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# FLAVOR CHANGING NEUTRL_L CURRENT TRANSITIONS ON THE LATTICE FOR HEAVY-LIGHT SYSTEMS* 

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#### Abstract

Flavor changing neutral current transitions are a very important test of the standard model. Some of the channels can also be very useful for extracting the Kobayashi-Maskawa mixing matrix elements. However, their potential applications are limited by the uncertainties in the relevant hadronic matrix elements. We show that these matrix elements are also amenable to lattice techniques. In this study, we focus on radiative transitions such as $B \rightarrow K^{*} \gamma$ and present preliminary results obtained by using $24^{3} \times 40$ lattices at $\beta=6.0$.


The rare, flavor changing, radiative decays, such as $B \rightarrow K^{*} \gamma$ are an important test of the higher order corrections in the standard model[1,2]. In particular, $B \rightarrow K^{*} \gamma$ results in a clear signature for experimental study, and it is expected to be the first loop decay of the B meson that will be seen. Unfortunately, as we will mention, current continuum calculations are plagued by large uncertainties in calculating the relevant hadronic matrix elements.

Currently, the heaviest quark one can reliably place on the lattice is the charm quark. This makes it difficult to do B physics directly with propagating bottom quarks. We present preliminary results which show that in the mass range accessible by the lattice, the relevant matrix element can be calculated. There appears to be no fundamental difficulty, given improved statistics and finer lattices, for extrapolation to the masses

* Presented by P. F. Hsieh. This work was supported in part by the U. S. Department of Energy under contract DE-AC02-76CH00016.


Fig. 1. The rare radiative decay (e.g. $B \rightarrow K^{*} \gamma$ ) proceeding though an electromagnetic penguin.
required for studying $B$ meson decays.
As an example of this class of processes, Figure 1 shows a heavy meson, $H$, decaying to a light vector meson, $V$, through a penguin graph. At the quark level, these decays are expected to oe controlled mainly by the operat or relevant for $h \rightarrow l \gamma$

Presently, the decay, $B \rightarrow K^{*} \gamma$ is attracting much phenomenological interest. The process, $b \rightarrow s \gamma$, has been reliably calculated in perturbation theory $[1,2]$. The branching ratio obtained is independent of mixing angle and varies only
from $2 \times 10^{-4}$ to $4 \times 10^{-4}$ as the top quark mass varies from 50 GeV to 200 GeV .

Current experiments are in the process of analyzing $O\left(10^{6}\right) B$ mesons. Proposed B-factories will produce a thousand times that amount. These numbers suggest that this decay will be measured soon. Unfortunately, the inclusive process, $B \rightarrow X_{s}+\gamma$ is extremely difficult to separate experimentally from background. Hence, the exclusive process $B \rightarrow K^{*} \gamma$, which has a clean experimental signature, has attracted the most attention in the study of loop decays.

In recent years, there have been many attempts to calculate the ratio [2]
$R=\frac{\Gamma\left(B \rightarrow K^{*} \gamma\right)}{\Gamma(b \rightarrow s \gamma)}$,
which monitors the branching ratio for the exclusive reaction $\left(B \rightarrow K^{*} \gamma\right)$. The results of these calculations span the range of $4 \%$ to $98 \%$ with clusters of results at $4 \%, 15 \%$, and $40 \%$. This large range of theoretical prediction underscores the difficulty that existing continuum methods have with calculating the necessary hadronic matrix element.

The crucial step in calculating the branching ratio for such flavor changing transitions is the non-perturbative evaluation of the matrix element $\langle V(k)| J^{\mu}|H(p)\rangle$, with the current,
$J^{\mu}=\bar{l} \sigma^{\mu \nu} q^{\nu} h_{R}$.
This matrix element can be parameterized as the sum of three form factors,
$\langle V(k)| J^{\mu}|H(p)\rangle=\sum_{i=1}^{3} c_{i}^{\mu}(k, p) T_{i}\left(q^{2}\right)$,
where $q$ is the transfer momentum, $p-k$. The conventional parameterization takes the coefficients of the form factors to be:

$$
\begin{equation*}
c_{1}^{\mu}=\epsilon^{\mu \nu \lambda \sigma} \epsilon^{\nu}(k) p^{\lambda} k^{\sigma} \tag{4}
\end{equation*}
$$

$$
\begin{align*}
c_{2}^{\mu}= & \epsilon^{\mu}(k)\left(m_{H}^{2}-m_{V}^{2}\right) \\
& -(\epsilon \cdot q)(p+k)^{\mu}  \tag{5}\\
c_{3}^{\mu}= & \epsilon \cdot q\left[(p-k)^{\mu}\right. \\
& \left.-\frac{q^{2}}{m_{H}^{2}-m_{V}^{2}}(p+k)^{\mu}\right] \tag{6}
\end{align*}
$$

where $\epsilon(k)$ is the polarization of the resulting vector meson $V$.

In this notation, the ratio, $R$ (eqn. 1), is a simple function of the form factor $\left|T_{1}\right|^{2}$ evaluated at $q^{2}=0$.

There is good reason to think that the lattice should be able to determine the needed matrix element for these processes. The matrix element is a simple three-point function which is very similar to the matrix element used in semi-leptonic decays. On the lattice, form factors for semi-leptonic decays have been calculated to within an $\sim 30 \%$ error.[3]

Our preliminary results were extracted from a set of eight lattice configurations generated originally under the Grand Challenge program. We use Wilson fermions in the quenched approximation. The dimensions of the propagators are $24^{3} \times 39$ and they were evaluated on gauge configurations of size $24^{3} \times 40$ generated at $\beta=6.0$. For the heavy quark, we use $\kappa_{H}=.118$ and .135. For the light quark, we use $\kappa_{L}=.152, .154$, and . 155.

We are currently in the process of analyzing previously generated propagators from $16^{3} \times 25$, $\beta=5.7$ lattices. We are also generating new propagators at $\beta=6.3$ and $\beta=6.4$ with a lattice size of $24^{3} \times 61$ and a $32^{3} \times 65$ respectively.
Figure 2 show the typical example of how one extracts the form factor from the lattice. One expects the calculated value for the form factor to reach a plateau at large distances away from the origin. Here, the error bars are deceptively large because all the measurements are highly


Fig. 2. $T_{1}\left(q^{2}=-0.2\right)$ vs. time plane. An example of the method for extrapole.cing the $T_{1}$ form factor. The jackknifed error bars are highly correlated. The horizontal lines represent values extracted from $\chi^{2}$ fits using selected points and the full correlation matrix.
correlated. We make a fit to a horizontal line using the full covariance matrix.

Once $T_{1}\left(q^{2}\right)$ has been determined for several values of $q^{2}$, we make an interpolation or extrapolation to $q^{2}=0$. Figure 3 shows this process for two selections of hopping constants.

The last step is to make the extrapolation to the physical masses for $B \rightarrow K^{*} \gamma$. Figure 4 shows both that more data is needed and that there is hope that we may be able to reach this goal in the coming years.

We have presented an exploratory calculation for a process to which lattize methods have not previously been applied. The problem is especially interesting phenomenologically as it appears to be very difficult for continuum methods. On the lattice, calculating the branching ratio involves calculating three-point functions so there are no complications such as final state interactions. The operator involved does not mix with lower dimension operators, and the form


Fig. 3. Values of the $\dot{T}_{1}$ form fictor as a function of $q^{2}$. Note that no extrapolation is needed to obtain $T_{1}\left(q^{2}=\right.$ 0 ). Several sets of hopping parameters, ( $\kappa_{h}, \kappa_{1}, \kappa_{11}$ ), are displayed. $\square:(.135, .152, .152), x:(.135, .152,154), \bigcirc$ : $(.118,152, .152)$, and $\diamond:(.118, .152, .154)$.


Fig. 4. Values of the $T_{1}$ form factor plotted against the ratio of the meson masses. The physical value of ( $M_{K} \cdot / M_{B}$ ) is approximately 0.2 .
factor of interest separates cleanly from the other two form factors. The lattice should be able to achieve high enough accuracy to make an important prediction for experiment. Of course, more work is necessary to directly deal with B mesons on the lattice. Simulations at higher $\beta$ for this purpose are currently underway. We also intend to apply the static and/or nonrelativistic heavy quark methods.

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[^0]:    * Research partially supported by U.S. Department of Energy under contract DE-AC0276 CH 00016.
    ** Presented at Lattice 91, International Conference on Lattice Field Theory, Tsukuba, Japan, 5-9 November 1991.

