LA-UR-91-992



LA-UR--91-992

DE91 009967

TITLE SIMULTANEOUS STABILIZATION USING GENETIC ALGORITHMS

AUTHOR(S) R. W. BENSON and W. E. SCHMITENDORF

SUBMITTED TO The Proceedings of the "1991 American Control Conference" held in Boston, MA, June 1991



Los Alamos National Laboratory
Los Alamos, New Mexico 87545

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

# Simultaneous Stabilization Using Genetic Algorithms

R. W. Benson, W. E. Schmitendorf
Department of Mechanical Engineering
University of California – Irvine
Irvine, CA 92717
714-856-8451

**Abstract**

This paper considers the problem of simultaneously stabilizing a set of plants using full state feedback. The problem is converted to a simple optimization problem which is solved by a genetic algorithm. Several examples demonstrate the utility of this method.

## 1 Introduction

This paper considers the problem of simultaneous stabilization of a finite number of plants via a single compensator. A number of researchers have presented results for this problem (see [1]-[7]) for both static and dynamic feedback controllers. In [1], a computational design is given for a collection of SISO plants with the assumption that each plant is minimum phase. The work presented in [2]-[5] are concerned with the conditions under which a solution which simultaneously stabilizes the set of plants exists, rather than the design of such a controller. In our work, we consider the case where the full state is available for feedback.

Several researchers have investigated the full state feedback problem. In [6], an analytical solution for single-input problems satisfying a given sufficient is presented. The technique, however, is difficult to apply in practice. An optimization approach was employed in [7] for simultaneously stabilizing a collection of single-input plants. In their work, a nonlinear programming algorithm was used to solve for the feedback gains.

We solve an optimization problem similar to that proposed in [7], but here we use a genetic algorithm (GA) to obtain the solution. This approach permits the consideration of multi-input problems. Another advantage gained by using a GA is that no auxilary information (such a gradients) is required. It suffices to specify the objective function and to place finite bounds on the controller gains. Additionally, the work presented in [7] was implemented on a Cray X-MP/24 computer while the work presented here was implemented on an Apollo workstation.

## 2 Problem Formulation

We consider the problem of finding a single linear control law which simultaneously stabilizes the family of $N$ plants

$$\dot{x}(t) = A_k x(t) + B_k u(t), \qquad k = 1, 2, \ldots, N \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector and $u(t) \in \mathbf{R}^m$ is the control. We assume that the full state is available for feedback.

A necessary and sufficient condition for the set of plants in Equation (1) to be simultaneously stabilizable with full state feedback is that there exists a $K \in \mathbf{R}^{m \times n}$ such that $A_k + B_k K$ is Hurwitz for $k = 1, \ldots, N$. This condition motivates the following approach for determining a set of gains $K$.

*Choose $K$ to minimize the following cost function:*

$$J = max\ Re(\lambda_{k,l}) \qquad k = 1, \ldots, N;\ l = 1, \ldots, n. \tag{2}$$

*where $\lambda_{k,l}$ is the $l^{th}$ eigenvalue of $A_k + B_k K$, subject to the constraints that $|K_{ij}| \leq c_{ij}$ ($i = 1, \ldots, m; j = 1, \ldots, n$) for an appropriate constant $c_{ij}$.*

Clearly if a solution is found such that $J < 0$, then the resulting $K$ simultaneously stabilizes the collection of plants. The existence of a solution is verified numerically by minimizing $J$.

In [7], this problem is converted to one which is computationally tractable via their numerical algorithm by imposing several constraints and by allowing the closed loop eigenvalues, $\lambda_{k,l}$ to be free parameters in the optimization scheme. Our approach is more straightforward since we can directly optimize $J$ in Equation (2) by employing a genetic algorithm approach.

## 3 Genetic Algorithms

The genetic algorithm (GA) is a global and robust optimization technique based on the principles of survival-of-the-fittest and basic genetic operators found in biological systems. The GA has been successfully applied to a variety of problems for many years [8] but has only recently been given much attention for use in control system applications. In [9], the GA is applied to an autopilot design problem and to the design of a windshear controller. An application of the GA to robust stability analysis is given in [10].

The GA begins with a randomly chosen group of candidate solutions referred to as a population (in our application, a specific set of controller gains $K$ would constitute one member of the population). The GA processes this population on a generation-to-generation basis by applying three operations: reproduction, crossover, and mutation. The reproduction operation ensures that members of the population which result in a lower cost (also referred to as *fitness*) have a higher chance of being reproduced for the next generation. The crossover and mutation operations help prevent the GA from converging to a local optima by creating new members in the search space.

The GA is suprisingly successful and efficient in solving a wide variety of problems. The GA does not suffer from many of the problems which calculus-based and enumerative schemes suffer from. For more details on the inner workings of the GA, we refer the interested reader to [8].

The GENESIS version of the GA (see [11]) was implemented on an Apollo workstation. From a practical vantage point, the user is only required to generate a subroutine for evaluation of the cost function (Equation 2), and to specify the bounds and resolution for each controller gain. This latter step is necessary since the GA maps the gains into binary strings for processing.

Within the prescribed bounds and resolution of the controller gains, the GA, in general, will find the global optimum, there is no guarantee that the GA will not converge to a local optimum. We note, however, that in this application, finding the global optimum is not critical. It suffices to find any controller gain K for which the cost function $J$ in Equation (2) is negative.

# 4 Examples

**Example 1:**

Consider the problem of stabilizing an F-4E fighter aircraft under different flight conditions [7]. Here we consider 4 operating conditions ($N = 4$). The system model has the form

$$\dot{x}(t) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & -30 \end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ 0 \\ 30 \end{bmatrix} u(t) \qquad (3)$$

where $x_1(t)$ corresponds to the normal acceleration, $x_2(t)$ corresponds to the pitch rate, and $x_3(t)$ represents the elevator angle. The unknown parameters are given in Table 1.

We applied the approach outlined in Section 2 under the constraint that the gains are restricted to be between $\pm 2$. Each gain is mapped to a binary string of bit length $l$, where $l$ is specified by the user depending on the desired resolution. For this example, a 9 bit string length was specified, with -2 corresponding to the string 000000000 and +2 corresponding to the string 111111111. Hence, the resolution for each gain is $4/2^9 = 0.00781$. The genetics algorithm was initialized with a population of 100 members. The most fit member within 50 generations was found to be

$$K = \begin{bmatrix} 0.1016970 & 1.875248 & 1.133013 \end{bmatrix}$$

with a resulting cost of $J = -1.886381$, indicating that this $K$ will simultaneously stabilize the four plants. This result is similar to the result of [7] where the globally minimum cost was found to be $J = -1.88643$ with the gains

$$K = \begin{bmatrix} 0.10831 & 2.00000 & 1.15230 \end{bmatrix}.$$

A plot of the best fitness and the average fitness over the population versus the number of generations is given in Figure 1. Note that the algorithm rapidly converges to a value close to the global optimum and that the average fitness eventually converges to values which are within a neighborhood of the global optimum.

**Example 2:**

Consider a ship steering problem given in [7]. The system model is of the form of Equation (1), with two operating conditions ($N = 2$). The system matrices for the two operat[illegible]

| Operating Point | $i=1$ | $i=2$ | $i=3$ | $i=4$ |
|---|---|---|---|---|
| Mach Number<br>Altitude (ft) | 0.5<br>5000 | 0.9<br>35000 | 0.85<br>5000 | 1.5<br>35000 |
| $a_{11}$ | -0.9896 | -0.6607 | -1.702 | -0.5162 |
| $a_{12}$ | 17.41 | 18.11 | 50.72 | 26.96 |
| $a_{13}$ | 96.15 | 84.34 | 263.5 | 178.9 |
| $a_{21}$ | 0.2648 | 0.08201 | 0.2201 | -0.6896 |
| $a_{22}$ | -0.8512 | -0.6587 | -1.418 | -1.225 |
| $a_{23}$ | -11.89 | -10.81 | -31.99 | -30.38 |
| $b_1$ | -97.78 | -272.2 | -85.09 | -175.6 |

Table 1: Values of parameters for F4E aircraft.

conditions are

$$A_1 = \begin{bmatrix} -0.298 & -0.279 & 0 \\ -4.370 & -0.773 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.116 \\ -0.773 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.428 & -0.339 & 0 \\ -2.939 & -1.011 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.150 \\ -1.011 \\ 0 \end{bmatrix}$$

Applying the GA, with a population of 100 members and with each gain constrained to be within $\pm 1$ and given 10 bits resolution, the most fit member within 100 generations was found to be

$$K = \begin{bmatrix} -1.0 & 0.9979190 & 0.9979190 \end{bmatrix}$$

with a corresponding cost of $J = -0.1151093$. This result also compares well with that found in [7], where the minimal cost was reported to be $J = -0.1158$ for the gains

$$K = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}.$$

A plot of best fitness and average fitness versus the number of generations is given in Figure 2.

Example 3:

In this example we consider a simple two mass system as shown in Figure 3 [12]. The states are $x_1$, $\dot{x}_1$, $x_2$, and $\dot{x}_2$ and the state equations are

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{m_1} & 0 & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & \frac{-k}{m_2} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u(t).$$

Let $m_1 = m_2 = 1$ and consider the stabilization of two plants corresponding to $k = 2$ and $k = 0.5$. Constraining the gains to values between $\pm 50$, the GA was executed with an initial population of 100 members and a resolution of 10 bits per gain. After 300 generations, the most fit member was found to be

$$K = \begin{bmatrix} -25.78131 & -8.984480 & -8.886824 & -50.0 \end{bmatrix}$$

with a corresponding cost of $J = -0.6831173$. Hence, this $K$ simultaneously stabilizes the two plants.

A plot of best fitness and average fitness versus the number of generations is given in Figure 4.

### Example 4:

We now onsider the simultaneous stabilization of a lateral autopilot for a remotely piloted vehicle [13]. The state vector for this system is

$$x(t) = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \delta_a \end{bmatrix}$$

where $v$ is the vehicle velocity parallel to the pitch axis, $p$ is the roll rate, $r$ is the yaw rate, $\phi$ is the roll angle, and $\delta_a$ is the aileron deflection. The state equations for this system are given by

$$\dot{x}(t) = \begin{bmatrix} 0.85 & 25.47 & -979.5 & 32.14 & 0 \\ -0.339 & -8.789 & 1.765 & 0 & 59.89 + q1.71 \\ 0.021 & -0.547 & -1.407 & 0 & 6.477 + q3.22 \\ 0 & 1 & 0.0256 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 20 \end{bmatrix} u(t)$$

where

$$q = \frac{C_{n\delta_a} - 1.99}{1.99}$$

The unknown coefficient $C_{n\delta a}$ varies in the range

$$-0.99 \leq C_{n\delta a} \leq 2.99$$

with a nominal value of $C_{n\delta a} = 1.99$. We considered stabilization of the three plants corresponding to $C_{n\delta a} = 1.99$ ($q = 0$), $C_{n\delta a} = -0.99$ ($q = -1.5$), and $C_{n\delta a} = 2.99$ ($q = 0.5$).

The feedback gain matrix for this example is row vector with five elements. These five gains were restricted to be within the ranges $\pm 0.01$, $\pm 0.1$, $\pm 1$, $\pm 0.1$, $\pm 1$ respectively. With a population of 200 members and a resolution of 12 bits per gain, the most fit member after 500 generations was found to be

$$K = \begin{bmatrix} 0.0074900 & 0.0581720 & -0.7389200 & -0.0997060 & 0.1058080 \end{bmatrix}$$

with a corresponding cost of $J = -1.894160$.

A plot of best fitness and average fitness versus the number of generations is given in Figure 5.

**Example 5:**

Consider the stabilization of a satellite on a circular, equitorial orbit [14]. The state vector is given by

$$x = \begin{bmatrix} r \\ \dot{r} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

where $r$ and $\theta$ are the polar coordinates of the satellite. The linearized equations of motion for small perturbations about an orbit are given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega r_0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-2\omega}{r_0} & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & \frac{1}{r_0} \end{bmatrix} u(t)$$

where $r_0$ is the nominal radius and $\omega$ is the nominal angular velocity. These two quantities are related by the expression $r_0^3\omega^2 = k$ where $k$ is constant whose units are normalized so that $k = 1$. The control $u(t)$ is applied by gas thrusters.

We consider the simultaneous stabilization of this system for two operating points with $r_0 = 1$ and $r_0 = 1.5$. For this problem we used a population of 100 members with a resolution of 10 bits per gain. The gains were constrained to be within ±12. The most fit member after 900 generations was found to be

$$K = \begin{bmatrix} -11.95312 & -6.632698 & -9.703076 & -4.781096 \\ 11.88332 & 5.367558 & -12.0 & -7.874912 \end{bmatrix}$$

with a corresponding cost of $J = -2.969937$.

A plot of best fitness and average fitness versus the number of generations is given in Figure 6.

**Example 6:**

In this example we consider the stabilization of a track guided bus, as shown in Figure 7 [13]. The state variables are $\alpha$, $\dot{\epsilon}$, $\epsilon$, $y$, and $\beta$. The state equations for the system are given by

$$\dot{x}(t) = \begin{bmatrix} -668ab & -1+181ab^2 & 0 & 0 & 198ab \\ 16.8a & -409ab & 0 & 0 & 67.3a \\ 0 & 1 & 0 & 0 & 0 \\ 1/b & 6.12 & 1/b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

where the parameters $a$ and $b$ depend on the vehicle velocity, mass, and the coefficient of friction. The values of $a$ and $b$ vary in the range

$$\frac{1}{32} \leq a \leq \frac{1}{9.95}, \quad \frac{1}{20} \leq b \leq \frac{1}{3}.$$

Here we consider endpoint stabilization; that is, we wish to simultaneously stabilize the four plants corresponding to the following four pairs of $a$ and $b$: $(a = 1/9.95, b = 1/3)$, $(a = 1/9.95, b = 1/20)$, $(a = 1/32, b = 1/20)$, and $(a = 1/32, b = 1/3)$.

The GA applied to this problem with a population of 100 members and 10 bits resolution per gain. Each gain was constrained to be within ±20. After 1350 generations, the most fit member was

$$K = \begin{bmatrix} -20.0 & -5.077934 & -0.5856890 & -13.39835 & -19.92187 \end{bmatrix}$$

where the resulting cost was found to be $J = -0.4991887$.

A plot of the best fitness and average fitness over the population is given in Figure 8

## 5 Conclusions

The problem of simultaneously stabilizing a finite set of plants under full state feedback was considered in this paper. The approach to finding a set of gains was to minimize an objective function with the constraint that the magnitude of each controller gain be bounded by some constant. Determination of these constants is one of the major difficulties of this approach. The easiest method is to find the gains which place the eigenvalues of each plant seperately to some desirable location in the complex plane, and then determine appropriate bounds on the gains by inspection.

With the approach outlined in this paper, it is rarely critical that the global optimum be found. If a set of controller gains is found which makes the cost $J$ in Equation (2) negative, then these gains will simultaneously stabilize the set of plants. By optimizing the gains, however, we determine those gains (within the prescribed bounds and resolution) which minimize the slowest rate of exponential decay for the set of plants under consideration and thereby improve performance.

Finally, we remark that the approach outlined in this paper can be modified to ensure that the closed loop system eigenvalues are sufficiently damped. For example, suppose that one pair of the closed loop eigenvalues of the system is $\lambda_{1,2} = -a \pm bj$. If the ratio $b/a$ is larger than some desirable value $\alpha$ (based on performance requirements), then a penalty term can be added to the cost $J$ given in Equation (2), thus penalizing gains which result in underdamped system eigenvalues.

## 6 Acknowledgements

This research was supported by Los Alamos National Laboratory under the INCOR Program. The authors thank Dr. Stephanie Forrest for her help in this research.

## References

[1] B. R. Barmish and K. H. Wei, "Simultaneous Stabilizability of Single-Input Single-Output Systems," in *Proc. 7th Int. Symp. Math. Theory Networks Syst.*, Stockholm, 1985

[2] R. Saeks and J. Murray, "Fractional Representations, Algebraic Geometry and the Simultaneous Stabilization Problem," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp.895-903, 1982.

[3] P.P. Khargonekar, A. Pascoal, and R. Ravi, "Simultaneous Stabilization of Linear Time-Varying Systems-Strong, Simultaneous and Reliable Stabilization," in *Proc. 1988 Amer. Contr. Conf.*, Atlanta.

[4] M. Vidyasagar and N. Viswanadham, " Algebraic Design Techniques for Reliable Stabilization," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 1085-1095, 1982.

[5] B.K. Ghosh and C.I. Byrnes,"Simultaneous Stabilization and Pole-Placement by Non-switching Dynamic Compensation," *IEEE Trans. Automat. Contr.*, vol. AC-28, pp. 735-741, 1983.

[6] W.E. Schmitendorf and C.V. Hollot, "Simultaneous Stabilization via Linear State Feedback Control," *IEEE Trans. Automat. Contr.*, vol. AC-34, pp.1001-1005, 1989.

[7] G.D. Howitt and R. Luus, "Simultaneous Stabilization of Linear Single-Input Systems by Linear State Feedback Control "

[8] D.E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading, Mass. 1989.

[9] K. Krishnakumar and D.E. Goldberg, "Genetic Algorithms in Control System Optimization," *AIAA Guidance, Navigation and Control Conf.*, pp. 1568-1577, Portland, 1990.

[10] T.M. Murdock, W.E. Schmitendorf, and S. Forrest, "Use of a Genetic Algorithm to Analyze Robust Stability Problems, " submitted to 1991 *Amer. Contr. Conf.*

[11] J.J Grefenstette, "GENESIS: A system for using genetic search procedures," *Proc. of the 1984 Conf. on Intelligent Systems and Machines*, pp. 161-165.

[12] K.W. Byun, B. Wie, and J. Sunkel, "Robust Control Synthesis for Uncertain Dynamical Systems," AIAA Paper No. 89-3516.

[13] W.E. Schmitendorf and C. Wilmers, "Simultaneous Stabilization via Low Order Controllers," *Control and Dynamic Systems*, vol. 35, pp. 165-184, 1990.

[14] T.E. Fortmann and K.L. Hitz, *An Introduction to Linear Control Systems*. New York: Marcel-Dekker, 1977.

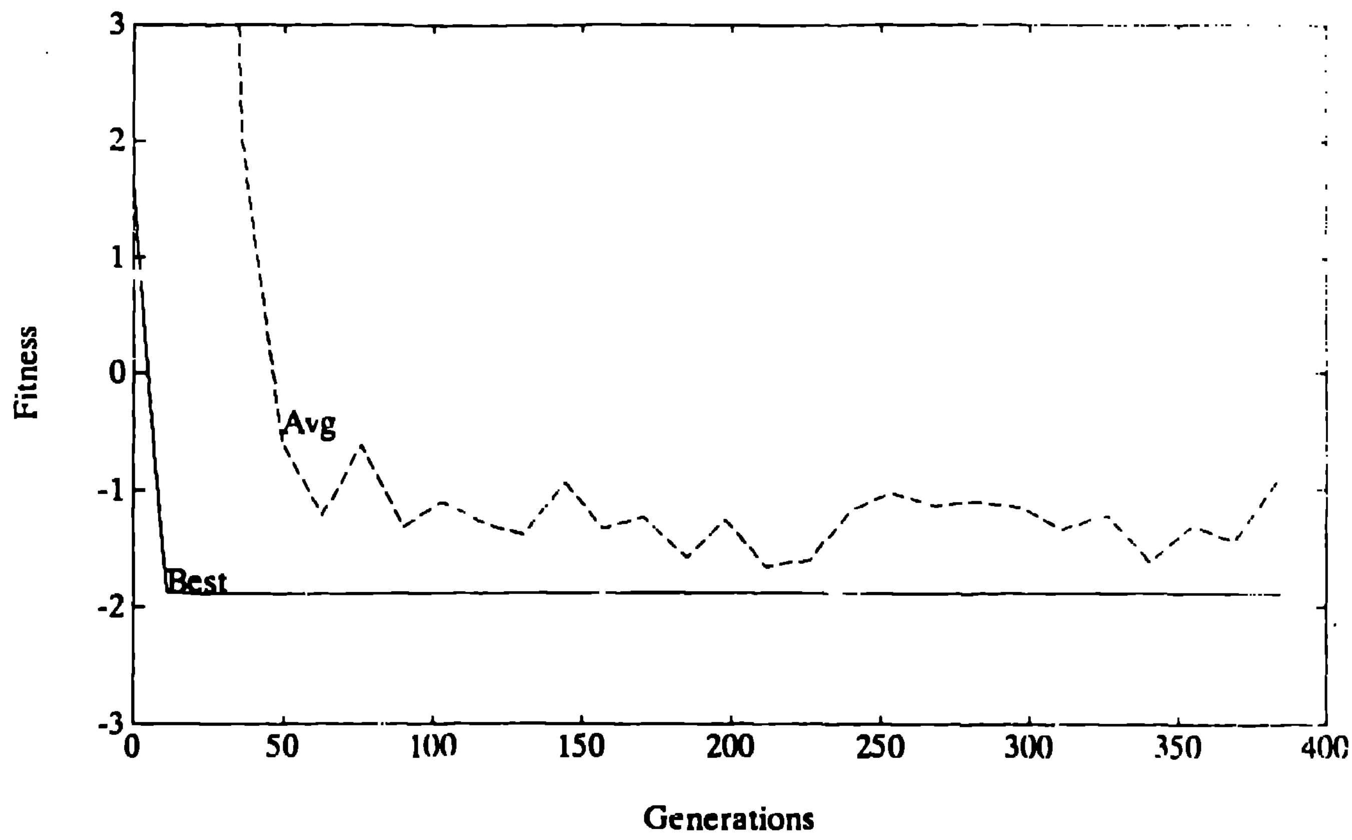


Figure 1: Average and best fitness (cost) vs. the number of generations for Example 1.

F4b

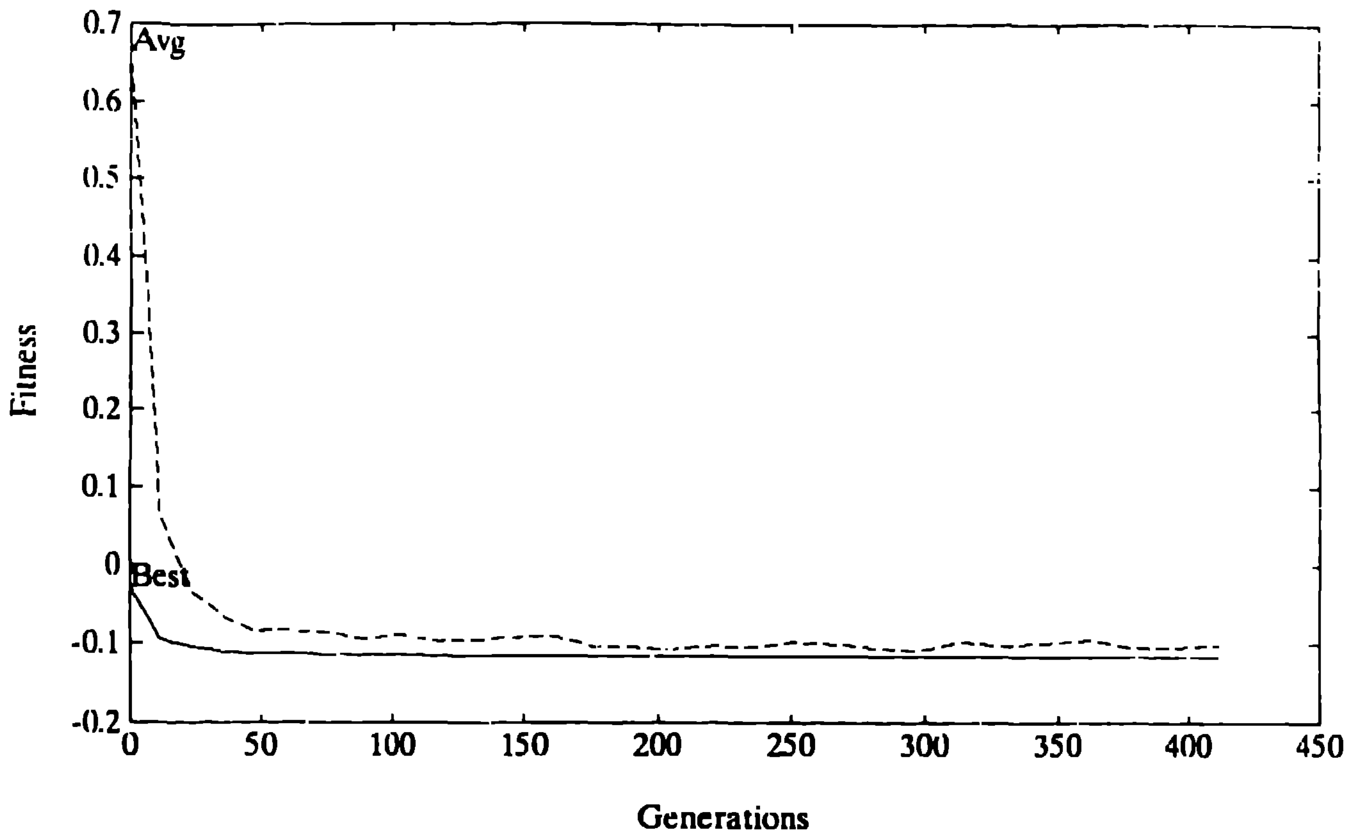


Figure 2: Average and best fitness (cost) vs. the number of generations for Example 2.

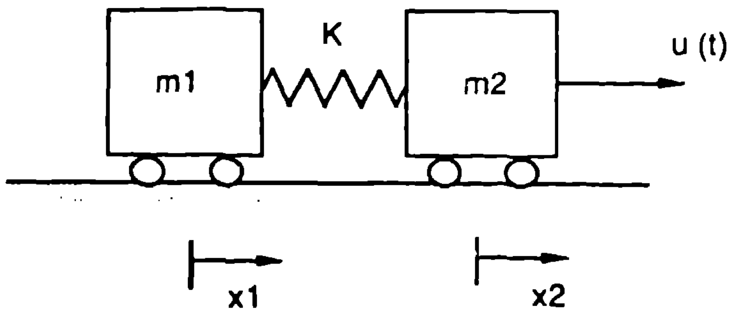


Figure 3: Two-mass system used in Example 3.

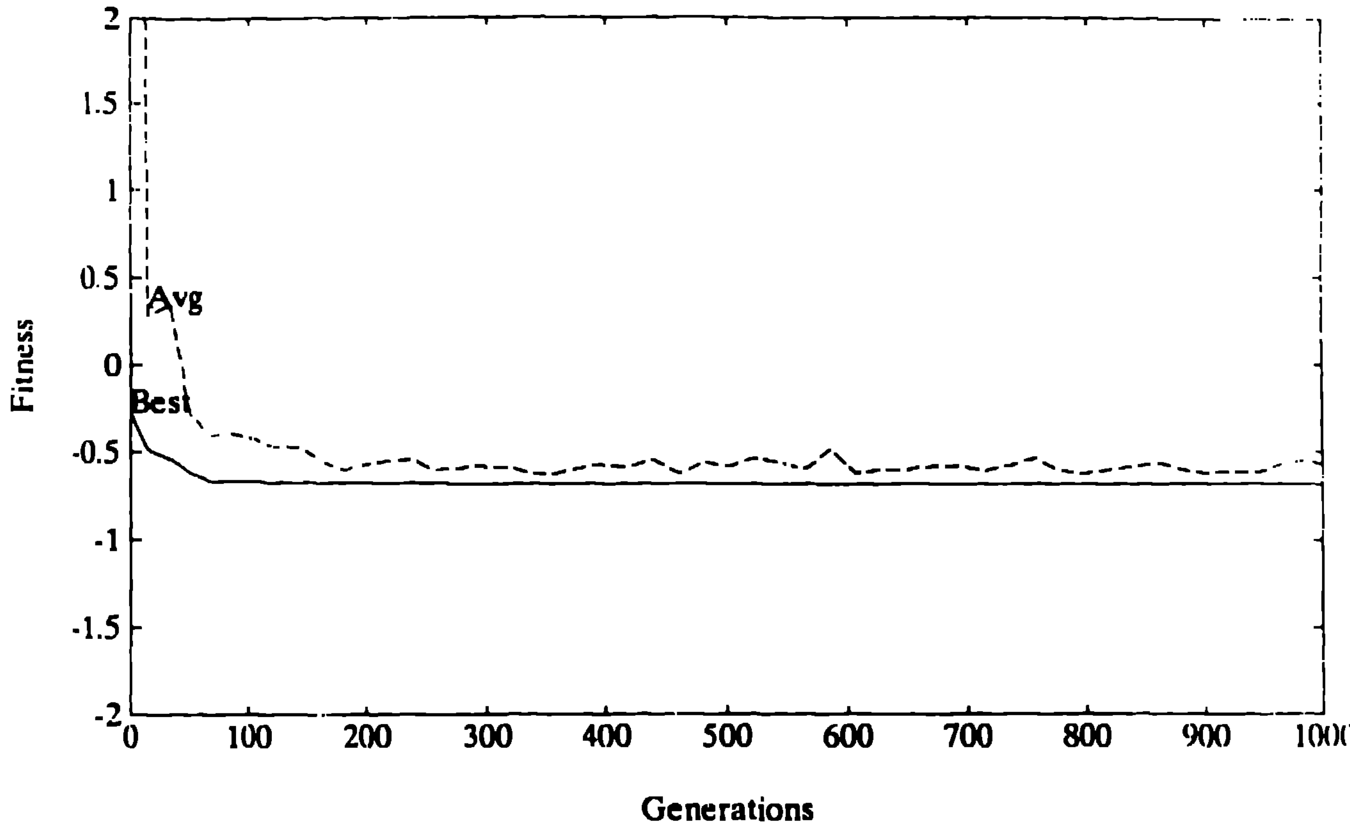


Figure 4: Average and best fitness (cost) vs. the number of generations for Example 3.

SPRING

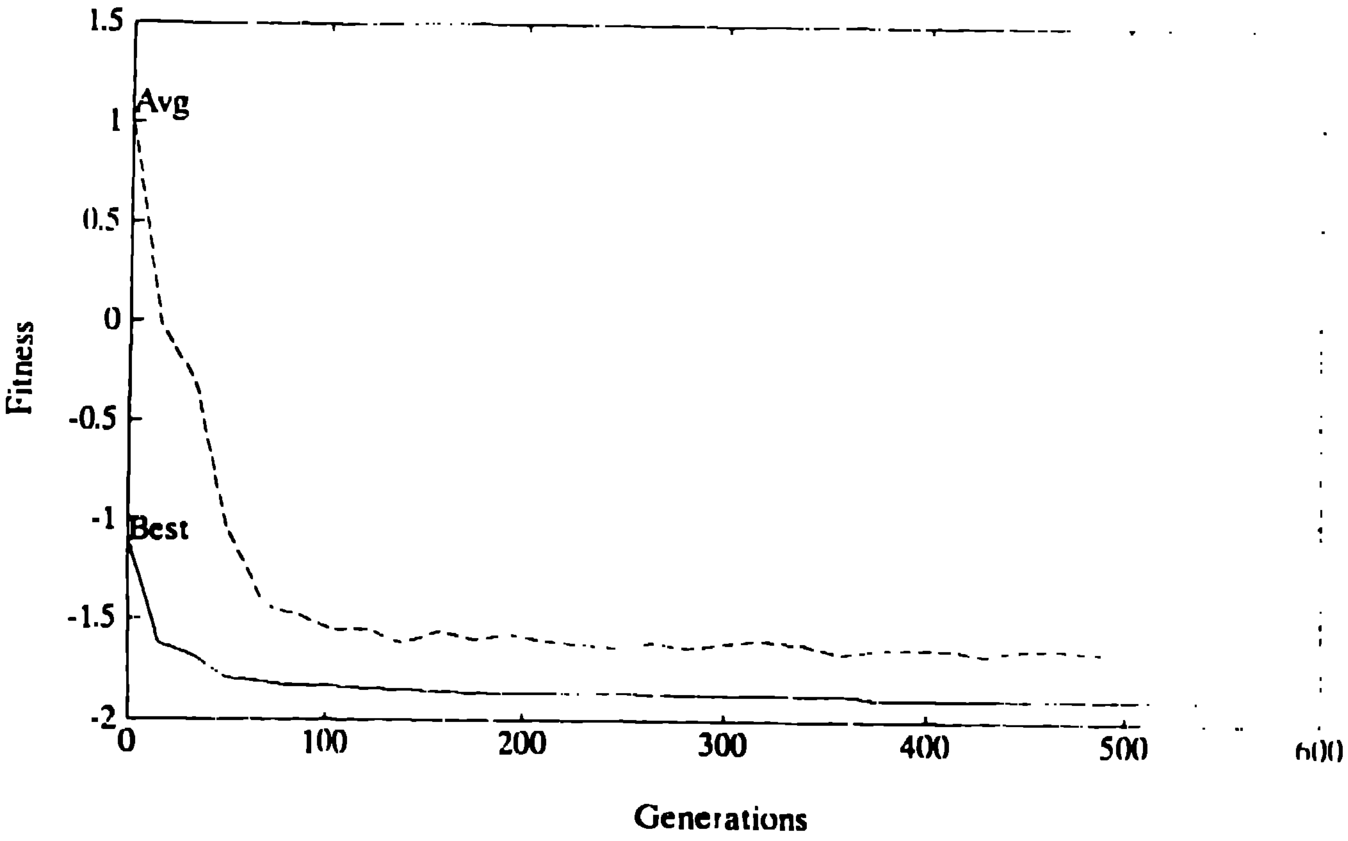


Figure 5: Average and best fitness (cost) vs. the number of generations for Example 4.

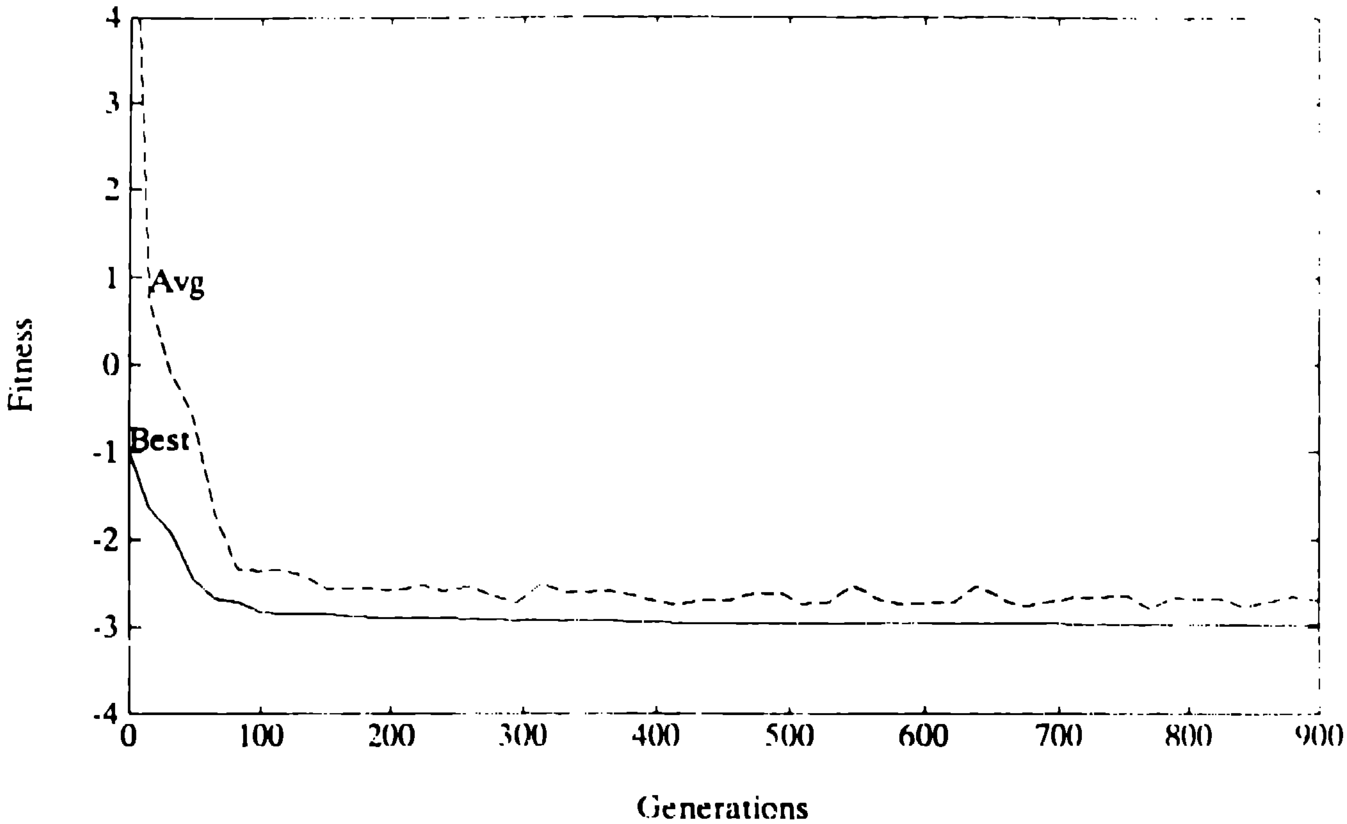


Figure 6: Average and best fitness (cost) vs. the number of generations for Example 5.

SATELLITE

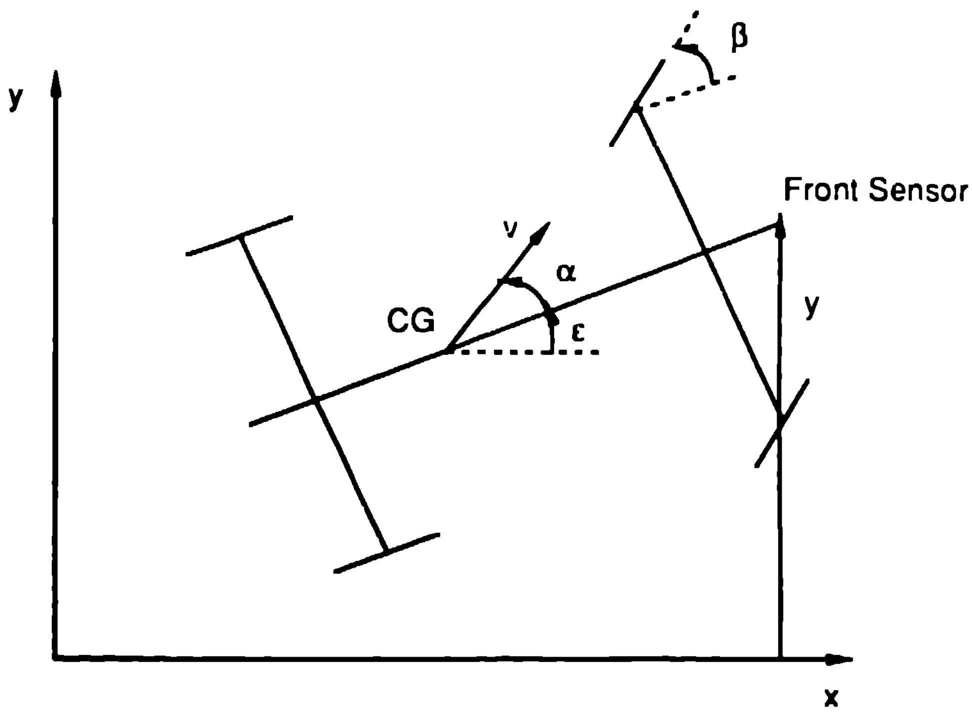


**Figure 7:** Schematic of track guided bus used in Example 6.

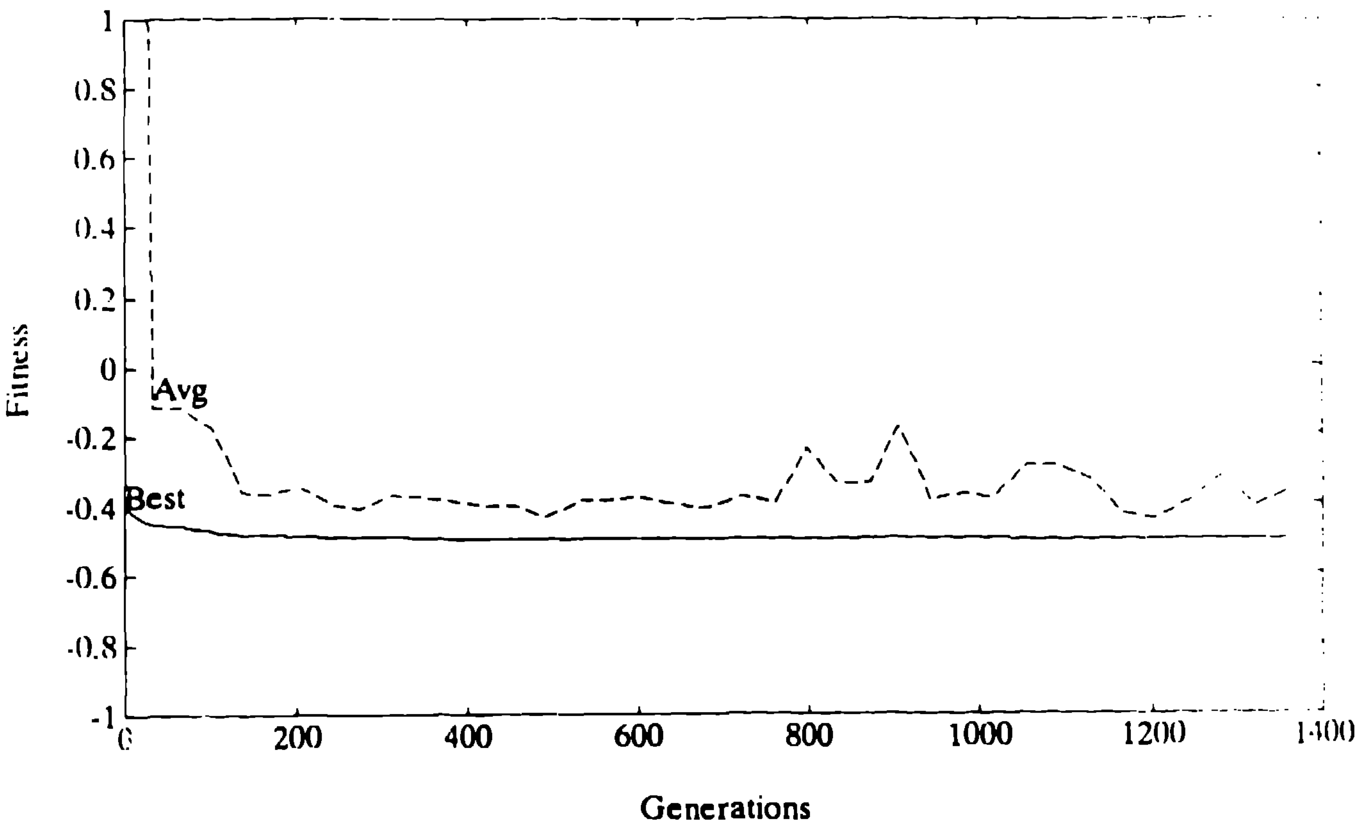


Figure 8: Average and best fitness (cost) vs. the number of generations for Example 6.