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3-D RESISTIVE MHD CALCULATIONS FOR TOKAMAK PLASMAS: BEYOND THE SIMPLE REDUCED SET OF EQUATIONS*

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ABSTRACT

Numerical studies of the resistive stability of tokamak plasmas in cylindrical geometry have been performed using: (1) the full set of resistive Magnetohydrodynamic (MHD) equations and (2) an extended version of the reduced set of resistive MHD equations including diamagnetic and electron temperature effects. In particular, the nonlinear interaction of tearing modes of many helicities has been investigated. The numerical results confirm many of the features uncovered previously using the simple reduced equations.

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INTRODUCTION

Since the last workshop, we have extended the scope of the three dimensional resistive MHD calculations, by moving in two directions simultaneously. First, we have incorporated more physics, like diamagnetic and viscous effects, and resistivity and density evolution, in the reduced set of resistive MHD equations [1]. Secondly, we have also started calculations using the full set of resistive MHD equations. Both types of calculations at present are limited to cylindrical geometry. We have made detailed comparisons with previous calculations using the standard reduced set of MHD equations. It was of special interest for us to study their potential impact on the basic disruption model which we developed on the basis of the interaction of multiple helicity tearing modes [2]. The present results do not modify the basic conclusions of the reduced set of equations, but they do show some quantitative differences.

II. FULL SET OF MHD EQUATIONS

The full set of resistive magnetohydrodynamic equations can be written as [3]

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B} - \frac{1}{S_A} \eta \vec{J}) , \quad (1)$$

$$\frac{\partial \vec{v}}{\partial t} = \vec{v} \times (\vec{\nabla} \times \vec{v}) - \frac{1}{2} \vec{\nabla} (v^2 + \beta_0 p) + \vec{J} \times \vec{B} + \frac{1}{R} \nabla_\perp^2 \vec{v} , \quad (2)$$

plus an equation of state for the pressure. The equation of state either assumes the form

$$\frac{\partial p}{\partial t} = -\vec{v} \cdot \vec{\nabla} p - \Gamma p \vec{\nabla} \cdot \vec{v} + \eta(\Gamma - 1) |\vec{J}|^2 (B_0 S_A)^{-1} \quad (3a)$$

which results in a compressible plasma model, or

$$\nabla^2 p = \frac{2}{\beta_0} \vec{\nabla} \cdot \left[\vec{v} \times (\vec{v} \times \vec{v}) - \frac{1}{2} \vec{\nabla} v^2 + \vec{J} \times \vec{B} + \frac{1}{2} \nabla_\perp^2 \vec{v}\right] . \tag{3b}$$

Equations (1)-(3) have been written in a dimensionless system of units with all lengths normalized to a (the plasma minor radius), the magnetic field $\vec{B}$ to B_0 (the equilibrium toroidal vacuum field at the plasma major radius R_0), the velocity $\vec{v}$ to the Alfvén velocity $v_A = (B_0^2/\mu\rho_0)^{1/2}$, the time to the Alfvén time $\tau_A = a/v_A$, the pressure p to p_0 (the equilibrium value at the magnetic axis), and the resistivity η to η_0 (the value at the magnetic axis). In terms of these quantities $S_A = \tau_r/\tau_A$ is the ratio of the resistive skin time $\tau_r = \frac{a^2\mu}{\eta_0}$ to the Alfvén time; $\beta_0 = 2\mu p_0/B_0^2$ is the equilibrium beta at the magnetic axis; and $R = av_A/\nu$ where ν is the viscosity in units of $[a^2/\tau_A]$. The unit vector $\hat{\zeta}$ denotes the toroidal (or axial in cylindrical geometry) direction, and the subscript $\perp$ denotes the poloidal (perpendicular to $\hat{\zeta}$) plane. The coefficient Γ in Eq. (3a) is the ratio of specific heats for the plasma.

Both the full and reduced sets of equations are solved using three-dimensional, nonlinear, initial value computer codes, CYL and RSF, [4] respectively. These codes are very similar: both are partially implicit, using finite differences in the radial coordinate r and in time, and a spectral representation with periodic boundary conditions in θ and ζ.

The first step in comparing both sets of equations has been to perform systematic linear stability studies. These results will be reported elsewhere [3]. We would like to emphasize here the reasonable agreement between the linear growth rates obtained from both calculations (Fig. 1) even for relatively small aspect ratios ($\varepsilon_L \simeq 0.2$). At very small aspect ratio ($\varepsilon_L \sim 1$) the main changes to the eigenfunctions come through pressure effects (Fig. 2). In this paper we will concentrate on the nonlinear three dimensional results, namely the nonlinear interaction of tearing modes.

In order to compare the results of calculations having interacting tearing modes, multihelicity calculations were carried out at $\varepsilon_L = 0.2$, $S_A = 10^5$, and $R = 1.67 \times 10^5$. The reduced set of equations, as well as the full set of equations with constant $B_\zeta^{eq} = 1$, were used. Sixteen components were included in the calculations according to the ordering scheme in Ref. [4]. In both the full and reduced equation calculations the 2/1 and 3/2 magnetic islands grow, overlap, and result in a stochastic magnetic field over a large portion of the plasma (Figs. 3 and 4). Consistent with the single helicity results [3], the islands display somewhat more rapid growth and larger amplitudes with the full equations than with the reduced equations, but the evolution as seen by the two systems of equations is quite similar. The (m=2;n=1) and (m=3;n=2) modes are both linearly unstable. These instabilities grow independently until their nonlinear interaction (facilitated by the (m=5;n=3) and the (m=1;n=1) modes) becomes strong. This occurs roughly at the time the 2/1 and 3/2 magnetic islands overlap. At this time the 2/1 drives the 3/2 instability through their nonlinear interaction leading to the stochastization of the magnetic field throughout a sizeable region of the plasma. We associate this process with certain major disruptions [2]. The effects of this process upon the magnetic field lines for the full equation calculation is shown in Fig. 4.

III. REDUCED SET OF MHD EQUATIONS WITH DIAMAGNETIC EFFECTS

We now consider a reduced set of nonlinear three dimensional resistive MHD equations including diamagnetic effects. These are a generalization of the ones used elsewhere [5,6] in a single helicity approximation, we have further included the resistivity evolution and the effect of the thermal force. The equations are

$$\frac{\partial\psi}{\partial t} + \vec{v}_\perp \cdot \vec{\nabla}_\perp\psi = \eta J_\zeta - S(\omega_{*e}T_e - \omega_{*i}T_i)\frac{1}{n}\nabla_\parallel n$$

$$- S\omega_{*e}(1 + \alpha)\nabla_\parallel T_e - \frac{\partial\phi}{\partial\zeta} \tag{4}$$

$$\frac{\partial U}{\partial t} + \vec{v}_\perp \cdot \vec{\nabla}_\perp U + \frac{1}{2}\zeta \cdot [\vec{\nabla} r \times \vec{\nabla} v_\perp^2]$$

$$= -S^2\nabla_\parallel J_\zeta + \mu\, S T_i\, \nabla_\perp^2 U +$$

$$\omega_{*i}\, S T_i\, \vec{\nabla}_\perp \cdot ([[\vec{\nabla}_\perp n \times \zeta] \cdot \vec{\nabla}_\perp]\, \vec{\nabla}_\perp\phi) \tag{5}$$

$$\frac{\partial n}{\partial t} + \vec{v}_\perp \cdot \vec{\nabla}_\perp n = \frac{S}{\tau_{hp}\Omega_{ci}}\nabla_\parallel J_\zeta \tag{6}$$

$$\frac{\partial T_e}{\partial t} + \vec{v}_\perp \cdot \vec{\nabla}_\perp T_e = \chi_\parallel\nabla_\parallel^2 T_e + \chi_\perp\nabla_\perp^2 T_e + \frac{2\alpha S}{3\tau_{hp}\Omega_{ci}}\frac{1}{n}\nabla_\parallel(T_e J_z) \tag{7}$$

and

$$\eta = T_e^{-3/2} \tag{8}$$

These equations are written in dimensionless form, and details on normalization and conventions can be found in Ref. [7]. They are solved by a time evolution scheme implemented in the computer code KITE, and are used to study drift-tearing [8], rippling [9] and drift-rippling modes. The numerical scheme is very similar to the ones described earlier for CYL.

In this paper we present only results relevant to the nonlinear interaction of drift tearing modes. One important consequence of adding the electron diamagnetic effects is the rotation of the magnetic islands (Fig. 5). This produces a time dependent phase between the different modes, which in turn might have an important effect on their nonlinear interaction. We have investigated this possibility for a case of a strong disruption, with an equilibrium q-profile $q = 1.344[1 + (r/r_0)]^{1/4}$ $r_0 = 0.56$, which was previously studied [7] in great detail with the simple reduced set of equations. We did the present numerical calculations for values of the electron diamagnetic frequency comparable to the linear tearing mode growth rate. In particular we show in Fig. 6, the results for $\bar{\omega}_{*e} = 1.3\gamma_T$ which is a reasonable value for present day experiments. The results are compared with the $\omega_{*e} = 0$ case. They show no apparent modification of the basic interaction mechanism leading to the strong destabilization of the (m=3;n=2) mode. More sensitive to the diamagnetic effects are the weakly interacting cases, in which magnetic island overlap is marginal. The diamagnetic effects are important in modifying the boundary between disruptive and nondisruptive equilibria.

CONCLUSION

Although the different effects studied in this paper quantitatively modify and certainly complicate the dynamics of the nonlinear tearing mode interaction, they do not change the nonlinear destabilization of the (m=3;n=2) mode by the (m=2;n=1). Therefore, the basic disruption model put forward in Ref. [2] is supported when a more complete physical model is incorporated in the calculations.

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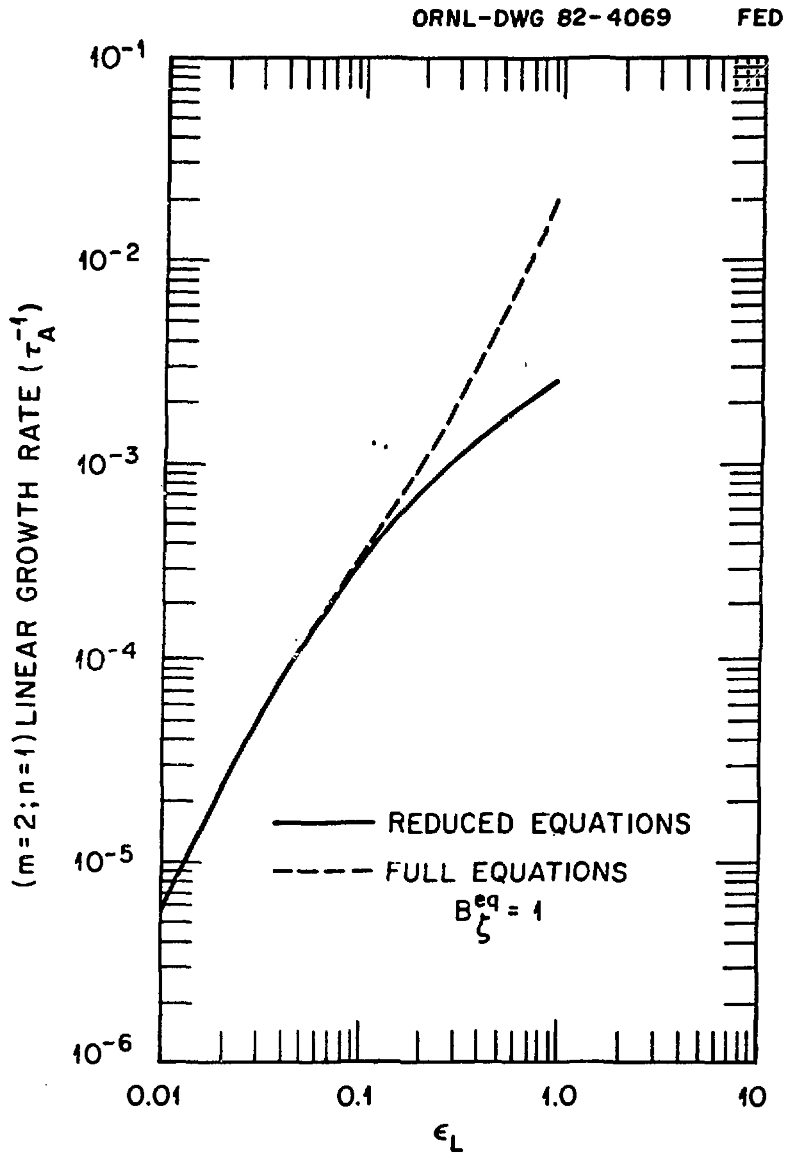

Fig. 1. (m=2;n=1) linear growth rate vs ϵ_L with $S_A = 10^5$.

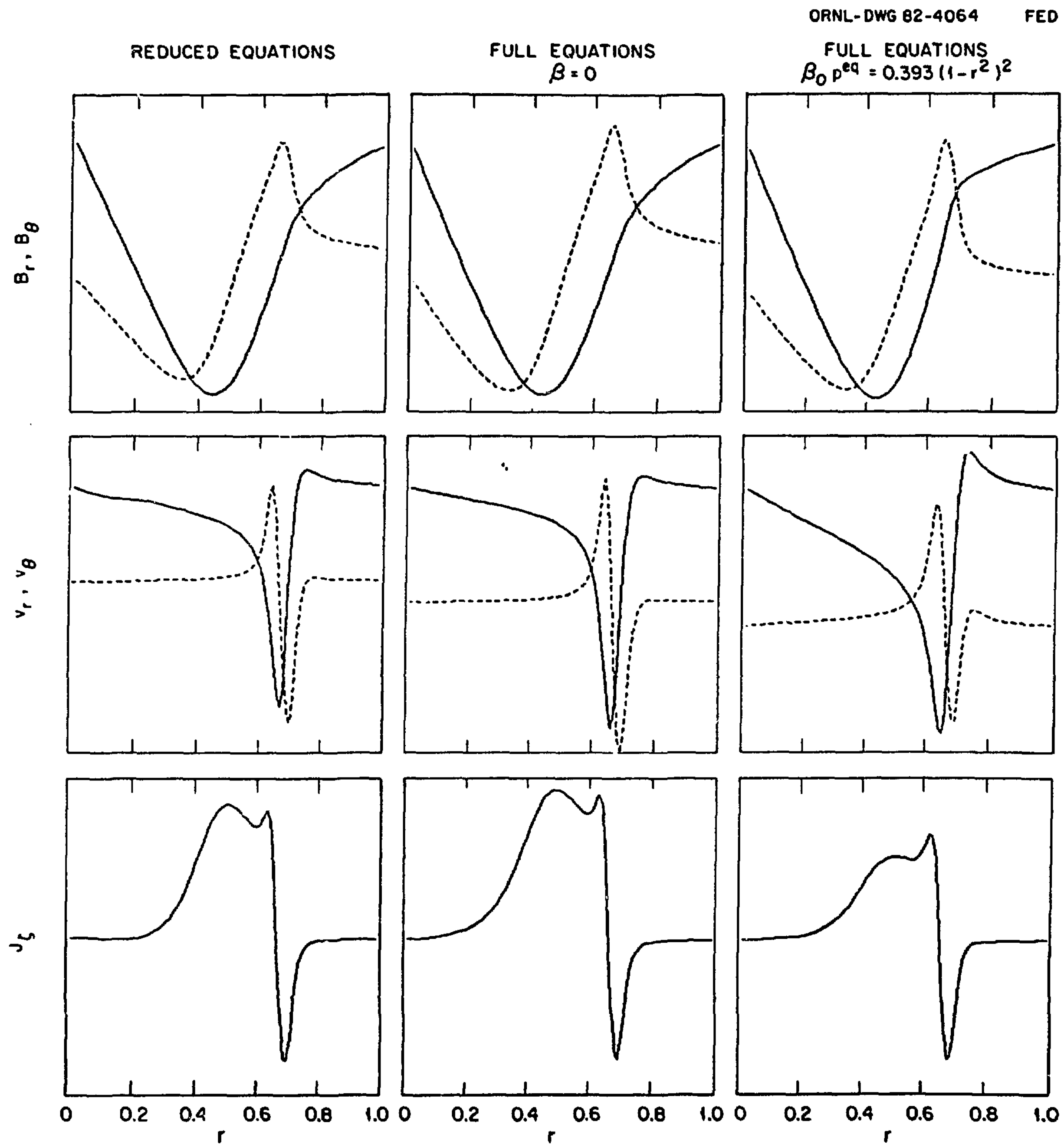

Fig. 2. (m=2;n=1) eigenfunctions showing B_r, B_θ, v_r, v_θ, and J_ζ for $\epsilon_L = 1$ and $S_A = 10^5$.

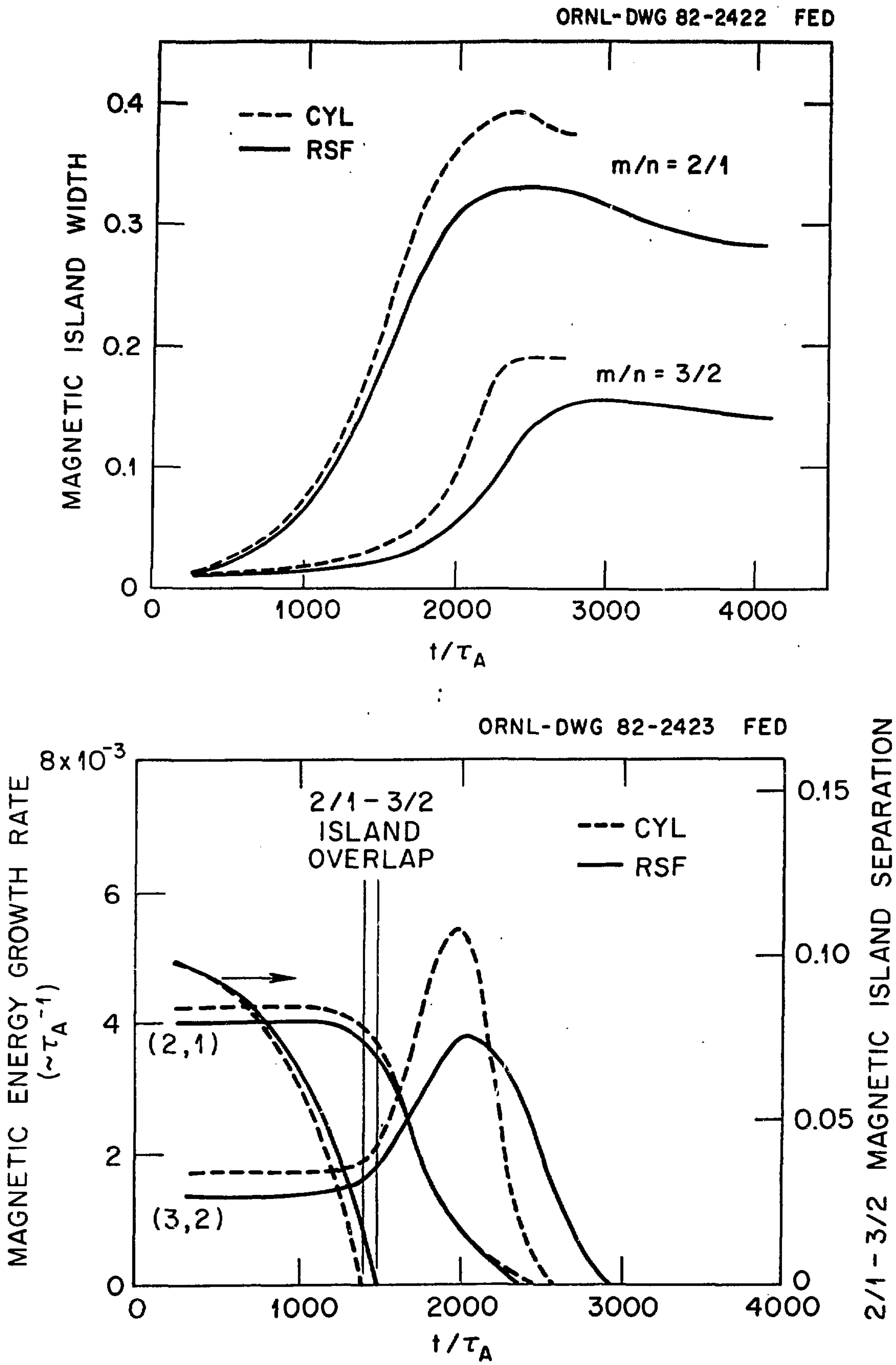

Fig. 3. m/n=2/1 and m/n=3/2 magnetic island widths (a) and magnetic energy growth rates (b) vs time for $\varepsilon_L = 0.2$ and $S_A = 10^5$. The arrows indicate the times at which the 2/1 and 3/2 magnetic islands overlap. The full set of equations was solved with $B_\zeta^{eq} = 1$.

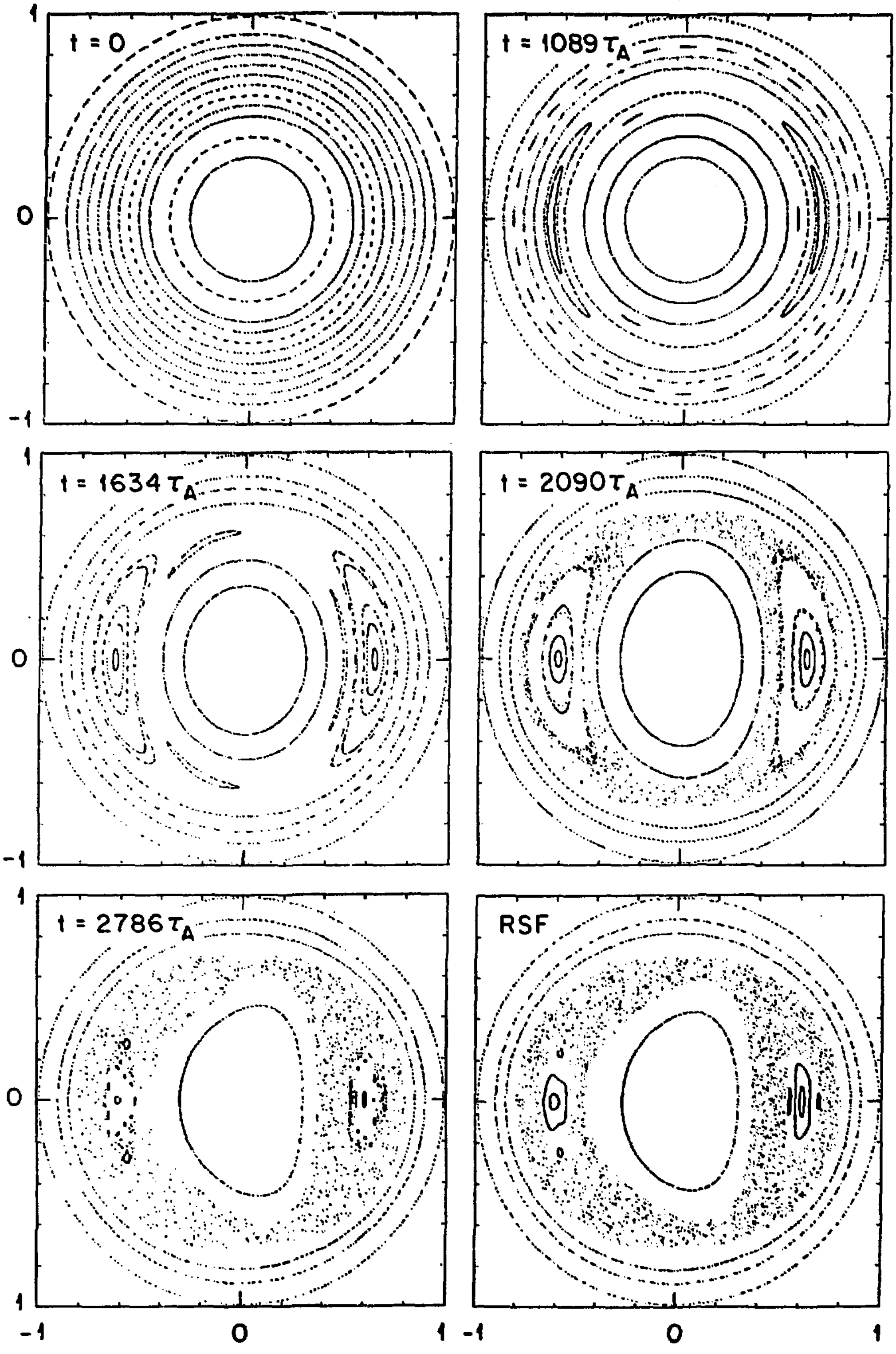

Fig. 4. Magnetic field line plots at several times for the case described in Fig. 3. The full set of equations was used.

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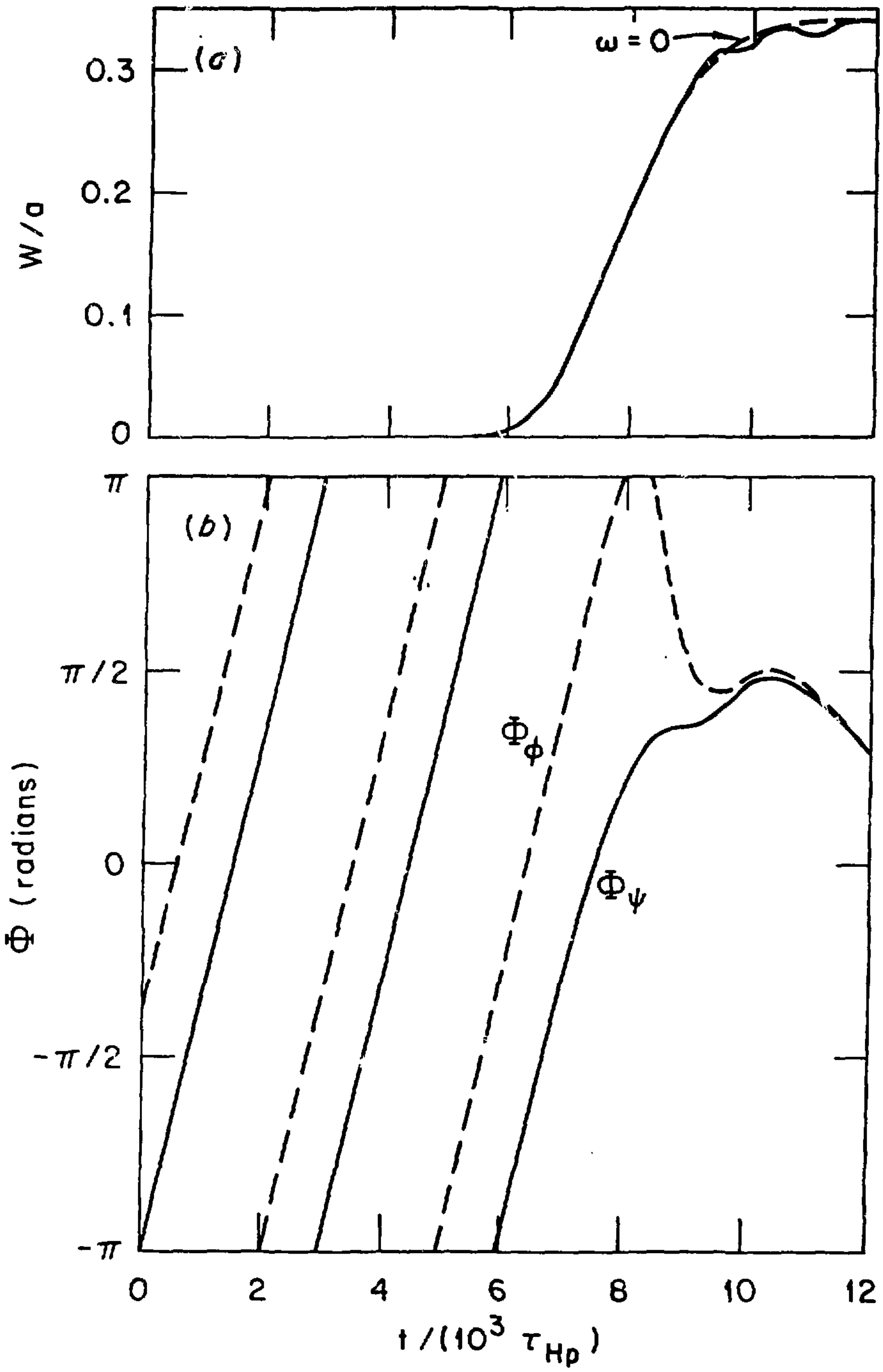

Fig. 5. Nonlinear evolution of the (m = 2;n = 1) drift tearing mode in the single helicity approximation: (a) magnetic island width and (b) phase versus time.

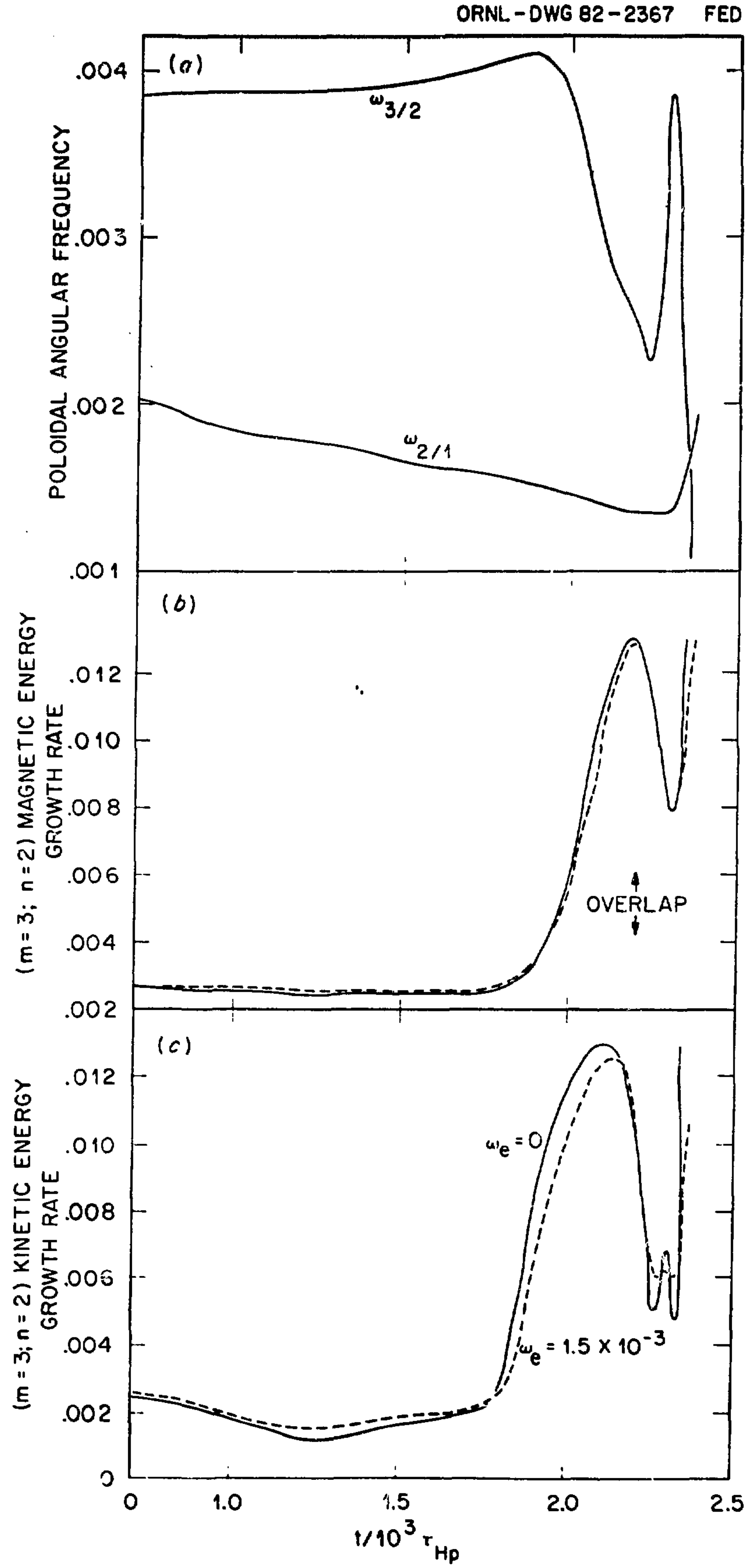

Fig. 6. Results of the nonlinear multiple helicity tearing mode calculation described in the text.