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# ION KINETIC EFFECTS ON THE TILT MODE IN FRCs

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Theory[1-3] and simulations[3-4] have shown that field reversed configurations (FRC's) should be unstable magnetohydrodynamically to the tilting mode, yet tilting seldom is seen in the experiments. Profile effects (within MHD) and ion finite larmor radius (FLR) effects have been proposed to explain the observed stability of FRC's. The present work seeks to test both of these effects.

## I. Model

Here we employ the dispersion functional[5] form of the Vlasov-fluid model, and then expand in two small quantities: a.) $\epsilon = r_L/a << 1$ (small larmor radius) and b) $\delta = a/b << 1$ (highly elongated equilibria). The two-dimensional nature of the equilibrium is retained exactly, and to leading order in $\epsilon$ and $\delta$ we retain FLR effects, parallel kinetic effects, and resonant particles.

The linearized equations of motion have the form[6]

$$\Delta(\vec{\xi}^*, \vec{\xi}) = -2\delta W + 2\omega^2 K + \omega F - R(\omega) = 0 \quad , \tag{1}$$

where $\delta W$ is the incompressible MHD potential energy, K is the Vlasov-fluid kinetic energy, F is the FLR term, and $R(\omega)$ contains the parallel kinetic effects and resonant particles. In the Vlasov-fluid model the displacement $\vec{\xi}$ has two components: $\vec{\xi} = \xi_\theta(s,\psi)\hat{\theta} + \xi_n(s,\psi)\hat{n}$, where $\hat{n} = \hat{\theta} \times \hat{b}$, $\hat{b} = \vec{B}/B$, $\psi$ is the poloidal flux function, and s is the arclength along a flux contour. A significant reduction in the dimensionality of the problem to be solved is accomplished by a) requiring that the largest term in $R(\omega)$ be made to vanish, and b) using the result from MHD[3] that $\delta W$ is minimized by an axial shift, $\xi_z(\psi)$, for highly elongated elliptical equilibria. These observations lead to the forms

$$\xi_n(s,\psi) = \hat{B}_r(s,\psi)\xi_z(\psi) \tag{2}$$

and

$$\xi_\theta(s,\psi) = ir^2 B_r(s,\psi)\xi_z'(\psi)/n \quad , \tag{3}$$

where $\hat{B}_r(s,\psi) = \sin\ \alpha = \hat{n}\cdot\hat{z}$, and the integer n in Eq. (3) is the toroidal mode number. The only unknown in Eq. (2) and Eq. (3) is the function $\xi_z(\psi)$. This is a vast simplification over the original problem.

The result of substituting Eq. (2) and Eq. (3) in Eq. (1) is

$$\Delta(\xi_z^*,\xi_z) = \int_{\psi_{vor}}^{\psi_{sep}} d\psi \ \{d_1(\psi,\omega)|\xi_z|^2 + d_2(\psi,\ \omega)|\xi_z'|^2\} \quad , \tag{4}$$

where both $d_1$ and $d_2$ are of the form

$$d_j(\psi,\omega) = \int ds\ \tilde{d}_j(s,\psi,\omega)/B(s,\psi). \tag{5}$$

The form Eq. (4) results when we take into account the leading order contribution of the parallel kinetic effects in $R(\omega)$, but drop the resonant particle terms in $R(\omega)$. The variation of $\Delta$ in Eq. (4) leads to the ordinary differential equation

$$d_2(\psi,\omega)\xi_z'' + d_2'(\psi,\omega)\xi_z' - d_1(\psi,\omega)\xi_z = 0 \quad , \tag{6}$$

where $\psi$ is the independent variable.

## II. MHD Results

Magnetohydrodynamic simulations show that the tilt mode is internal, $\xi_z(\psi_{sep}) = 0$. Therefore, we only require equilibrium solutions that are realistic inside the separatrix. Convenient control over equilibrium profiles is provided by the Berk, Hammer, Weitzner[7] solution for $\psi(r,z)$. These solutions are designed so that the "flatness" of the flux contours can be adjusted by a parameter p, where

$$\lim_{\substack{z\to o\\ r\to r_{sep}}} \psi(r,z) \sim z^p.$$

$p = 2$ is a Hill's vortex and $p \gg 1$ is a race track-like equilibrium.

The MHD portion of our code has been benchmarked against the linear MHD simulation code of Shestakov. For a particular run of Shestakov,[8] we have the following comparison of the e-folding time of an unstable tilt mode:

$\gamma^{-1}$ = 2μsec (Shestakov)

$\gamma^{-1}$ = 2.3μsec (present work)

The main effect to be studied in an MHD context is how the flatness of the flux contours affects the gr   of the tilt mode. Figure 1 shows the growth rate from MHD versus the flatness parameter p of the Berk, Hammer, Weitzner solution. The conclusion is the stability of the tilt mode is enhanced significantly by making the equilibrium more race track-like. Additional runs have to be made for larger p, and verification of the current results have to be performed to determine whether or not the tilt mode can be stabilized by profile effects alone.

III. FLR Effects

With MHD regularity conditions in the current model, the FLR term in Eq. (1) diverges at the vortex due to a breakdown in the assumption that $\varepsilon \ll 1$. For instance, near the field null the ion orbits are no longer cycloidal with $\Omega_{ci} \gg \Omega_d$, where $\Omega_d$ is the (cross field) azimuthal drift frequency. The model is being ammended to make the transition from cycloidal to betatron orbits, and soon we will be able to investigate more correctly the effect of FLR.


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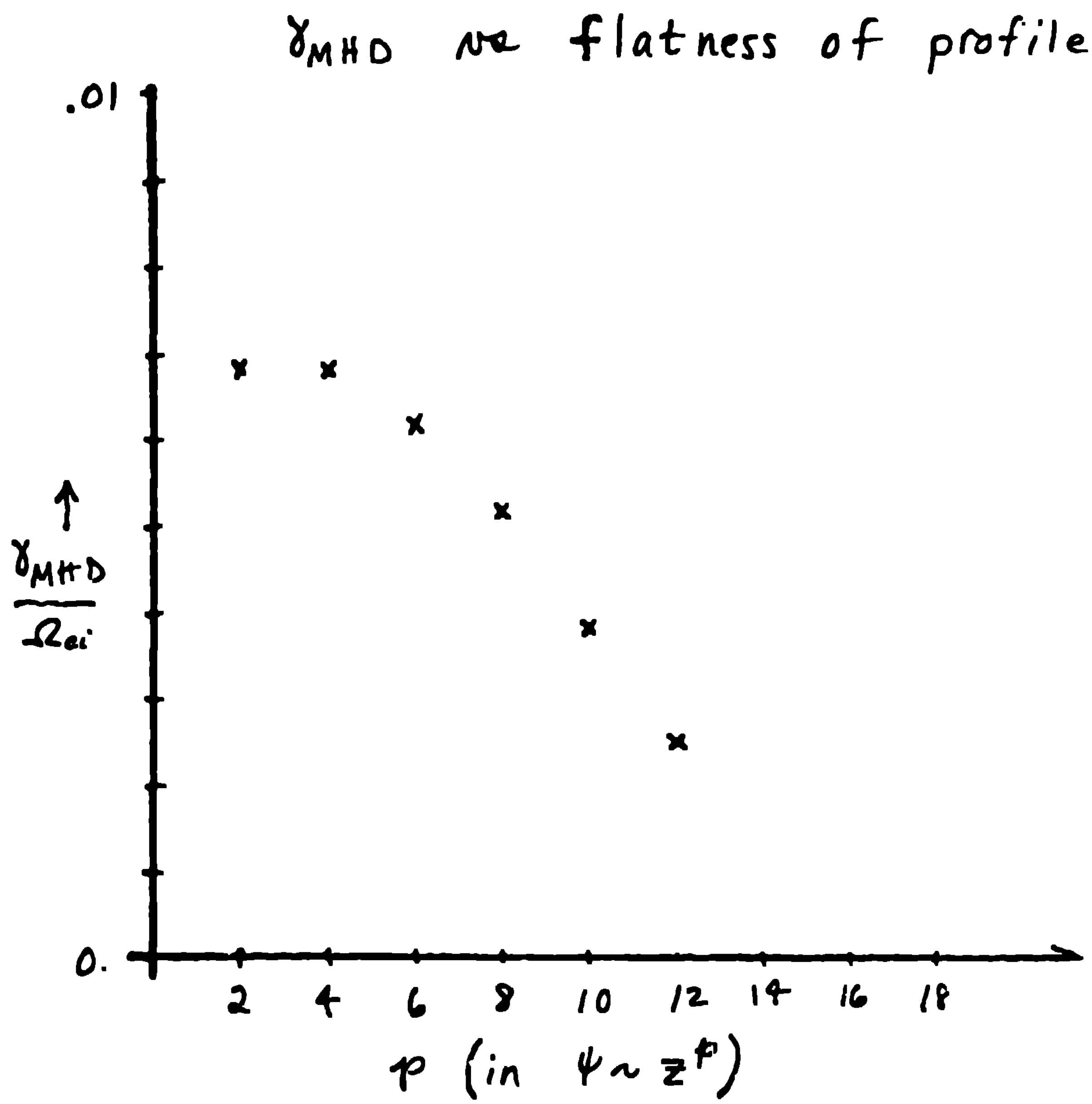


Figure 1

MHD growth rate versus flatness of the equilibrium profile.