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NEW RESULTS IN PERTURBATIVE QCD.*

R. K. Ellis,
Fermi National Accelerator Laboratory,
P. O. Box 500, Batavia, IL 60510.

ABSTRACT

Three topics in perturbative QCD important for Super-collider physics are reviewed. The topics are:

1. $(2 \rightarrow 2)$ jet phenomena calculated in $O(\alpha_S{}^3)$.
2. New techniques for the calculation of tree graphs.
3. Colour coherence in jet phenomena.

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2. New techniques for the calculation of tree graphs.
3. Colour coherence in jet phenomena.

TWO JET EVENTS AND CALORIMETRIC CROSS-SECTIONS.

Solid evidence for jet structure in $p\bar{p}$ collisions has been presented by the UA1 and UA2 collaborations working at the CERN $Sp\bar{p}S$ collider[1,2]. At large transverse energy events displaying a clear two jet structure are the most copious[3]. This is in accordance with the simple parton parton scattering mechanism for jet production. To pass from this qualitative observation of jets to a more quantitive description of jet cross-sections we must choose a definition of a jet, both experimentally and theoretically. For the purposes of this discussion we shall define a jet in terms of an idealised version of the UA1 jet finding algorithm[4]. A jet of energy E is said to exist when an energy E is deposited within a solid angle ΔR defined such that,

$$\Delta R = \left((\Delta y)^2 + (\Delta\phi)^2\right)^{\frac{1}{2}} \leq 1 \tag{1}$$

where Δy and $\Delta\phi$ define the angular size of the cone in rapidity and azimuth respectively.

Such jet cross-sections are calculable theoretically because they are insensitive to the emission of soft and collinear radiation. The insensitivity to collinear radiation follows in an obvious way because radiation which remains inside the cone does not change the amount of energy deposited. In lowest order, the cancellation of the soft singularity can be illustrated as follows[5]. Let us consider the case of a jet initiated by a quark which may or may not be accompanied

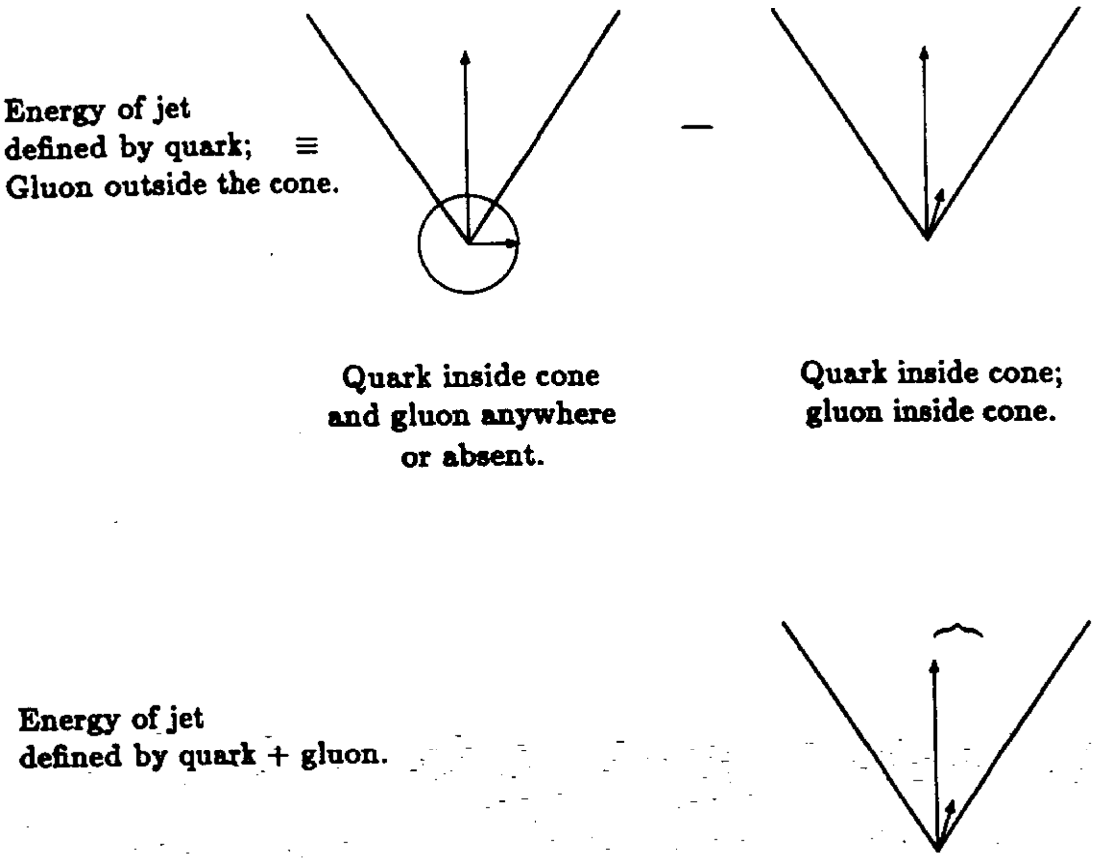

Figure 1: Illustration of the cancellation of soft divergences. The long arrow represents the quark and the short arrow represents the gluon.

by a single gluon. If there is no gluon inside the cone the energy of the jet is defined by the energy of the quark alone. This is shown in the top row of Fig. 1 where, in order to illustrate the cancellation of the soft singularity, the case in which only the quark lies inside the cone (and thus the gluon either lies outside the cone or is totally absent) is represented as the difference of two terms. If the gluon and the quark both lie inside the cone the energy of the jet is given by the sum of the gluon and quark energies. This is indicated in the second row of Fig. 1. The soft singularities of the two terms in column 2 of Fig. 1 cancel. Column 1 of Fig. 1 is also free from soft singularities because it is the full inclusive cross-section for the observation of a quark. The calorimetric cross-section is thus free from soft singularities in lowest order. These simple arguments make it plausible that such calorimetric cross-sections are free of singularities to all orders in perturbation theory.

They are therefore calculable perturbatively. For example, the inclusive one jet cross-section may be written as,

$$\frac{d\sigma}{dE_T dy d\phi} = \left(\frac{\alpha_S(Q^2)}{2\pi}\right)^2 \left\{r^{(0)}(E_T, y, \phi, Q^2) + \left(\frac{\alpha_S}{2\pi}\right) r^{(1)}(E_T, y, \phi, Q^2, \Delta R) + O(\alpha_S{}^2)\right\} \tag{2}$$

At present only the lowest order term $r^{(0)}$ has been calculated completely. Monte Carlo programs do include certain effects of initial and final state radiation in the leading logarithmic approximation. However their estimates of the size of jet cross-sections only become exact in the limit,

$$\ln\left(\frac{1}{\Delta R}\right) \gg 1. \tag{3}$$

Comparison with Eq.(1) shows that this condition is badly violated, if we use the normal definition of a jet. Q^2 in Eq.(2) is a large scale which is formally required to be of the same order as E_T^2 or S. Changes in the scale Q modify the function $r^{(1)}$ in a calculable way.

$$\begin{aligned}\frac{d\sigma}{dE_T dy d\phi} = \left(\frac{\alpha_S(Q^{*2})}{2\pi}\right)^2 \Bigg\{ & r^{(0)}(E_T, y, \phi, Q^{*2}) \\ & + \left(\frac{\alpha_S}{2\pi}\right)\left[r^{(1)}(E_T, y, \phi, Q^{*2}, \Delta R) + r^{(1)\prime}(E_T, y, \phi)\ln\left(\frac{Q^{*2}}{Q^2}\right)\right]\Bigg\}\end{aligned} \tag{4}$$

We see that the optimum choice for the scale Q^* is intimately connected with the calculation of the higher order terms.

Very little is known theoretically about the higher order corrections to jet cross-sections. Almost all calculations of higher order corrections discuss one hadron inclusive cross-sections[6,7]. As described above the calculation of one jet inclusive cross-sections can be organised so that one parton inclusive cross-sections form part of the answer, but they need not be the dominant part. In ref.(5) certain contributions to one jet inclusive cross-sections are calculated in the limit of small ΔR.

There is clearly a pressing need for a full $O(\alpha_S{}^3)$ calculation of the one and two jet inclusive cross-sections with jets defined using a prescription such as Eq.(1). This would allow a determination of the special value of Q^* in Eq.(4), (and in the corresponding equation for the two jet inclusive cross-section) such that the higher order corrections are minimised. In the literature it is often assumed that this special scale Q^* is some multiple of the transverse energy and is independent of the subprocess and the parameters used to define the jet, $(E_T, y, \phi, \Delta R)$.

$$Q^{*2}(E_T, y, \phi, \Delta R) = \eta E_T^2 \tag{5}$$

η is taken to be less than one; in the literature the value $\eta \approx \frac{1}{4}$ has received special favour[7,8,9]. Since no complete calculations of jet cross-sections exist a scientific critique of these assertions is impossible. If we make the assumption that information about jet cross-sections may be gleaned from calculations of one parton inclusive cross-sections, the evidence indeed suggests that η is less than one. However η is not expected to be independent of angle[10].

To resolve this unsatisfactory state of affairs a complete calculation of the $O(\alpha_S{}^3)$ contributions to jet cross-sections including all parton subprocesses has been initiated in ref.(10). Specifically, the invariant matrix elements squared of the following parton sub-processes in $O(\alpha_S{}^3)$ have been calculated.

$$\begin{array}{ll} (a) & q_j + q_k \to q_j + q_k \qquad j \neq k \\ (b) & q_j + q_j \to q_j + q_j \\ (c) & q_j + \bar{q}_j \to g + g \\ (d) & g + g \to g + g \end{array} \tag{6}$$

and

$$\begin{array}{ll} (A) & q_j + q_k \to q_j + q_k + g \qquad j \neq k \\ (B) & q_j + q_j \to q_j + q_j + g \\ (C) & q_j + \bar{q}_j \to g + g + g \\ (D) & g + g \to g + g + g \end{array} \tag{7}$$

All other matrix elements for parton parton scattering processes in $O(\alpha_S{}^3)$ can be obtained from the above by time reversal and crossing. The results are given for massless quarks and in n dimensions in order to regulate divergences.

Separately the $(2 \to 2)$ and $(2 \to 3)$ matrix elements contain singularities in the regions of soft and collinear emission which are regulated by continuation to n dimensions. When the $(2 \to 2)$ and $(2 \to 3)$ cross-sections are combined to form physical quantities these singularities either cancel or can be factored into the parton distribution functions. In ref.(10) this last step has been performed only in a very limited number of cases.

As an example of the results given in ref.(10) consider the result for the process (D). The four momenta of the gluons p_i are assigned such that,

$$p_1 + p_2 = p_3 + p_4 + p_5 \tag{8}$$

The matrix element squared for process (D) summed and averaged over the $(N^2 - 1)$ gluon colours and $(n - 2)$ gluon spins is given by,

$$\overline{\sum} |M|^2 = \frac{1}{(n-2)^2(N^2-1)^2} \left[D(p_1, p_2, -p_3, -p_4, -p_5)\right] \tag{9}$$

The result for the base function D which describes the five gluon transition probability is most conveniently written by introducing a compact notation for the dot-product

$$p_i.p_j = (ij) \qquad (10)$$

The function D is a completely symmetric function of the momenta of the five gluons. It can be expressed as the sum over all 5 factorial permutations of the arguments of the function F^D,

$$D(p_1,p_2,p_3,p_4,p_5) = g^6(\mu)^{(12-3n)}\,\frac{1}{10}\sum_{permutations} F^D(1,2,3,4,5) \qquad (11)$$

where,

$$\begin{aligned}F^D(1,2,3,4,5) = {} & \frac{4(N^2-1)N^3}{(12)(23)(34)(45)(51)} \\ & \Big[\frac{(n-2)^2}{8}\Big\{(12)^4+(13)^4+(14)^4+(15)^4+(23)^4+(24)^4+(25)^4+(34)^4+(35)^4+(45)^4\Big\} \\ & -\frac{3(n-4)(n-10)}{2}\Big\{\frac{1}{2}\Big((12)^2(23)^2+(23)^2(34)^2+(34)^2(45)^2+(45)^2(51)^2+(51)^2(12)^2\Big) \\ & -(12)(23)^2(34)-(23)(34)^2(45)-(34)(45)^2(51)-(45)(51)^2(12)-(51)(12)^2(23) \\ & +(12)(23)(34)(45)+(23)(34)(45)(51)+(34)(45)(51)(12) \\ & +(45)(51)(12)(23)+(51)(12)(23)(34)\Big\}\Big]\end{aligned} \qquad (12)$$

Because of the great symmetry this matrix element squared has an extremely compact form even in n dimensions. Notice that the terms which vanish in four dimensions also vanish in ten dimensions. We find that the results for the matrix element squared D in four and ten dimensions are proportional. We have no explanation for this simplicity. For the results for all the other $(2\to2)$ and $(2\to3)$ matrix elements in $O(\alpha_S{}^3)$ we refer the reader to ref.(10).

NEW TECHNIQUES FOR TREE GRAPHS.

Planning for the SSC has given a new impetus to the study of tree graphs in the Standard model. Even at the comparatively low energies of the CERN $Sp\bar{p}S$ collider, four jet events have been observed at a rate roughly consistent with the QCD prediction. At the higher energies of the SSC multi-jet events will be copiously produced. These events are interesting in their own right as tests of QCD. However at the SSC they assume special importance as the principal source of background to the production of a heavy object H, which decays into jets of quarks and gluons. The discovery of such a heavy object is one of the physics objectives of the SSC. Thus for efficient background rejection it is important to have accurate estimates of QCD multi-jet cross-sections. If the mass of H is not small with respect to the incoming energy of the partons, the leading pole approximation (as used in Monte Carlo programs) will not provide a good description of the QCD background processes[11], and the correct matrix element is required.

The brute force method of calculation of these transition probabilities leads to an extremely large number of terms which become unwieldy even with sophisticated algebraic manipulation techniques. For example, the matrix element squared for process (D) of Eq.(7), calculated in a straightforward way, leads in intermediate stages to at least $6^6 \approx 46,000$ terms, even in four dimensions. Many of these terms cancel in the final answer as a consequence of gauge invariance as demonstrated by the simple answers of ref.(12).

The key to resolving this impasse has been the development of techniques for gauge theories[13,14] - originally for QED - which calculate helicity amplitudes rather than matrix elements squared. Exploiting simplifications due to the gauge invariance of the theory and the masslessness of the quanta extremely compact expressions have been obtained for the helicity amplitudes. In some cases it has also proved expedient to relate amplitudes involving vector particles to amplitudes with scalars or spinors which are analytically more tractable. This is acheived by introducing an unbroken supersymmetry[15].

To illustrate these techniques, consider the process

$$e^+(p_1) + e^-(p_2) \to \gamma(k_1) + \gamma(k_2) \tag{13}$$

The simplicity of the helicity amplitudes follows if the polarisation vectors are expressed in terms of vectors already present in the problem. Thus for the photon with momentum k_1 we may choose polarisation vectors[13],

$$\begin{aligned} \epsilon_{\parallel}^{\mu}(k_1) &= \sqrt{8}\mathcal{N}(p_1 \cdot k_1\, p_2^{\mu} - p_2 \cdot k_1\, p_1^{\mu}) \\ \epsilon_{\perp}^{\mu}(k_1) &= \sqrt{8}\mathcal{N}\epsilon^{\mu\alpha\beta\gamma} p_{1\alpha} p_{2\beta} k_{1\gamma} \end{aligned} \tag{14}$$

where,

$$\mathcal{N} = \frac{1}{\sqrt{16\, p_1 \cdot k_1\, p_2 \cdot k_1\, p_1 \cdot p_2}} \tag{15}$$

These polarisation vectors make a gauge choice for the photon; both polarisation vectors are orthogonal to a definite linear combination of the vectors p_1 and p_2. From these polarisation vectors we may form states of definite helicity.

$$\epsilon_{\pm}^{\mu} = \frac{1}{\sqrt{2}}(\epsilon_{\parallel}^{\mu} \pm i\epsilon_{\perp}^{\mu}) \tag{16}$$

In QED we are always interested in the quantity $\not\epsilon$ so we may write,

$$\not\epsilon_{\pm}(k) = \mathcal{N}\left\{\not k_1 \not p_1 \not p_2 (1 \pm \gamma_5) - \not p_1 \not p_2 \not k_1 (1 \mp \gamma_5) \mp 2 p_1 \cdot p_2 \not k_1 \gamma_5\right\} \tag{17}$$

If we work with massless fermions the last term proportional to $\not k_1 \gamma_5$ can frequently be omitted due to the conservation of an axial current. In the massless limit, left-handed and right-handed fermions interact separately so only one of the remaining two terms contributes to a given helicity amplitude. If the photon is next to an external fermion line only one of these two terms can ever contribute because,

$$\not p_2 u(p_2) = 0 \quad \bar{v}(p_1) \not p_1 = 0 \tag{18}$$

The remaining term which is next to a free particle spinor cancels a denominator factor leading to a further simplification

$$-\frac{1}{\not p_2 - \not k_1} \not p_1 \not p_2 \not k_1 (1 \mp \gamma_5) u(p_2) = (\not p_2 - \not k_1) \not p_1 (1 \mp \gamma_5) u(p_2) \tag{19}$$

A further generalisation of these techniques suitable for QCD has been presented in ref.(14). In QCD one is interested not only in $\not\epsilon$, but also in the four-vector ϵ^μ. If the last term in Eq.(17) proportional to $\not k_1 \gamma_5$ is dropped, Eq.(17) is no longer the contraction of the four-vector $\epsilon^\mu_\pm$ with γ^μ. For the simplifications illustrated above it was essential that the polarisations were chosen in terms of the fermion momentum to which the photon (or gluon) attaches. If we have more than one fermion line in the problem with external momenta p_i and q_i the best polarisation vectors for the two lines will be related by

$$\epsilon_p^{\pm\mu}(k) = e^{i\phi^\pm} \epsilon_q^{\pm\mu}(k) + \beta^\pm k^\mu \tag{20}$$

The technical improvements introduced in ref.(14) are:

(a) ϵ^μ can always be written as a four vector and is chosen relative to a single light-like four momentum for every fermion line. This means the polarisation is chosen in a light cone gauge whereas the polarisations of Eq.(14) can be considered to be in an axial gauge.

(b) By adroit choice of the normalisation factor $\mathcal{N}$, the phase factor $\phi^\pm$ in Eq.(20) is set equal to zero, thus simplifying the treatment of the problem in which there is more than one fermion line.

(c) The formalism is simplified by introducing a bra and ket notation incorporating many of the simplifications of working with massless fermion lines. For further details of these powerful techniques we refer the reader to ref.(14).

In ref.(15) an interesting new technique has been presented for the calculation of tree graphs in QCD. The basic idea is to embed QCD in a minimal $N = 2$

supersymmetric extension such that, to tree level, the two theories are identical for the physical quarks and gluons. In the extended theory there are simple relationships between vector gluon scattering amplitudes and scalar scattering amplitudes when expressed in terms of the helicities of the external particles. Thus a vector scattering amplitude can be deduced from the calculation of the appropriate scalar scattering amplitude. The calculation of amplitudes with scalar particles are considerably less onerous, firstly, because thay contain fewer three gluon vertices and secondly, because the problems associated with polarisation vectors for the external gluons are circumvented.

By way of example we illustrate the calculation of the amplitude for $(g+g \to g+g)$. In terms of $N=1$ superfields the $SO(2)$ gauge hypermultiplet of the extended theory contains one gauge vector superfield and one chiral superfield. In addition the extended theory includes a matter hypermultiplet containing - inter alia - the quark fields. By supersymmetric rotation we find that,

$$\left|M(g^1_+, g^2_+; g^3_+, g^4_+)\right| = \left|M(\phi^1_+, \phi^2_+; \phi^3_+, \phi^4_+)\right| \tag{21}$$

where ϕ is the complex scalar in the adjoint representation contained in the chiral superfield$(\phi_- = \phi^*_+)$. The subscripts on the gluon field denote the helicity and for this process all non-vanishing helicity amplitudes can be obtained from $\left|M(g^1_+, g^2_+; g^3_+, g^4_+)\right|$ by crossing.

The result for the scalar amplitude is

$$\left|M(\phi^1_+, \phi^2_+; \phi^3_+, \phi^4_+)\right| = 2ig^2\left\{\left[f_{X13} f_{X24} \frac{p_1 \cdot p_2}{p_1 \cdot p_3}\right] + \left[1 \leftrightarrow 2\right]\right\} \tag{22}$$

f is the structure constant of $SU(3)$. Squaring this amplitude and adding the squares of all other non-vanishing helicity amplitudes we recover the standard $O({\alpha_S}^2)$ result for this transition probability.

Using a combination of these techniques, numerical results (and in some cases analytic results) have been obtained for all $(2 \to 4)$ processes in QCD[14,15,16,17].

COLOUR COHERENCE IN JET PHENOMENA.

The standard description of hadronic jets has two components. In the first stage partons, which have been produced far from their mass-shells by a hard interaction, radiate cascades of quarks and gluons of decreasing virtuality. This first phase of jet evolution is well described by perturbative QCD. At some lower virtuality Q_0, the quark gluon interactions become strong, and the further development of the quarks and gluons into the observed hadrons is controlled by a non-perturbative mechanism. Note however that perturbation theory continues

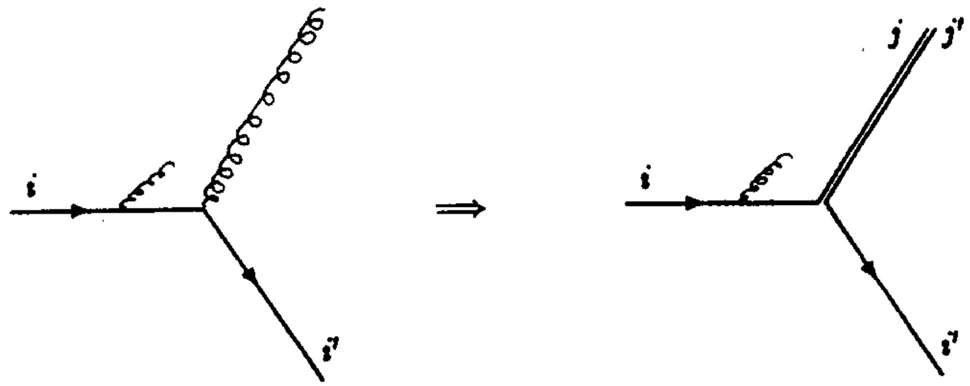

Figure 2: Emission of a soft gluon in the $1/N$ approximation.

to provide clues about the nature of the hadronisation mechanism. In particular, it predicts the phenomenon of pre-confinement[18]. It is therefore reasonable to assume that the hadronisation is local in momentum space so that the collimated structure of the parton showers is preserved. The observed jets of hadrons are therefore a consequence of the softness of the hadronisation stage. More recently, it has been realised that detailed features of the parton shower, such as the flow of colour quantum numbers, influence significantly the distribution of colour singlet hadrons in the final state[19,20,21].

To examine these phenomena in more detail consider the case of e^+e^- annihilation into jets. Three jets consisting of a quark, an antiquark and a gluon are produced by a colour singlet photon. As these three partons separate from one another they form a colour "antenna" which gives rise to a characteristic pattern of associated radiation[21]. A complete analysis of the radiation associated with these three separating partons would be extremely difficult, but fortunately the radiation is dominated by soft emission which can be easily calculated.

Consider the radiation of a single gluon of energy E, in the limit in which E is very much less than the energy of the three hard partons. The radiation pattern may be written as,

$$\frac{dn}{d\Omega} \approx \frac{\alpha_S N}{8\pi^2} W(\Omega) \frac{dE}{E} \tag{23}$$

where $W(\Omega)$ describes the angular distribution of the soft radiation and N is the number of colours. Neglecting terms of order $1/N^2$ the hard gluon can be represented as a quark anti-quark combination as shown in Fig. 2. In this approximation each external quark line is uniquely connected to an external antiquark line of the same colour. In calculating the resultant soft radiation pattern we need only consider the sets of colour connected lines, because the in-

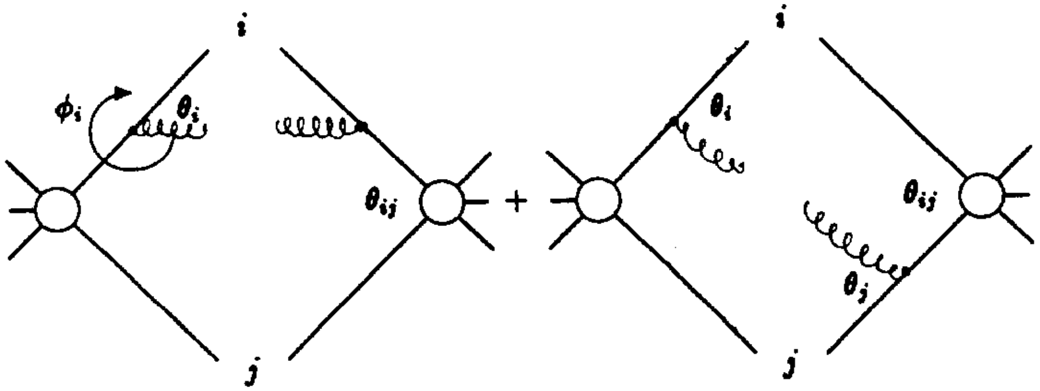

Figure 3: Direct and interference terms for soft gluon emission.

terference between gluons emitted from non-colour connected lines is suppressed by powers of $1/N^2$.

In this approximation the full radiation pattern may be written as,

$$W(\Omega) = \sum_i W_i(\theta_i, \theta_j, \theta_{ij}) \tag{24}$$

where W_i is the radiation pattern due to a single external quark line i, and the sum runs over all quark and antiquark lines. Thus the hard gluon line in Fig. 2 contributes to the sum as both a quark and an antiquark, (j and j'). We denote by θ_i and θ_j the angles between the soft gluon and the lines i and j respectively. The angle between the lines i and j is θ_{ij}. The soft radiation from each quark line is determined by the classical colour current and may be written as,

$$W_i(\theta_i, \theta_j, \theta_{ij}) = \frac{1}{(1-\cos\theta_i)} + \frac{(\cos\theta_i - \cos\theta_{ij})}{(1-\cos\theta_i)(1-\cos\theta_j)} \tag{25}$$

We shall refer to the two terms in this equation as the incoherent and the interference terms. These two terms are illustrated in Fig. 3. We define W_i^* to be the incoherent part of W_i given by the first term on the left of Eq.(25). This identification of the two terms in Eq.(25) is somewhat arbitrary and is chosen to facilitate the the physical interpretation given below. Note particularly the differing behaviour of the incoherent and interference terms as the soft gluon rotates in azimuth angle ϕ_i about line i. At fixed θ_i, the incoherent term is independent of ϕ_i. The interference term depends on ϕ_i through the angle θ_j,

$$\cos\theta_j = \cos\theta_i \cos\theta_{ij} + \sin\theta_i \sin\theta_{ij} \cos(\phi_i - \phi_{ij}) \tag{26}$$

When $\phi_i = \phi_{ij}$ the soft gluon lies in the plane defined by i and j and the

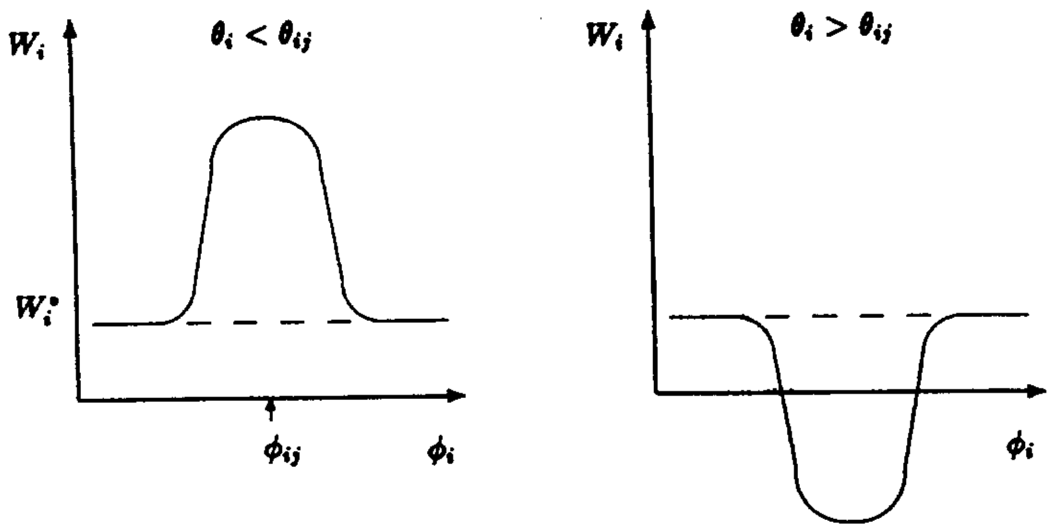

Figure 4:

interference term is largest. The sign of its contribution depends on whether the soft gluon lies inside or outside the cone defined by the quarks i and j. The azimuthal dependence of W_i, the radiation pattern associated with line i, is shown in Fig. 4. If the soft gluon lies within the cone described by i and j the interference is positive. If the gluon lies outside this cone the interference is negative. After integration over ϕ it turns out that the total contribution of the incoherent and interference terms are equal, so that after azimuthal averaging we find[21],

$$\begin{aligned} \langle W \rangle_{\phi_i} &= 2\,W_i^* \quad for\ \theta_i < \theta_{ij} \\ \langle W \rangle_{\phi_i} &= 0 \qquad\ \ for\ \theta_i > \theta_{ij} \end{aligned} \tag{27}$$

This extremely elegant result allows one to incorporate some of the effects of interference into a Monte-Carlo program in a probabilistic fashion. We replace the full W_i for soft gluons by,

$$W_{AO} = \sum_i \langle W \rangle_{\phi_i} = 2 \sum_{\theta_i < \theta_{ij}} W_i^* \tag{28}$$

By restricting the phase space for soft gluon emission using this angular ordering criterion interference effects are included - on the average - as a sum of probabilities.

The accuracy of the angular ordering approximation is investigated in Fig. 5 taken from ref. 22. A quark, an antiquark and a gluon are produced in a plane with relative angles $\theta_{q\bar{q}} = 155^0$, $\theta_{\bar{q}g} = 75^0$ and $\theta_{gq} = 130^0$. The angular distribution W of soft radiation in the plane of the event coming from these partons is shown in Fig. 5. The solid curve is the full $q\bar{q}g$ prediction and the dashed curve is the angular ordering approximation to this curve. The dotted curve displays the radiation from a quark-anti-quark system in a colour singlet state, (with no hard gluon). Note the net destuctive interference in the region between the

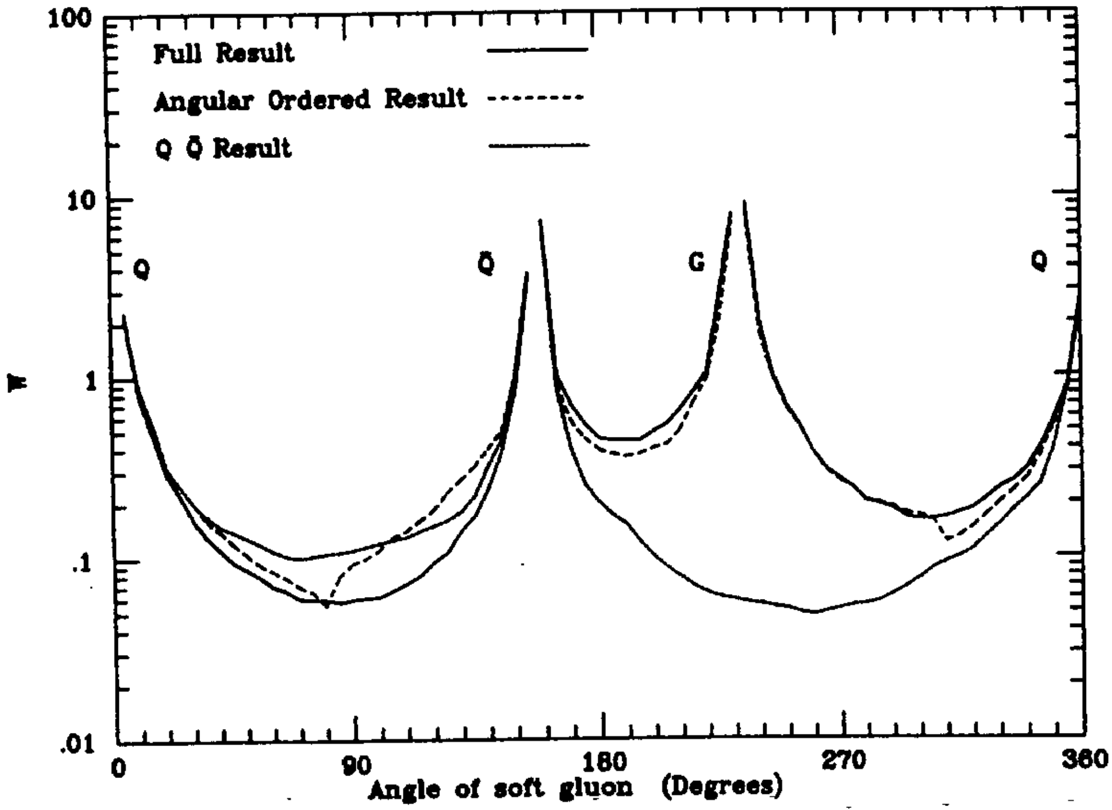

Figure 5: Soft gluon radiation pattern from $q\bar{q}g$.

quark and antiquark. The soft radiation in the presence of the gluon jet, (solid and dashed lines) is *less* than the radiation in the absence of the gluon jet. This is the string-like effect occurring in lowest order perturbation theory as pointed out by Azimov *et al.*[21] and as observed in the data[23,24].

In summary, it appears that the string-like effect is present in perturbative QCD in the region opposite the gluon jet as a consequence of interference effects. These coherence effects can be well approximated by assuming incoherent emission in limited region of phase space determined by angular ordering. It is of course evident that interference effects are present in the perturbative stage of jet evolution and that they make definite predictions for the form of the parton shower. It is perhaps more surprising that these interference effects survive the hadronisation stage and have observable consequences for the distributions of observed hadrons. The depletion of the radiation between the quark and tha anti-quark jets can also be taken to provide evidence in favour of the string effect as implemented in the Lund model[25]. Note that the approximation of Fig. 2 already distinguishes the soft radiation lying between the quark and anti-quark from the radiation lying between the gluon and the quark or the anti-quark.

This suggests that lowest order perturbation theory will provide an explanation for the string effect observed in 3-jet events in e^+e^- annihilation.

By ordering emission angles we can also include coherence effects in the calculation of the growth of the average parton multiplicity. The average multiplicity can be shown to vary with scale of the hard interaction which produces the parton jet as follows[26],

$$\bar{n}(Q) \propto \tau^{-\epsilon} \exp\sqrt{\frac{12\tau}{\pi b_0}} \tag{29}$$

where $b_0 = (33 - 2n_f)/12\pi$, $\tau = \log(Q/\Lambda)$ and $\epsilon = (11 + 22n_f/27)/16\,\pi b_0$. If coherence effects were neglected (as is done in some Monte Carlo programs) the growth of the parton multiplicity would be overestimated. The leading term would be modified by a factor of $\sqrt{2}$ in the exponent. We can model the multiplicity growth predicted by these incoherent models using the formula,

$$\bar{n}(Q) \propto \tau^{-\epsilon} \exp\sqrt{\frac{24\tau}{\pi b_0}} \tag{30}$$

Assuming that these asymptotic formulae hold already at $Q = 15$ GeV, and normalising to the quark jet multiplicity $\langle n_q \rangle = 4.7$ observed at this energy in e^+e^- annihilation[27,28], we obtain results for the hadron multiplicity of a quark jet in Fig. 6. The results use five flavours of quarks and assume $\Lambda = .1$ Gev. For a jet produced at a hard interaction scale of 1 TeV the incoherent Monte Carlo programs over-estimate the hadron multiplicity by more than a factor of two. Note that 1 TeV jet is not a rare occurrence at the SSC; in every 100 GeV bin they can be expected to occur at a rate of about one per second[8]. The inclusion of these coherence effects is a necessary requirement for the accurate description of jet structure at TeV energies.

Comparing the mean multiplicities in quark and gluon jets, one finds that the ratio is a series in $\sqrt{\alpha_S}$,which has now been calculated[29] up to $O(\alpha_S)$,

$$\frac{\bar{n}_g}{\bar{n}_q} = \frac{9}{4}(1 - 0.27\sqrt{\alpha_S} - 0.07\alpha_S) \tag{31}$$

The order α_S correction is about 1%, so the ratio of gluon and quark multiplicities should be considered a firm prediction of QCD. Unfortuanately, it is hard to test experimentally, because in $p\bar{p}$ collisions which are the most plentiful source of gluon jets, there is a serious background from soft partons coming from spectators.

These coherence effects are most conveniently included in Monte Carlo programs using the angular ordering approximation described above. Recent work[30]

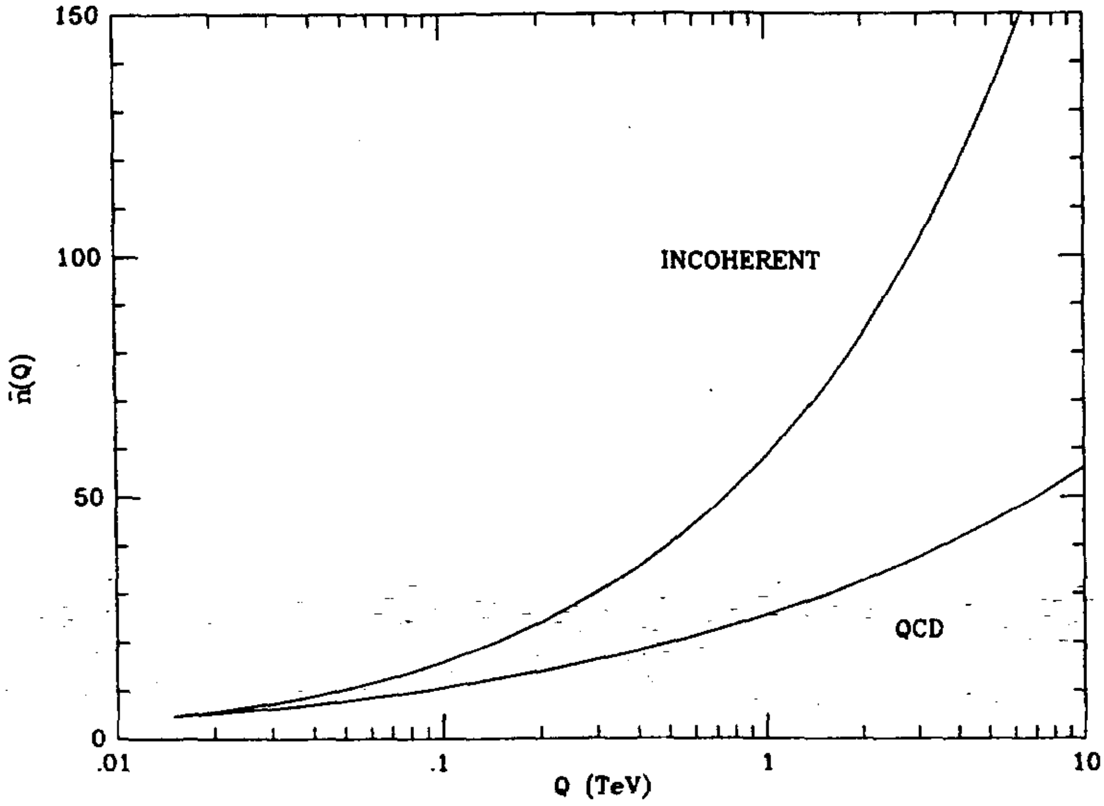

Figure 6: Growth of the average hadronic multiplicity of a quark jet.

has examined the validity of this approximation for the exclusive process $g \rightarrow g + g + g$. Regions of phase space were found in which the angular ordered result differed substantially from the full QCD matrix element. The angular ordered approximation must hold in the strongly ordered region $x_i \ll x_j \ll x_k$ but breaks down outside these regions. x_i is defined to be the light cone momentum fraction of the i^{th} final state parton with respect to the initial parton. In particular, for $x_i \ll x_j \approx x_k$ the discrepancy betwen the angular ordering approximation and the exact result was found to be of order 100%.

This large discrepancy is attributable to soft emissions from the incoming coloured gluon. It is therefore to be expected that the angular ordering prescription will be a much better approximation for colour singlet sources from which such an emission cannot occur. This has been demonstrated to be the case in ref.(31). This suggests that the angular ordering approximation is always reliable for physical processes that are colour singlet initiated. However this mechanism will presumably not be operative in the region of phase space in which a gluon is soft with respect to the scale of the hard interaction, yet harder than the confinement scale on which the colour cancellation occurs. The fate of the angular ordered approximation in this region of phase space is not yet known.

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