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Applications of a 2-D Moving Finite Element Formulation to Elastic/Viscoplastic Dynamic Fracture Analysis

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1 INTRODUCTION

Current efforts to resolve the near crack tip fields in elastic/viscoplastic materials using finite element methods have failed to achieve a finite non-zero energy flow to the crack tip (see e.g. Brickstad 1983; Bass et al. 1987; Brickstad 1987, and Bass et al. 1989). Motivated by these difficulties, a moving element formulation incorporating a variable order singular element to enhance the local crack tip description is presented. The moving mesh zone is embedded in a finite global mesh to provide a functional tool for the analysis of dynamic crack growth experiments.

The necessary elasto-dynamic formulations have been previously implemented in a transient finite element program DYNCRACK and checked against known analytical solutions (Thesken and Gudmundson 1987; Thesken and Gudmundson 1990; and Thesken 1991). These results have encouraged an attempt to include Perzyna's (1966) elastic/viscoplastic model in the formulation. However, the introduction of non-linear history dependent material behavior into a moving element scheme requires a method to interpolate related field quantities to new Gauss point positions for each time step.

The following summary of numerical procedures outlines the approach taken to develop a transient elastic/viscoplastic moving finite element formulation. Results for a standard test problem are then compared to those obtained using the nodal relaxation technique. Further development of the code is discussed with respect to applications to dynamic fracture experiments.

2 NUMERICAL METHODS

Numerous authors have employed moving singular elements in the analysis of elasto-dynamic fracture mechanics (Nishioka and Atluri 1986). Those contributions utilizing the complete convective formulation have been discussed by Thesken and Gudmundson (1990) in comparison with the unique aspects of DYNCRACK, e. g.: the use of variable order singular elements (Akin 1975), explicit time integration with a lumped mass matrix, a convecting G-integral, and an exact formulation to accommodate instantaneous jumps in crack speed. Figure 1 illustrates a moving region of elements in DYNCRACK that translate with special crack tip elements and convecting contours for calculating the energy-release rate G-integral. Periodic remeshing minimizes disortion of the mesh, but additional computational effort is required to handle the history dependent fields.

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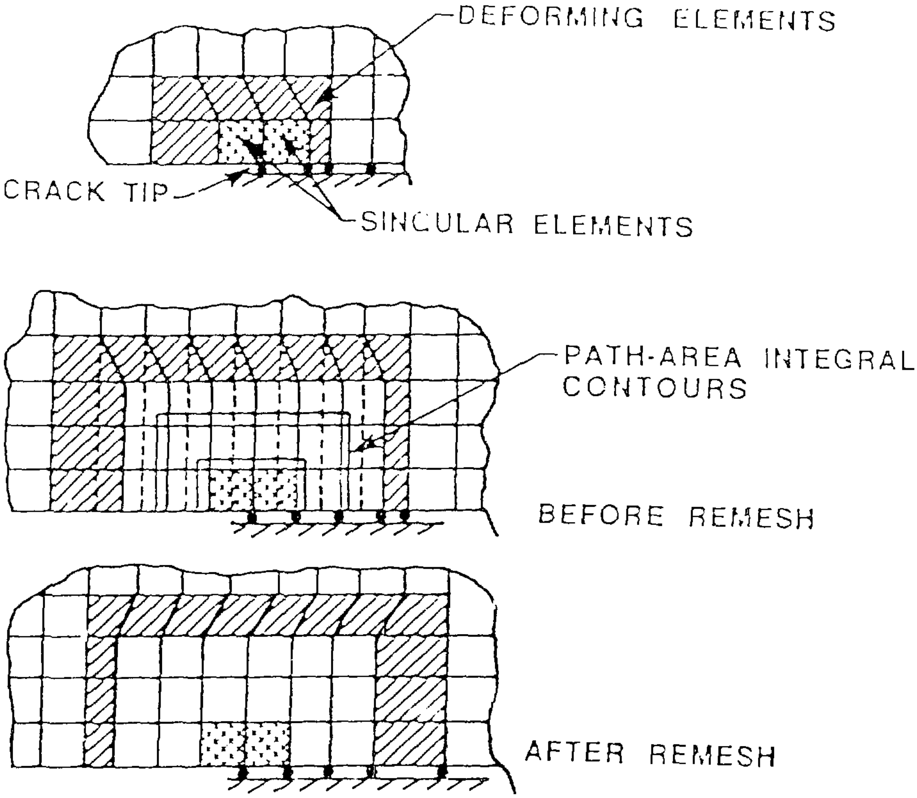


Fig. 1 Adjustable moving mesh zone incorporating convecting integral contour.

2.1 Moving element formulations

Complete derivation of the moving finite element formulations was described by Thesken and Gudmundson (1990); the final system of matrix equations is given here to explain extension to viscoplastic material behavior. Moving finite element formulations are recovered by introducing the discretized total material acceleration, the constitutive laws and the strain-displacement relations into the variational equation of motion. The resulting system of equations for dynamic equilibrium, expressed in terms of nodal displacements U, velocities $\dot{U}$, and accelerations $\ddot{U}$ is given by

$$M\,\ddot{U} + C'\,\dot{U} + (K + C'')\,U = P, \tag{1}$$

which involves mass and stiffness matrices M and K, along with nonsymmetric matrices C′ and C″ associated with the convection of momentum. Integration in time is achieved using the central difference scheme, where the well known kinematic approximations for $\dot{U}$ and $\ddot{U}$ are inserted into Eq. (1) to obtain

$$\left[\frac{M_t}{(\Delta t)^2} + \frac{C_{t'}}{\Delta t}\right] U_{t+\Delta t} = p_t^{app} - p_t^{int} + \left[\frac{2M_t}{(\Delta t)^2} - C_{t''}\right] U_t + \left[\frac{C_{t'}}{\Delta t} - \frac{M}{(\Delta t)^2}\right] U_{t-\Delta t} \tag{2}$$

The subscripts t and t ± Δt indicate the time stepping sequence with respect to time step Δt. The definitions of the internal system of forces p^{int} are

$$p_t^{int} = K_t U_t \tag{3}$$

$$p_t^{int} = \int_{V_e} B^T E \left[B\ U_t - \varepsilon_t^{vp} \right] dv \tag{4}$$

for the elastic and the elastic/viscoplastic cases, respectively. In Eq. (4), B is the strain-displacement matrix and E is the matrix of elastic constants, and ε_t^{vp} is the vector of current elastic/viscoplastic strains. Equation (4) reflects that the elastic strains and internal stress state can be determined by subtracting the nonlinear component from the total strains given by the product B U_t. If a lumped mass matrix is used, then Eq. (2) can be solved directly without recourse to matrix inversion for regions where C′ and C″ are identically zero, i.e., outside the moving mesh zone of Fig. 1.

2.2 Implementation of elastic/viscoplastic model

The plastic strain rates in the following computational procedure are characterized by the power-law overstress model of Perzyna (1966). Owen and Hinton (1980) have given procedures for implementing this model into an explicit transient finite element code and Brickstad (1983) has demonstrated its application to dynamic crack growth using nodal relaxation. In the same manner, DYNCRACK has been adapted to elastic/vicoplastic material behavior and performs identically for problems not involving moving elements. A discussion of the relevant principles is described by Bass et al. (1989), where DYNCRACK was used to duplicate Brickstad's (1983) vibrating beam problems for comparison to results from ADINA.

The adaptation of the model to a moving element scheme involves a parallel algorithm for updating ε^{vp} with respect to time and position. Figure 2 gives a simple flow chart for the procedure. At current time t and Gauss point location 1, the current state of stress σ_t is evaluated for input to the updating algorithm described in Fig. 2a. The current viscoplastic strain rate is evaluated from Perzyna's model using an associative flow law and von Mises yield criterion.

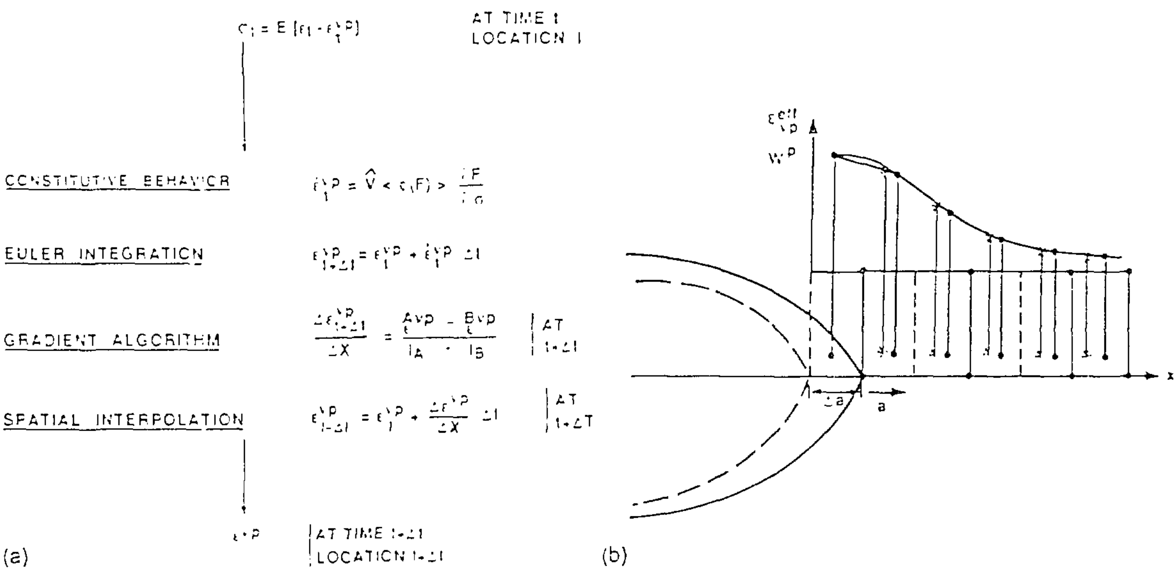


Fig. 2 Procedure for updating viscoplastic strains with respect to time and space: (a) flow chart for algorithm; (b) spatial interpolation.

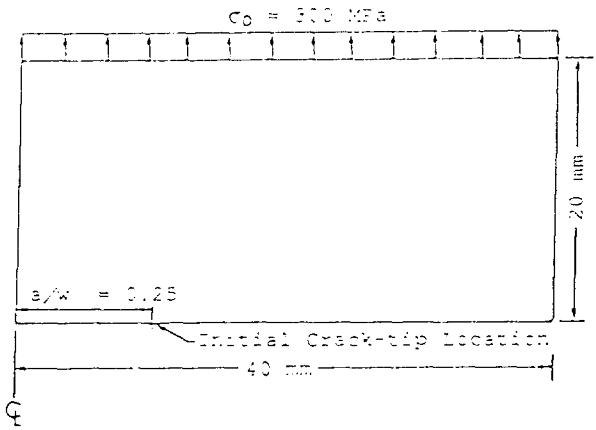
Euler time integration is then used to update $\varepsilon^{vp}_{t+\Delta t}$. This is accomplished for each Gauss point location in a loop over all elements in the region of plastic deformation. Note that a stability limit must be imposed on Δt according to Eq. (11) of Brickstad (1983).

Next, a gradient algorithm computes the gradient of the viscoplastic strains in the x_j direction between each pair of adjacent Gauss points in the plastic zone. With knowledge of the change in Gauss point position Δl and a simple spatial interpolation, the new viscoplastic strains at the new Gauss point locations are obtained. Figure 2b illustrates the necessary spatial interpolation.

3 TEST PROBLEM: RESULTS AND DISCUSSION

A simple center-cracked plate geometry was chosen to test the elastic/viscoplastic formulation in DYNCRACK. Figure 3 gives the geometry and material properties of the problem. Initial conditions were determined from an elasto-static solution. Values of crack opening displacement (COD), energy-release rate G and energy flow to the crack tip γ have been converted to pseudo-stress intensity K for comparison. Results are compared to solutions from the nodal relaxation code CRACK 1 developed by Brickstad (1983).

The full series of mesh refinements specified in Fig. 3 has been analyzed with the order of the singular elements set to zero, i.e., identical to normal isoparametric elements. Figure 4 gives results for the coarsest (Fig. 4a) and finest (Fig. 4b) meshes to illustrate convergence characteristics. The complete matrix of mesh sizes has not yet been analyzed for the order of the singular element set to the square root singularity, but Fig. 4c presents results for the coarsest mesh size.



(a)

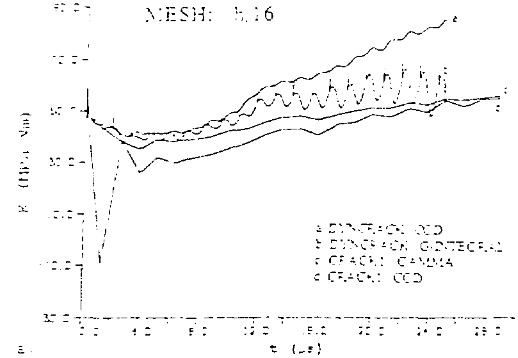
(b) Material Properties:

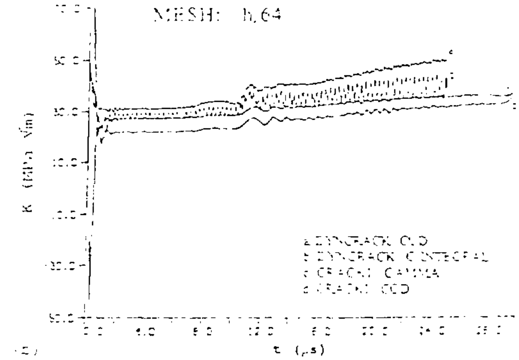
Young's Modulus, E = 206.84 GPa;
Poisson's Ratio, ν = .3
Tangent Modulus, E_T = E/75
Perzyna's Model: Yield Stress, σ_y = 449 MPa,
Fluidity Parameter, γ = 10,000/sec , n = 2.5

(c) Finite Element Meshes:

No. of Elements	Element Size (mm)
512	h/16=2.5000
2048	h/32=0.6250
8192	h/64=0.3125

Fig. 3 Center-cracked panel: (a) geometry; (b) material properties; (c) mesh refinement.





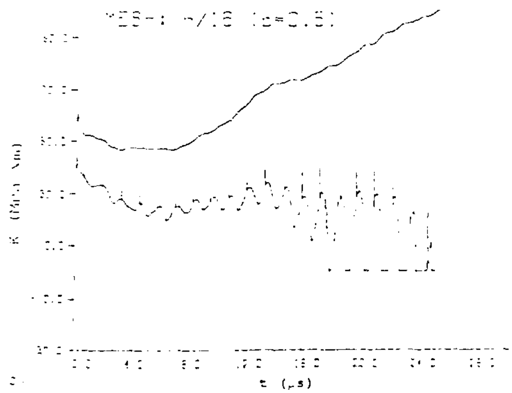


Fig. 4 Time histories of pseudo-K values: (a) and (b) nodal relaxation compared with moving element (without singularity) for coarse and fine mesh; (c) moving element with activated singularity.

In general, the results in Fig. 4 show a distinct oscillatory behavior in the DYNCRACK results in comparison to CRACK1 results. It should be kept in mind that γ and COD can only be computed from CRACK1 after the crack has transversed each discrete element. If the sampling in DYNCRACK was as coarse as the CRACK1 analysis, the data from DYNCRACK would also appear smooth. The existence of these oscillations is a common feature of moving element formulations with periodic remeshing (Thesken and Gudmundson 1990).

Although results for G and γ appear in close agreement, the COD results from both analyses appear as upper and lower bounds to the solution. This aberration could be explained with respect to the assumption of zero plastic strains with the initial conditions. With the release of the first node by the CRACK1 program, an extreme local yielding causes an unrealistic negative displacement of the crack surface and a corresponding negative spike in the K from COD. This has the effect of producing a permanent bias in the COD results. With mesh refinement, Fig. 4b shows that this aberration is not so apparent.

Figure 4c shows that the results from the G-integral and the COD seem to diverge when the square root singularity in the variable order singular element is activated. Now the G-integral is lower in magnitude and the COD values are higher than any of the results in Fig. 4a. The presence of extreme oscillations in the G-integral calculations after 12 μs seems to indicate an instability in the solution as large plastic strains accumulate. However, the COD results from this solution are reasonable, indicating that the finite element solution itself maybe acceptable, while the method for computing the G-integral may not perform properly when the singular element is introduced.

4 CONCLUSIONS

The close agreement between CRACK1 and DYNCRACK with zero singularity gives confidence that the general approach could prove to be a functional tool with further development. Despite the questionable behavior of the G-integral in conjunction with the activated singular element, the COD results appear acceptable, thus indicating that further work with the G-integral algorithm is necessary. Presently, the employment of the G-integral requires a moving element zone of four layers of elements around the crack tip, so that an extremely large zone of elements are involved in the spatial interpolation of the viscoplastic strain, plastic strain energy density, etc.

An alternative approach would be to shrink the moving zone to two crack tip elements and forgo the use of the convecting G-integral in favor of the COD as a crack tip parameter. The errors due to interpolation could be reduced over a smaller region, allowing a more accurate model of the general plastic zone. This approach is currently being implemented and applications to experimental data will be reported in a future publication.

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