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A NEW TECHNIQUE FOR BACKSCATTERING ANALYSIS*

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We suggest a new technique for analyzing Rutherford backscattering spectra. By determining the ratio of the range of a particle of energy E, to that of a particle whose energy is a fraction of E, we convert two basic integral equations into a pair of linear equations that are easy to solve and manipulate. We demonstrate the applicability of this formalism by using it to measure the stopping power of gold for alpha particles with energies 1.2. 2.2, 3.7, 5.2 and 7.1 MeV.

1. INTRODUCTION

Backscattering spectrometry is being used for a wide variety of applications as indicated by the range of topics discussed at this conference. Many of the basic analytic tools that have been developed for this field are discussed in a recent book by Chu et al.[1] In formulating the problem addressed in the present paper, we have used the definitions, and to some extent the notation as found in Ref. 1.

A beam of particles, of incident energy E_i, bombard a target at angle Θ_i with respect to its normal. As shown in Fig. 1, incident particles elastically scattered from the surface of film atoms at angle Θ_f, in the plane of the incident beam and the target normal, will be reflected with an energy KE_i, where K is the kinematic factor.[1] When the incident particle penetrates the film to a depth x and then elastically scatters from a target atom, the relationship between x, E_i, Θ_i, Θ_f, and the final energy E_f with which the particle emerges, can be written

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$$\frac{x}{\cos \Theta_i} = \int_{E}^{E_i} \frac{dE'}{S(E')} = R(E_i) - R(E) \quad ,$$

$$\frac{x}{\cos \Theta_f} = \int_{E_f}^{KE} \frac{dE'}{S(E')} = R(KE) - R(E_f) \quad . \qquad \ldots(1)$$

In Eq. (1), E is the energy of the beam particle at depth x before elastically scattering, $S(E) = -dE/dx$ is the stopping power of the particle in the target, and

$$R(E) = \int_{0}^{E} dE'/S(E')$$

is the total range of the particle.

For typical applications such as deducing the depth, x, when E_i and E_f are given, Eq. (1) contains two nonlinear equations and two unknowns, x and E. The limits of integration have a gap between E and KE, which makes it difficult to eliminate either x or E. Normally, this obstacle is overcome[1] by one of three methods: (1) assuming that S(E) varies only slightly over the integration limits so that it can be accurately represented by a constant (thin-film approximation), (2) mathematically slicing the thick target into thin films and deducing E from a summation process, or (3) some iteration technique. In this paper we take a different approach.

2. ANALYTICAL PROCEDURE

Noticing in Eq. (1) that range is a monotonically increasing function with respect to energy, and that $0 < K < 1$, we define

$$R(KE) = \lambda_K(E)\, R(E) \qquad \ldots(2)$$

and it follows that $0 < \lambda_K(E) < 1$. If $\lambda_K(E)$ can be found, Eqs. (1) have been reduced to two linear equations with two unknowns [x and R(E)], namely

$$\begin{aligned} x &= R(E_i) - R(E) \quad , \\ C \cdot x &= \lambda R(E) - R(E_f) \quad , \end{aligned} \qquad \ldots(3)$$

where for convenience we have set $\cos \Theta_i = 1$, $\lambda_K(E) = \lambda$, and defined $C = \cos \Theta_i / \cos \Theta_f$.

The solutions are:

$$x = \frac{\lambda R(E_i) - R(E_f)}{C + \lambda} \quad \text{and} \quad R(E) = \frac{C \cdot R(E_i) + R(E_f)}{C + \lambda} \quad . \qquad \ldots(4)$$

Equation (4) gives directly the depth of a film of any thickness x, if E_i and E_f are known.

The parameter λ in Eq. (2) can be shown to vary slowly with E. For values of K above 1/2, λ is well represented by $\lambda = K^B$, where $B = E/[R(E) \cdot S(E)]$. This follows from the observation that range varies as E^B over intervals of energy that easily span the gap between E and KE.

The parameter λ can be measured in the normal course of many backscattering experiments such as that shown in Fig. 1, if the atomic weight of the substrate atoms is sufficiently less than that of the film atoms so that the film backscattering peak is well isolated. If the incident energy is increased (or decreased) by an amount ΔE_i, the final energy edge will shift by ΔE_f accordingly. If changes in λ can be neglected over the interval $E + \Delta E$, differentiation of Eq. (4) (holding x constant) shows that

$$\lambda = \frac{S(E_i)}{S(E_f)} \cdot \left(\frac{\Delta E_f}{\Delta E_i}\right)_x \quad , \qquad \ldots(5)$$

where ΔE_f and ΔE_i are easily measured in the experiment and the ratio $S(E_i)/S(E_f)$ is rather insensitive to various published sources of stopping powers that may not agree among themselves. We have used this technique to measure λ for 2-, 3-, 4.5-, and 6-MeV alphas incident on a 4000 Å gold film deposited on a silicon substrate, as part of the stopping power measurements mentioned in the next section.

3. EXPERIMENTAL PROCEDURE

One consequence of the formalism developed here is a new technique for measuring stopping power. Differentiating Eq. (4) for x as before, but this time holding E_i constant, results in the relation

$$S(E_f) = \frac{-1}{C + \lambda}\left(\frac{\Delta E_f}{\Delta x}\right)_{E_i} \qquad \ldots(6)$$

This suggests that by increasing (decreasing) the target film thickness an amount Δx, and by measuring the corresponding shift in the edge at E_f, one can calculate the stopping power for energy E_f.

We consider two ways to change the target thickness. One is to rotate the target angle about an axis, which is the intersection of the plane of the target and the plane containing the incident and final beams. This leaves Θ_i and Θ_f unchanged, while effectively changing the thickness x. A second way is to deposit three different depths of film on the substrate such that the medium-thickness layer can be called x, the thickest layer can be called $x + \Delta x$, and the thinnest layer $x - \Delta x$. In this approach, x need not be measured at all; only $+\Delta x$ and $-\Delta x$ come into the equation. We chose the latter "step" target method to make our measurements.

In a preliminary experiment to test these concepts, we have used the technique suggested by Eq. (6) to measure the stopping power of alpha particles in gold at five energies. The results are given in Table I and compared with values of stopping power recently compiled by Ziegler.[2]

The target was prepared by evaporating three layers of ~ 2000 Å each of gold onto an optically flat single crystal of silicon in such a

way that 1/3 of the target surface area contained ~ 4000 Å of gold film, 1/3 contained ~ 2000 Å of gold film ($x - \Delta x$), and 1/3 contained ~ 6000 Å of gold film ($x + \Delta x$). The step thicknesses were measured with an optical interferometer using light of effective wavelength 5892 Å. The step thicknesses, $\pm\Delta x$, were measured to be 2236 ± 22 Å and 2235 ± 27 Å, respectively, where the quoted uncertainties are relative ones indicative of the scattering of numerous measurements. The absolute uncertainty in the areal density depends on the thickness of the step, density of the evaporated gold, and contaminant atoms deposited in the evaporation process. We quote a preliminary value of ±2% absolute. In the experiment, the target was physically translated so as to expose the appropriate thickness of gold film to the alpha beam.

The shift, ΔE_f, was measured by fitting the low-energy edge of the alpha peak backscattered from gold with an error function curve. A computer program was devised to adjust the height, width, and centroid of the error function to the data by minimizing the Chi-squared function. Relative errors in fitting the edges were less than 1/4 channel on a multichannel analyzer. Typical energy shifts were 40 channels corresponding to ~ 260 keV. Absolute errors in fitting the edges were not addressed since only edge shifts are relevant to this experiment.

Other parameters in this experiment are $\Theta_i = 0°$, $\Theta_f = 37.5°$, $C = 1.26$, and $K = 0.93$. The experiment was performed at the Sandia Laboratories EN Tandem Van de Graaff accelerator facility. The beam energy was found to have relative and absolute uncertainties of 0.5% and 1.0%, respectively. Values of λ were measured directly and checked by calculation as described above. Based on an estimate taken from variations between theory and experiment, we conservatively assign an absolute error of ±2% to λ. Note in Eq. (6) that this introduces an uncertainty of less than 1% into $S(E_f)$.

And finally, in deriving Eq. (6) from Eq. (4), if λ is allowed to vary with E, then it can be shown that λ in Eq. (6) is replaced by $\lambda' = \lambda(1 + \varepsilon)$ and $\varepsilon = \ln K \cdot [1-(B + b)] \cdot [R(E_i) - x/R]$, where the arguments of B, b, and R are evaluated at $(1/2) \cdot (1 + K)E$, and B and b are defined in the Appendix. However, the same considerations show that the

experimental measurement suggested by Eq. (5) actually measures $\lambda'' = \lambda(1 + \epsilon \cdot \lambda)$ rather than λ. It turns out that λ'' and λ' are so nearly equal that λ'' can replace λ in Eq. (6) and resulting errors in $S(E_f)$ are 0.2% for values given in Table I. The corrections ϵ do not need to be made here except to compare $\lambda'' = \lambda_{exp}$ in Table I with $\lambda = K^B$. The agreement is typically within ±1%.

4. CONCLUSION

In conclusion, we have explored some consequences of the equation $R(KE) = \lambda\ R(E)$. A formalism results, which allows the direct calculation of depth x from knowledge of E_i and E_f. Also a formula is derived for measuring stopping power. This technique transfers most of the experimental uncertainty from knowledge of beam characteristics to knowledge of step thicknesses, which can be measured with interferometry techniques. To demonstrate these techniques, the stopping power in gold of million-electron-volt alpha particles was measured and reported in Table I.

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APPENDIX

The development of this formalism has implicitly assumed that the data analyst has readily available values of range vs energy for arbitrary energy. This is not always the case. In the course of this work we have found an emperical formula for stopping power and range that is sufficiently accurate to be useful over the stopping power maximum. When stopping power is plotted as a function of ln E, the curve often shows striking symmetry about the energy, E_0, at which it reaches its maximum value, S_0. Introducing a "width parameter," n, the function can be represented in the vicinity of its peak by a Lorentzian in semilog space:

$$S(E) = \frac{S_o}{1 + n^2\left(\ln \frac{E}{E_o}\right)^2} \qquad \ldots(A1)$$

Fortunately, the inverse of this function is exactly integrable, so that the range is given by:

$$R(E) = \int_o^E \frac{dE'}{S(E')} = \frac{E}{S_o}\left[1 + n^2\left(\ln \frac{E}{E_o}\right)^2 + 2n^2\left(1 - \ln \frac{E}{E_o}\right)\right] \qquad \ldots(A2)$$

The parameters adjusted to stopping power values for alpha particles in gold as published by Ziegler[2] and Ziegler and Chu[3†] are $S_o = 123.5/10^{15}$ atoms/cm^2, $E_o = 1071$ keV, and $n = 0.5150$, and $S_o = 128.6$ eV/10^{15} atoms/cm^2, $E_o = 980$ keV, and $n = 0.4581$, respectively.

Of specific interest to the present work is the observation that stopping power varies with energy as E^b and range as E^B over limited energy intervals. Using the Lorentzian model for stopping power,

$$b(E) = \frac{E}{S(E)}\frac{dS}{dE} = \frac{-2n^2\left(\ln \frac{E}{E_o}\right)}{1 + n^2\left(\ln \frac{E}{E_o}\right)^2}, \qquad \ldots(A3)$$

and

$$B(E) = \frac{E}{R(E)}\frac{dR}{dE} = \frac{E}{R(E)\,S(E)} = \frac{1}{1 + b - b/\ln(E/E_o)}. \qquad \ldots(A4)$$

The Lorentzian model for stopping power is very useful in determining design parameters for setting up an experiment and, with care, can be used in Eq. (4) for routine analyses. It is most useful in the present formalism for calculating small, but not negligible, correction terms as discussed at the end of Sec. 3.

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† As parameterized by Brice.[4]

References

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TABLE I

Measured stopping power in gold of ^4He

E_f(MeV)	E_i(Mev)	λ_{exp}	$S(E_f)_{exp}$ [b]	$S(E_f)_z$ [c]
1.169	2.0	0.918	127.2	123.7
2.208	3.0	0.909	105.8	105.1
3.726	4.5	0.906	84.9	85.8
5.164	6.0	0.881	74.6	74.4
7.099	8.0	0.895[a]	63.3	63.9

[a]Calculated, not measured.
[b]Relative uncertainty ± 1.1%, absolute uncertainty ±2.6%.
[c]From Ziegler, Ref. 2.

FIGURE CAPTION

Fig. 1. Schematic and notation for a typical backscattering experiment as discussed in text.

BACKSCATTERING
EXPERIMENT

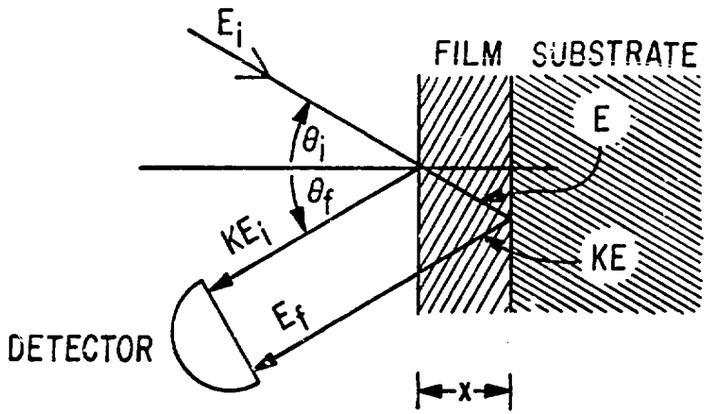

Figure 1