PROBABILISTIC ASPECTS OF FAULTING IN ANISOTROPIC MEDIA

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I. INTRODUCTION

When considering the possibility of release of radioactive waste from the confines of a waste repository in layered geologic media, it is generally accepted that faulting presents a potential threat to the long term isolation of this waste from circulating groundwater. A fault can penetrate either the isolating barrier media or the depository itself to form a potential conduit for groundwater to make contact with the radioactive waste.

Three primary aspects of faults which relate to their potential for degradation of a repository site are: 1) the probability of an existing but undetected fault intersecting the repository site, 2) the potential for a new fault occurring and propagating through the repository area, and 3) the ability of any such fault to transmit groundwater. Given that a fault might be present in the region surrounding this site, the probability that it intersects the repository site area depends primarily on its orientation (or the orientation of the anisotropy relative to the principal stresses) and the density of faulting in the area. Once these parameters are known, a model can be developed to determine the probability that an existing but undetected fault will intersect the repository site area.

Similar techniques can be used to estimate the potential for new faults occurring and intersecting the repository site, or intersection from propagation along existing faults. However, additional data including in situ stress measurements and records of seismic activity would be needed. One can determine the stress level at which the strength of the surrounding media is exceeded, and thus determine a time-dependnt probability of movement along a pre-existing fault or of a new fault occurring, from a predicted rate of change in local stresses. In situ stress measurements taken at intervals of time could aid in determining the rate of stress change in the surrounding media, although measurable changes might not occur over the available period of observation. In situ stress measurements might also aid in assessing the ability of existing faults to transmit fluids.

II. PROBABILISTIC TREATMENT

Assume that initially (at time of depository closure), the mean density of faults existing in a region R surrounding the repository site is λ_0 per unit area. Assume further that new faults appear in this region according to a nonstationary Poisson process with mean rate $\lambda_1(t)$ per unit area per year, where t indicates the time-dependent rate of formation of new faults. Then, the mean density of faults existing in the region R at some time t following closure of the repository site can be represented by

$$\lambda(t) = \lambda_0 + \int_0^t \lambda_1(\tau)\, d\tau \qquad (1)$$

From equation (1) the probability of exactly N faults existing in the region R by time t is given by

$$P(N, t) = \frac{[\lambda(t)A]^N}{N!} \exp[-\lambda(t)A], \qquad (2)$$

where A is the area of region R. Let p denote the conditional probability that, if a fault exists in the region R, it will intersect the repository site. Then the probability that at least one fault intersects the repository site in the time interval (0, t) is given by

$$P = 1 - e^{-\lambda(t)pA} \qquad (3)$$

From equation (3) we see that to determine a value of P we need to know values of $\lambda(t)$ and p. $\lambda(t)$ can be evaluated once a specific site for the waste repository has been selected. Geologic and historic records of tectonic and seismic activity, determination of ages of existing faults, and in situ measurements of local stresses can all be used to arrive at a representation for $\lambda(t)$.

To arrive at a representation for p which takes into account the presence of anisotropy in the surrounding media, we consider faults in the region R as anisotropic random lines in a plane (Santalo, 1976) and examine the measure of those lines intersecting the repository site. The position of a line is described in terms of the coordinates (r, α), where r is the signed distance from the origin to the line and α is the angle between a reference axis and the perpendicular to the line (see Fig. 1 below). The position of a random line is thus determined by the values of the random variaables r and α. We can then denote by $F(\alpha)$ the cumulative distribution function of α and assume that the distribution of the orientation is translation-invariant. That is, the orientation of a random line is invariant under translations of the coordinate axes. Then for any given α the density of r is dr, and the joint density of r and α is $drdF(\alpha)$. The measure m(W) of all positions of a random line satisfying some condition W can be written as

$$m(W) = \iint drdF(\alpha) \tag{4}$$

where the integration covers all positions of the random line such that condition W is satisfied. Note that in the isotropic case this measure becomes $\frac{1}{\pi}\, drd\alpha$. We can define the thickness $T(\alpha)$ of a set S in the direction to be the length of the projection of S onto a line with direction . For example, the thickness of a circle of radius r is 2r, and the thickness of a rectangular region of dimensions s_1 and s_2 is $T(\alpha) = s_1|\cos| + s_2\ |\sin\alpha|$. From the definition of $T(\alpha)$ we have that the integral dr of all positions of a line with perpendicular direction α such that it intersects the set S is $T(\alpha)$. From this and equation (4) the measure m(S) of all positions of a

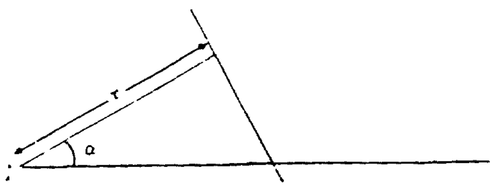

Figure 1

random line such that it intersects a (measurable) set S in the plane is given by

$$m(S) = \int T(\alpha)\, dF(\alpha) \tag{5}$$

Equation (5) can be used to obtain an expression for the probability that a random line intersects a set S_1 given that it intersects another set S_0. It is necessary to consider conditional probabilities because the unconditional probability that a line, random over the whole plane, will intersect any bounded set S is zero.

If we assume that a random line intersects a measurable set S_0, then from equation (5) we have that the probability that it also interesects a subset S_1 of S_0 is given by

$$P[\text{line intersects } S_1 \mid \text{intersects } S_0] = \frac{T_1(\alpha)\, dF(\alpha)}{\int T_0(\alpha)\, dF(\alpha)} \tag{6}$$

where $T_0(\alpha)$ and $T_1(\alpha)$ are the thicknesses of S_0 and S_1, respectively, in the direction α.

Viewing the repository site area as a region R_1 which is a subset of a larger region R_0 containing a collection of faults with a definite orientation, we can use the above arguments to arrive at an estimate of the parameter p.

III. APPLICATION

Within the upper few hundred meters of the earth's crust, at depths at which a depository for high-level nuclear waste might be located, one of the three principal stresses is likely to be vertical or very steeply plunging. Therefore, there are three ideal orientations of potential planes of faulting, defined by which of the three principal stresses is vertical (Anderson, 1951, see Fig. 2). A recent review of the state of stress in the earth's crust (McGarr and Gay, 1978) indicates that at depths less than 1 km the maximum principal stress commonly is horizontal and the minimum principal stress is vertical. This stress orientation favors the development of low-angle reverse faults (see Fig. 2(b) which, if unaffected by the presence of anisotropy, would dip about 30 degrees, since in isotropic media this is the inclination of a fault to the maximum principal stress responsible for it. However, the presence of planar anisotropy can cause this angular relationship to vary from a few degrees to more than 45 degrees (Donath, 1963).

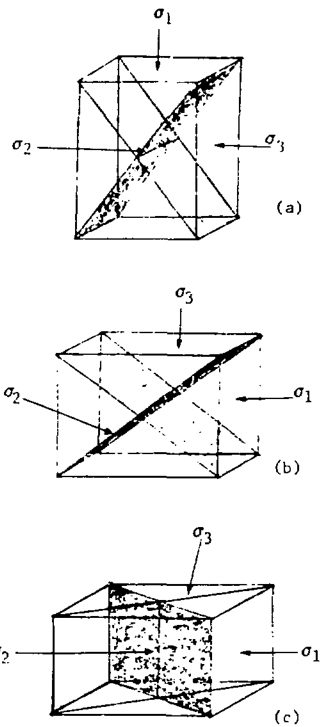

Figure 2. Fault planes (shaded) and principal stress directions for one principal stress vertical.
(a) High angle-normal faults with σ_1 vertical. (b) Low angle-reverse faults with σ_3 vertical. (c) Strike-slip faults with σ_2 vertical.

For purposes of illustration we assume that, because of anisotropy, the dip angle of existing faults or of newly formed faults in a region R_0 containing the repository region R_1 tend to be uniformly distributed over the range from 10 degrees to 60 degrees. We will assume further that the regions R_0 and R_1 are rectangular regions with dimensions s_{01}, s_{02} and s_{11} and s_{12}, respectively. Because the dip angle of the faults are uniformly distributed over the range from 10 degrees to 60 degrees, the perpendicular direction α of these fault lines is uniformly distributed over the range from 30 degrees to 80 degress. Thus,

$$m(R_0) = \int T_0(\alpha)dF(\alpha) = \int_{30}^{80} \frac{1}{50}(s_{01} \cos\alpha + s_{02} \sin\alpha)\, d\alpha$$

$$= \frac{1}{50}[s_{01}(.48) + s_{02}(.7)].$$

Similarly,

$$m(R_1) = \int T_1(\alpha)dF(\alpha) = \frac{1}{50}\ [s_{11}(.48) + s_{12}(.7)].$$

Thus, the probability that a fault intersects the repository site R_1, given that it intersects the region R_0, is

$$\rho = \frac{m(R_1)}{m(R_0)} = \frac{\Delta_{11}(.48) + \Delta_{12}(.7)}{\Delta_{01}(.48) + \Delta_{02}(.7)}$$

IV. FORMATION OF NEW FAULTS

Whether a new fault will form or renewed movement will occur along a pre-existing fault in anisotropic media is determined by a linear failure criterion of the form

$$\tau = \tau_0 + \sigma\mu \qquad (7)$$

where τ and σ are the shear and normal stresses on the potential fault planes or on a pre-existing fault plane, and μ is the tangent of the angle of internal friction for intact anisotropic rock or the coefficient of sliding friction for displacement on an already present fault surface. The term τ_0 is the cohesive strength for intact rocks or the shearing resistance on an existing fault.

For intact anisotropic rock, τ_0 and μ vary systematically with the inclination β of the anisotropy to the direction of maximum principal stress according to the relationship

$$\tau_0 = \underline{a} - \underline{b} \cos 2(\gamma - \beta) \tag{8}$$

and

$$\mu = \underline{c} - \underline{d} \cos 2(\gamma - \beta) \tag{9}$$

where γ is the orientation of β for which τ_o is a minimum (Donath, 1972). For faults that develop parallel to the anisotropy or for pre-existing faults the values of τ_o and μ are constant.

The formation of a new fault or the displacement along an existing fault can thus be predicted to occur when the existing state of stress in the rocks satisfies one or the other of the stated criteria for fault formation or for sliding, i.e., when

$$\tau - \tau_0 \geq \mu\sigma$$

where

$$\tau = (\sigma_1 - \sigma_3) \sin\theta \cos\theta \tag{10}$$

and

$$\sigma = \sigma_1 \sin^2\theta + \sigma_3 \cos^2\theta \tag{11}$$

and θ is the angle between the fault and the direction of $_1$, the maximum principal stress. Thus, once the statistical distribution of τ and σ are determined, one can arrive at an estimate of the probability of a new fault forming or of movement occurring along a pre-existing fault. The methods describe in the earlier part of this paper can be used to arrive at an estimate of the probability that the fault will intersect the waste repository site.

Since there is a functional relationship between τ and σ, one need only determine the distribution function for one of the variables, say σ. Once this is known, the distribution function for τ can easily be determined in the following manner. Assume that σ is a continuous random variable with density $f(\sigma)$ and that $f(\sigma) > 0$ for $a < \sigma < b$. Then, the density $g(\tau)$ of τ is given by

$$g(\tau) = f\left(\frac{\tau - \tau_0}{\mu}\right) \frac{1}{\mu} .$$

Similar techniques apply if is a discrete random variable.

For a given state of stress in the earth's crust, the criterion for development of a new fault might not be satisfied by the existing state of stress but the criterion for sliding could be met for certain orientations of the sliding surface. Thus, for this situation the problem is to identify the range of orientations that satisfy the sliding criterion and to determine the probability that an existing fault surface falls within this range. This probability could be determined from knowledge of the existing stress field and orientation of the anisotropy in the region being considered, or from collected data concerning the surface deformation of the fault being considering (J. B. Walsh, 1969).

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