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RADIAL MODE STRUCTURE OF CURVATURE-DRIVEN  
INSTABILITIES IN EBT

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**MASTER**

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\* Research sponsored by the Office of Fusion Energy, U.S. Department of Energy, under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Incorporated.

250

RADIAL MODE STRUCTURE  
OF CURVATURE - DRIVEN  
INSTABILITIES IN EBT

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Oak Ridge National Lab.

Hot Electron Physics Miniworkshop

Jan. 11, 1983

## FEATURES OF THIS CALCULATION:

- RETAINS NONLOCAL STRUCTURE OF MODES.
- CONNECTS INNER AND OUTER RING REGIONS TOGETHER IN ONE TREATMENT.
- A SELF-CONSISTENT FINITE  $\beta$  EQUILIBRIUM  $B$  FIELD IS USED.  
(including  $dB/dr$  and  $d^2B/dr^2$ )
- A WIDE RANGE OF EBT PARAMETERS HAVE BEEN EXAMINED.
- RELATIVISTIC EFFECTS ARE INCLUDED FOR THE HOT ELECTRON RING.

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## ASSUMPTIONS OF PRESENT CALCULATION

● FINITE LARMOR RADIUS EFFECTS NEGLECTED.

● BALLOONING EFFECTS NEGLECTED AND

$$\vec{B} \cdot \vec{\nabla} (\text{EQUILIBRIUM QUANTITIES}) = 0$$

● Z - PINCH GEOMETRY LOCALIZED TO RING  
REGION USED WITH OUTGOING ENERGY BOUNDARY  
CONDITIONS (NATURAL CURVATURE DRIFT).

● DELTA - FUNCTION HOT ELECTRON DISTRIBUTION:

$$F_{\text{hot}} = \frac{\delta p_{\perp H}}{\mu_0 B^2} \delta(p_{\parallel}) \delta(\mu - \mu_0)$$

● WARM CORE ELECTRONS, COLD IONS.

BASIC EQUATIONS

● MOMENTUM BALANCE:

$$\rho_i \mathbf{v}_i = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla \cdot \mathbf{P}$$

$$\mathbf{P} = \sum_i \int F_i \mathbf{v} \rho d^3 \rho$$

$$\mathbf{P} = n_e T \mathbf{v}$$

● THE HOT ELECTRON PRESSURE TENSOR MAY BE

WRITTEN AS:

$$\mathbf{P} = \sum_i \int \frac{dH d\mu B}{|p_{\parallel}|} F(r, H, \mu) [\mu B (\mathbf{I} - \mathbf{b}\mathbf{b}) + p_{\parallel}^2 \mathbf{b}\mathbf{b}]$$

where  $c^2 p_{\parallel}^2 = H^2 - 2\mu B c^2 - m_e c^4$

$$\mu = \frac{p_{\perp}^2}{2B}$$

●  $F$  is obtained from the drift kinetic equation:

$$\frac{\partial F}{\partial t} + v_{\parallel} \mathbf{b} \cdot \nabla F + \mathbf{v}_D \cdot \nabla F + H \frac{\partial F}{\partial H} = 0$$

where  $v_{\parallel} = \frac{p_{\parallel}}{\gamma} = \pm \frac{[H^2 - 2\mu B^2 c^2 - m^2 c^4]^{1/2}}{\gamma c^2}$

$$\mathbf{v}_D = \frac{\mu \mathbf{b} \times \nabla B}{q_j m_j B \gamma} + \frac{p_{\parallel}^2}{q_j m_j B \gamma} \mathbf{b} \times (\mathbf{b} \cdot \nabla) \mathbf{b} + \frac{E \times \mathbf{b}}{B}$$

$$H = q_j E_{\parallel} v_{\parallel} + q_j E_{\perp} \cdot \mathbf{v}_D + \frac{\mu}{\gamma m_j} \frac{\partial B}{\partial t}$$

● These equations are then combined, transformed to  $z$  - pinch geometry, and linearized about perturbed fields  $E_{\perp}$  and  $B_{\parallel}$ . These may be characterized by a "displacement"  $\xi$ :

$$\underline{\xi} = i \frac{E \times \mathbf{b}}{\omega B} \exp[-i\omega t + ikz]$$

THIS RESULTS IN A SECOND ORDER EQUATION FOR  $\xi_r$ :

$$\frac{d}{dr} \left( r P \frac{d\xi_r}{dr} \right) - Q \xi_r = 0$$

where  $P = \frac{\lambda B^2 (1 + G_1)}{D v_A^2}$

$$Q = \frac{r B^2}{v_A^2} \left[ \frac{k^2 \lambda (1 + G_1)}{D} - \frac{\omega^2 \lambda}{v_A^2 D} \right]$$

$$- \frac{k^2 \mu_0 v_A^2}{r B^2} \frac{d}{dr} (P_H + P_C) - \frac{k^2}{r^2} v_A^2 (\sigma + G_3)$$

$$+ \frac{2 \lambda \omega k (1 - G_2)}{r \omega_{ci} D} + \frac{k^2 v_A^2 (1 - G_2)^2}{r^2 D}$$

$$+ \frac{v_A^2}{r B^2} \frac{d}{dr} \left( \frac{r \lambda B^2 S_j}{v_A^2 D} \right)$$

$$S = \frac{1 - G_2}{r} + \frac{\omega k (1 + G_1)}{\omega_{ci}}$$

$$D = 1 + G_1 - \frac{\lambda}{k^2 v_A^2}$$

$$\lambda = \frac{\omega^2 \omega_{ci}^2}{\omega_{ci}^2 - \omega^2}$$



$G_1, G_2, G_3$  ARE KINETIC INTEGRALS WHICH INVOLVE THE HOT ELECTRON DISTRIBUTION FUNCTION:

$$G_1 = -B \sum_i \int \frac{dp_{\perp} d\mu \mu^2}{T m_i} \left[ \frac{1}{B} \frac{\partial F}{\partial \mu} + \frac{L_i^* F}{T \Omega} \right]$$

$$G_2 = - \sum_i \int \frac{dp_{\perp} d\mu \mu}{T m_i} p_{\parallel}^2 \left[ \frac{1}{B} \frac{\partial F}{\partial \mu} + \frac{L_i^* F}{T \Omega} \right]$$

$$G_3 = - \sum_i \int \frac{dp_{\perp} d\mu p_{\parallel}^4}{B T m_i} \left[ \frac{1}{p_{\parallel}} \frac{\partial F}{\partial p_{\parallel}} + \frac{L_i^* F}{T \Omega} \right]$$

where  $\Omega = \omega - \omega_{DB} - \omega_{cv}$

$$L_i^* = \left[ \frac{k}{m_i q_i B} \frac{\partial F}{\partial r} + \frac{T \omega_{cv}}{p_{\parallel}} \frac{\partial F}{\partial p_{\parallel}} \right]$$

THE INTEGRALS  $G_1$ ,  $G_2$ ,  $G_3$  DEPEND ON THE MODEL USED FOR THE HOT ELECTRON DISTRIBUTION.

FOR THE CASE OF A DELTA FUNCTION WITH NO PARALLEL ENERGY:

$$F_{\text{Hot}} = \frac{\tau_0 p_{\perp H}}{\mu_0 B^2} \delta(p_{\parallel}) \delta(\mu - \mu_0)$$

$$1 + G_1 = (\omega - \omega_{DB})^{-1} \left\{ \omega \left[ 1 + \beta_{\perp} - \beta_{\perp H} \frac{\mu_0 B}{2 \tau_0^2 m_e^2 c^2} \right] - \omega_{cvl} \left[ 1 + r \frac{d}{dr} \left( \frac{p_{\perp c}}{B^2} \right) \right] \right\}$$

$$\text{where } \omega_{cvl} = \frac{k \mu_0}{e n_e \tau_0 r}$$

$$\omega_{DB} = \frac{k \mu_0}{e n_e \tau_0 B} \frac{dB}{dr}$$

$$G_2 = \frac{\beta_{\perp c}}{2}$$

$$G_3 = \frac{3 \beta_{\perp H c}}{2}$$

Using these forms for  $G_1, G_2, G_3$  which are accurate for arbitrary  $\Delta B/R_c$  and  $w/w_{db}$  one can make a local approximation:

$$\frac{d^2 \xi_r}{dr^2} = -k_r^2 \xi_r, \quad k_r = \frac{2}{\Delta}$$

$$\frac{d \xi_r}{dr} = 0$$

$$\frac{1}{n_c} \frac{dn_c}{dr} = -\frac{1}{\Delta}$$

To obtain a fifth order dispersion relation:

$$y^5 + Ay^4 + By^3 + Cy^2 + Dy + E = 0$$

$$\text{where } y = \frac{\omega}{\omega_{cvt}}$$

## A. High Frequency Modes

Compressional Alfvén

Hot Electron interchange ( $q_0 \gg 1$ )

Retaining A, B and C terms in the 5th order eqn.,  $B^2 = 4AC$  gives the marginal stability boundary:

$$P \left\{ P^2 \left( 1 + \frac{2\tilde{\beta}_c}{\tilde{\beta}_H} \right)^2 - 2P \left[ \left( 1 + \frac{1}{q} + \tilde{\beta}_c \right) \left( 1 + \frac{2\tilde{\beta}_c}{\tilde{\beta}_H} \right) - \frac{4\tilde{\beta}_c}{\tilde{\beta}_H} \right] + \left( 1 - \frac{1}{q} - \tilde{\beta}_c \right)^2 \right\} = 4 \left( \frac{\Delta}{R} \right)^2 (\tilde{\beta} - 1) \tilde{\beta}_H \left[ P \tilde{\beta}_c \left( 1 + \frac{2\tilde{\beta}_c}{\tilde{\beta}_H} - \frac{2}{\tilde{\beta}_H} \right) + \frac{1}{q} (1 - \tilde{\beta}_c) \right]$$

→ Cubic in  $p = n_H / n_i$

2 roots near  $p \approx 1$  → hot electron interchange

1 root at  $p < (\Delta/R)^2 \ll 1$  → compressional Alfvén

$$\left( \tilde{\beta} = \frac{R_c}{2\Delta} \beta, \quad p = \frac{n_H}{n_i}, \quad q = \left( \frac{k}{k_\perp} \right)^2 \frac{V_{cv\perp}}{\Delta \omega_{ci}} \right)$$

## Hot Electron Interchange

$p \approx 1 \rightarrow$  neglect R.H.S.  $\rightarrow$  quadratic  
in  $p$ : only one physical  
solution (i.e. with  $p < 1$ ):

$$p < p_1 = \left[ 1 - \left( \frac{1}{q} + \tilde{\beta}_c \right)^{1/2} \right]^2$$

note: if  $q < (4 - \tilde{\beta}_c)^{-1} \rightarrow p > 1$   
and this mode stabilizes  
(actually one goes over to  
low freq. hot interchange)

## Compressional Alfvén Mode

$p \ll 1$  root  $\Rightarrow$  neglect  $p^2$  and  $p^3$   
terms

$$p > p_2 = \frac{1}{q} \left( \frac{2\Delta}{R} \right)^2 \tilde{\beta}_H (\tilde{\beta} - 1) (1 - \tilde{\beta}_c) \left( 1 - \frac{1}{q} - \tilde{\beta} \right)$$

note: when  $\tilde{\beta}_c = 1$  mode stabilizes

$$\text{also, when } 1 - \frac{1}{q} - \tilde{\beta}_c = 0 \quad p_2 \rightarrow \infty$$

$$\left( q = \left( \frac{k}{k_L} \right)^2 \frac{v_{cv}}{\Delta \omega_{ci}} \right) \quad p_1 \rightarrow 0$$

## Low Frequency Modes ( $\omega < \omega_{ci}$ )

- A. Low-frequency hot electron interchange  
( $q \ll 1$ )

Keep B, C, D terms

(justified when  $q \ll 1$  and  $\beta_c \ll \beta_H$ )

$C^2 = 4BD$  gives:

$$p < p_3 = \frac{1}{4} (k_{\perp} \Delta)^2 q_0 (1 - \tilde{\beta}_c)^2$$

- B. Low-frequency background interchange

Keep C, D, and E terms ( $\beta_c \ll \frac{2\Delta}{R_c}$ )

Stability achieved in 2 ways:

$$\beta_H > \frac{4\Delta}{R_c} - 2\beta_c \quad (\text{drift reversal})$$

or

$$\frac{n_H}{n_i} \gtrsim 8q_0 (k_{\perp} \Delta)^2 \left( \frac{\beta_c}{\beta_H} \right) \quad \text{for } \tilde{\beta}_H \gg 1, \beta_c / \beta_H < 1$$

(charge uncovering,  $n_{ce} \neq n_{ci}$ )

frequency shift  $\propto (n_{ci} - n_{ce}) / n_{ce} = p$

C. Interacting Background Interchange  
 ( $\beta_c \sim 2\Delta/R_c$ )

Keeping B, C, D and E terms:

$$[1 + q(1 - \tilde{\beta}_c)]y^3 - (1 - \tilde{\beta}_c)y^2 + \tau \left(1 + \frac{2\tilde{\beta}_c}{\tilde{\beta}_H}\right)y + \tau \tilde{\beta}_c \left(1 + \frac{2\tilde{\beta}_c}{\tilde{\beta}_H} - \frac{2}{\tilde{\beta}_H}\right) = 0$$

where  $\tau = P/q_0 (k_\perp \Delta)^2$

note:  $\tau < 1$  is stability condition for low-freq. hot electron interchange

assuming  $\tau \ll (1 - \tilde{\beta}_c)^2/3$  one finds that for  $(1 - \tilde{\beta}_c)$  small, 2 roots of the cubic coalesce and lead to instability if:

$$1 - \tilde{\beta}_c < 3 \left[ \frac{\tau}{4} \left(1 + \frac{2\tilde{\beta}_c}{\tilde{\beta}_H} - \frac{2}{\tilde{\beta}_H}\right) \right]^{1/3}$$

## NUMERICAL SOLUTION METHOD

- Radial equation:  $\frac{1}{r} \frac{d}{dr} \left( r P \frac{d\psi}{dr} \right) - Q\psi = 0$
- Equivalent to 2 coupled first order equations:

$$\begin{cases} \frac{dy_1}{dr} = \frac{y_2}{r \cdot P} \\ \frac{dy_2}{dr} = r Q y_1 \end{cases}$$

where  $y_1 = \psi$ ,  $y_2 = r P \frac{d\psi}{dr}$

- Solve as 2-point boundary value problems on intervals  $r_{min}$  to  $r_m$  and  $r_m$  to  $r_{max}$  using the SUPPORT code.
- Powell's hybrid method is used to solve for  $w$  by matching:

$$\left. \frac{dy_1}{dr} \right|_{r=r_m^+} = \left. \frac{dy_1}{dr} \right|_{r=r_m^-}$$



PLASMA AND HOT ELECTRON PROFILES

ARE CHOSEN WHICH ARE FLAT INSIDE

AND OUTSIDE THE RING AND WHICH

HAVE CONTINUOUS DERIVATIVES.

$$P_{LH}(r) = P_{LH0} \begin{cases} 0 & x < -x_0 \\ \frac{(x-x_0)^4 (x+x_0)^4}{x_0^8} & -x_0 < x < x_0 \\ 0 & x > x_0 \end{cases}$$

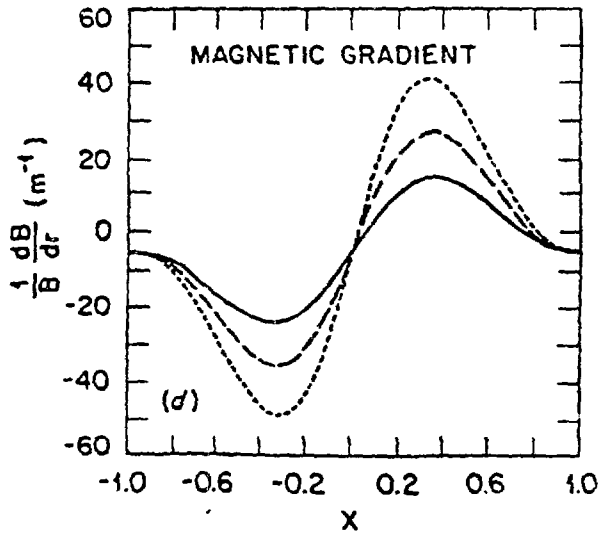
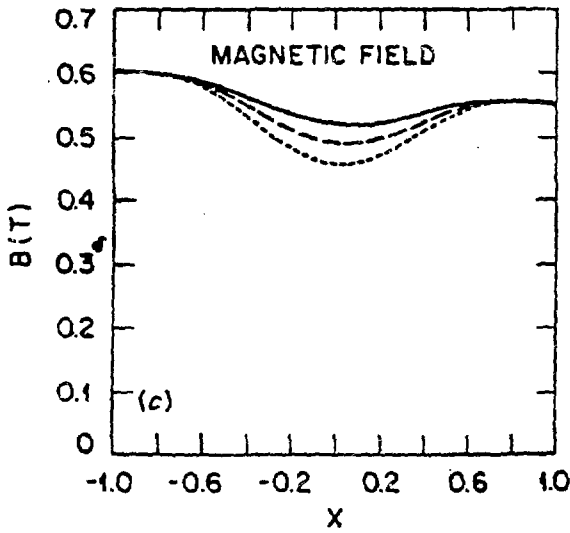
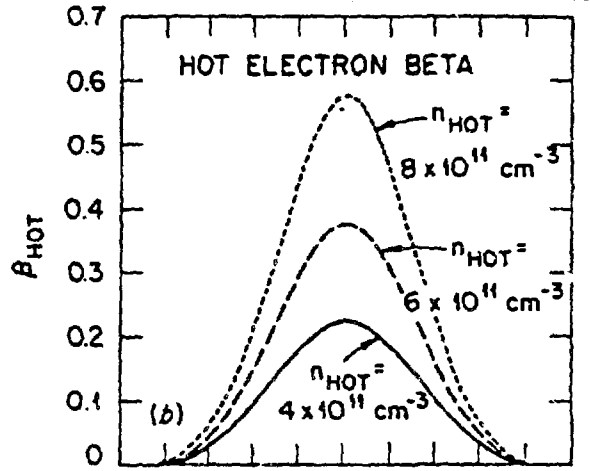
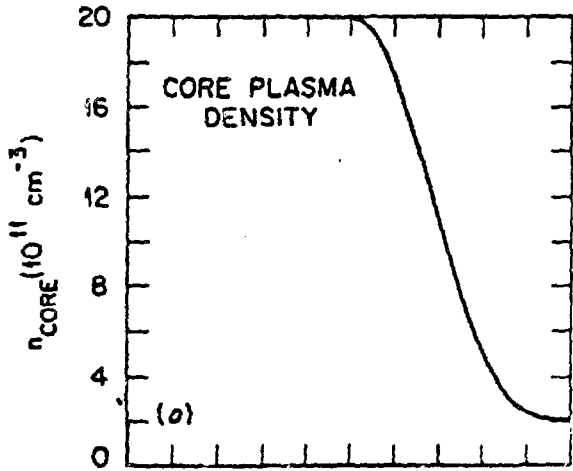
$$P_c(r) = P_{c0} \begin{cases} 1 & x < 0 \\ (1-s) \frac{(x-x_0)^4 (x+x_0)^4}{x_0^8} + s & 0 < x < x_0 \\ s & x > x_0 \end{cases}$$

where  $x = \frac{r-r_0}{\Delta}$

$\Delta$  = hot electron annulus half-width

$x_0$  = parameter which controls extent of pressure profile

$s$  = density shelf factor



A MAGNETIC EQUILIBRIUM MODEL  
IS USED WITH FIELD LINE  
CURVATURE APPROPRIATE TO  
Z-PINCH GEOMETRY ( $\kappa = -1/r$ ).

FROM:

$$\vec{\nabla} \left( p_{\perp} + \frac{B^2}{2\mu_0} \right) = \vec{\kappa} B^2 \left[ 1 - \frac{\mu_0 (p_{\parallel} - p_{\perp})}{B^2} \right]$$

ONE HAS:

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{B^2}{2\mu_0} \right) = - \frac{dp_{\perp}}{dr} - \frac{p_{\perp H}}{r}$$

WHICH CAN BE INTEGRATED:

$$B = \frac{1}{r} \left\{ r_i^2 B_i^2 - 2r^2 \mu_0 (p_{\perp H} + p_{\perp C}) - 2\mu_0 \int_{r_i}^r r' dr' (p_{\perp H} + 2p_{\perp C}) \right\}^{1/2}$$

100-

OUTGOING ENERGY / EVANESCENT  
BOUNDARY CONDITIONS ARE USED  
AT INSIDE AND OUTSIDE OF  
ANNULUS.

$$\frac{d}{dx} \left( P \frac{d\zeta}{dx} \right) - Q\zeta = 0$$

OUTSIDE ANNULUS REGION,

$P, Q \approx \text{constant to } O\left(\frac{1}{r}\right)$

$\therefore$  solution is matched onto plane waves  $\zeta \sim e^{ik_x x}$

$k_x = \pm i \sqrt{Q/P}$

$\pm$  sign determined by:

(1)  $k_r > k_i$  outgoing energy

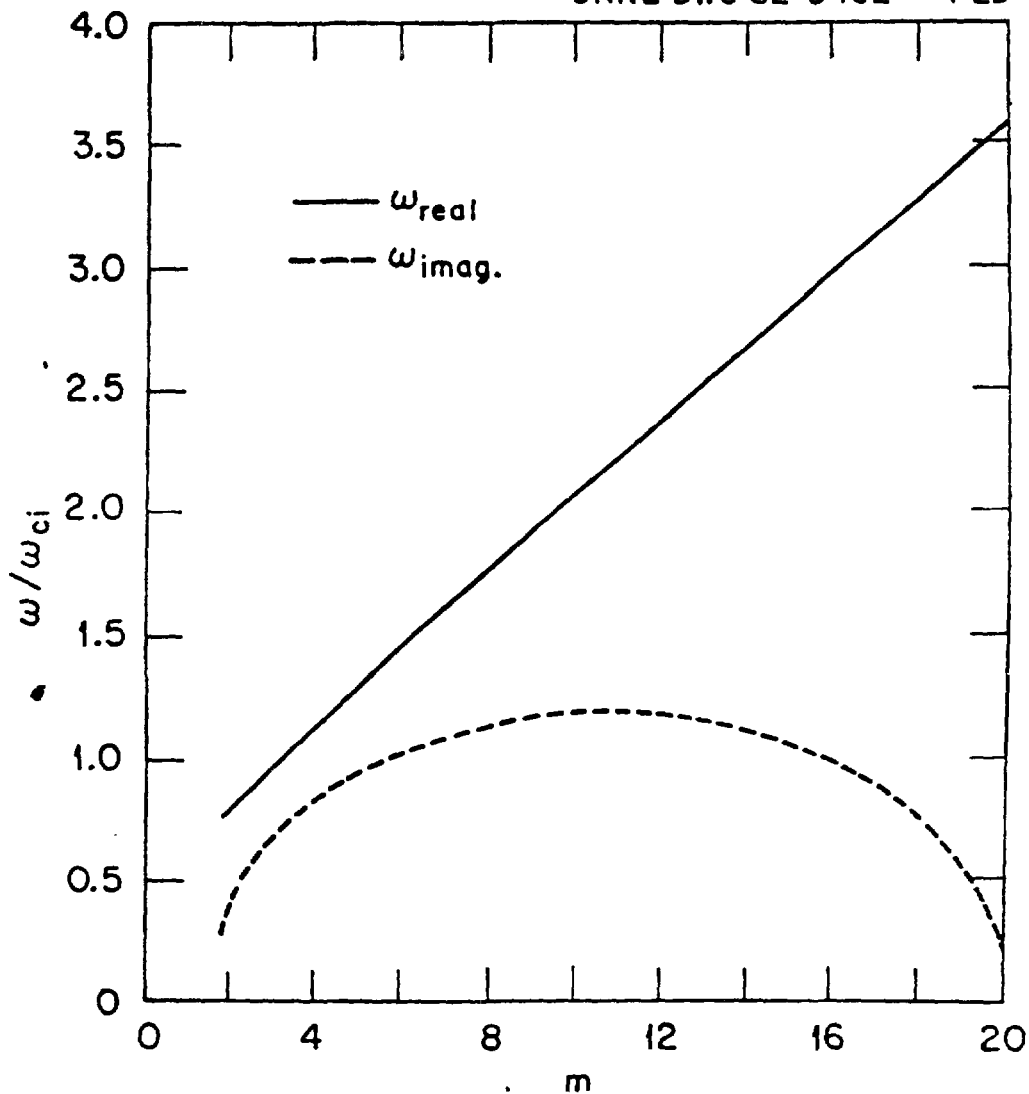
$$\frac{\partial \omega}{\partial k} \geq 0 \quad \text{for } x \geq 0$$

(2)  $k_i > k_r$  evanescent

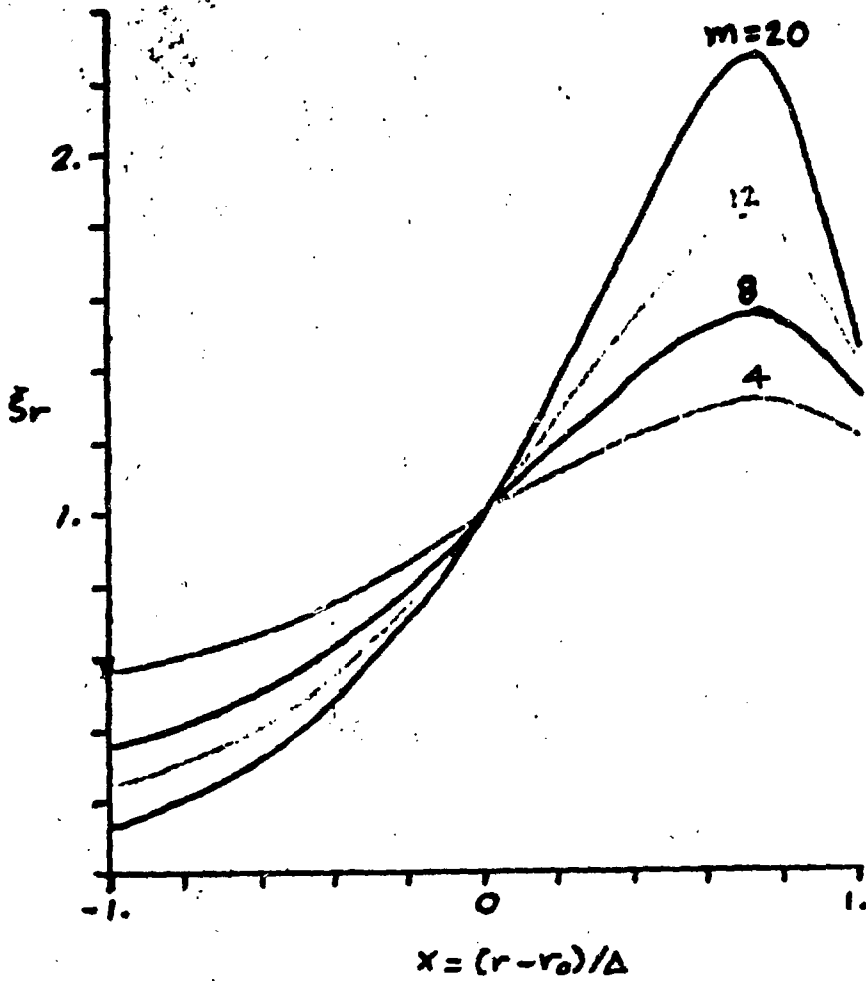
$$k_i \geq 0 \quad \text{for } x \geq 0$$

DEPENDENCE OF FREQUENCY ON AZIMUTHAL MODE NUMBER  
FOR HOT ELECTRON INTERCHANGE ( $N_{\text{core, elec.}} = 5 \times 10^{11} \text{ cm}^{-3}$ ,  
 $N_{\text{hot}} = 5 \times 10^{11} \text{ cm}^{-3}$ ,  $T_{\text{core}} = 0$ ,  $T_{\text{hot}} = 500 \text{ keV}$ )

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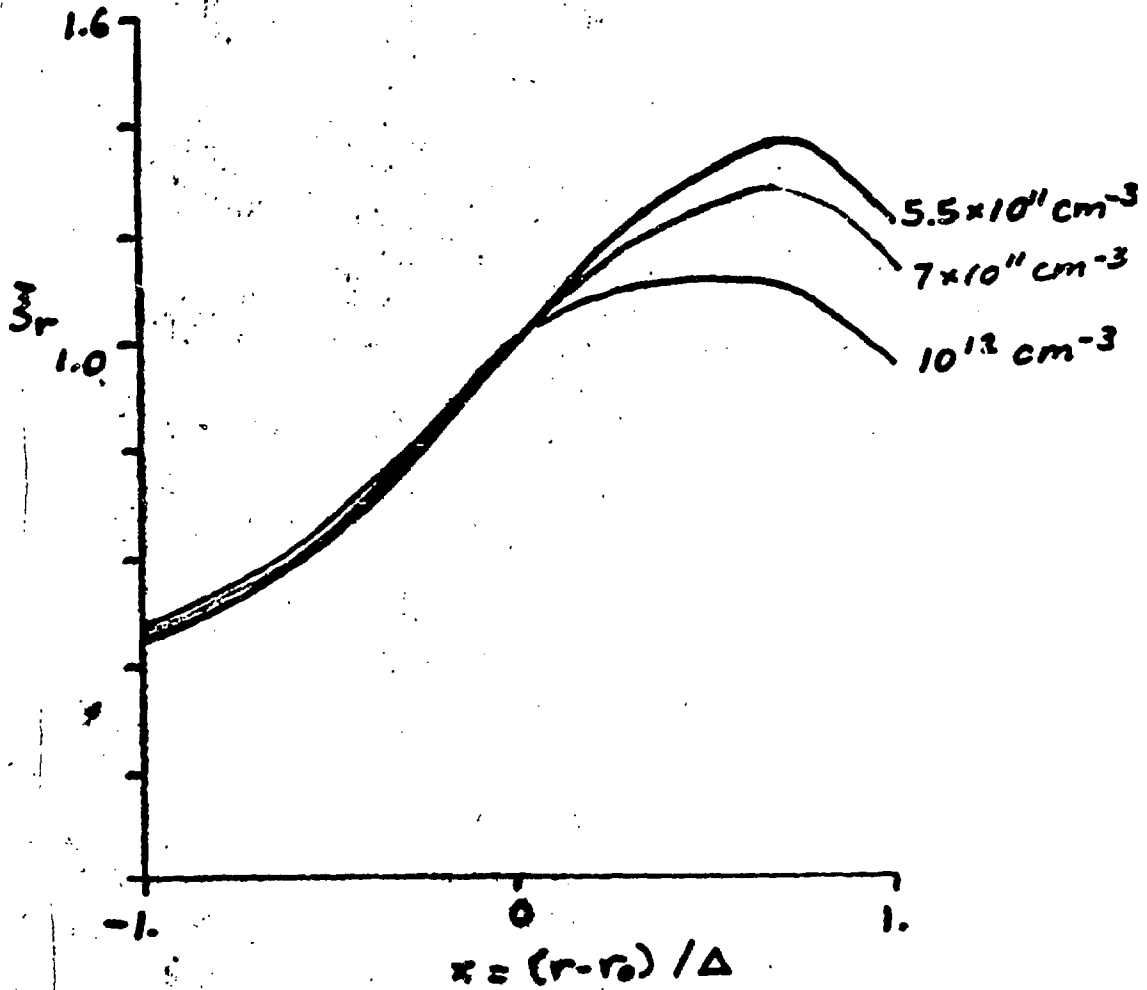


STRUCTURE OF HOT ELECTRON INTERCHANGE VS. MODE NO.



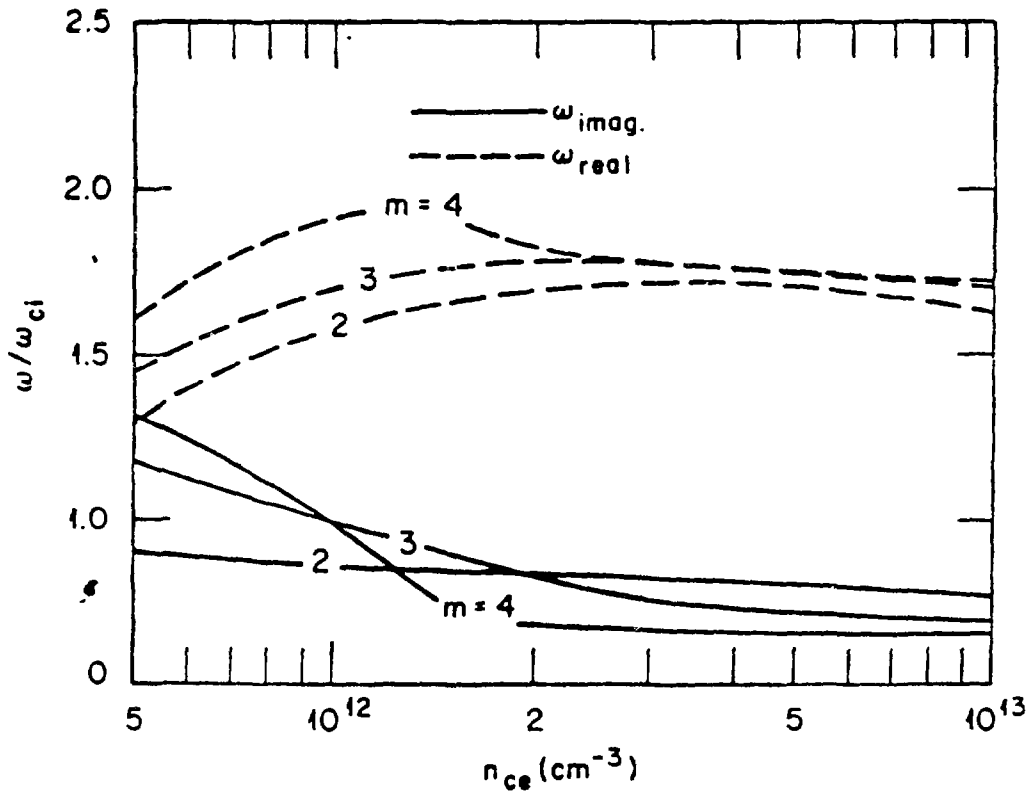
HOT ELECTRON INTERCHANGE MODE  
STRUCTURE VS. CORE DENSITY

( $m = 6$ )



### HOT ELECTRON INTERCHANGE ( $m = 2-4$ )

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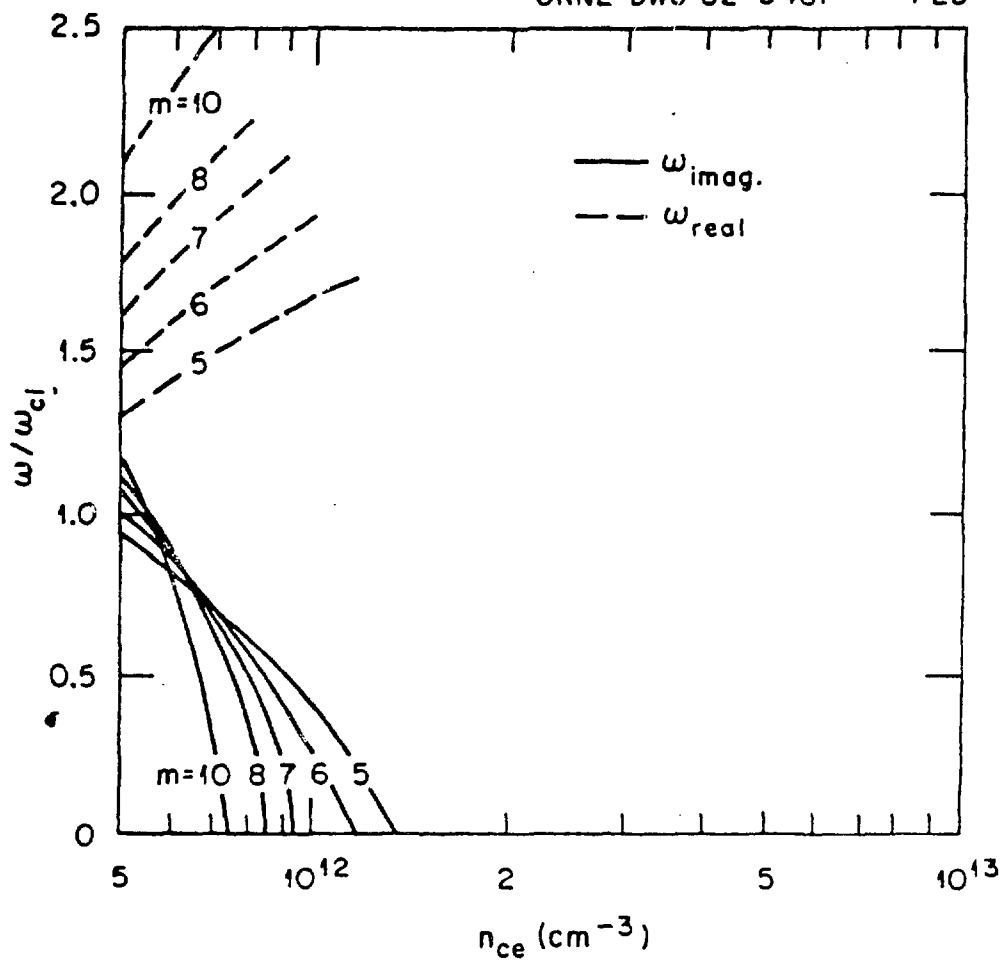


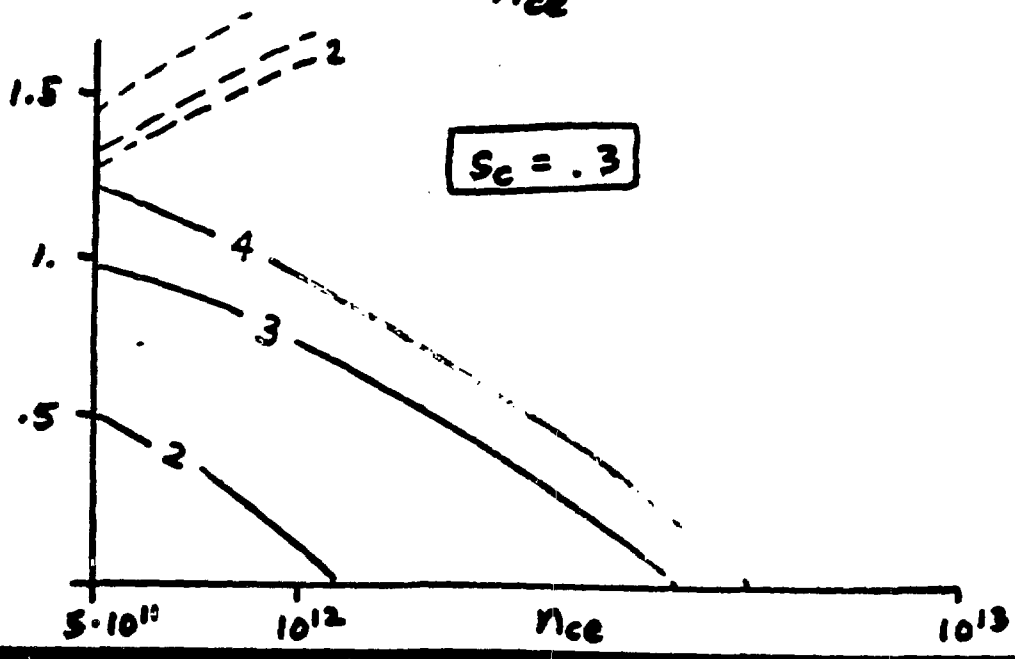
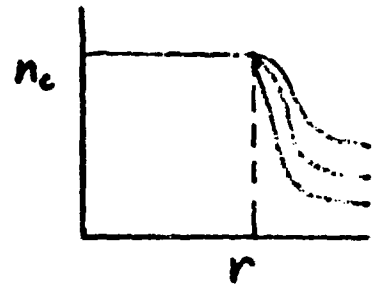
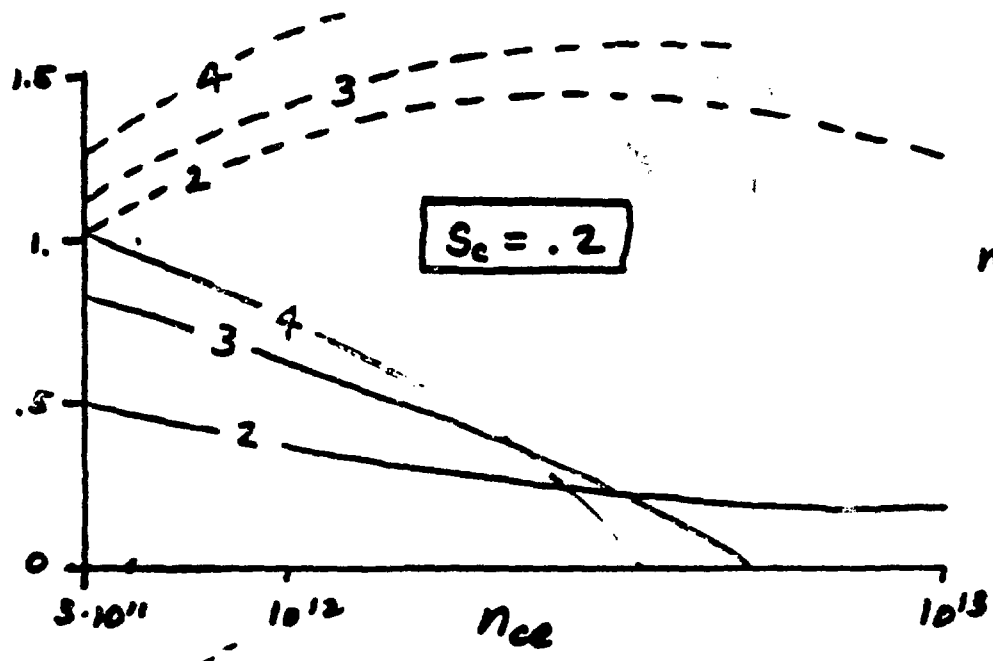
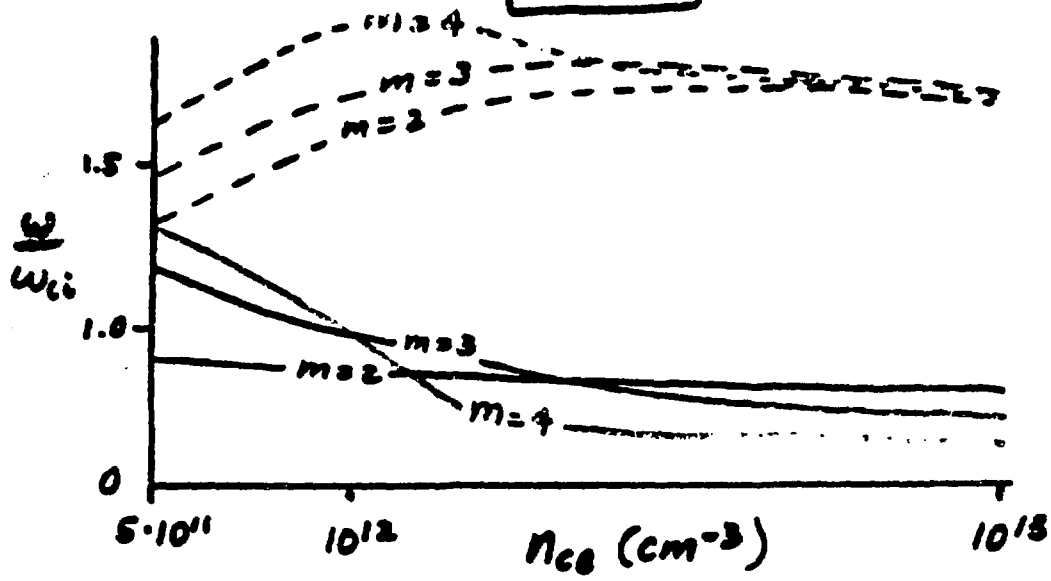


### HOT ELECTRON INTERCHANGE ( $m = 5-10$ )

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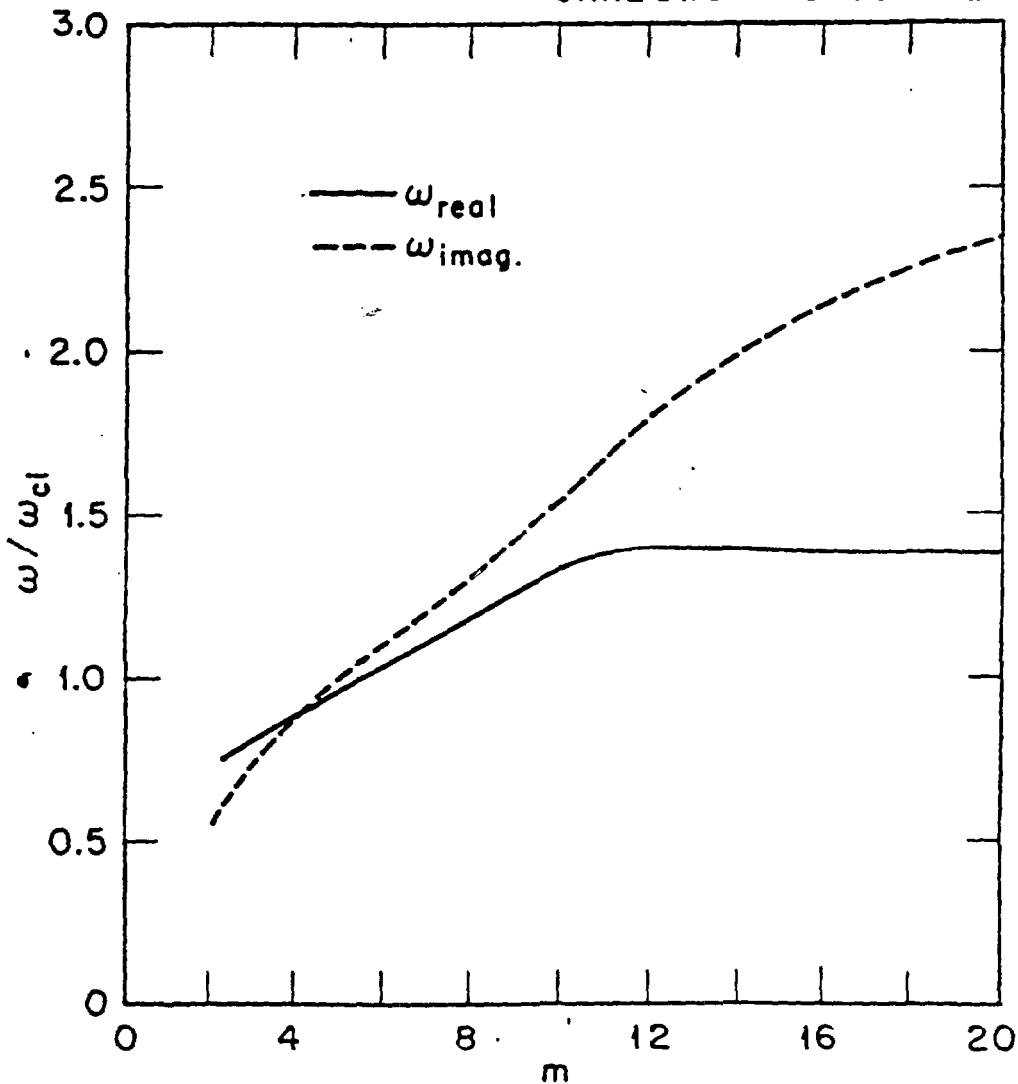
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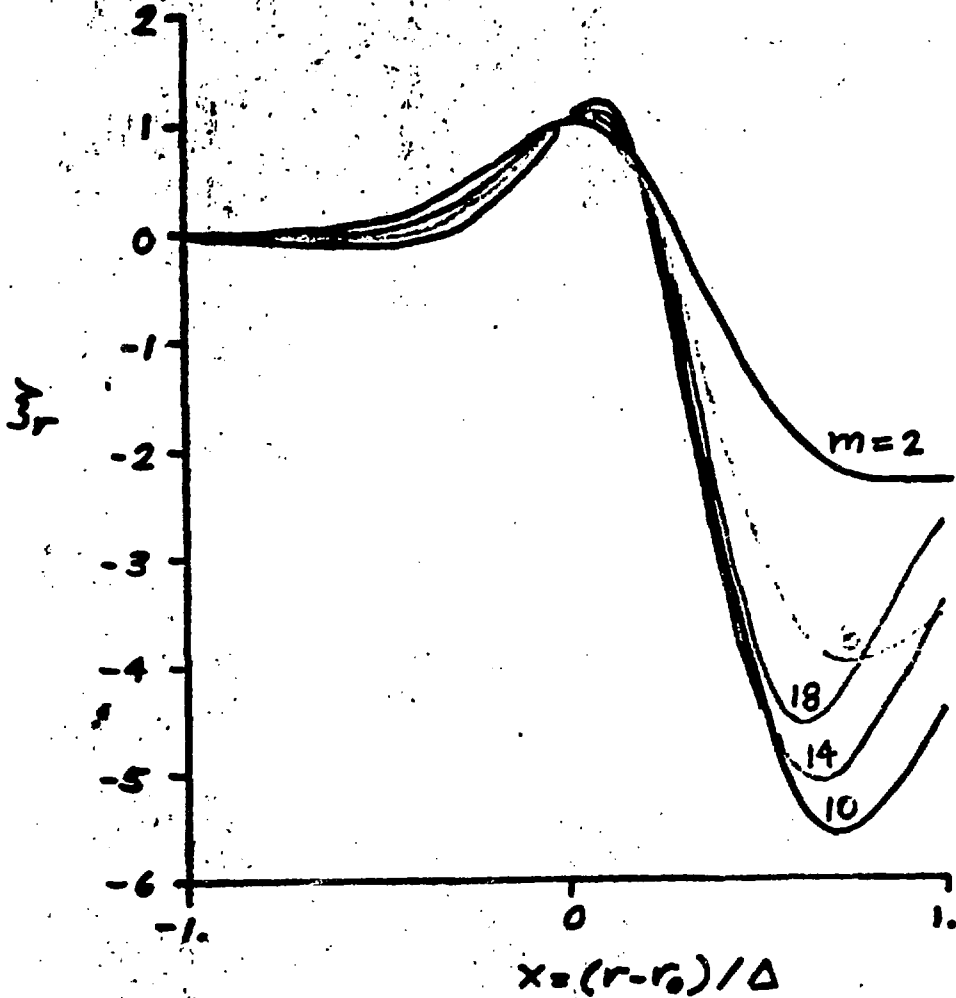


DEPENDENCE OF FREQUENCY ON AZIMUTHAL MODE NUMBER  
FOR COMPRESSIONAL ALFVEN MODE ( $N_{\text{core}} = 7 \times 10^{14} \text{ cm}^{-3}$ ,  
 $T_{\text{core}} = 0$ ,  $N_{\text{hot}} = 5 \times 10^{11} \text{ cm}^{-3}$ ,  $T_{\text{hot}} = 500 \text{ keV}$ )

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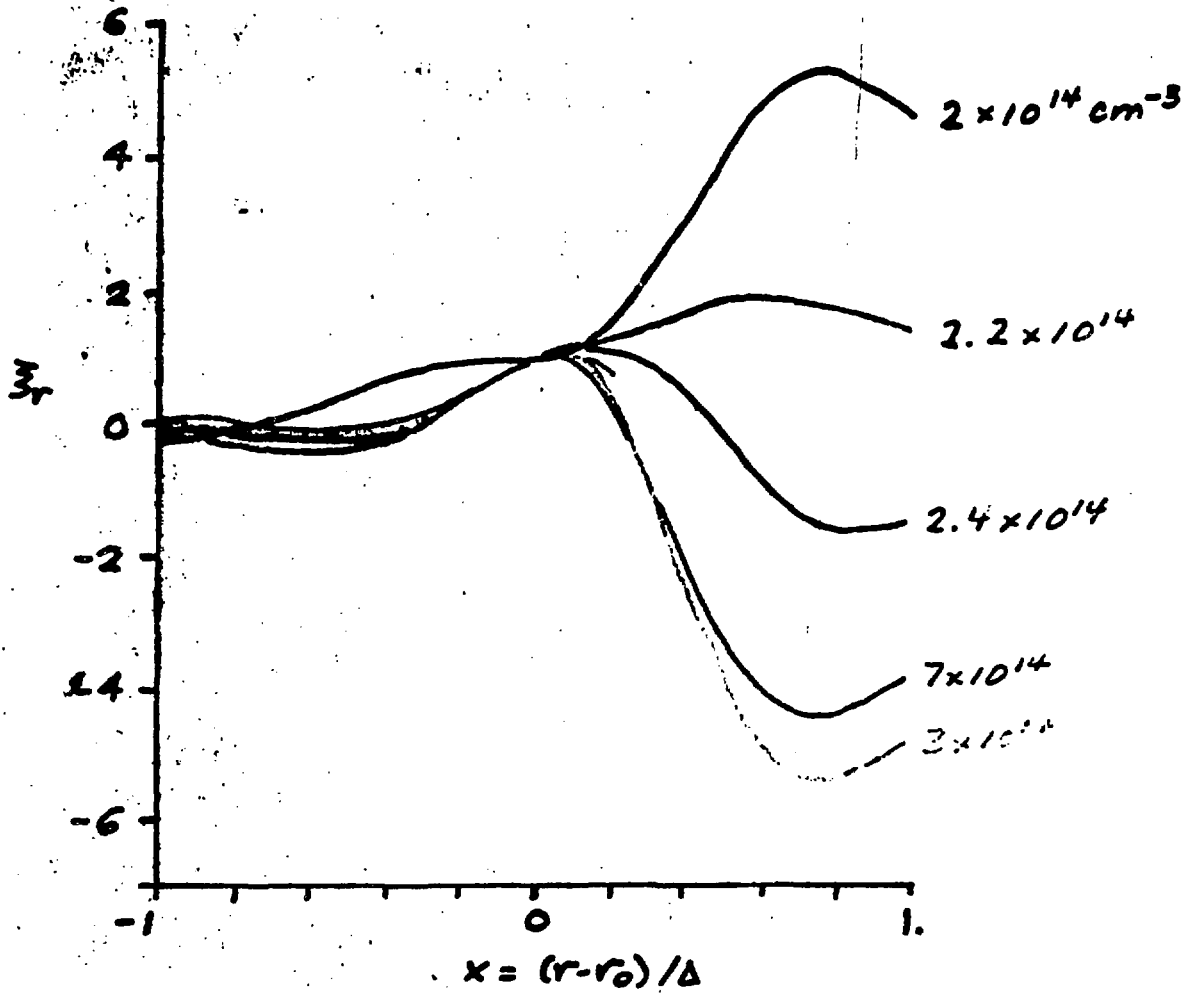


COMPRESSIONAL ALFVEN MODE  
STRUCTURE VS. MODE NO.



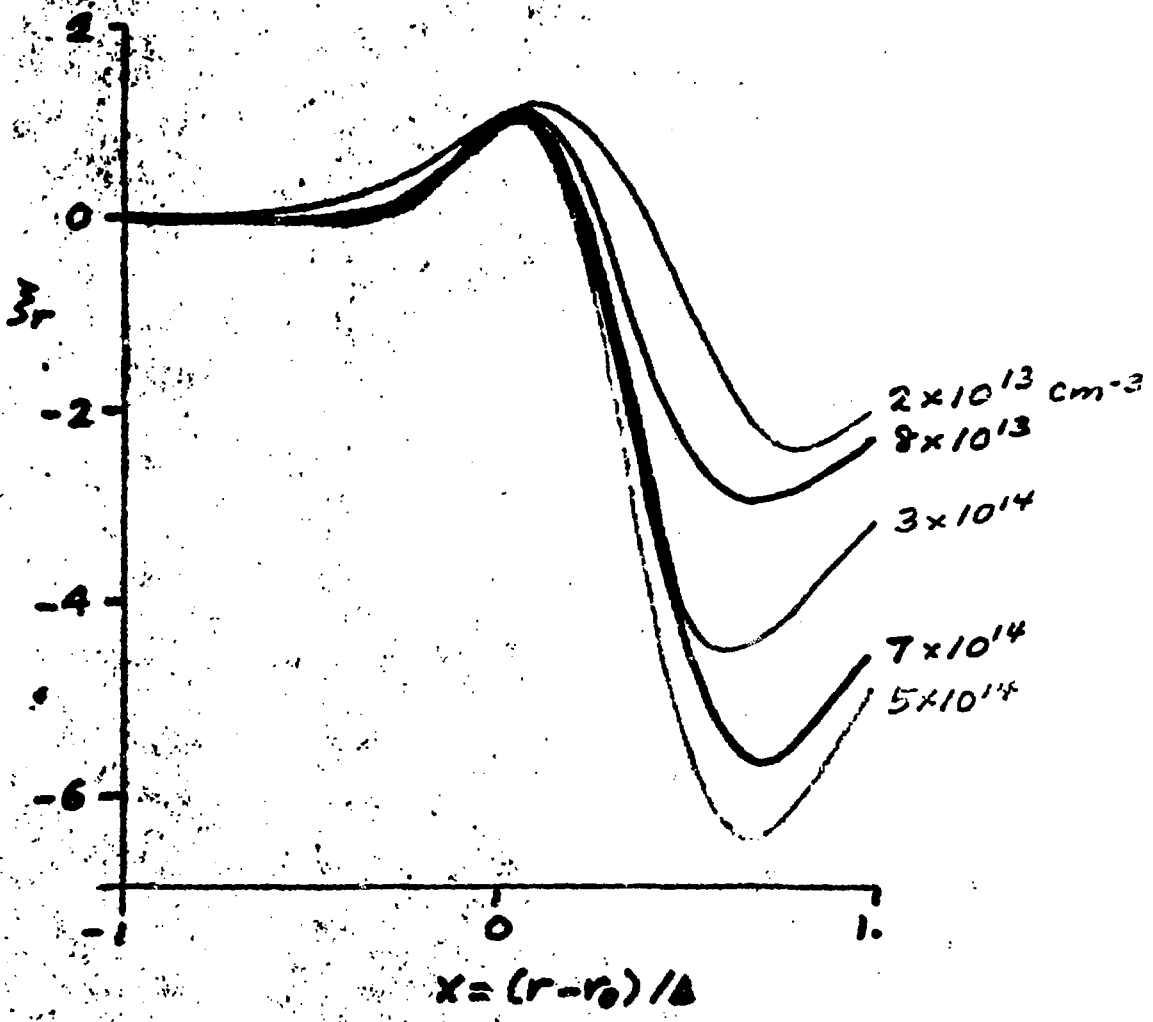
COMPRESSIONAL ALFVEN MODE  
STRUCTURE VS. CORE DENSITY

( $m=7$ )



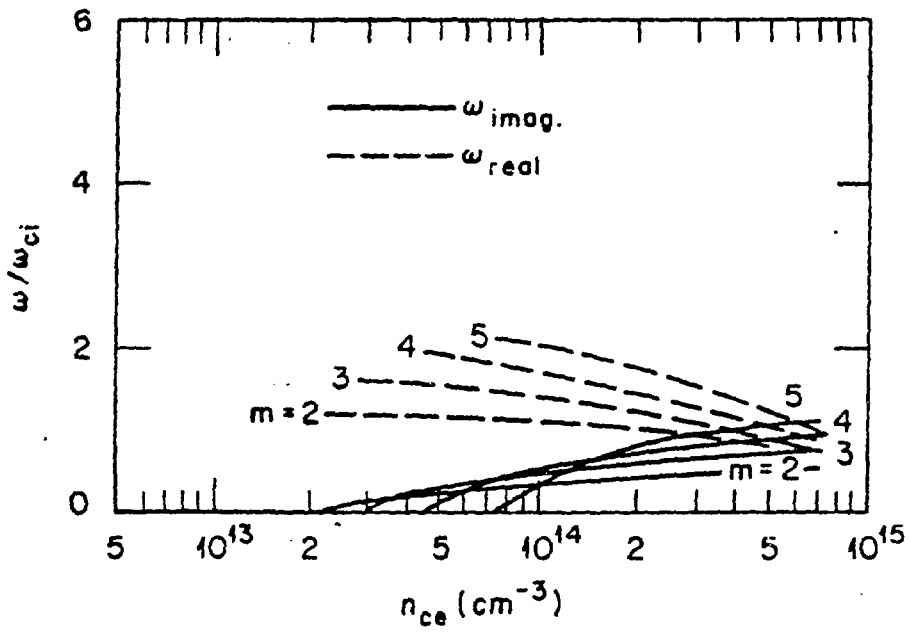
# COMPRESSIONAL ALFVEN MODE STRUCTURE VS. CORE DENSITY

( $m = 10$ )



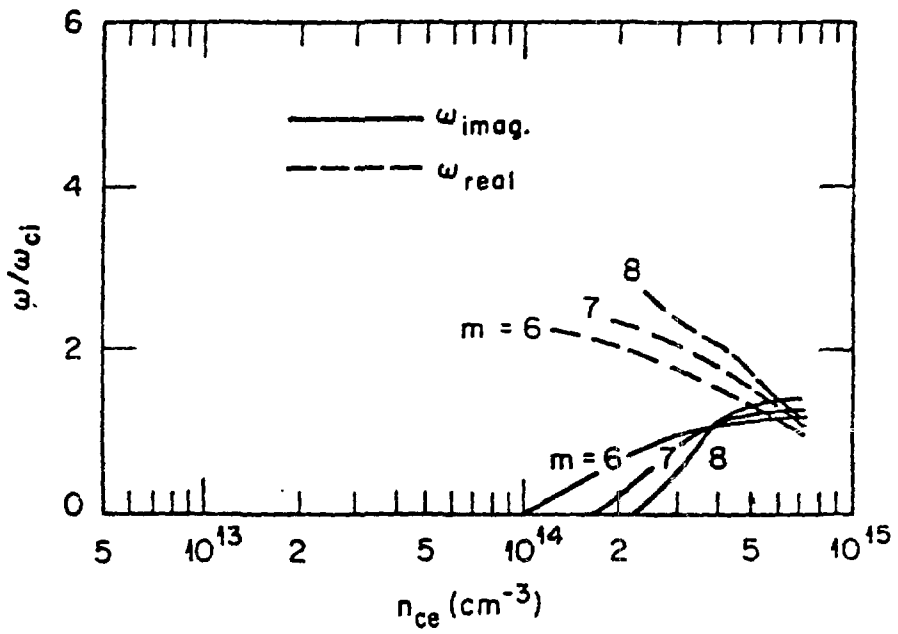
### COMPRESSIONAL ALFVÉN WAVE ( $m = 2-5$ )

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# COMPRESSIONAL ALFVÉN WAVE (m = 6-8)

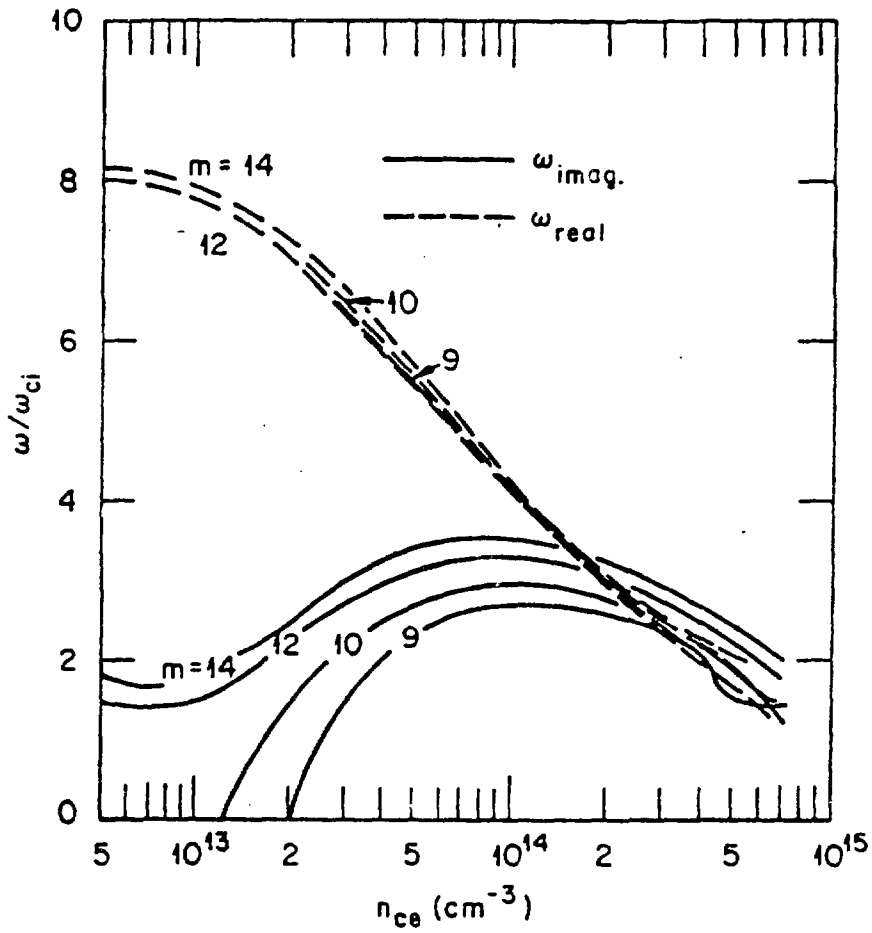
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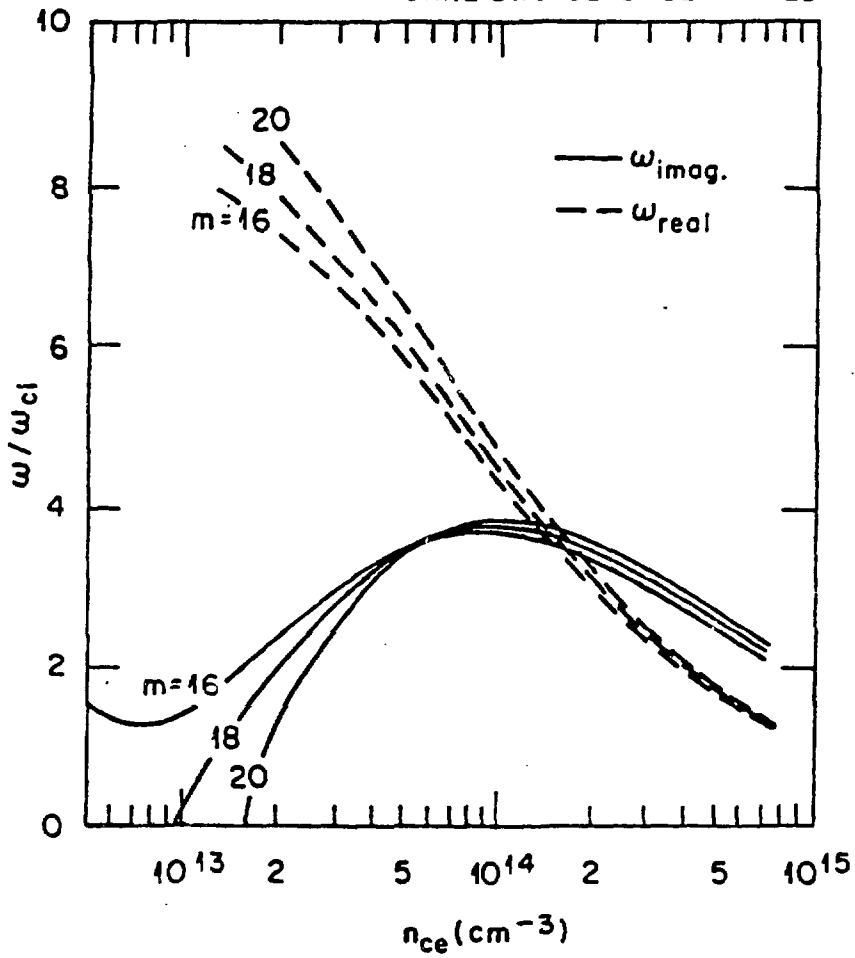
### COMPRESSIONAL ALFVÉN WAVE (m = 9-14)

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### COMPRESSIONAL ALFVÉN WAVE ( $m = 16-20$ )

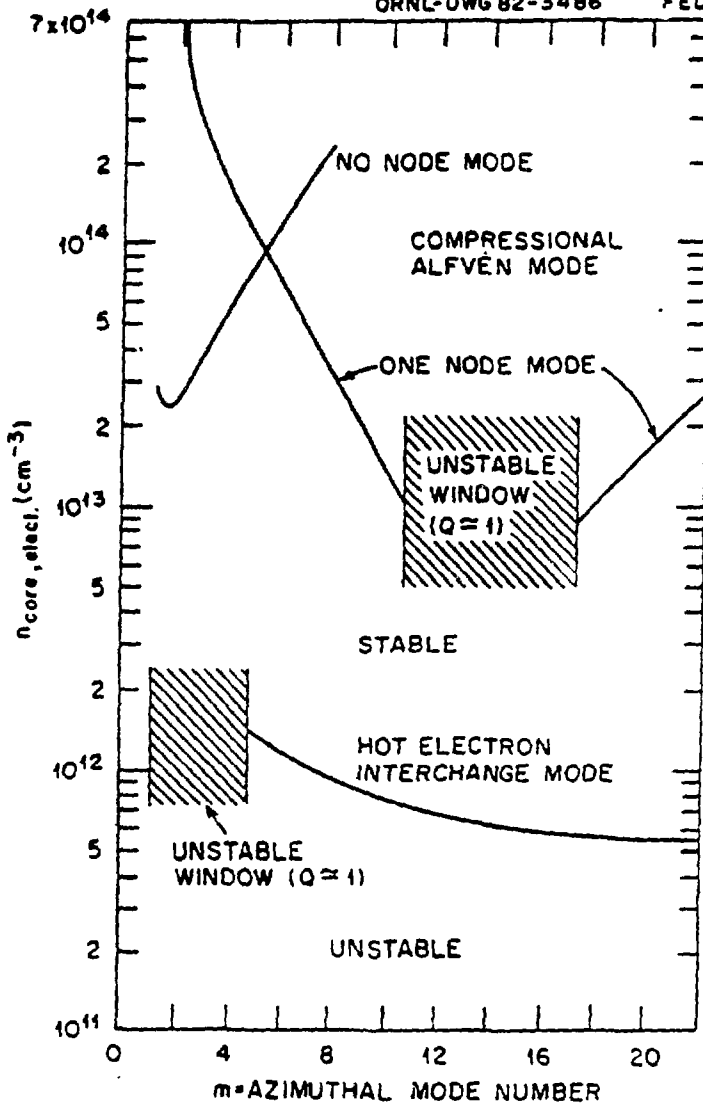
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HIGH FREQUENCY MODE STABILITY BOUNDARIES  
AS A FUNCTION OF  $m$  FOR  $T_{core} = 0$ ,  
 $N_{hot} = 5 \times 10^{11} \text{ cm}^{-3}$ ,  $T_{hot} = 500 \text{ keV}$

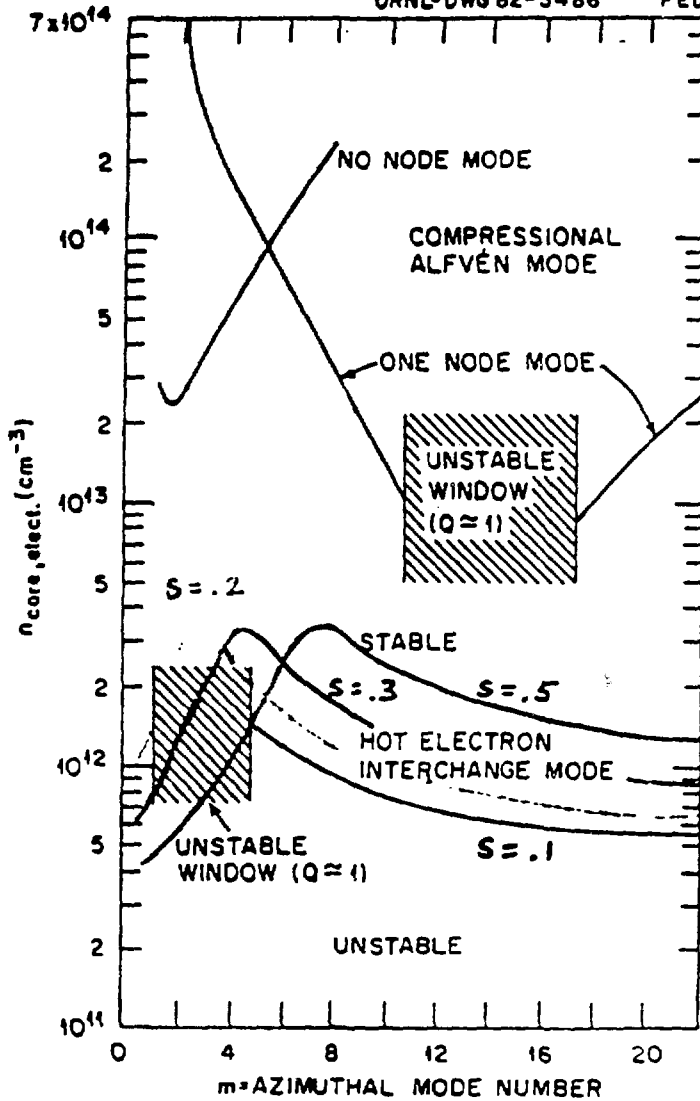
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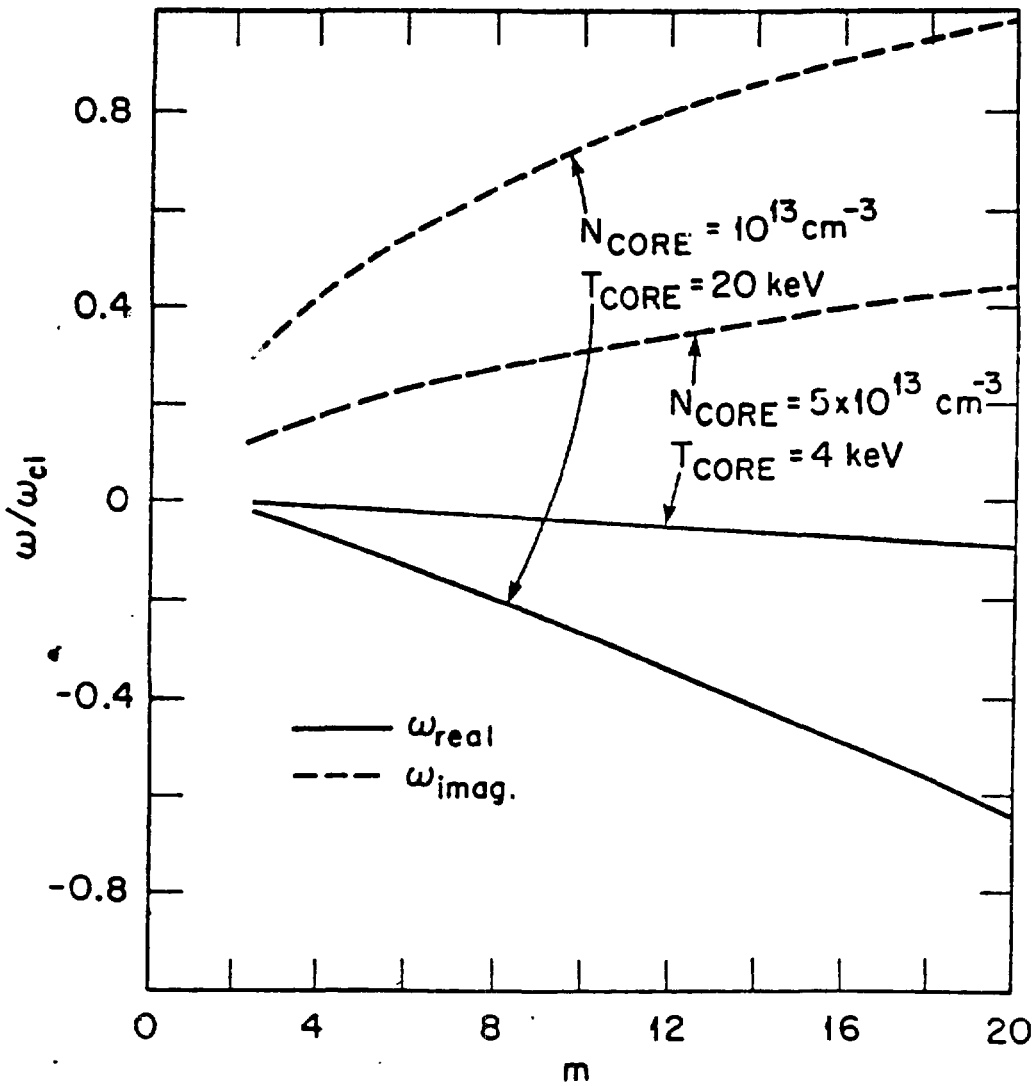
HIGH FREQUENCY MODE STABILITY BOUNDARIES  
 AS A FUNCTION OF  $m$  FOR  $T_{\text{core}} = 0$ ,  
 $N_{\text{hot}} = 5 \times 10^{11} \text{ cm}^{-3}$ ,  $T_{\text{hot}} = 500 \text{ keV}$

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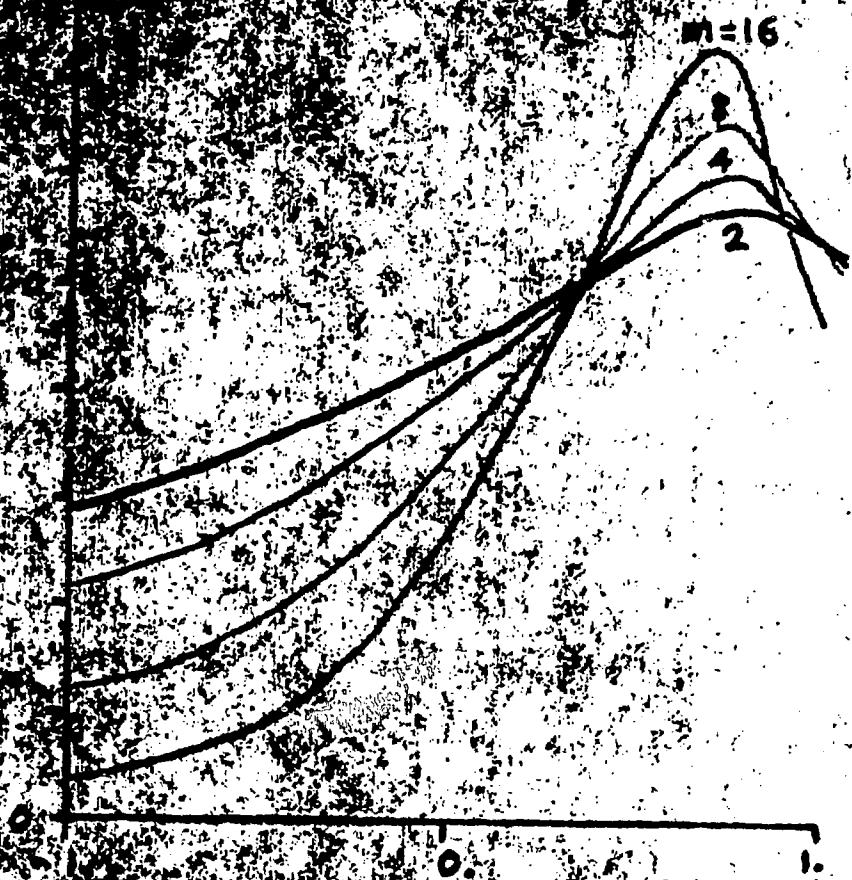


DEPENDENCE OF FREQUENCY ON AZIMUTHAL MODE NUMBER  
FOR INTERACTING CORE INTERCHANGE  
( $N_{\text{hot}} = 5 \times 10^{11} \text{ cm}^{-3}$ ,  $T_{\text{hot}} = 500 \text{ keV}$ )

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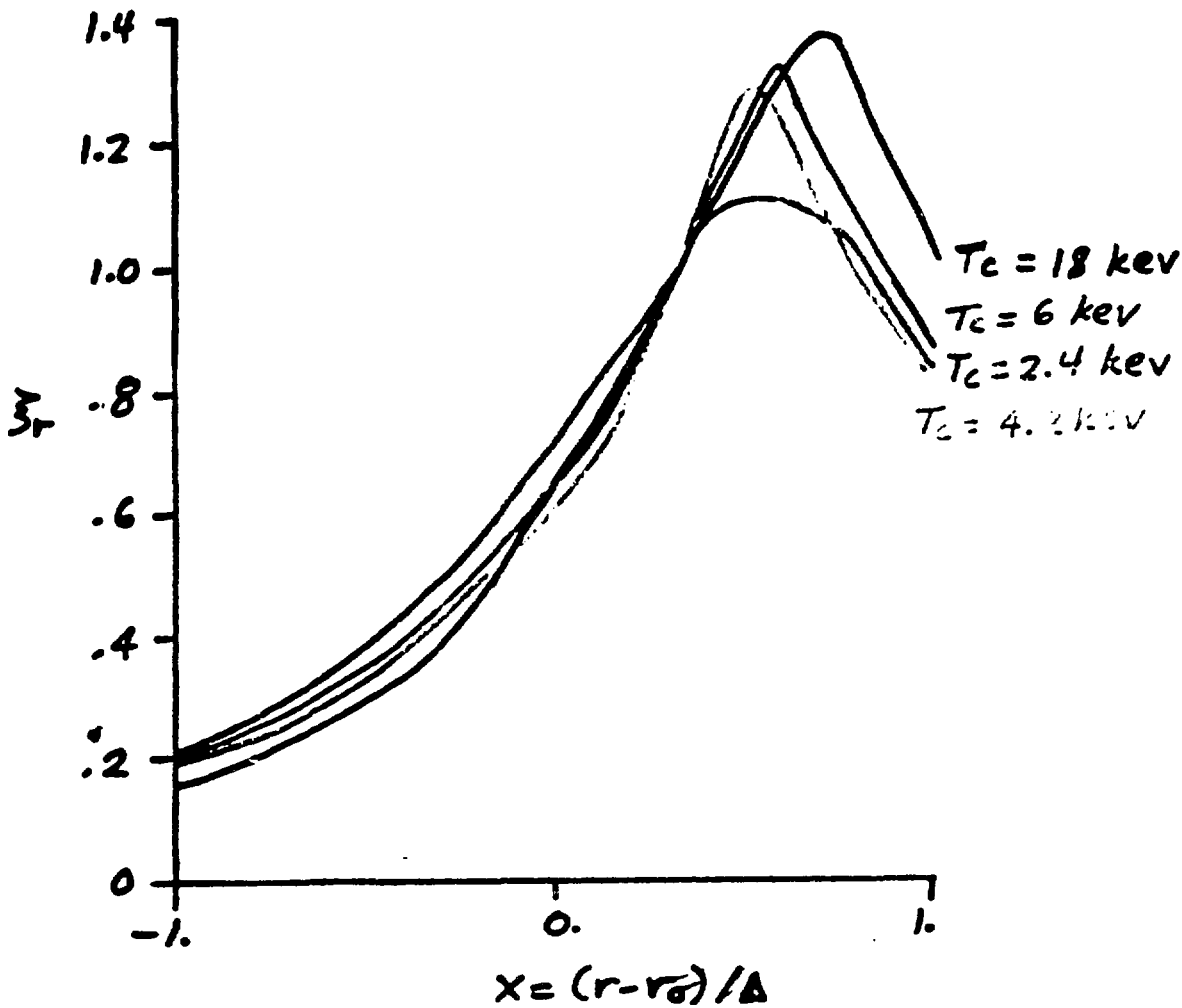
INTERMITTENT PRESSURE-DRIVEN INTERCHANGE  
PIPE STRUCTURE VS. M



$$x = (r - r_0) / \Delta$$

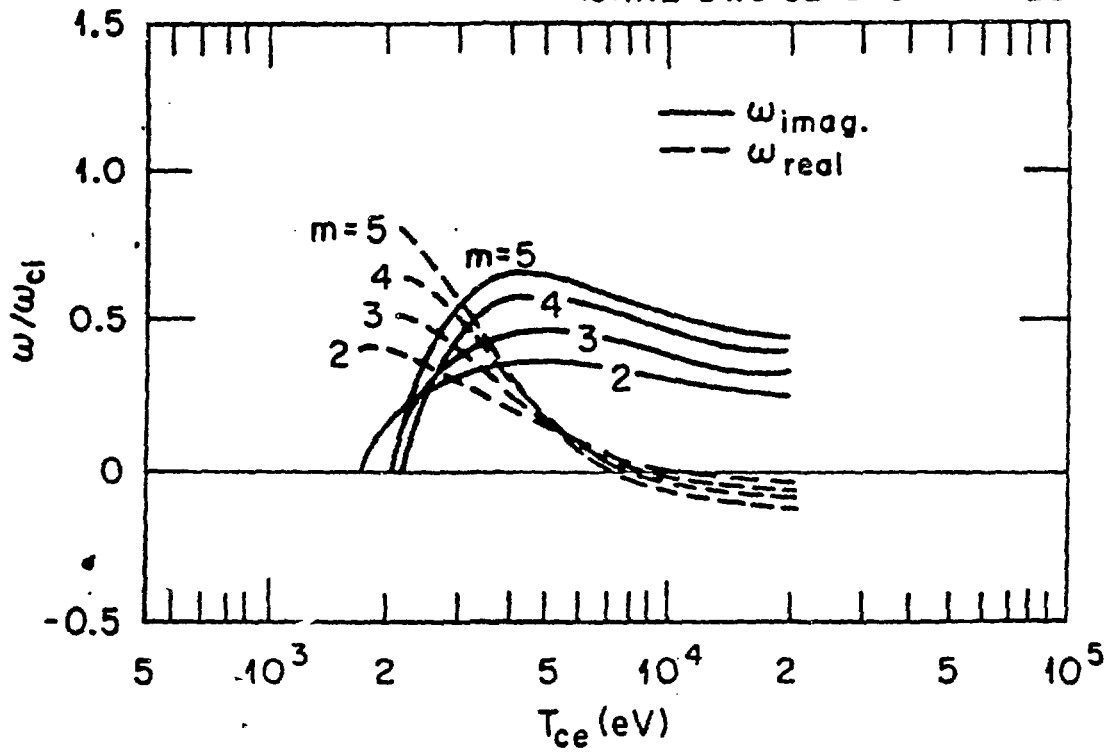
# INTERACTING PRESSURE-DRIVEN INTERCHANGE MODE STRUCTURE VS. CORE TEMPERATURE

( $m=11$ )



INTERACTING CORE INTERCHANGE MODE  
( $N_{\text{core}} = 10^{13} \text{ cm}^{-3}$ ,  $m = 2-5$ )

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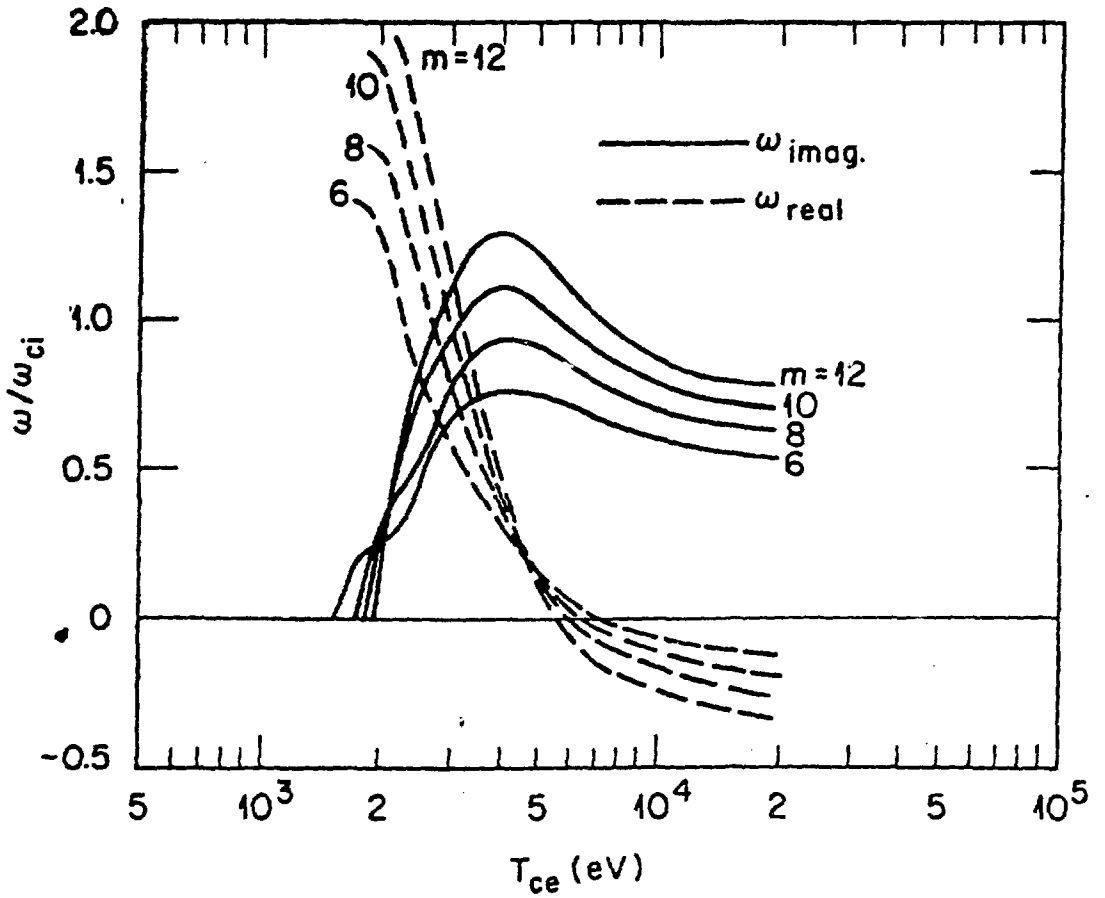


# INTERACTING CORE INTERCHANGE MODE

( $N_{\text{core}} = 10^{13} \text{ cm}^{-3}$ ,  $m = 6-12$ )

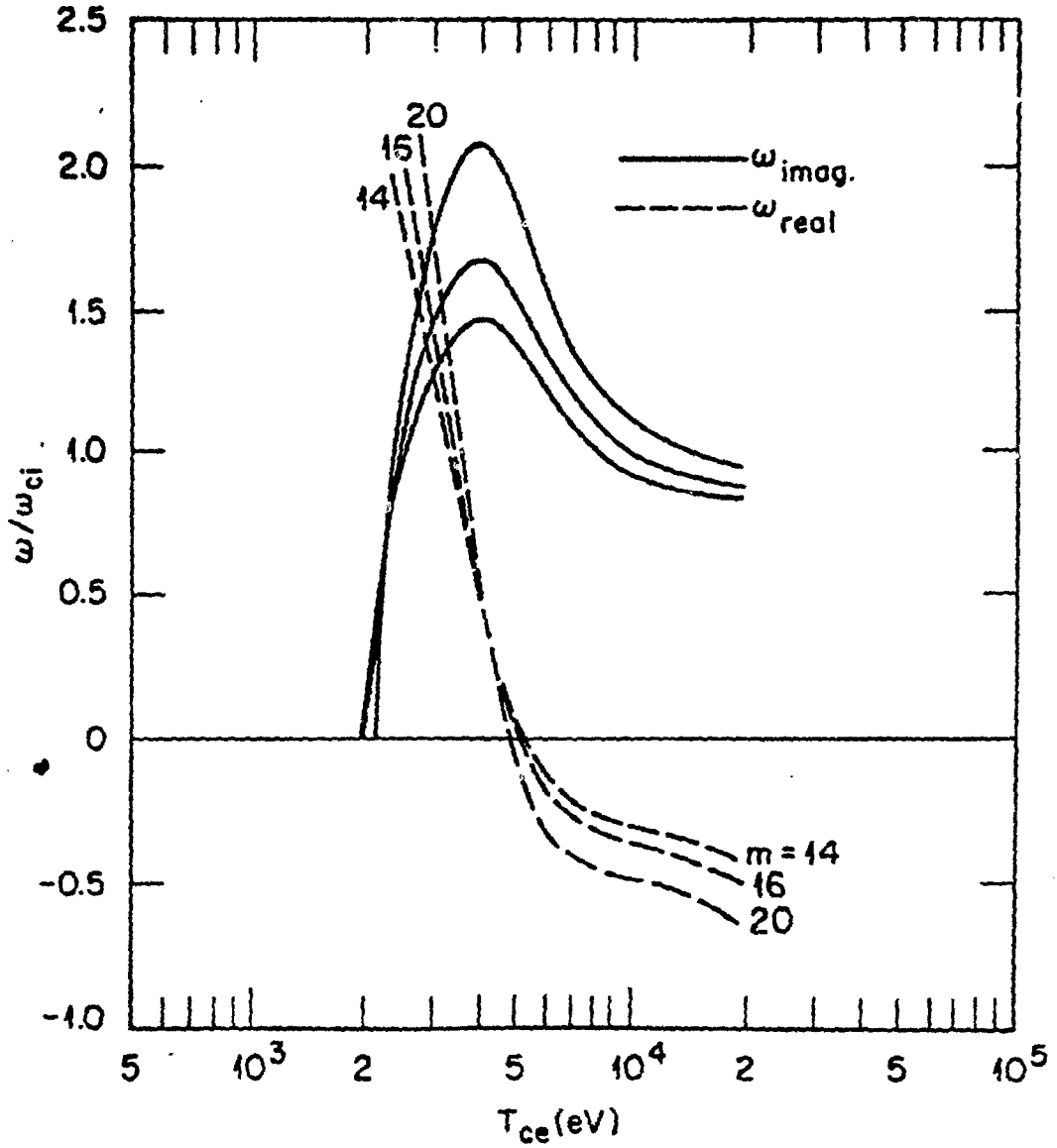
ORNL-DWG 82-3489

FED



INTERACTING CORE INTERCHANGE MODE  
( $N_{\text{core}} = 10^{13} \text{ cm}^{-3}$ ,  $m = 14-20$ )

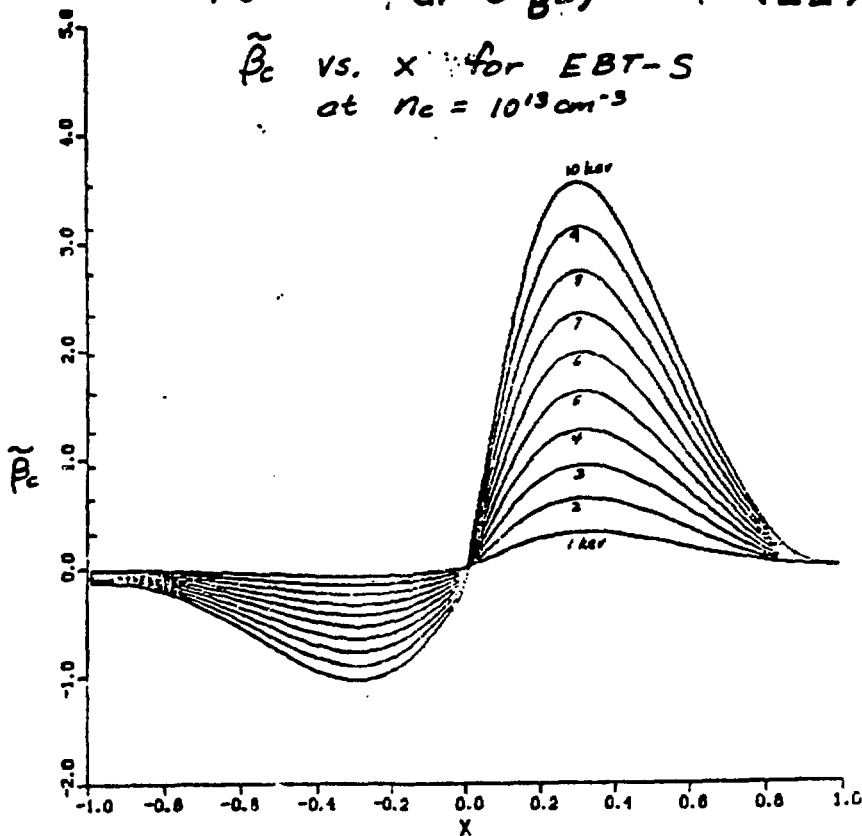
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TYPICAL  $\tilde{\beta}_c$  PROFILES USED  
IN RADIALLY - DEPENDENT  
CALCULATIONS

$$\tilde{\beta}_c = -r \frac{d}{dr} \left( \frac{\rho_c}{B^2} \right) \approx \beta_c \left( \frac{r}{2\Delta} \right) (1 + \beta_H)$$

$\tilde{\beta}_c$  vs.  $x$  for EBT-S  
at  $n_c = 10^{13} \text{ cm}^{-3}$



# HOT PLASMA DECOUPLING CONDITIONS:

## • WKB ANALYSIS:

$$1 - \tilde{\beta}_c < 3 \left[ \frac{5}{8} \tilde{\beta}_c \left( 1 + \frac{2\tilde{\beta}_c}{\tilde{\beta}_H} - \frac{2}{\tilde{\beta}_H} \right) \right]^{1/3}$$

defining  $\tilde{\beta}_{cv} = \frac{u_{cv}}{v_{thD}} = \sqrt{\frac{k a_0}{\rho}}$

$$\Omega_{cv} < \frac{3 (k a_0)^{1/2} \tilde{\beta}_c \sqrt{1 + 2\tilde{\beta}_c / \tilde{\beta}_H - 2 / \tilde{\beta}_H}}{(k a_0) (1 - \tilde{\beta}_c)^{3/2}}$$

## • LAYER ANALYSIS:

$$\tilde{\beta}_c \ll 1$$

$$\Omega_{cv} < \max \left[ 2 \left( \frac{k}{K} \delta \right)^{1/2}, 4 \left( \frac{k}{K} \tilde{\beta}_c \right)^{1/2} \right]$$

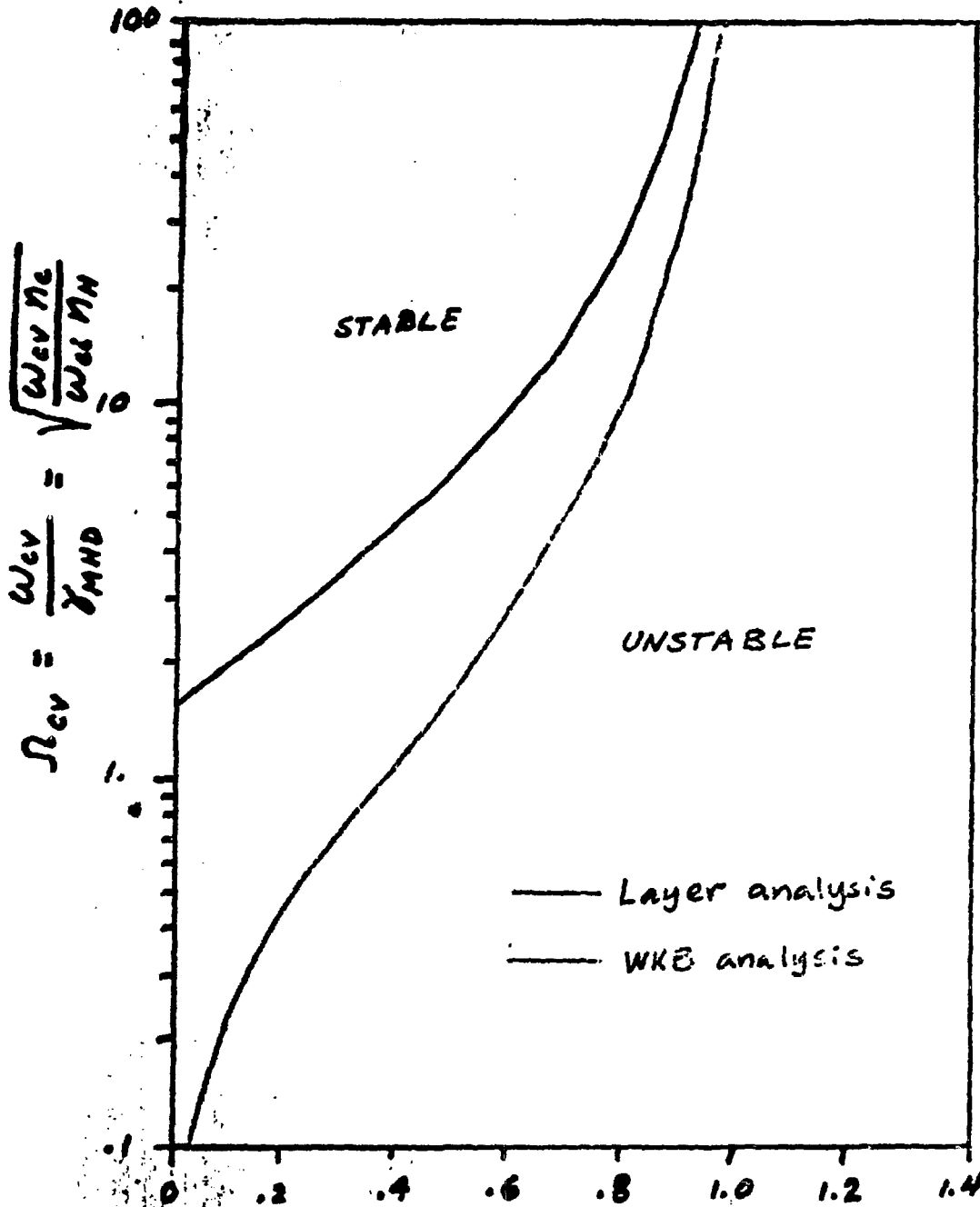
$$\delta \approx |k| a \quad K = \frac{1}{5(R-a)} \frac{d5(R-a)}{dr} \sim |k|$$

$$\tilde{\beta}_c \lesssim 1$$

$$\Omega_{cv} > \frac{\delta^{3/2} |k|}{2 K^{1/2} (1 - \tilde{\beta}_c)^{3/2}}$$

# HOT PLASMA DECOUPLING CONDITION FOR PRESSURE-DRIVEN INTERCHANGE

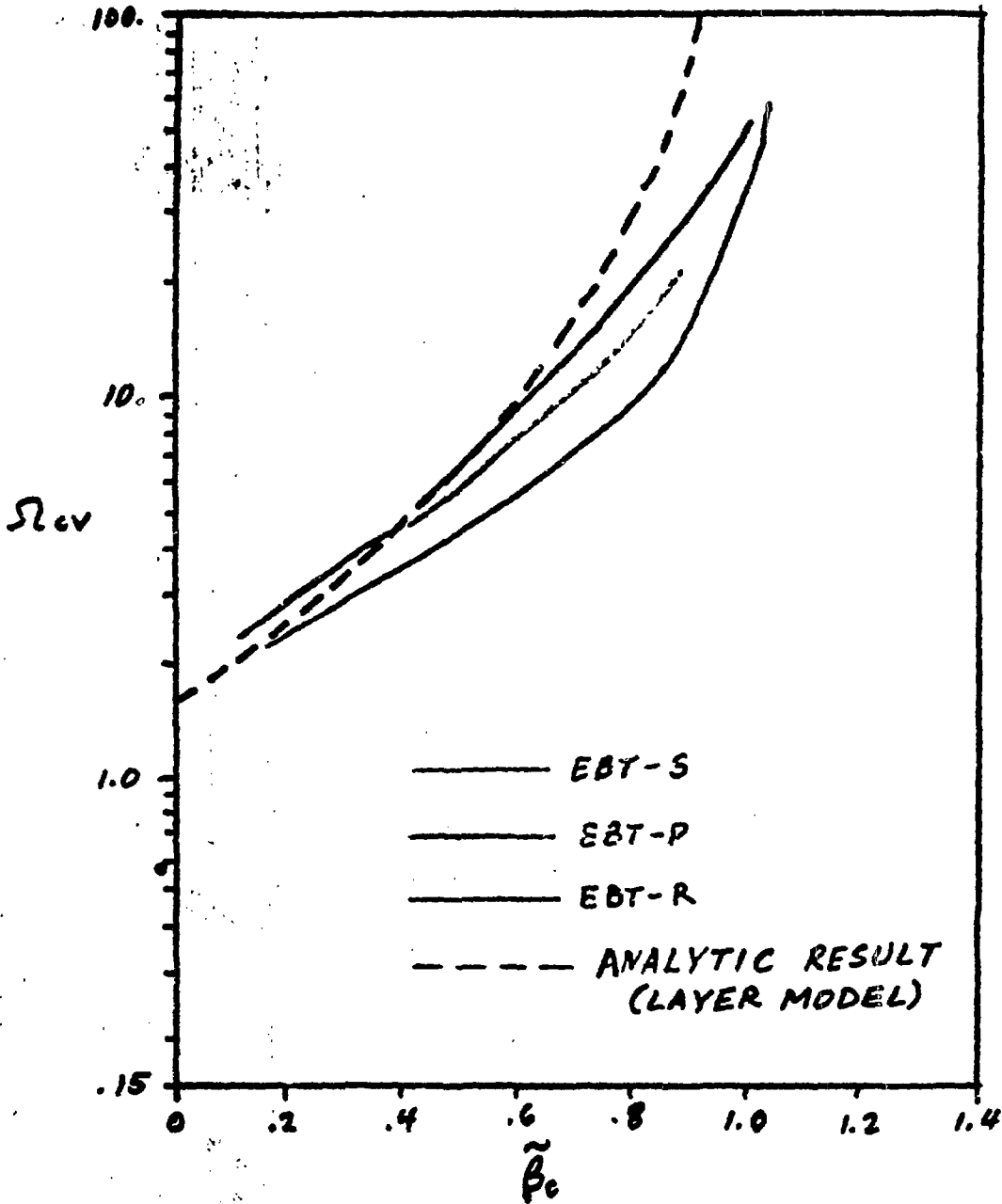
$m = 4$



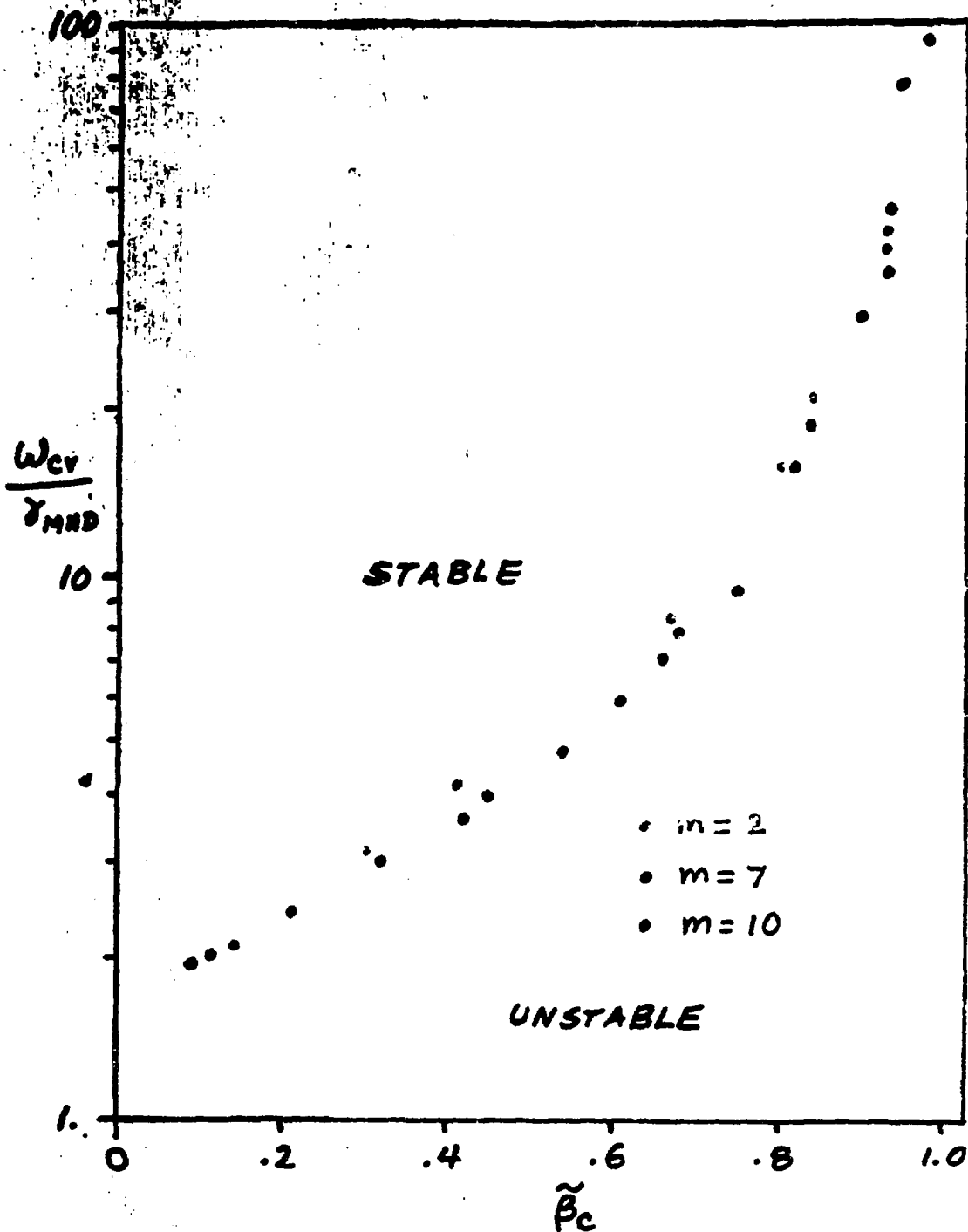
$$\tilde{R}_c = -r \frac{d}{dr} \left( \frac{\rho c}{B^2} \right) \approx \beta_c (1 + \beta_H) \frac{R}{2\Delta}$$

# DECOUPLING CONDITION FROM NONLOCAL CALCULATION

$m = 4$

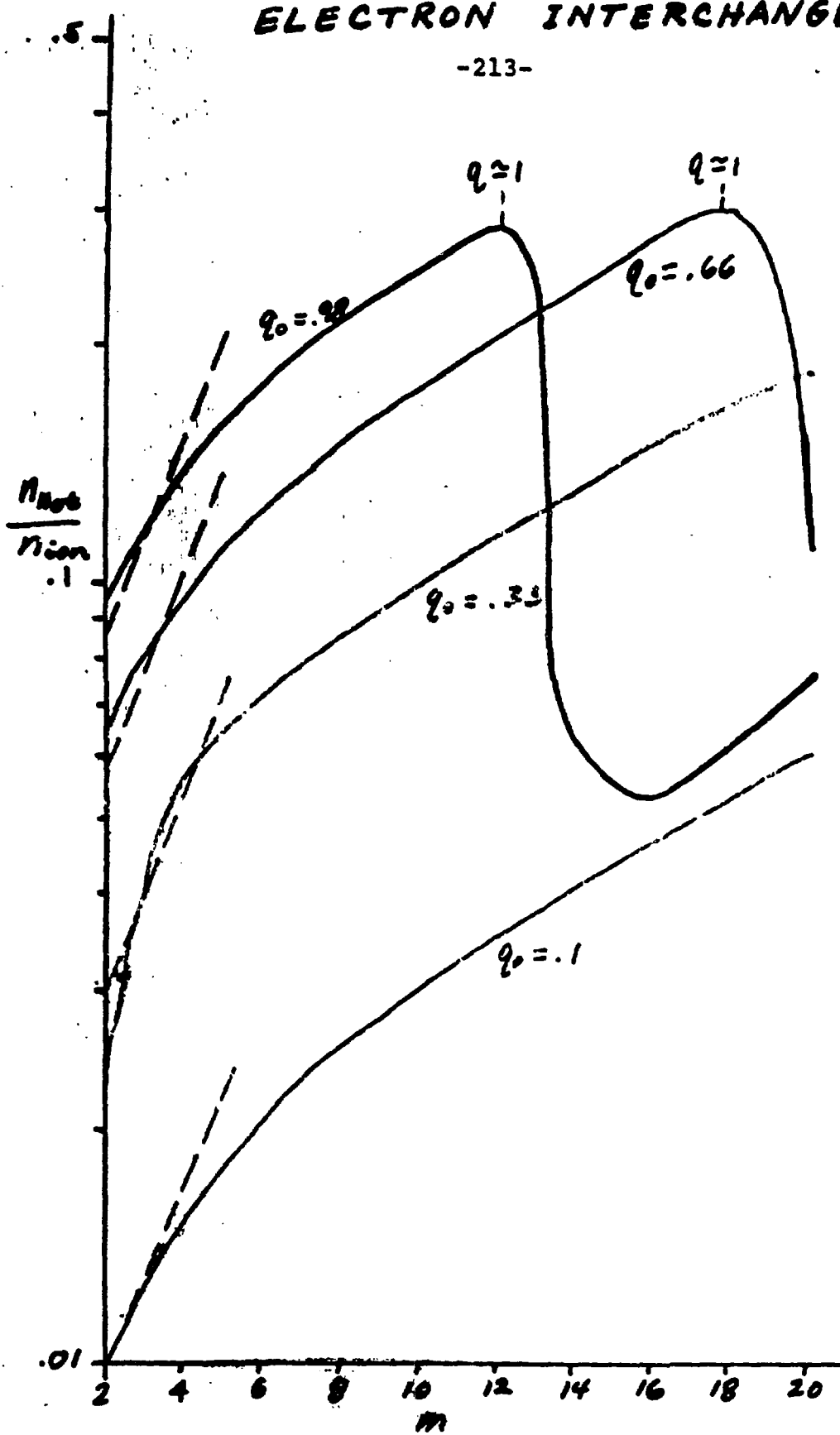


THE HOT PLASMA DECOUPLING  
CONDITION IS NOT STRONGLY  
DEPENDENT ON  $m$  (except through  $\frac{\omega_c}{\delta m}$ )



# ELECTRON INTERCHANGE

-213-





CONCLUSIONS:

● HOT ELECTRON INTERCHANGE

$m \geq 6$  somewhat more optimistic than local theory due to  $k_r \Delta < 2$

$m \leq 6$  instability band present ( $q < 1$ ) can be moved to  $m < 1$  by profile change (surface plasma frequency and  $(n_H/n_c)_{crit}$  in reasonable agreement with expt.

● COMPRESSIONAL ALFVEN

stability determined by  $\left\{ \begin{array}{l} \text{no node mode for } m \leq 9 \\ \text{one node mode for } m \geq 9 \end{array} \right.$

$(k_r)_{eff.}$  higher than local theory  
→ larger density threshold

decoupling condition at different  $m$  than local theory due to radial mode structure

one node unstable band at  $\sim 80 \text{ MHz}$  seen in expt. throughout T mode

CONCLUSIONS: (cont'd.)

- LOW FREQUENCY HOT ELECTRON INTERCHANGE

SIMILAR TO HIGH FREQUENCY BRANCH,  
 BUT WITH LOWER  $q_0$   
 LAYER SCALING FITS LOW  $m$  RESULTS.

- INTERACTING RING-CORE INTERCHANGE

RELEVANT PARAMETER IS:  $\tilde{\beta}_c = -r \frac{d}{dr} \left( \frac{P_c}{B^2} \right)_{MAX}$

EBT-S , EBT-P       $\tilde{\beta}_c \leq .2$   
 (20 keV)      (1-2 keV)

RING-CORE DECOUPLING LOST AT:

$$\frac{\omega_{cv}}{\delta_{MHD}} \approx 1.5$$