test source and a method to estimate its parameters; these items were treated in the present investigation.

Other elements are also needed. Specifically, a method for assessing the quality of the parameter estimates for the shock source and a means for generating the excitation on an electrodynamic shaker are required. The former requirement can be satisfied when a statistical confidence analysis is performed on the estimated parameters of the shock signal. The latter requirement can be satisfield when existing software is modified or new software is developed to generate nonstationary random signals on an electrodynamic shaker.

The elements provided in this study describe part of a procedure to be used in specifying a shock test. It is possible to take the estimated parameters of the random process and use these to generate a test excitation by generating the band-limited white noise random process in (2) and superimposing components to form (1). Generally, though, some degree of conservatism will be sought, achievable by increasing the parameters S and b in each bandwidth. A systematic way for increasing S and b must be specified.

The technique established in this paper for estimating the parameters of a nonstationary random process assumes that the analyst will specify a number of parameters of the random process by inspection of the signal. For example, the start and end times of the superimposed pulse must be specified. To apply a source specification technique, a method must be developed for specifying almost all the parameters of the random process source, automatically. Automatic specification of model parameters can be done with the present model through the use of other statistical techniques.

The model proposed in this study is one of the simplest available for a nonstationary random process. Particularly, the superimposed pulse in (2) is characterized by only two parameters. The use of only two parameters for description of the pulse limits the spectrum of pulse shapes that can be modeled, but for preliminary purposes the present model is adequate. Other two-parameter models are available, but these are similarly limited. Models with more than two parameters can certainly be defined, and this objective should be pursued in the future. The problem with increasing the number of parameters in the model is that as the number of parameters estimated using a particular collection of data increases, the confidence interval of each estimate widens. Therefore, the parametric description should be limited.

Finally, note that the fundamental approach to shock source specification proposed in this paper is different from others currently in use. No assumption regarding the potential modes of damage in structures is made. An attempt is simply made to probabilistically characterize the shock source, then generate an excitation based on the source. If this approach is used, then excitations that directly simulate the probabilistic character of the actual source will be generated and the environments to which structures are subjected will be realistic.

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NOMENCLATURE

a,b	amplitude parameters of random process
i,j	time and frequency indices
n	number of points in a measured data sequence
p(·)	probability density function
t	time
t_c	central time in measured data sequence
t_s	start time of pulse
x_i	ith measured random process realization
H(·)	heaviside unit step function
L	likelihood function
N	number of components in random process
R	autocorrelation function
S	spectral density
X(t)	nonstationary random process
Z(t)	band-limited white noise random process
α	decay parameter of random process
σ^2	mean square of random process
ω	frequency
Ω	central frequency of random process component
$\Delta\Omega$	frequency band of random process component