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UNDERSTANDING THE BOILING WATER REACTOR LIMIT CYCLE

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This paper presents an interpretation of the physical mechanisms involved in the development of limit cycle oscillations in boiling water reactors (BWRs). Based on this interpretation, approximate correlations for some oscillation parameters are developed and shown to be largely independent of the particular reactor operating condition. The stability of the limit cycle is also studied in this paper. It is shown that the BWR limit cycle may become unstable and bifurcate. The bifurcation process leads to aperiodic (chaotic) behavior of the reactor power and causes the peak oscillation powers to be larger than those from a nonbifurcated limit cycle.

BACKGROUND

It is a well known fact that BWRs are susceptible to instabilities when operated at relatively low flow and high power. At least three instability types have been recognized in commercial BWRs:

(1) Control system instabilities, that may be caused by out of tune controllers.

(2) Channel thermohydraulic instabilities, that are purely thermohydraulic instabilities related to the momentum dynamics of heated channel in a two-phase flow regime.

(3) Reactivity instabilities, that originate from the coupling of the reactor neutronics with the channel thermohydraulics, including momentum dynamics. Two types of reactivity instabilities are recognized:

   (a) In-phase or core-wide instabilities, when the neutron dynamics of the fundamental mode dominates resulting in core-wide oscillations.

   (b) Out-of-phase or regional instabilities, when parallel channel momentum dynamics is the dominant feedback loop and is reinforced by a subcritical mode of the neutronics.

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The above four instability modes have been observed in either special tests or as a result of normal reactor operation at low flows. Typically, reactivity instability modes are of more relevance to safe commercial BWR operation than the other two types because the control system and the channel thermohydraulics are designed to be stable under normal conditions.

Reactivity instabilities result in power oscillations that diverge from the steady state equilibrium point following a spiral trajectory in phase space. If BWRs behaved as linear systems, the oscillation trajectory would diverge indefinitely. The nonlinearities in the system, however, cause the oscillation to remain bounded as the trajectory converges to a limit cycle. The limit cycle is a particular periodic trajectory in phase space that attracts all other trajectories. If the oscillation trajectory is perturbed away from the limit cycle, it will eventually converge back to the limit cycle when the cause of the perturbation is removed. An example of a limit cycle in phase space can be seen in Fig. 1. In this figure, phase space is represented by the neutron flux and the excess fuel temperature. It can be observed that the trajectories diverge (i.e., spiral away) from the unstable equilibrium point but stay bounded and are attracted by the limit cycle trajectory. Mathematically, the limit cycle is caused by the nonlinearities in the system dynamics that have an stabilizing effect on the divergent oscillation. The main nonlinearity affecting BWR dynamic behavior was determined to be related to the power-dependence of the cross sections[1,2] (i.e, the void reactivity coefficient). In the point kinetics approximation, this nonlinearity is represented by the term ρ times n.

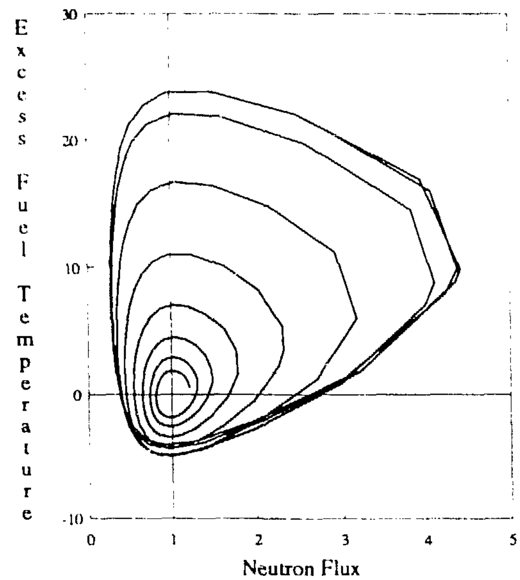

Fig. 1. Illustration of the development of a typical limit cycle in phase space

REACTIVITY FEEDBACK DURING LIMIT CYCLE OSCILLATIONS

Boiling water reactors are extremely complex nonlinear devices. To understand its behavior we must simplify their dynamics to a bare minimum. With this goal in mind, we should attempt to understand first their linear dynamic behavior. Figures 2 and 3 show the open loop transfer functions of a typical BWR

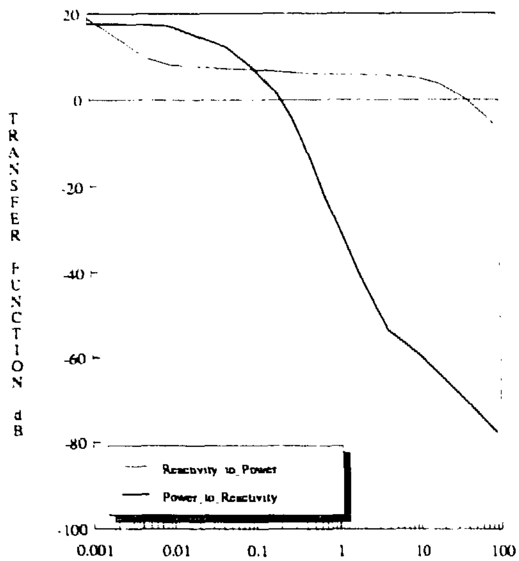

Fig. 2. Typical BWR reactivity-to-power and power-to-reactivity transfer functions

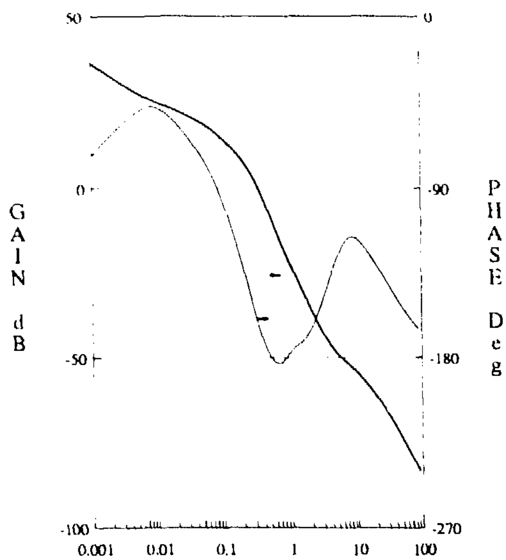

Fig. 3. Typical BWR open loop transfer function

computed using the LAPUR code.[3] A reactor is unstable if the open-loop gain is greater than 0 dB at the point where the phase is -180°. Observation of Fig 3 shows that the phase is only below -180° for a small range of frequencies around 0.4 Hz; therefore, unstable oscillations in BWRs must have a period of approximately 2.5 s. This period is related to the bubble residence time in the core. We also observe in Fig. 2 that the magnitude of the reactivity feedback transfer function has a steep slope beyond 0.3 Hz. This implies that the reactivity feedback acts as a strong low-pass filter, and damps high frequencies from the power oscillation. Thus, we can reach two conclusions from these simple observations:

(1) If a limit cycle is developed by a BWR, the reactor power should follow a periodic oscillation with a period of about 2.5 s (i.e., a frequency of approximately 0.4 Hz).

(2) Regardless of the time-shape of the power oscillation, the reactivity oscillation should be essentially sinusoidal. This is due to the filtering effect of the reactivity feedback transfer function. No mater how many harmonics the power oscillation has, the reactivity feedback transfer function will filter out their contribution to the reactivity.

These two findings might seem rather irrelevant at first, but they allows us to simplify the limit cycle analysis dramatically. Typically, most of the modelling complications arise from the thermohydraulic feedback. If these two findings are applicable to any limit cycle case, then the feedback reactivity, $\rho(t)$, is approximately given by an expression of the form

$$\rho(t) = -\rho_0 + \rho_1 \sin(wt) , \tag{1}$$

where ρ_0 is the average reactivity value, ρ_1 is the amplitude of the oscillations and w is the oscillation frequency that is approximately equal to 0.4 Hz.

The reduction of the complexity of a thermohydraulic model to a simple equation such as Eq. (1) allows us to perform very simple analysis that yield general correlations applicable with some degree of approximation to any BWR as long as Eq. (1) holds. The parameters of these correlations become now ρ_0 and ρ_1, instead of the physical geometry and cross sections of a particular reactor.

Equation (1) can be justified mathematically in the following manner: First. We know that a limit cycle oscillation is periodic. Thus, the power oscillation, $n(t)$, can be expanded in Fourier series without loss of generality as

$$n(t) = \Sigma A_k \cos(kwt) + \Sigma B_k \sin(kwt) . \tag{2}$$

Due to the filtering effect of the fuel transfer function, large oscillations in power result in fairly small oscillations in heat flux from fuel to coolant, and the reactivity feedback transfer function can be considered to behave linearly.[2] Thus, the reactivity feedback can be approximated with good accuracy by the following expression

$$\rho(t) = \Sigma H_k A_k \cos(kwt + \psi_k) + \Sigma H_k B_k \sin(kwt + \psi_k) , \tag{3}$$

where H_k and ψ_k are the gain and phase, respectively, of the feedback transfer function. Typical power feedback magnitudes (H_k) are: 0.14 dollars per percent power change at 0 Hz, 0.007 \$/% at 0.4 Hz, and 0.0014 \$/% at 0.8 Hz. Since the first two harmonics (k = 0, and k = 1) account for 99% of the feedback energy, higher harmonics can be neglected and Eq. (3) reduces to Eq. (1) with the proper selection of initial phase lag. We shall use Eq. (1) to study the BWR limit cycle and to obtain some general correlations.

REACTOR RESPONSE TO SINUSOIDAL REACTIVITY PERTURBATIONS

The response of a nuclear reactor to sinusoidal reactivity perturbations has been well studied.[4,5] It is well known that, for the solution to exhibit bounded periodic solutions (i.e., limit cycles), a negative bias (i.e., $-\rho_0$) is required. For each value of ρ_1, there is only one value of ρ_0 that results in a limit cycle: if ρ_0 is too small, the oscillations diverge exponentially; if ρ_0 is too large, the oscillations

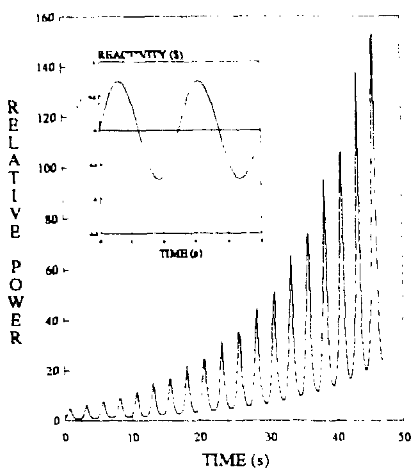

Fig. 4. Power response to sinusoidal reactivity with small bias

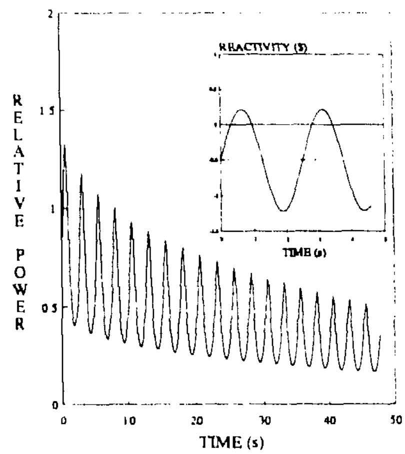

Fig. 5. Power response to sinusoidal reactivity with large bias

converge to zero. This effect can be observed in Figs. 4 and 5, that shows the response of a point kinetics model (with typical BWR parameters) to a sinusoidal reactivity perturbation of the type of Eq. (1). In Fig. 4, the bias, ρ_0, is too small and the resulting power oscillation diverges. In the case of Fig. 5, the bias is too large and the power oscillation is damped and converges towards zero.

If the right amount of bias, ρ_0, is present for the particular amplitude of reactivity oscillation, ρ_1, we obtain the situation represented in Fig. 6, where the power oscillation reaches and maintains a constant amplitude. The reactor behavior in this case is similar to that of a limit cycle caused by an instability. Thus, we conclude that in order to obtain periodic bounded power oscillations in a nuclear reactor, a relationship must be satisfied between the reactivity bias, ρ_0, and the reactivity oscillation amplitude, ρ_1. Physically, the reactivity bias is caused by an increase in average reactor power that increases with the oscillations until the reactor is subcritical enough to compensate for the divergent tendency imposed by the reactivity oscillation term, $\rho_1 \sin(wt)$.

To study the relationship between ρ_0 and ρ_1, the model of ref. 2 was used to compute a large number of limit cycles with different oscillation amplitudes. In the model, which is described and validated in refs 1 and 2, the fuel dynamics are represented based on a single node approximation, the channel thermohydraulics are

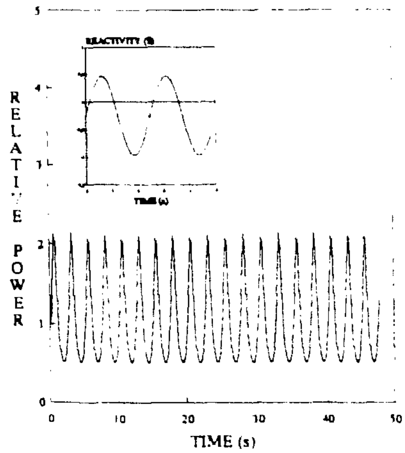

Fig. 6. Power response to sinusoidal reactivity with the right bias

modeled using a two-node representation, and the neutron dynamics are based on the point kinetic approximation. The model parameters used are those presented in ref. 2 with the exception of using 6 groups of delayed neutrons instead of 1 and $K_0 = -3.2769\times10^{-3}$. Different limit cycle conditions were modeled by changing the feedback gain, K.

Table 1. Calculated limit cycle parameters as function of peak magnitude

K[a]	DR[b]	N_{peak}[c]	ρ_0[d]	ρ_1[e]
1.0	1.00	1.00	0.00	0.00
1.02	1.02	1.45	0.063	0.36
1.05	1.08	1.82	0.16	0.59
1.1	1.15	2.35	0.32	0.87
1.2	1.30	3.30	0.65	1.32
1.3	1.47	4.24	0.98	1.72
1.4	1.62	5.15	1.32	2.10
1.5	1.78	6.2[f]	1.65	
1.6	1.93	9.8[f]	1.90	
1.7	2.08	13.5[g]		
1.8	2.24	18.[g]		
1.9	2.39	24.[g]		
2.0	2.53	32.[g]		

[a] Inverse gain margin (see ref. 2)
[b] Decay ratio
[c] Peak oscillation value as fraction of equilibrium power
[d] Reactivity bias in dollars
[e] Reactivity oscillation in dollars
[f] Solution has bifurcated (see ref. 2). Value reported is highest peak
[g] Solution is chaotic (see ref. 2). Value reported is highest peak

A summary of the results of this analysis is presented in Table 1, that contains the values of ρ_0 and ρ_1 required to obtain a limit cycle of amplitude N_{peak}. The linear decay ratio and inverse gain margins are also shown in Table 1. From these data, we can obtain the following correlation:

$$\rho_1^2 = 2\,\rho_0 + \rho_0^2 \qquad (\pm .006,\ \rho_1 < \$2)\ , \qquad (4)$$

that relates reactivity bias required to maintain a bounded limit cycle with a particular reactivity oscillation amplitude. Equation (4) compares well with the analytic solution for small oscillations reported in ref 4 ($\rho_1^2 = 2\rho_0$, $\rho_0 << \$1$).

From the results of Table 1, we observe that the peak value of the neutron flux oscillations, N_{peak}, is related to ρ_1 through a correlation of the form

$$(N_{peak} - N_0)/N_0 = \rho_1 + 0.75\,\rho_1^2 - 0.14\,\rho_1^3 \qquad (\pm .001,\ \rho_1 < \$2)\ , \qquad (5)$$

where N_o is the equilibrium flux. Note that Eq. (5) is only valid for reactivities satisfying Eqs (1) and (4), i.e., for fully developed periodic limit cycles, because Eq. (5) obviously can not be applied to cases like those of Figs 4 or 5. Equation (5) can be interpreted as the nonlinear reactor transfer function at the frequency studied (w = 0.4 Hz), and reverts to the linear gain (i.e., 1.0) for small values of ρ_1, as expected.

Numerical simulations appear to indicate that Eqs. (4) and (5) are independent of reactor parameters and, thus, general for any nuclear system where the point kinetics approximation is applicable.

PHYSICAL MECHANISM OF THE BWR LIMIT CYCLE

As seen in the previous section, in order to have a self-sustained, periodic bounded oscillation of the power in a nuclear reactor, the reactivity feedback must oscillate sinusoidally and have a negative bias of the appropriate magnitude. Indeed, this bias can be understood as the stabilizing mechanism for an unstable divergent oscillation. To understand this mechanism, let's follow, step by step, the development of a limit cycle in a BWR from the point of inception of the linear instability:

(1) A moment before the instability event starts, the reactor can be assumed to be operating in a steady state condition with some particular power and flow. Since the reactor is critical at this point, the net reactivity is zero.

(2) If by some change of conditions, the reactor suddenly becomes unstable in the linear sense, any small perturbation will result in diverging power oscillations that, in phase space, will spiral away from the equilibrium point similarly to the case presented in Fig. 1. Initially, the perturbation around the equilibrium point will be small enough so that the reactor will behave linearly and the oscillation will grow exponentially with a time constant equal to DR/w, where DR is the decay ratio and w is the oscillation frequency.

(3) As the oscillation becomes larger, the nonlinearities in the system, and specially the ρ-times-n term begin to grow in importance. These nonlinearities have the effect of "leaking" power between otherwise linearly-orthogonal modes. In effect, the power that the instability is generating on the fundamental oscillation mode (sin(wt)) is distributed among other modes. In particular, some of the power of the fundamental mode "leaks" into the steady

state mode (i.e, the average power). This increase in average power feeds back to the reactivity and generates a negative reactivity bias, ρ_o.

(4) As the reactivity bias increases, the reactor becomes more and more subcritical and tends to damp out the unstable oscillation. When a sufficiently large degree of subcriticality is reached to cancel out the growth tendency imposed by the instability, a dynamic equilibrium is established and the limit cycle oscillation remains at a constant level.

Thus, we see that the underlying cause of the appearance of limit cycles that bound the oscillations of an unstable BWR is the increase in the average power that accompanies the oscillations.

THE AVERAGE POWER INCREASE

We have seen that the average reactor power must increase during oscillations. Unfortunately, general correlations of the type of Eqs. (4) and (5) can not be found for this increase because it depends on the particular reactor conditions. The procedure to estimate the average power increase for a given peak oscillation power is as follows:

(1) Estimate, from Eq. (5), the value of ρ_1 required to establish the particular peak power, N_{peak}. Note that this procedure depends on the initial (or steady state) power, N_o. For instance, almost twice as much reactivity, ρ_1, is required to obtain a peak power of 100 MW over the initial power when operating at an initial power of 1000 Mw than at a power of 2000 MW. Thus, the value of ρ_1 depends not only on N_{peak}, but also on N_o.

(2) Determine, from Eq. (4), the reactivity bias, ρ_o, required to maintain a periodic bounded oscillation with the particular value of ρ_1 obtained from step 1.

(3) Determine the average power increase required to produce a reactivity bias of ρ_o. This is accomplished by dividing ρ_0 by the steady state power reactivity coefficient, which depends on the particular reactor and operating conditions.

This procedure results in an approximate correlation for the average power increase, N_{ave}, of the form

$$N_{ave} = F_{ave} \ (N_{peak} - N_o) \ , \tag{6}$$

where F_{ave} is a proportionality factor that, for typical BWR parameters, is of the order of 0.015 to 0.02. In other words, the average power increase is typically 1.5% to 2% of the value of the peak power minus the steady state power.

LIMIT CYCLE STABILITY

Numerical simulations[2,6] have shown that as a parameter controlling the reactor's linear instability is increased, the limit cycle may become unstable and degenerate, through a cascade of period-doubling *pitchfork bifurcations*,[7] into a chaotic (i.e., aperiodic) sequence of power oscillations. In this regime, the power oscillations are aperiodic but remain bounded by a *strange attractor*.[7] It appears, however, that the oscillation amplitude (i.e., peak power) is increased by this phenomenon, and that the bifurcated limit cycle exhibits larger peak powers than extrapolation of the nonbifurcated limit cycle would predict. Thus, the understanding of the bifurcation phenomenon in BWRs is relevant to accident analysis where large peak powers may affect fuel integrity.

As seen in previous sections, if a positive disturbance is imposed on a limit cycle, the system nonlinearities create a positive average power increase that, through its reactivity effect, damps out the disturbance. An instability in this mechanism can occur if the average power increases too much during a power peak causing the next peak to decrease by an amount larger than the original disturbance. Thus, if a limit cycle should become unstable, an oscillation of double the original period will be established because the mechanism involves two full oscillation periods. In first approximation, the average power ($<N>$) and the reactivity bias, ρ_o, can be assumed proportional to the peak disturbance value, N_{peak}.

$$\frac{\Delta\rho_o}{\rho_o} = \frac{\Delta<N>}{<N>} = \frac{\Delta N_{peak}}{N_{peak}} \quad . \tag{7}$$

Poincare maps were used in ref. 2 to study numerically the stability of the limit cycle and the bifurcation process. These maps are obtained by plotting the value of a power oscillation peak, N_2, versus the previous oscillation peak value, N_1. It was shown in ref. 2 that this procedure results in a quadratic map similar to those studied by Feigenbaum.[7] An analytic expression for this map is the following:

$$N_2 = N_1 \, DR(N_1) \ , \tag{8}$$

where DR is the nonlinear decay ratio, which is a function of the oscillation amplitude. The equilibrium limit cycle is defined by $N_2 = N_1$, or $DR = 1.0$.

The stability of the limit cycle is guaranteed as long as the derivative of N_2 with respect to N_1 evaluated at the equilibrium point ($DR = 1.0$) is greater than -1.0 (i.e., the slope of the Poincare map is smaller than 45°). Taking derivatives in Eq. (8), we obtain

$$\frac{dN_2}{dN_1} = DR + N_1 \frac{dDR}{dN_1} = 1.0 + \rho_o \frac{dDR}{dK} \frac{dK}{d\rho_o} , \tag{9}$$

where K is the inverse gain margin,[2] and we have made use of Eq. (7). Thus, the stability condition is

$$\rho_o > -2 / (\frac{dDR}{dK} \frac{dK}{d\rho_o}) , \tag{10}$$

From Table 1 we observe that the term dDR/dK evaluated around DR = 1.0 is approximately constant and equal to 1.43. The term $dK/d\rho_o$ is physically the change in gain of the neutron dynamics caused by a reactivity increase. For small perturbations around the limit cycle, this second term is approximately 1.0 dollars^{-1}. Thus, the condition for stability of the limit cycle can be approximated by

$$\rho_o > \$-1.40 . \tag{11}$$

Using the correlations developed above, the stability condition can be expressed in terms of peak power. The result is that BWR limit cycles should be stable for peak power values of approximately less than 500% of equilibrium (i.e, initial) power. Beyond these limits, the bifurcation and chaotic regimes are established and result in larger peak powers than a nonbifurcated limit cycle study would predict.

SUMMARY

It has been shown in this paper how a limit cycle bounds the power oscillation in BWRs when they become unstable. The underlying cause for the BWR limit cycle is an increase in the average power that result in a negative reactivity bias. This average power increase is above the required equilibrium power taking into account all system effects such as water level or inlet subcooling changes. Thus, BWRs under limit cycle conditions are actually subcritical.

Two general correlations have been developed that relate the reactivity bias with the amplitude of the oscillation. Based on reactor dependent parameters, the average power increase can be calculated from these correlations. Typical values of the average power increase are 1.5% to 2% of the peak power oscillation.

The stability of the BWR limit cycle has been studied. It has been shown that the limit cycle can become unstable and bifurcate into an aperiodic regime. The bifurcation process increases the amplitude of the expected peak powers, and it is estimated that should occur for limit cycles with peak powers greater than 500% of the steady state power.

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