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IMPLICATIONS OF TAE MODES FOR THE DESIGN OF ITER

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ABSTRACT

A simple mixing-length estimate of diffusion of alpha particles by toroidicity-induced shear Alfvén eigenmodes (TAE) is used, in zero and one-dimensional models, to evaluate the importance of diffusion on meeting ignition requirements for ITER and other next-generation burning plasma experiments. It is found that, depending on a number of assumptions, diffusion could reduce the effectiveness of alpha heating in the core as much as an order of magnitude. However, the effect would be less if only alphas resonant with the Alfvén waves diffuse. Also, in the Appendix it is argued that the mixing length diffusion formula, though qualitatively reasonable, may be an over estimate.

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1. INTRODUCTION

Theory has long suggested that energetic alphas may cause new classes of instabilities that could impact requirements to achieve ignition.[1,2] However, despite strenuous recent efforts[3,4], there is as yet no theoretical model of alpha transport by instabilities in a form suitable to be incorporated in systems design codes, such as those being used to design ITER and other next-generation machines.

The purpose of this paper is to assess what if anything can be said, based on the present state of knowledge, about the effect of alpha-driven instabilities on the design of ignition experiments. As a specific example, we focus on the toroidicity-induced shear Alfvén eigenmode (TAE) discussed by Fu and Van Dam[5]. While they emphasized long-wavelength modes (m = 1) that might transport alpha particles to the walls in a coherent manner, we suggest a *different* scenario. Namely, we note that instability occurs first for shorter wavelength (higher m) modes. We estimate that incoherent, diffusive transport by these short wavelength modes could be sufficient to prevent the build-up of the alpha density to the threshold for the coherent m = 1 mode to become unstable. Thus diffusive transport by short wavelength modes, which spreads the alphas away from the plasma core where they are created, would be the dominant process. However, spreading the alphas in this way greatly reduces their effectiveness in heating the core plasma to ignition temperatures, so alpha instabilities would still have a deleterious effect.

Qualitative arguments in support of the diffusion scenario are given in Section 2. Sensitivity to theoretical uncertainties is explored by means of an ad hoc diffusion model in Sections 3 and 4. Section 5 discusses the potential consequences of alpha diffusion on machine design.

2. QUALITATIVE CONSIDERATIONS

The TAE growth constant calculated by Fu and Van Dam can be written in the form

$$\gamma = \omega_0 q^2 \left\{ R\beta_\alpha \left(\frac{\omega_{*\alpha}}{\omega_0} - \frac{P}{R} \right) - \sqrt{\frac{\pi}{2}} \;\; \beta_e \frac{v_A}{v_e} \right\} \tag{1}$$

where P and R are functions of order unity for alpha velocities near the Alfvén speed v_A, ω_0 is the real frequency ($\omega_0 \simeq k_{\|} v_A$), β_e is the beta for the bulk plasma electron and v_e is their thermal speed, β_α is the beta of the alphas alone, and

$$\omega_{*\alpha} = \frac{k_\perp}{e} \frac{T_\alpha}{B} \frac{1}{n_\alpha} \nabla n_\alpha \tag{2}$$

where n_α and T_α are the density and temperature of the energetic alpha population, and B is the toroidal field. Instability occurs at specific values of q given by

$$q = \frac{2m + 1}{2n} \tag{3}$$

where m and n are integers, $k_\perp = m/a$ and $k_{\|,m,n} = (n - m/q)/R$, a and R being the minor and major radii of the tokamak plasma. Equation (3) follows from the so-called "gap" condition, $k_{\|m,n} = - k_{\|}m + 1$, $m = (2qR)^{-1}$ and $\omega_0 \simeq k_{\|}v_A = (v_A/2qR)$.[5]

Note that, since ω_* is proportional to $k_\perp$, Eq. (1) yields instability ($\gamma > 0$) for sufficiently large $k_\perp$ for any value of $\beta_\alpha (\propto n_\alpha)$ however small, limited only by effects such as the finite Larmor radius of the alphas that were not included in the calculation. Thus, as the alpha density builds up by fusion reactions, instability occurs first at large $k_\perp$ (large m values).

Suppose that these short-wavelength modes cause the alpha particles to diffuse with a coefficient D_α. Then the density of energetic alphas near the plasma core would reach a steady-state value given by

$$n_\alpha = \tau_\alpha \left(\frac{1}{4} n^2 \sigma v\right) \simeq \left(\frac{D_\alpha}{a^2} + \tau_s^{-1}\right)^{-1} \left(\frac{1}{4} n^2 \sigma v\right). \tag{4}$$

Here n is the density of DT plasma undergoing fusion reactions at a rate σv, and τ_α is the lifetime of energetic alphas with respect to spreading away from the core by diffusion; or by slowing down by collisions with electrons in a time τ_s, after which they no longer contribute to instability and therefore they no longer count as energetic alphas for our purposes.

Qualitatively, the effect of alpha transport on ignition is already apparent in Eq. (4). The steady-state heating rate P_α at the core is just

$$P_\alpha = n_\alpha n E_\alpha \tau_s^{-1} \tag{5}$$

where E_α= 3.5 MeV is the alpha energy at birth. Thus the reduction in heating as a consequence of diffusion is just proportional to the reduction in n_α relative to the steady-state value $n_{\alpha c}$ without diffusion, given by Eq. (4) with $D_\alpha = 0$; or

$$f_\alpha \equiv \frac{n_\alpha}{n_{\alpha c}} = \frac{a^2}{D_\alpha \tau_s + a^2} \tag{6}$$

where f_α is the fraction of the actual heating power relative to what it would be without diffusion (f_α being unity if D_α were zero).

Whether diffusion matters depends on the magnitudes of D_α and τ_s. As was noted in the Introduction, as yet there is no theoretical formula for D_α available to the designer. For want of a better value, for estimating purposes we shall adopt a mixing length estimate,

$$D_\alpha = \left(\frac{\gamma}{k_\perp^2}\right)_{max} \tag{7}$$

where γ is given by Eq. (1). Here "max" denotes the maximum value for any $k_\perp$, which occurs at

$$k_\perp a\alpha = 2\left\{\frac{P}{R} + \sqrt{\frac{\pi}{2}}\quad \beta_e\,\frac{v_A}{v_e}\,\frac{1}{R\beta_\alpha}\right\} \tag{8}$$

where $\alpha = (\omega_{*\alpha}/k_\perp a\omega_0)$. Then

$$D_\alpha = \frac{\omega_0 q^2}{k_\perp^2}\left\{R\beta_\alpha\,\frac{P}{R} + \sqrt{\frac{\pi}{2}}\quad \beta_e\frac{v_A}{v_e}\right\} \tag{9}$$

with $k_\perp$ given by Eq. (8). For a Maxwellian distribution of alphas, the functions P and R are given by Fu and Van Dam to be[5]:

$$P_{max} = 1/2\,R_{max} \tag{10}$$

$$R_{max} = \sqrt{\frac{\pi}{2}}\quad x\,(1 + 2\,x^2 + 2\,x^4)e^{-x^2} \tag{11}$$

where $x = v_A/v_\alpha$, v_α being the speed of the energetic alphas[6].

Accepting this estimate of D_α for the moment, we may calculate the heating fraction f_α from Eq. (6), with $n\tau_s = 10^{18}\,T_e^{3/2}$ in M.K.S. units, T_e being the electron temperature in KeV. To do so, since D_α depends on n_α through β_α, we must first solve Eq. (4) to obtain a self-consistent value. As an example, we take input parameters representative of ITER to obtain the results shown in Table I. Note that we find $ka = 7$, consistent with our hypothesis that short wavelength modes dominate. At the resultant density of alphas, longer wavelengths have lower growth rates and modes with $ka < \underline{4}$ would be stable, by Eq. (1). Though not necessarily corresponding to TAE modes, wavelengths in the range $ka \sim 5$ or so are typical of experiments in which neutral beams may be exciting Alfvén waves in various experiments.[7]

Taken literally, the results in Table I say that diffusion could reduce core heating by alphas by an order of magnitude and prompt us to further analysis in the next section.

3. AD HOC DIFFUSION MODEL

In this section we extend the zero dimensional model of Section 2 to a 1-D diffusion model for circular cross-section, given by

$$-\frac{1}{r}\frac{d}{dr} r D_\alpha \frac{dn_\alpha}{dr} = \frac{1}{4} n^2 \sigma v - \frac{n_\alpha}{\tau_s} K \tag{12}$$

where as before slowing down (the last term on the right) is treated as a loss of energetic alphas. (The factor K in this term is unity to represent alpha-electron collisions only. We shall make other uses of K later on.) Such an equation, coupled to the Supercode[8], would form a closed set of equations sufficient to determine whether ignition would occur, with heating P_α given by Eq. (5). However, as noted above no formula for D_α is yet available.

Despite the lack of information about D_α, we shall proceed using an ad hoc model with the aim of finding out what really matters for design. To this end, we shall again adopt the mixing length formula given by Eq. (9), now treating D_α as a local quantity varying continuously with radius through its dependence on P, R and so on. In this, we assume localization on the grounds of short wavelengths, and we neglect the quantization of m and n in Eq. (3) and instead treat q and D_α as continuous functions of r , on the grounds of spatial overlap of the resonant eigenmodes. Most questionable is the underlying mixing length formulation, Eq. (7). Also, in the absence of finite Larmor radius corrections, γ does not have a maximum (since $\gamma \propto k_\perp$). This prompted us to take instead the maximum of $(\gamma/k_\perp^2)$ itself, which gives a maximum in the range $k_\perp \rho_\alpha$ of a few tenths, ρ_α being the alpha Larmor radius.

For our purposes, the virtue of choosing the mixing length estimate for D_α is its simplicity and the fact that, independent of its magnitude, the dependence of D_α on γ incorporates the main qualitative feature anticipated from theory; namely, instability and transport by Alfvén waves should be greatest at the core where v_A is small and least at the edge where v_A increases as the fuel density decreases. Such a diffusion coefficient would flatten the alpha heating profile, as anticipated in the original work on kinetic Alfvén instabilities by Rosenbluth and Rutherford.[2]

Moreover, at least crudely the mixing length formula may be appropriate for the gradient-driven TAE mode as it appears to be for electrostatic gradient-driven modes. For such modes we might expect saturation when $k_\perp^2 D \sim \gamma$ (flattening of the profile), which leads to Eq. (7).[9] This topic is discussed further in the Appendix.

4. MODEL CALCULATIONS

We first calculate the model as defined thus far and later add other features including loss of alphas due to banana excursions, cut off of D_α within the confinement volume where $k_\perp \rho_\alpha$ becomes large, and a slowing-down alpha distribution rather than a Maxwellian. In the absence of the latter effects, the proper boundary condition is that there be no net loss of alpha particles, or equivalently,

$$\int_0^a r\,dr \left(\frac{1}{4} n^2 \sigma v - \frac{n_\alpha}{\tau_s} \right) = 0 \quad . \tag{13}$$

To see this, note the rapid decrease of D_α approaching $r = a$ where the fuel density n is vanishing or negligible and v_A becomes very large in the exponential factor in Eq. (11). It can be shown that, without the slowing-down term, Eq. (12) has no solutions with the usual boundary conditions that $D_\alpha\,(dn_\alpha/dr)$ vanish at the origin and n_α vanish at $r = a$. Physically, the rapidly vanishing diffusion coefficient could not push particles across the $r = a$ boundary and hence there is no steady state (for the time-dependent problem, n_α would grow indefinitely).

With the boundary condition in Eq. (13), the solution flattens out as expected. This is shown in Figure 1, which plots the solution of Eq. (12) versus radius ($x = r/a$). The resulting profile depends on the fuel density and temperature profile shapes, characterized by α_n and α_T,

$$n = n_o (1 - x^2)^{\alpha_n} \tag{14}$$

$$T = T_o (1 - x^2)^{\alpha_T} \quad , \tag{15}$$

where $\alpha_n = \alpha_T = 1$ in Figure 1. The figure also plots $n_{\alpha c}$, which is the solution without diffusion ($D_\alpha = 0$).

As in Section 2, we take the ratio $f_\alpha = n_\alpha/n_{\alpha c}$, which is also the ratio of local alpha heating with and without diffusion, to be a measure of the effect of diffusion on achieving ignition. That is, given a design that achieves ignition for given n and T profiles, ignition would presumably not occur for that design if $f_\alpha << 1$ at the core (small x) where the fuel density and temperature are concentrated. Figure 2 shows f_α versus radius for different profiles (different α's), all for $\alpha_n = \alpha_T$. As we might expect, f_α at the core is least for large α's (steep profiles) and f_α approaches unity for flat profiles ($\alpha = 0$) since alpha heating without diffusion is already constant for this case. Figure 3 plots f_α for fixed $\alpha_T = 1$ and various α_n. Then $f_\alpha << 1$ at the core for all cases.

Figures 4 and 5 examine other aspects of the model. Since n_α and n appear to different powers in Eq. (12), n_α depends on the magnitude of n_o. Figure 4 tests this dependence by plotting f_α for various values of n, with $\alpha_n = \alpha_T = 1$. The model also depends, through the P and R functions in γ, on the fraction of alpha particles with velocity parallel to the field exceeding v_A. To test this dependence, in Figure 5 we compare f_α for the Maxwellian distribution, that might represent turbulence, with that for purely classical processes dominated by slowing-down by alpha-electron collisions. The slowing-down distribution yields different functional forms for P and R in Eq. (1).[10] As can be seen from the figures, neither the magnitude of n nor details of the alpha velocity distribution affect our results appreciably. Therefore, in the following we take $n_o = 10^{20} m^{-3}$ and the slowing-down distribution for all cases.

We now turn to five effects that could make a significant difference. These are:

(1) Resonant versus non-resonant diffusion of the alphas. If only alphas resonant with the Alfvén waves diffuse as rapidly as our model indicates, our f_α applies only to this resonant portion and the overall effect on achieving ignition is much less (see Appendix).

(2) Diffusion of the fuel itself by the TAE turbulence. We have neglected any such feedback on the n and T profiles, on the grounds that the TAE free energy, proportional to β_α, is much less than the fuel plasma free energy.

(3) A significant reduction in magnitude of D_α compared to our model (see Appendix). An increase in D_α would be less important, since already $f_\alpha << 1$ at the core for cases of most interest.

(4) A cut-off of D_α at larger radii where $v_A > v_{\alpha o}$ (birth energy) or where $k_\perp\rho_\alpha$ becomes large, ρ_α being the alpha Larmor radius, neglected in Eq. (1).

(5) Loss of some alphas at an inner radius x_D where a $(1 - x_D)$ becomes equal to their banana width, so that upon reaching x_D they are lost in one banana drift time.

The first four points are beyond the scope of this paper, but are typical of issues receiving attention by the theory community.[4] We shall return to these points, qualitatively, in the next section. In the remainder of this section, we consider the last point as follows.

To simulate banana excursions, in Eq. (12) we now take $K = 1$ if $x < x_D$ and $K = K_2$ if $x > x_D$. To include also a possible cut-off on D_α as discussed in item (4) above, we also introduce another quantity x_A beyond which D_α vanishes abruptly. To take account of x_A, we simply replace the upper limit of integration by x_A in the boundary condition, Eq. (13). Here we define x_A as the position where $v_A = v_{\alpha o}$. For x_D we take a range of values characteristic of high energy alphas trapped at the field minimum on the outboard side of the plasma profile.

As noted, for $x > x_D$ the parameter K is intended to represent rapid loss by banana orbit excursions. For this purpose, we simply choose K small enough so that results are independent of K but large enough to avoid numerical problems in integrating Eq. (12) with different K values in the two ranges $x < x_D$ and $x > x_D$. This is shown in Figure 6, which plots f_α for $x_D = 0.6$ and a range of K values.

Finally, Figure 7 presents our main result, showing f_α for a range of x_D values, for the case $\alpha_n = \alpha_T = 1$. Again, $n = 10^{20} m^{-3}$, the alphas have a slowing-down distribution, and $x_A = 0.84$, which has little effect on the results.

Figure 7 confirms the concerns raised by the zero-D model in Section 2. Namely, with our model, diffusion of the alphas reduces their effectiveness in heating by a factor of 3 to 10, the larger value applying to all alphas with large banana excursions widths. Moreover, even a "heat" pinch pumping heat from the edge to the interior would be of little help for the latter class of particles that escape before slowing down. Whereas all of the alpha heating is deposited either in the core or in the edge if $x_D = 1$ (no excursion), if alphas escape at radii where D_α is still large, their heating is lost completely. This is illustrated by the quantity called "heating" labelling each curve, meaning the ratio of heat deposited in $0 < x < x_D$ to the total alpha energy produced. Even for $x_D = 0.8$, only 11% of the heat is deposited. To calibrate, for ITER a typical banana width $\Delta = (q\varepsilon^{-1/2})\, \rho_\alpha$ gives, for a 3.5 MeV alpha, $q = 3$ and $\varepsilon = 1/3$, $\Delta/a = .25$ and $x_D = 1 - \Delta/a = .75$. Since the diffusion time is $<<\tau_s$, alphas that diffuse remain near their birth energy of 3.5 Mev.

5. DISCUSSION

Of the various effects examined, we conclude that the most important missing pieces of information, in order to determine the effect of kinetic Alfvén instabilities in the design of ITER and other burning plasma experiments, are:

(1) The actual magnitude of D_α. To help, D_α must be about an order to magnitude less than our mixing length estimate (which is 10 times χ for the fuel).

(2) Whether strong diffusion (at our rate) applies to all alphas or only to a select resonant class.

Both points are discussed in the Appendix, in relation to the mixing length model.

Other details, including the question of actual loss of the alphas ($x_D < 1$) versus merely spreading their profile out to some radius where diffusion peters out ($x_A < 1$), is less relevant since heating in the core is presumably of greatest importance. Specifically, to be effective, finite Larmor radius stabilization would have to eliminate or drastically reduce diffusion in the core itself. In this regard, note that $k_\perp \rho_\alpha = .35$ for the parameters in Table I.

If diffusion is large only for resonant alphas, our results suggest that alpha heating is for practical purposes limited to alphas generated in the core with pitch angles such that they do not resonate with Alfvén modes ($v_{\alpha\parallel} < v_A$). Such particles would not diffuse and would heat the fuel on the inner flux surfaces. Moreover, since classical slowing-down of the alphas by electrons reduces their speed without changing their pitch angles, those born as non-resonant particles remain non-resonant.

Finally, our work suggests that it would be worthwhile to incorporate an anomalous particle diffusion term, initially of mixing-length form, in the alpha transport equation incorporated in the Supercode.

TABLE I. Approximate steady-state values for ITER-like device according to the zero-dimensional model of Section II. Symbols are defined in the text.

Input Parameters	Results
$v_\alpha = 10^7 ms^{-1}$	$n_\alpha = 3 \times 10^{16} m^{-3}$
$v_A = 7 \times 10^6 ms^{-1}$	$\beta_\alpha = 10^{-3}$
$\omega_o = 4 \times 10^5 s^{-1}$	$\tau_\alpha = 0.03s$
$\alpha = .9$	$\tau_s = 0.6s$
$\beta_e = 0.025$	$ka = 7$
$T_e = 15$ keV	$f_\alpha = 0.05$
$n = 10^{20} m^{-3}$	
$B = 5T$	

Figure 1
Alpha Density Profiles
Maxwellian Model

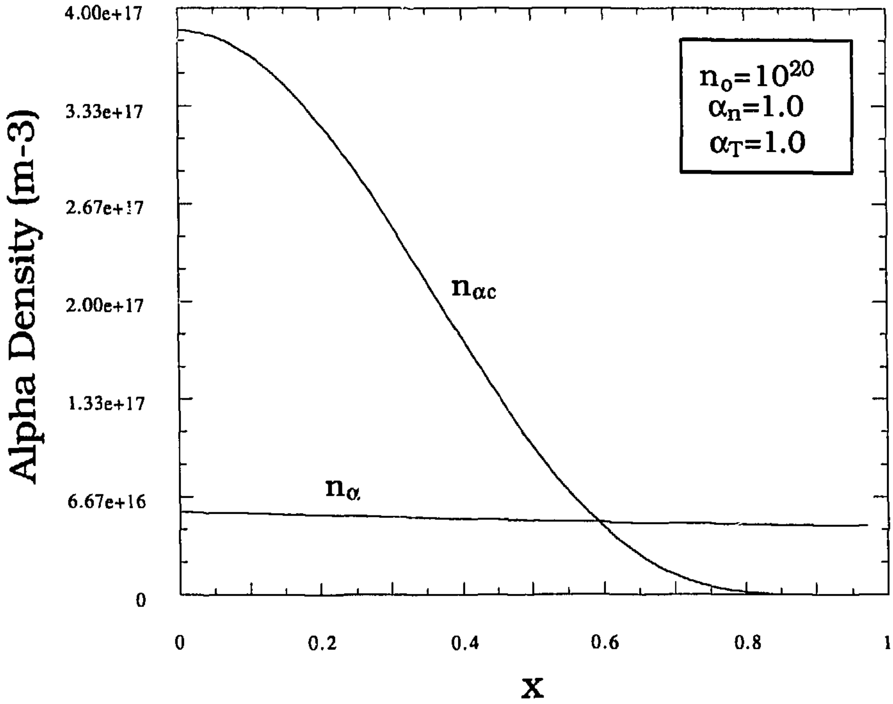


Figure 2

Effect of TAE Mode on Heating Profile

Variation with Fuel Profile, Maxwellian Model

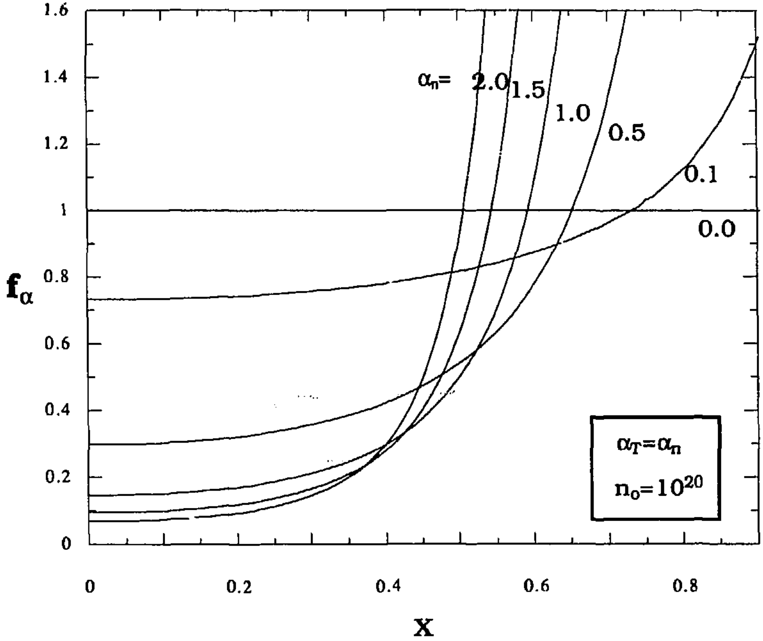


Figure 3

Effect of TAE Mode on Heating Profile

Variaion with Fuel Profile, Maxwellian Model

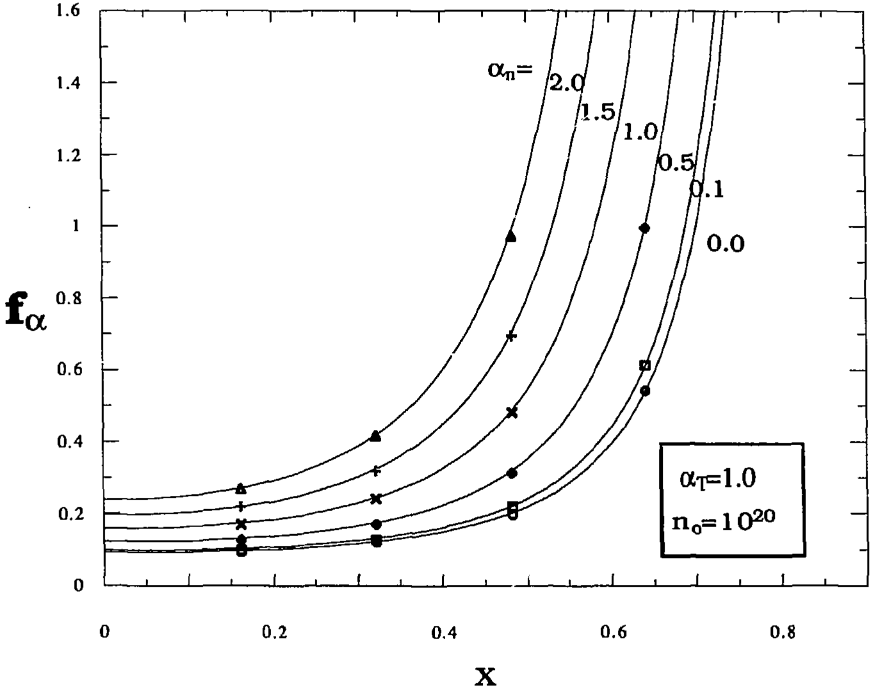


Figure 4

Effect of TAE Mode on Heating Profile

Variation with Fuel Density, Maxwellian Model

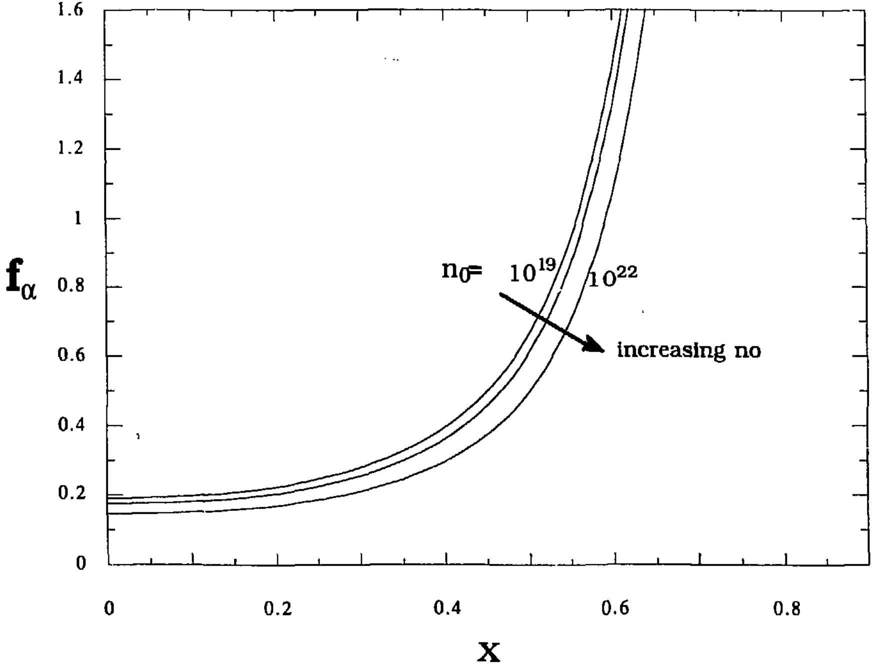


Figure 5

Effect of TAE Mode on Heating Profile

Slowing Down and Maxwellian Models

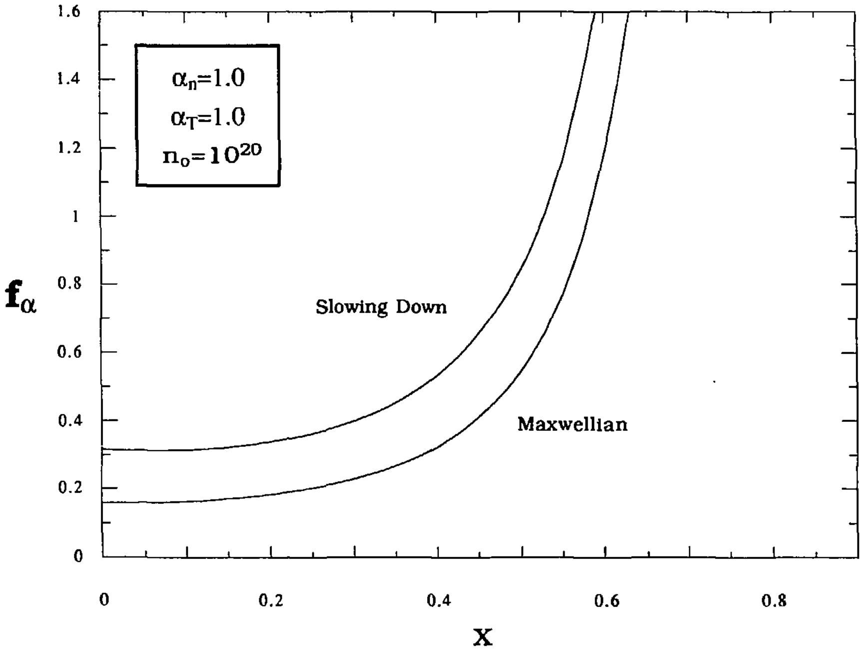


Figure 6

Effect of TAE Mode on Heating Profile

Variaion with K, Slowing Down Model

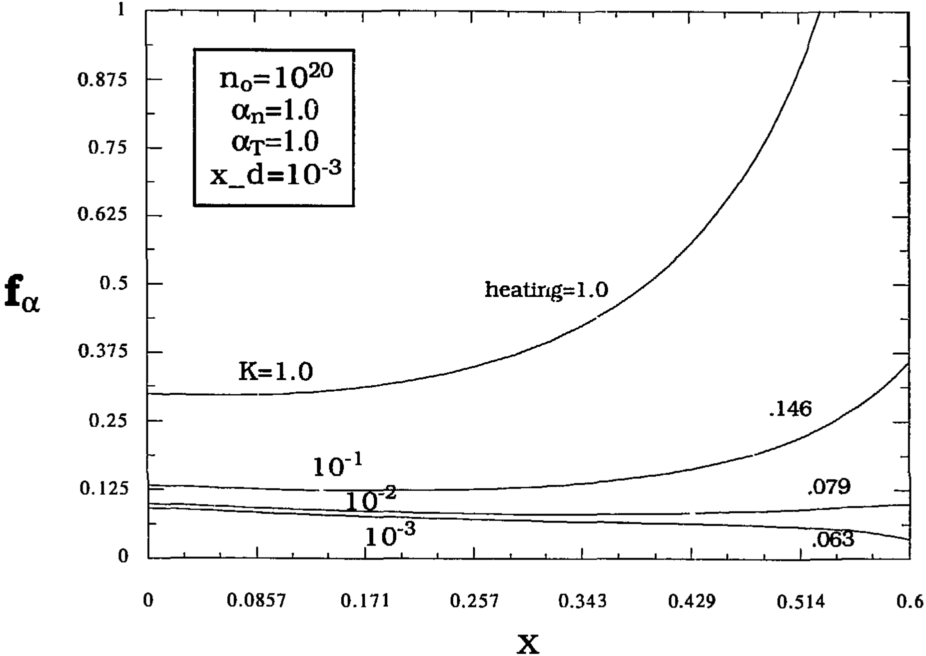
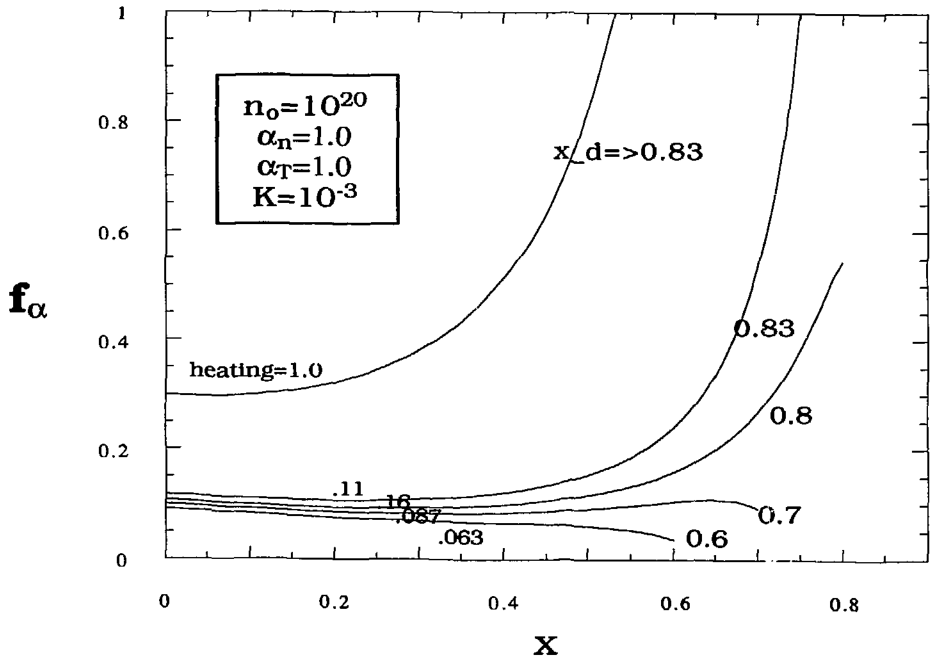


Figure 7

Alpha Heating Profile

Heating vs x_d, slowing down model



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6. Actually $P_{max} = R_{max}$ (Z. Guo, private communication) but the factor 1/2 from Ref. 1 does not significantly affect our conclusions. Also, in their Eq. (10), they take m = n = 1 (q = 3/2) and they drop the factor $\sqrt{\pi/2}$.

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APPENDIX

Here we shall attempt to justify the mixing length estimate for diffusion by TAE modes.

Applied to gradient-driven modes, the mixing-length conjecture assumes saturation when the density (or temperature) profile flattens over a wavelength $k_\perp^{-1}$. Roughly, saturation occurs ($\delta f/\delta t \sim o$) when the non-linear diffusion $\nabla \cdot D \cdot \nabla f \sim k_\perp^2 Df$ becomes comparable to the linear driving term of order γf (real part). Hence $D \sim \gamma/k_\perp^2$.[9] This is also consistent with a quasilinear correlation time, $t_c \sim \gamma/\omega^2$, and the assumption that at saturation the electric field perturbation can move particles a distance $k_\perp^{-1}$ in one cycle ω^{-1},

$$\omega^{-1}\delta E/B \sim k_\perp^{-1} \tag{A1}$$

$$D \sim (t_c \delta E/B)^2 t_c^{-1} \tag{A2}$$

$$t_c \sim \gamma/\omega^2 \quad . \tag{A3}$$

Combining again gives

$$D \sim \gamma/k_\perp^2 \quad . \tag{A4}$$

To be consistent, it is necessary that the free energy driving the modes be sufficient to produce electric field perturbations satisfying Eq. (A1). For electrostatic drift waves, with $\gamma \sim \omega \sim \omega_*$, this is the expansion energy, $(k_\perp a)^{-2} nT$, the condition on δE being

$$\varepsilon \delta E^2 \lesssim (k_\perp a)^{-2} nT \tag{A5}$$

with $\varepsilon \sim (k\lambda_D)^{-2}$.[11] Then $\delta E \sim T/e\, a$ and $k_\perp \delta E/B \sim \omega_* \sim \omega$, which does satisfy Eq. (A1). The free energy for the gradient-driven TAE mode is also presumably the expansion energy, for the alphas. In this case δE is the inductive field associated with magnetic perturbations δB, and the equation corresponding to Eq. (A5) is:

$$nm_i\left(\frac{\delta E}{B}\right)^2 + \frac{(\delta B)^2}{\mu_o} \approx 2\,\frac{(\delta B)^2}{\mu_o} \lesssim \frac{n_\alpha T_\alpha}{k_\perp{}^2 a^2} = \frac{\beta_\alpha}{k_\perp{}^2 a^2}\,\frac{B^2}{2\mu_o} \tag{A6}$$

On the left we have used Maxwell's curl E equation, whereby $\delta E \sim (\omega/k_{||})\delta B$, $\omega \sim k_{||}v_A$ and Eq. (A1) becomes

$$\omega^{-1}\delta E/B \sim \frac{1}{k_{||}}\,\frac{\delta B}{B} \sim k_\perp{}^{-1} \quad . \tag{A7}$$

Again this is an identity if there were sufficient free energy to bend the field lines a distance $k_\perp{}^{-1}$ at saturation, that is, if at saturation $\delta B \sim (k_{||}/k_\perp)B$. Using Eq. (A6), we find that actually

$$\omega^{-1}\,\frac{\delta E}{B} \lesssim \left(\frac{\beta_\alpha}{k_{||}{}^2 a^2}\right)^{1/2} k_\perp{}^{-1} \tag{A8}$$

Hence Eq. (A1) is satisfied only if β_α is sufficiently large. By our estimate, we need $\beta_\alpha > k_{||}{}^2 a^2$, a ballooning-like criterion, but one not dependent on the radius of curvature since it is the parallel energy of the alphas that drives the mode.[12] For parameters in Table I, $\beta_\alpha / k_{||}{}^2 a^2 \sim 0.1$ (not unity), suggesting that the mixing length formula may be an overestimate.

Finally, we note that it is reasonable to assume that the correlation time would be even less for alphas not resonant with the Alfvén waves. Then, since the mixing length estimate itself gives an alpha lifetime $\tau_\alpha \sim 0.1\,\tau_s$, it would seem reasonable to neglect diffusion in comparison with slowing-down for the non-resonant alphas.

Note that t_c in this discussion represents different stochastic effects for electrostatic modes, for which ExB is stochastic, and Alfvén waves, for which other stochastic processes may be involved that break field lines or cause transport when oscillating field lines approach each other sufficiently closely. For the latter, the physical picture is sometimes described as flow along displaced field lines (rather than ExB), but this is equivalent to Eq. (A1) through the relation of δE and δB, by Eq. (A7).