2.

DOE/ER/40614--10

DE92 004266

QCD BASED STATIC POTENTIAL BETWEEN HEAVY QUARKS

M. BAKER
Department of Physics, FM-15, University of Washington, Seattle, Washington 98195, USA

JAMES S. BALL
Department of Physics, University of Utah, Salt Lake City, Utah 84112, USA

and

F. ZACHARIASEN,
California Institute of Technology, Pasadena California 91125, USA

ABSTRACT

We calculate the static potential between a quark-anti quark pair using dual potentials to describe long-distance Yang-Mills theory.

1. Introduction

In this talk we show how the dual superconducting picture of confinement arises out of the long-distance dynamics of Yang-Mills theory and we calculate the resulting quark anti-quark potential. Our method involves the use of dual potentials C_μ to describe long-distance Yang-Mills theory.

In the case of the electrodynamics of a source free relativistic dielectric medium having a dielectric constant $\epsilon(q)$ and magnetic permeability $\mu(q) = \frac{1}{\epsilon(q)}$, the electric displacement vector $\vec{D}$ and the magnetic $\vec{H}$ vector are expressed in terms of the dual potentials by the equations:

$$\vec{D} = -\vec{\nabla} \times \vec{C} \quad , \quad \vec{H} = -\frac{\partial \vec{C}}{\partial t} - \vec{\nabla} C_0 \, . \tag{1}$$

Maxwell's equations then become differential equations for C_μ that are generated by the Lagrangian

$$\mathcal{L}_M = \frac{1}{2}(\vec{H}\mu(q)\vec{H} - \vec{D}\mu(q)\vec{D}) \equiv -\frac{1}{4}G_{\mu\nu}\mu(q)G^{\mu\nu} \, . \tag{2}$$

where $G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$. The C_μ propagator is $\Delta_C = \frac{1}{q^2\mu(q)}$.

In the non-Abelian case dual potentials can be defined,[1] but the explicit form of the Lagrangian expressed in terms of dual potentials is not known. We only know

that it must be invariant under non-Abelian gauge transformations, so in order to construct this Lagrangian in the long-distance regime we must use some information about long-distance Yang-Mills dynamics.

Our choice is motivated by the work of many authors who have obtained a solution of a set of truncated Dyson equations of Yang-Mills theory for the A_μ propagator, Δ_A, which has the behavior $\Delta_A \sim -M^2/(q^2)^2$ as $q^2 \to 0$. (M is an undetermined mass scale.). It follows that $\epsilon \to -q^2/M^2$ as $q^2 \to 0$, and consequently $\mu \to -M^2/q^2$ as $q^2 \to 0$. By choice of normalization we can write $\mu = -\frac{M^2}{q^2}+1$ and hence, $\Delta_C \to \frac{1}{q^2-M^2}$. Thus in the dual language the truncated Dyson equations give rise to a mass for the C_μ field: that is, for the dual gluon.

Since $\mu = -\frac{M^2}{q^2} + 1$, the Lagrangian, $\mathcal{L}^{(0)}(C)$, describing this solution is

$$\mathcal{L}^{(0)}(C) = -\frac{1}{4}G^{\mu\nu}\left(\frac{M^2}{\partial^2}+1\right)G_{\mu\nu}. \qquad (3)$$

$\mathcal{L}^{(0)}(C)$ can be written in the following local form:

$$\mathcal{L}^{(0)} = \frac{M}{2}\tilde{F}^{\mu\nu}G_{\mu\nu} + \frac{1}{4}\tilde{F}^{\mu\nu}\partial^2\tilde{F}_{\mu\nu} - \frac{1}{4}G^{\mu\nu}G_{\mu\nu}, \qquad (4)$$

where $\tilde{F}_{\mu\nu} = -\tilde{F}_{\mu\nu}$ is a set of auxiliary anti-symmetric tensor fields. The original form, Eq. (3), of $\mathcal{L}^{(0)}$ obtains after eliminating $\tilde{F}_{\mu\nu}$ from Eq. (4) via the equations of motion.

We assume[2] that the Lagrangian $\mathcal{L}(C)$, describing long-distance Yang-Mills theory in terms of dual potentials, is the minimal extension of $\mathcal{L}^{(0)}(C)$ which is invariant under non-Abelian gauge transformations of the dual potentials C_μ. Thus we obtain $\mathcal{L}(C)$ by making the following substitutions in $\mathcal{L}^{(0)}(C)$:

$$\partial_\mu \to \partial_\mu - ig[C_\mu, \;] \equiv \mathcal{D}_\mu, \qquad G_{\mu\nu} \to \partial_\mu C_\nu - \partial_\nu C_\mu - ig[C_\mu, C_\nu] \equiv G_{\mu\nu}, \qquad (5)$$

where $g \equiv \frac{2\pi}{e}$ and where e is the usual Yang-Mills coupling constant: i.e. $\alpha_s = \frac{e^2}{4\pi}$. The Lagrangian $\mathcal{L}$ then contains interaction terms of the form $\tilde{F}\tilde{F}C$ and $\tilde{F}\tilde{F}CC$. These give rise to divergences in $\tilde{F}$ self energy graphs and in $\tilde{F}\tilde{F}$ scattering. We then add a counter term $-W$ to $\mathcal{L}$, where

$$W = -\mu^2\tilde{F}^2 + \lambda(\tilde{F}^2)^2. \qquad (6)$$

W has a minimum at a non-vanishing value $\tilde{F}_0$ of $\tilde{F}$. We take $\mu^2 < 0$ so that $-\tilde{F}_0^2 > 0$. This non-vanishing value of $\tilde{F}_0^2$ produces (via the $\frac{1}{4}\tilde{F}_{\mu\nu}\mathcal{D}^2 F^{\mu\nu}$ term in $\mathcal{L}$) a dual gluon mass term, $-g^2\tilde{F}_0^2\frac{C^\mu C_\mu}{2}$. We do not then need the $M\tilde{F}_{\mu\nu}G^{\mu\nu}$ term in $\mathcal{L}$ in order to generate a dual gluon mass. We then set $M = 0$ in $\mathcal{L}$ and write $-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} \equiv \frac{1}{2}(\vec{H}^2 - \vec{D})$ to end up with the following form for the dual QCD Lagrangian:

$$\mathcal{L} = \frac{1}{4}\tilde{F}^{\mu\nu}\mathcal{D}^2\tilde{F}_{\mu\nu} + \frac{1}{2}(\vec{H}^2 - \vec{D}^2) - W. \qquad (7)$$

The components of $\tilde{F}_{\mu\nu}$ are independent dynamical variables which can be interpreted as magnetic and electric fields. $\vec{B}$ and $\vec{E}$ respectively $(-\tilde{F}^2/2 = \vec{B}^2 - \vec{E}^2)$. Since $-\tilde{F}_0^2 > 0$ we can choose $\vec{E}_0 = 0$, so that the symmetry breaking vacuum is purely magnetic. Furthermore, we can take $\vec{E} = 0$ also for non-vacuum solutions.

2. The Static Potential Between Heavy Quarks

We now determine the color fields surrounding a heavy quark-anti quark pair on the z axis and separated by a distance R. We choose a gauge in which $\vec{C}$ lies in the ϕ direction and is proportional to the hypercharge matrix $Y \equiv \lambda_8/\sqrt{3}$, and in which the components of $\vec{B}$ lie along the 2, 5 and 7 directions in color space respectively.

The quark charge density $\rho(\vec{x})$, must also lie along the Y direction in order to absorb the flux of $\vec{D}$. Thus,

$$\rho(\vec{x}) = eY\delta(x)\delta(y)\left[\delta(z - R/2) - \delta(z + R/2)\right] . \tag{8}$$

We couple the dual potentials to $\rho(\vec{x})$ by inserting into $\mathcal{L}$, Eq. (7), the expression

$$\vec{D} = -\vec{\nabla} \times \vec{C} + D_s , \tag{9}$$

where

$$\vec{D}_s = eY\delta(x)\delta(y)\left[\theta(z - R/2) - \theta(z + R/2)\right] \hat{e}_z , \tag{10}$$

so that

$$\vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot D_s = \rho . \tag{11}$$

and then solve the resulting classical equations for $\vec{C}$ and $\vec{B}$. We find that for $R \to 0$, $\vec{C} \to \vec{C}_D$, the usual Dirac monopole potential, and Eq. (9) yields a pure Coulomb field for $\vec{D}$. The field $\vec{D}$ evolves from a squashed dipole at small R to a flux tube for large separations. The integrated energy density then yields a potential $V(R)$ which changes rapidly from a Coulomb potential to a linear potential as R increases. This is similar to the potential between magnetic monopoles in a relativistic superconductor.[3]

Next we calculate the spin dependent contribution to the potential assuming that the quarks have Dirac magnetic moments. We obtain an explicit form for the tensor force and the $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ interaction. For small R these interactions have their usual electromagnetic form, while for $R \to \infty$ they vanish exponentially. We are not yet able to calculate the spin orbit force from the fundamental parameters, λ and $\tilde{F}_0^2$, of dual QCD, so we use the Breit-Fermi spin orbit potential. Then, choosing $\lambda = 1.78$, $\sqrt{-\tilde{F}_0^2} = 434$ MeV in our static potential, we fit the sixteen lowest lying states of $c\bar{c}$ and $b\bar{b}$ system to an accuracy of 1 percent.

References

1. S. Mandelstam, *Phys. Rev.* **D20** (1979) 3323.
2. M. Baker, J.S. Ball and F. Zachariasen, *Phys. Rev.* **D41** (1990) 2612.
3. J.S. Ball and A. Caticha, *Phys. Rev.* **D37** (1988) 524.

END

DATE FILMED

2 /07/92